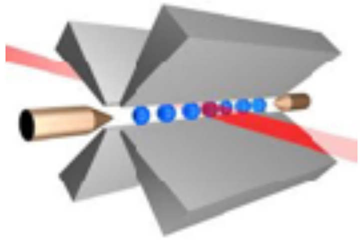
SFT 2024 - Lectures on Statistical Field Theories, GGI, Florence
Feb 05-16, 2024

Trapped ions



UIBK & IQOQI

Lectures 1-4: **Theoretical Quantum Optics**

Part I: Hamiltonian engineering & quantum optical toolbox

Part II: quantum noise & open quantum systems

… basic concepts & minimal models

… how we "think" about quantum noise in quantum optics

Seminar: **Programmable Quantum Simulators with Atoms and Ions**

SFT 2024 - Lectures on Statistical Field Theories, GGI, Florence
Feb 05-16, 2024

Programmable Analog Quantum Simulators

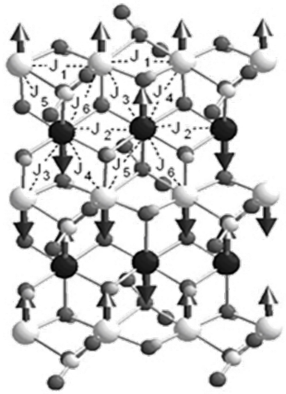
Learning large-scale entanglement in quantum many-body systems

Peter Zoller

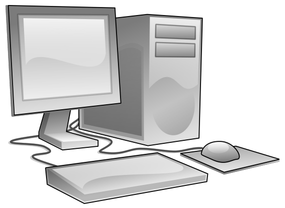
AFOSR MURI (JILA)

Quantum Simulation

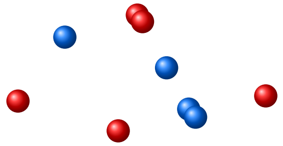
Problem: `solving’ a quantum many-body problem



$$\hat{H} = \sum_{i,j} \left[J_1 \hat{S}_i^z \hat{S}_j^z + J_2 \left(\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+ \right) \right]$$



Classical Computing

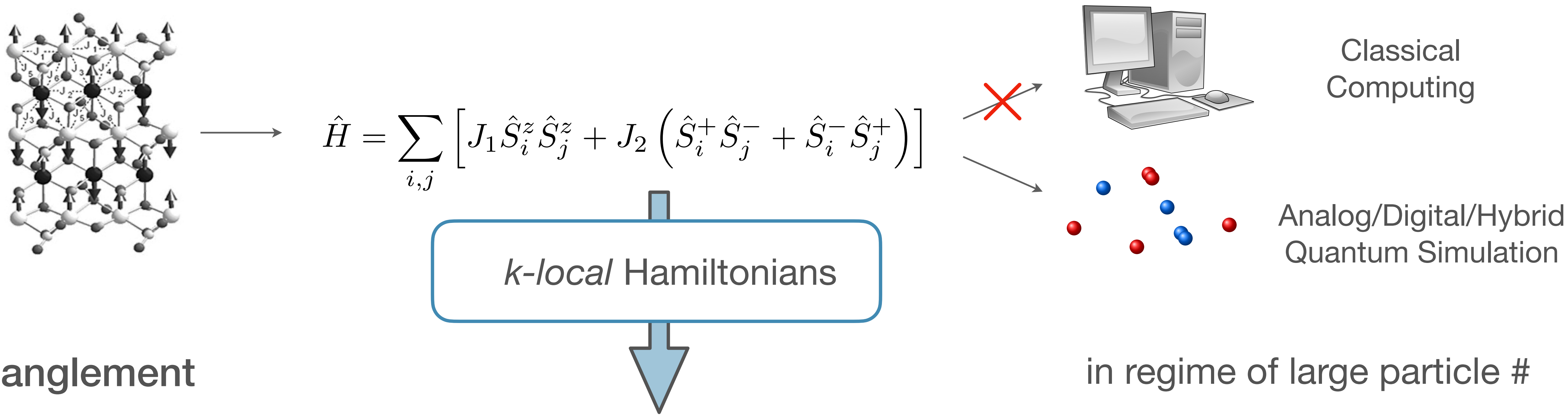


Analog/Digital/Hybrid Quantum Simulation

`quantum advantage’

Frontier of *large-scale entanglement / large-particle number*

Problem: `solving' a quantum many-body problem



$$\hat{H} = \sum_{i,j} \left[J_1 \hat{S}_i^z \hat{S}_j^z + J_2 \left(\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+ \right) \right]$$

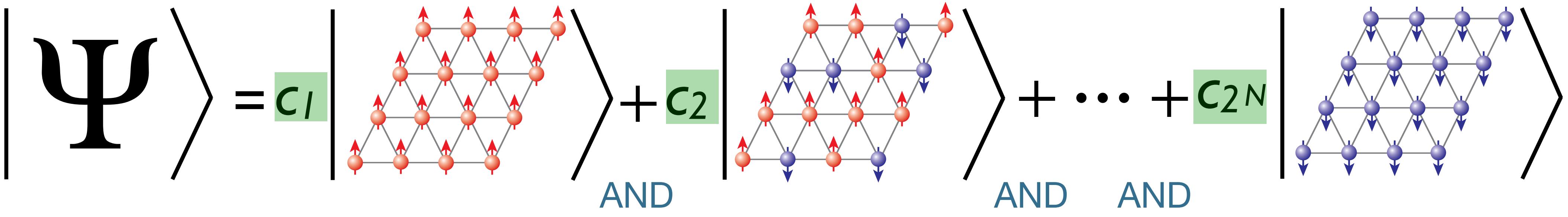
Classical Computing

Analog/Digital/Hybrid Quantum Simulation

k-local Hamiltonians

Entanglement

in regime of large particle #



$$|\Psi\rangle = c_1 |\;\rangle + c_2 |\;\rangle + \cdots + c_{2^N} |\;\rangle$$

AND

AND AND

Goal/Challenge: *Learn* large-scale entanglement structure of many-body quantum state

Outline:

Introduction & Background Material

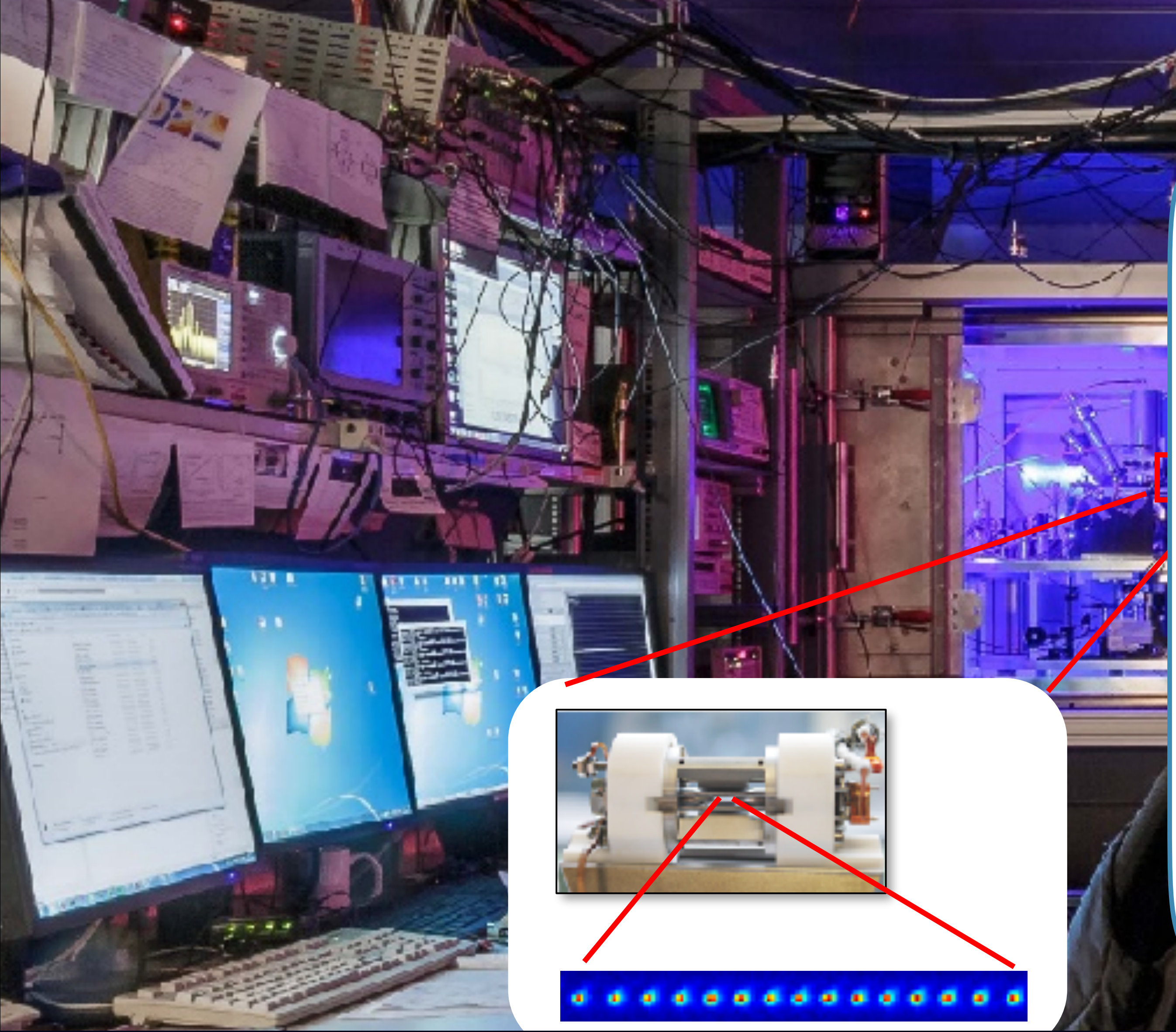
- ⇨ Programmable Quantum Simulators - Atomic Platforms
- Characterizing Entanglement in Many-Body Systems

How to measure Entanglement

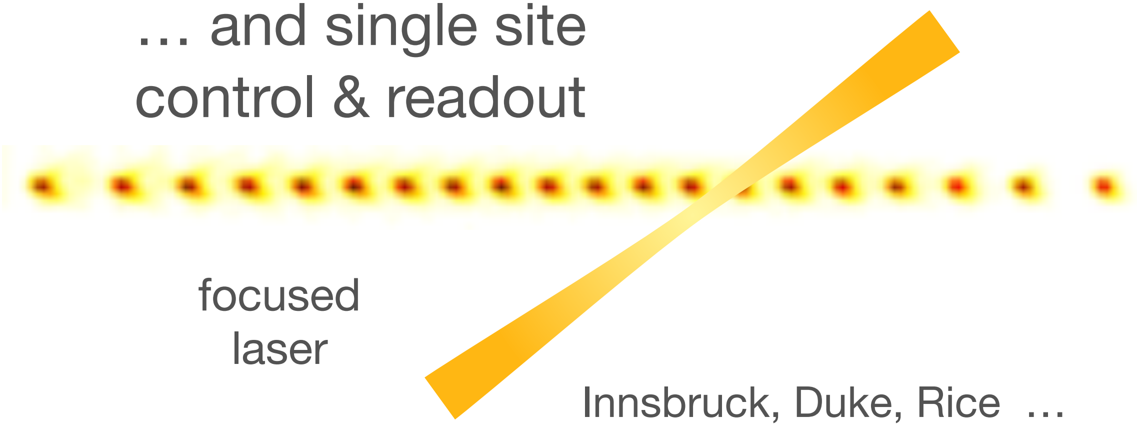
- Randomized Measurement Toolbox
- Renyi Entanglement Entropy
- …

A Elben, ST Flammia, HY Huang, R Kueng, J Preskill, B Vermersch & PZ, Nature Review Physics (2022)

10, 20 … 50 Qubit Trapped-Ion Programmable Quantum Simulator @ IQOQI-Labs



Transverse long-range Ising model



… and single site control & readout

focused laser

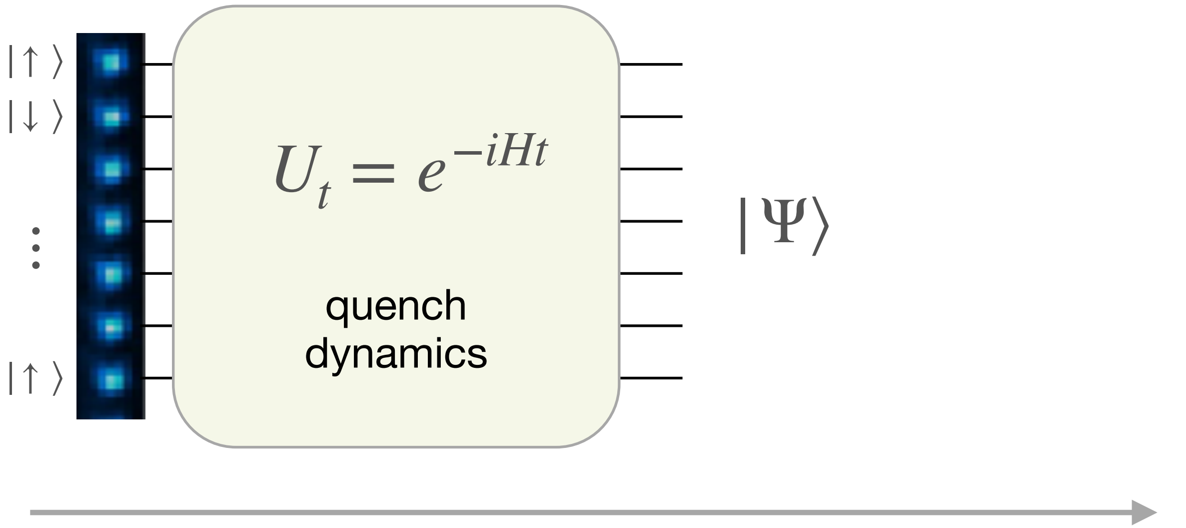
Innsbruck, Duke, Rice …

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

C. Monroe et al., *Programmable quantum simulations of spin systems with trapped ions. RMP* (2021)

Analog Quantum Simulators

What physics can we do … ?



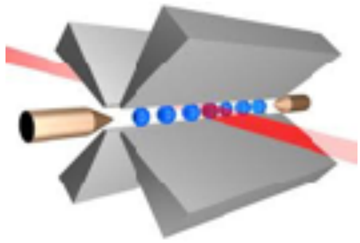
$$U_t = e^{-iHt}$$

quench
dynamics

$$|\Psi\rangle$$

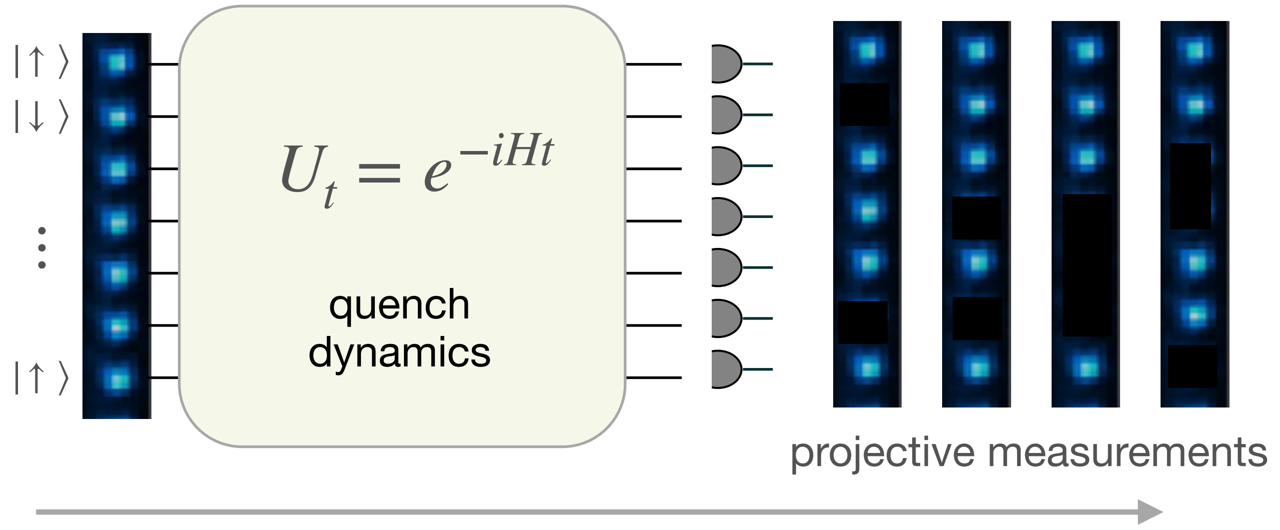
product state → entangled state

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x + B\sum_i \hat{\sigma}_i^z$$



Analog Quantum Simulators

What physics can we do … ?



Repeat

Probability to see a specific configuration of bit string $\mathbf{s}$

$$P_\Psi(\mathbf{s})$$

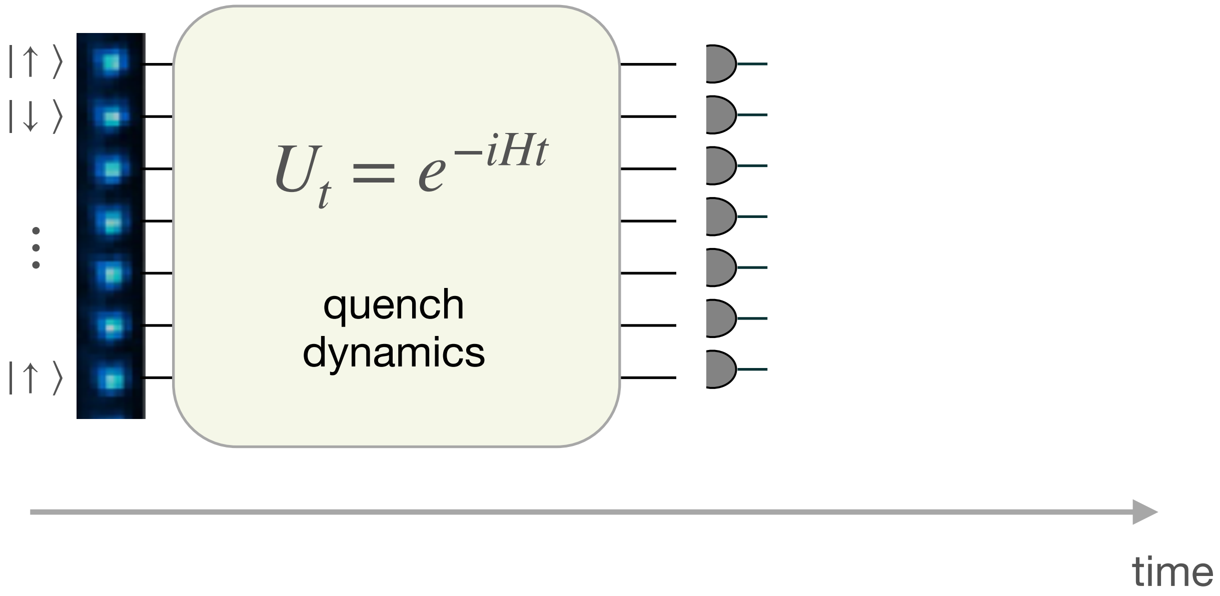
quantum
expectation values

Native Hamiltonian

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij} \hat{\sigma}_i^x \hat{\sigma}_j^x + B \sum_i \hat{\sigma}_i^z$$

Analog Quantum Simulators

What physics can we do … ?

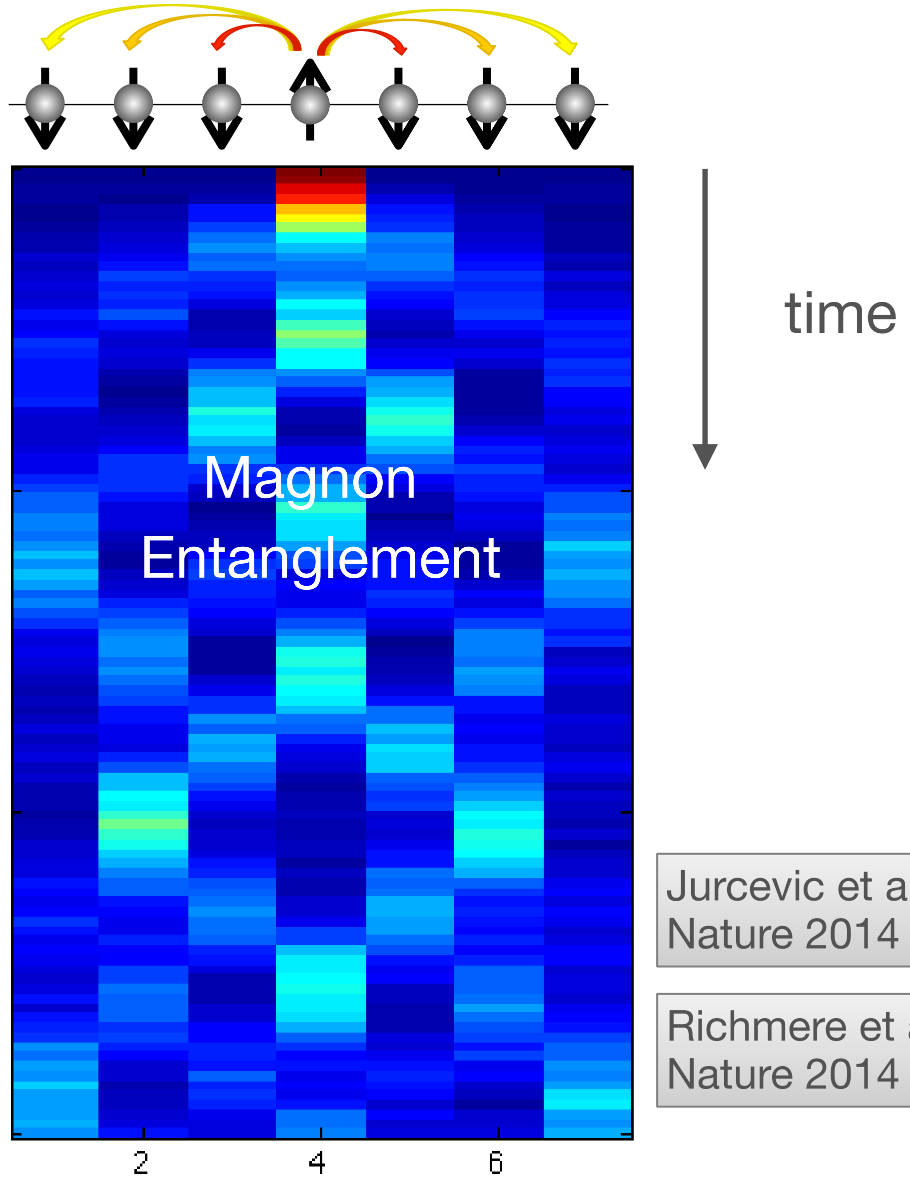


Native Hamiltonian

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x + B\sum_i \hat{\sigma}_i^z$$

Entanglement in quench dynamics

$$|\psi(t)\rangle = e^{-i\hat{H}_{XY}t}|\psi(0)\rangle$$

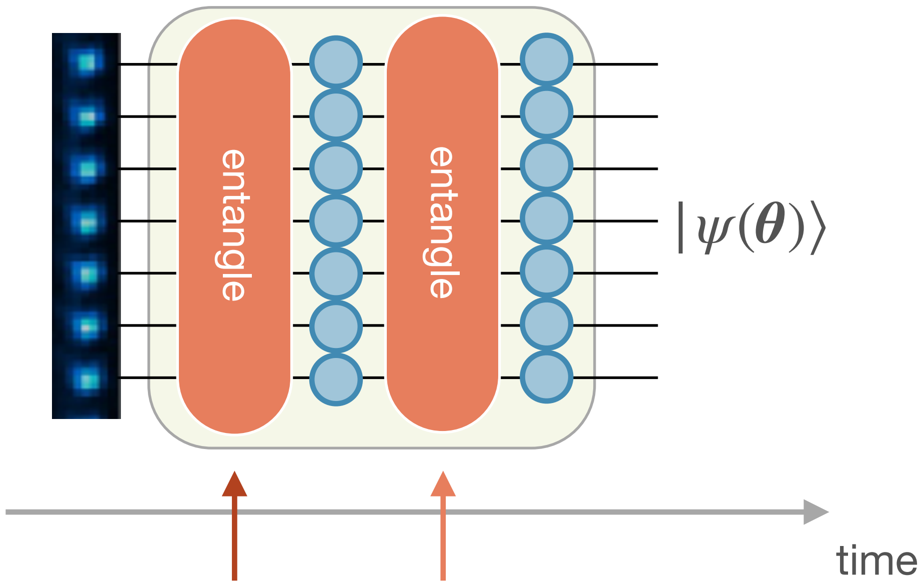


Jurcevic et al., Nature 2014

Richmere et al., Nature 2014

`Programming’ Quantum Simulators

programming entangled states



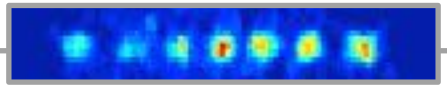
Native Hamiltonian

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x + B\sum_i \hat{\sigma}_i^z$$

… as resource for high-fidelity N-body gate

family of entangled states

$$|\psi(\boldsymbol{\theta})\rangle = \hat{U}_N(\theta_N)\ldots\hat{U}_2(\theta_2)\hat{U}_1(\theta_1)|\psi_0\rangle$$



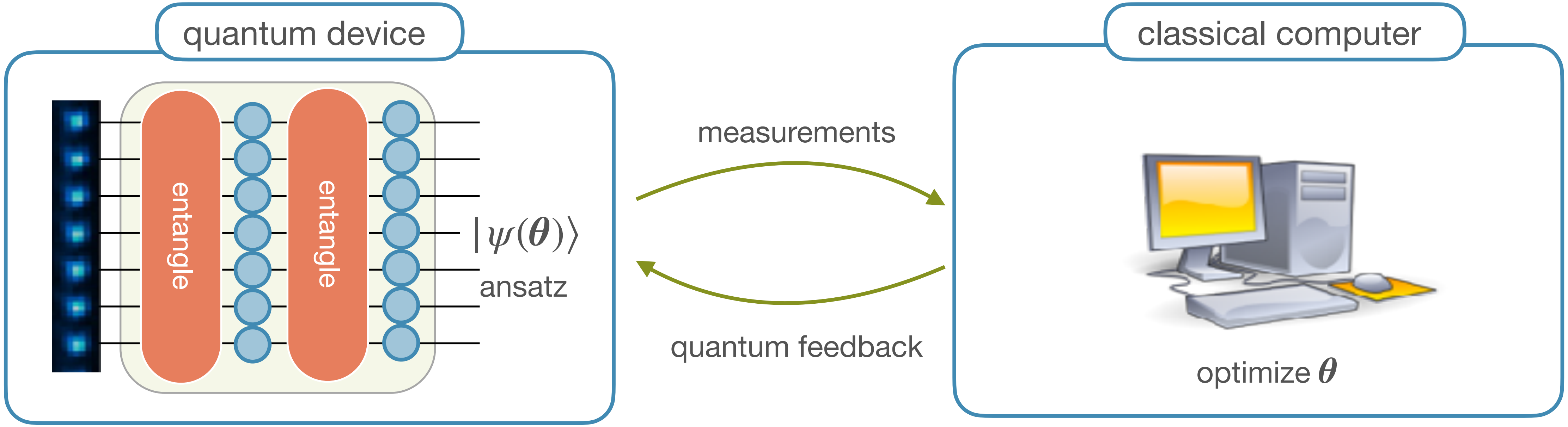
trapped ion quantum resources

$\hat{U}_1(\theta) = e^{-i\theta\sum_{ij}J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x}$ entangle (Ising)

$\hat{U}_{2,i}(\theta) = e^{-i\theta\mathbf{n}\cdot\hat{\sigma}_i}$ local rotations

- in general not universal gate set
- scalable

`Programming’ Quantum Simulators



Variational Classical-Quantum Algorithms

target Hamiltonian (e.g. lattice model)

$$\hat{H}_T = \sum_{n\alpha} h_n^\alpha \hat{\sigma}_n^\alpha + \sum_{n\ell\alpha\beta} h_{n\ell}^{\alpha\beta} \hat{\sigma}_n^\alpha \hat{\sigma}_\ell^\beta + \ldots$$

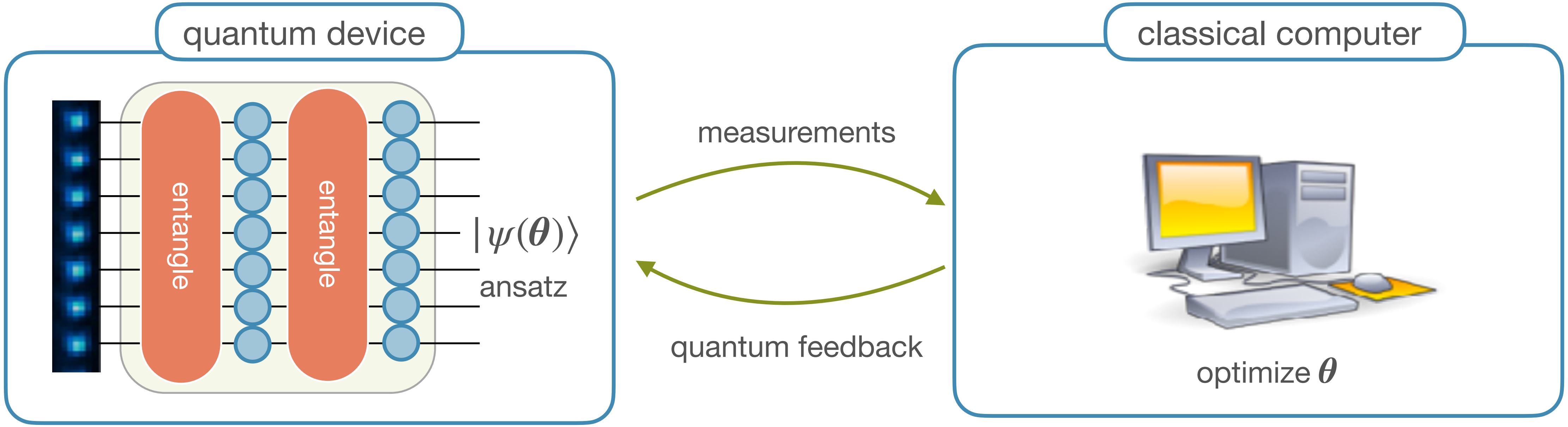
Variational Quantum Eigensolver (VQE)

$$\text{Energy}(\theta) = \langle \psi(\boldsymbol{\theta}) | \hat{H}_T | \psi(\boldsymbol{\theta}) \rangle \to \text{min}$$

… computing ground states

QAOA, E Farhi, J Goldstone, S Gutman, arXiv:1411.4028,
Review: M Cerezo, A Arrasmith, R Babbush, SC Benjamin, S Endo, K Fujii, JR McClean, K Mitarai, X Yuan, L Cincio, PJ Coles, Nature Reviews Physics 3, 625 (2021)

`Programming' Quantum Simulators



Variational Classical-Quantum Algorithms

target Hamiltonian (e.g. lattice model)

$$\langle \hat{H}_T \rangle = \sum_{n\alpha} h_n^\alpha \langle \hat{\sigma}_n^\alpha \rangle + \sum_{n\ell\alpha\beta} h_{n\ell}^{\alpha\beta} \langle \hat{\sigma}_n^\alpha \hat{\sigma}_\ell^\beta \rangle + \ldots$$

measured on quantum device

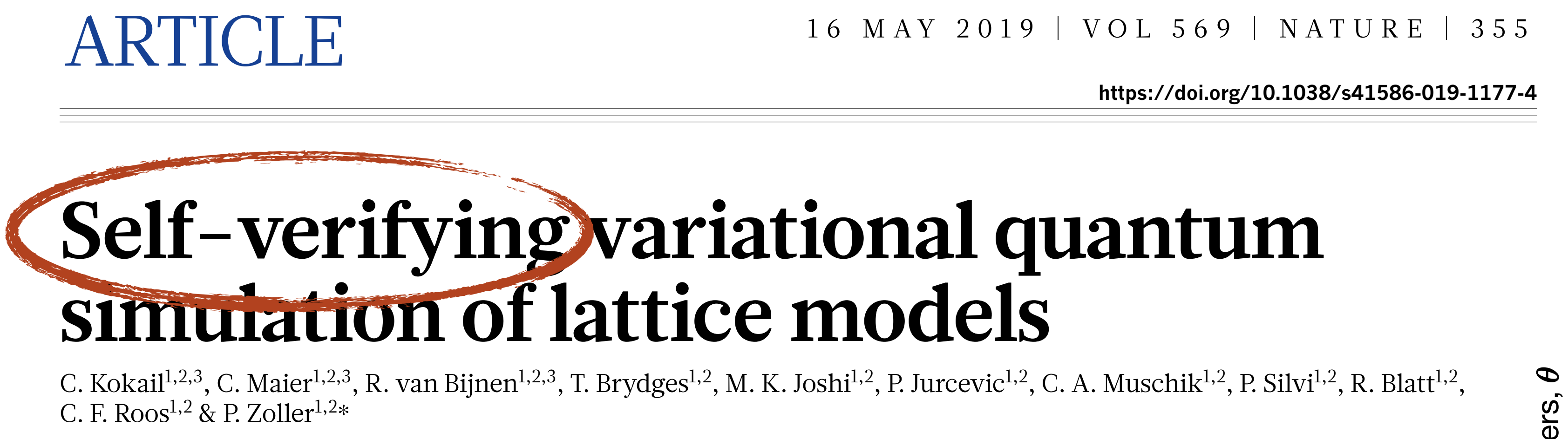
Variational Quantum Eigensolver (VQE)

$$\text{Energy}(\theta) = \langle \psi(\boldsymbol{\theta}) \,|\, \hat{H}_T \,|\, \psi(\boldsymbol{\theta}) \rangle \rightarrow \text{min}$$

… computing ground states

robust to design errors in resource Hamiltonians!

Classical - Quantum Feedback Loop

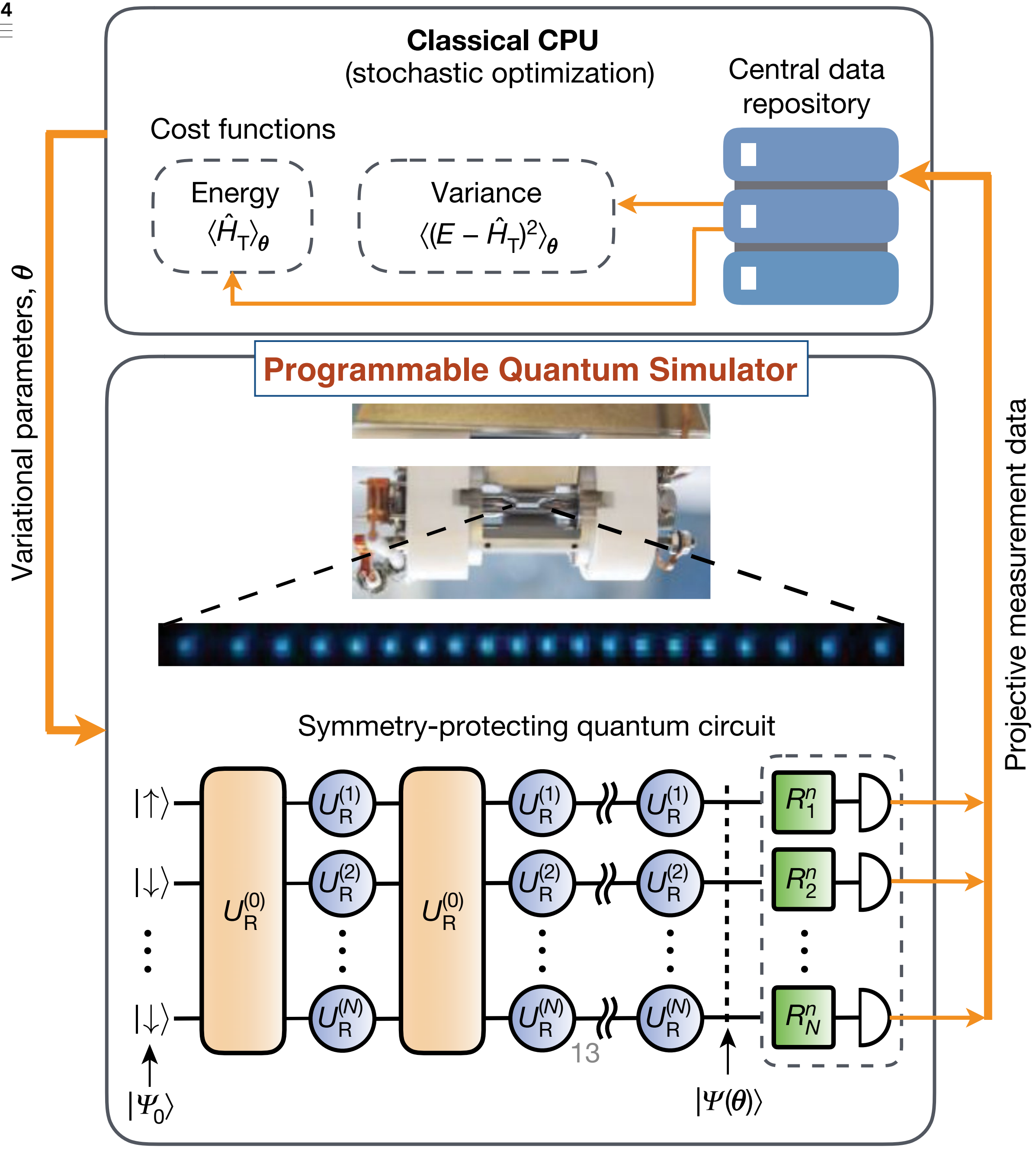


ARTICLE
16 MAY 2019 | VOL 569 | NATURE | 355
https://doi.org/10.1038/s41586-019-1177-4

Self-verifying variational quantum simulation of lattice models

C. Kokail[1,2,3], C. Maier[1,2,3], R. van Bijnen[1,2,3], T. Brydges[1,2], M. K. Joshi[1,2], P. Jurcevic[1,2], C. A. Muschik[1,2], P. Silvi[1,2], R. Blatt[1,2], C. F. Roos[1,2] & P. Zoller[1,2]*

Rick van Bijnen (th-postdoc), Christine Maier (exp-PhD), Christian Kokail (th-PhD)



20 (now: 50) qubits, 10^5 call of PQS, circuit depth 6

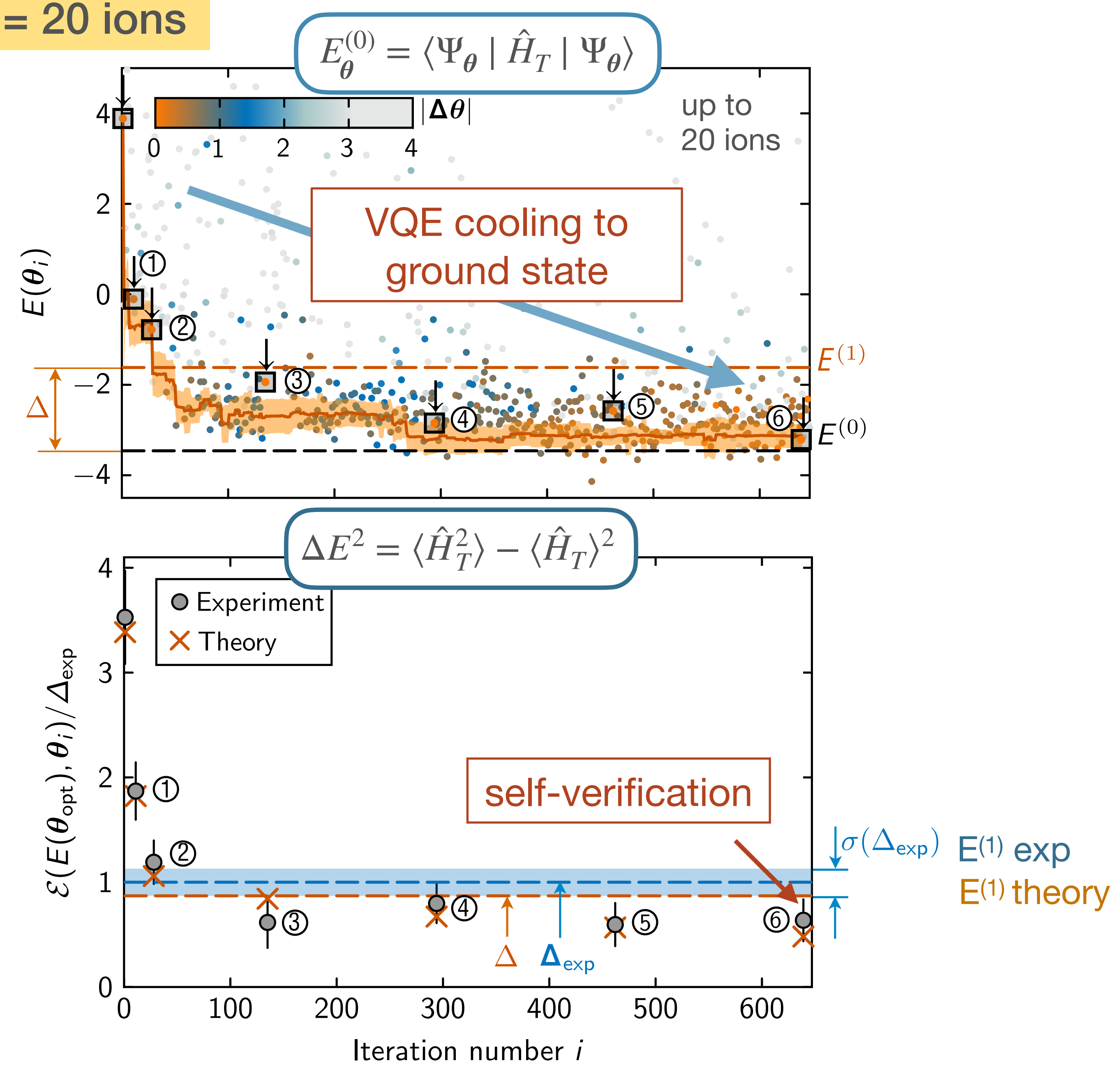
Energy Optimization Trajectory for Ground State (VQE)


C Kokail


C Maier


R. van Bijnen

N = 20 ions



$$E_\theta^{(0)} = \langle \Psi_\theta \mid \hat{H}_T \mid \Psi_\theta \rangle$$

VQE cooling to ground state

$$\Delta E^2 = \langle \hat{H}_T^2 \rangle - \langle \hat{H}_T \rangle^2$$

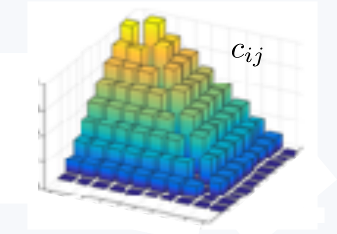
self-verification

Quantum machine provides energy *and* error bar

Lattice Schwinger Model (1+1D QED)

$$H_T = J \sum c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

long - range interaction



$$+w \sum \left(\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^- \right)$$

particle - antiparticle creation/annihilation

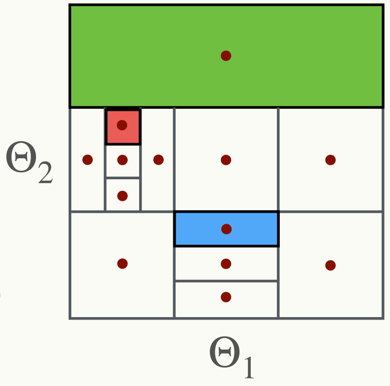
$$+m \sum c_i \hat{\sigma}_i^z + J \sum \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses

DIviding RECTangles (DIRECT)

global optimization
in noisy landscape

- 15 parameters
- circuit depth = 6
- budget: 10^5 calls to simulator

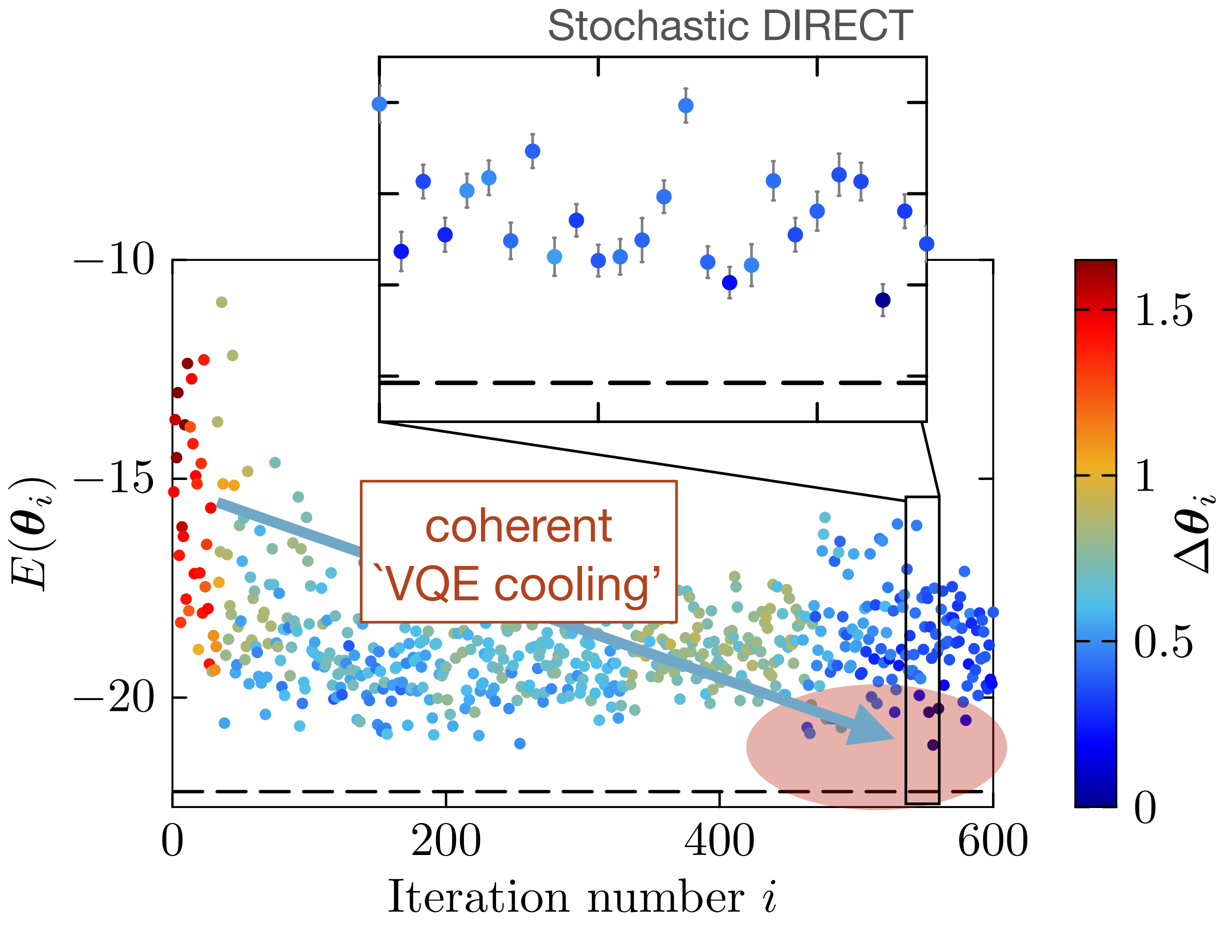


C Kokail, C Maier, R van Bijnen, T Brydges, MK Joshi, P Jurcevic, CA Muschik, P Silvi, R Blatt, CF Roos & P.Z., Nature (2019)

Experimental Energy Optimization Trajectory for Ground State (VQE)

MJ Joshi, C Kokail, R van Bijnen,
F Kranzl, TV Zache,R Blatt,CF Roos, & PZ,
Nature online Nov 29 (2023)

N = 51 ions

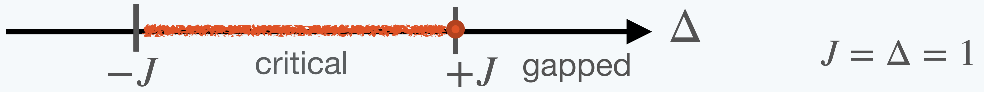

Stochastic DIRECT

prepare pure quantum state $|\Psi\rangle$
~ low `temperature' $T \sim$ few J

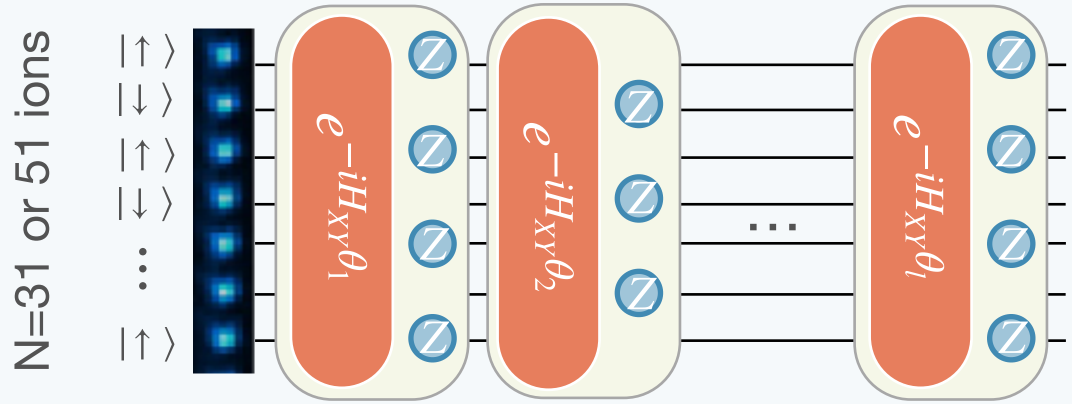
explore low energy physics

Target Model: XXZ (Heisenberg spin-½)

$$\hat{H}_T = J\sum_{i=1}^{N-1}\left(\hat{S}^x_i\hat{S}^x_{i+1}+\hat{S}^y_i\hat{S}^y_{i+1}\right)+\Delta\sum_{i=1}^{N-1}\hat{S}^z_i\hat{S}^z_{i+1}$$



Short VQE Circuit with Ion Resources



5 layers and 10 parameters

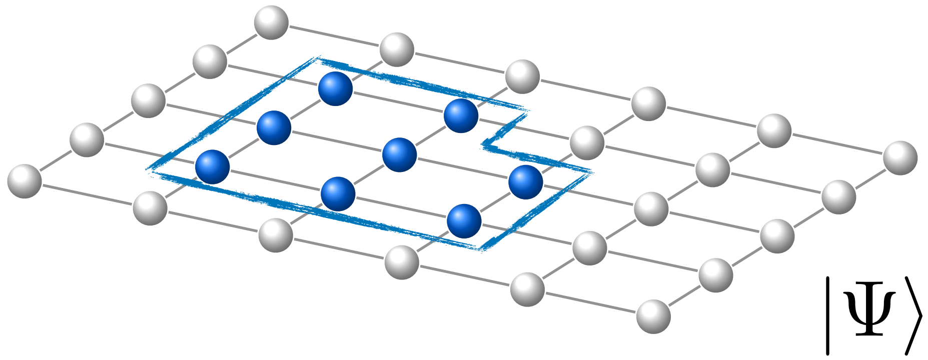
New Physics …?

- Basic Quantum Science

`Learning’ Entanglement

C. Kokail et al, Nat. Phys. 17, 936–942 (2021)

MK Joshi, C Kokail, R van Bijnen et al., Nature online Nov 28 (2023)



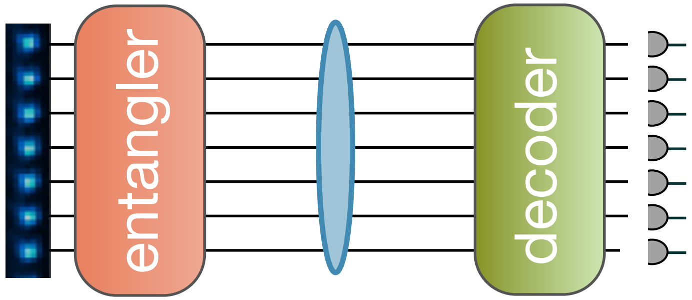
- Applications

Optimal Quantum Metrology

R. Kaubruegger et al., Phys. Rev. X. 11, 041045 (2021)

CD Marciniak et al., Nature. 603, 604 (2022).

Ramsey Interferometer



variational quantum circuits

[beyond squeezing]

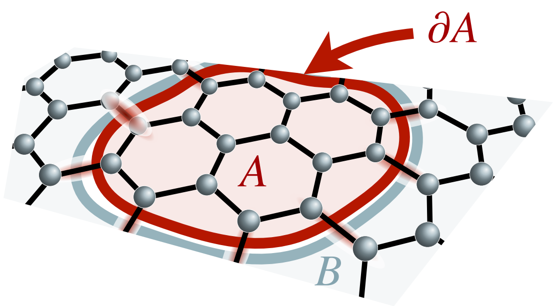
… enabled by Programmable Q-Simulator

Outline:

Introduction & Background Material

- Programmable Quantum Simulators - Atomic Platforms
- ➡ Characterizing Entanglement in Many-Body Systems

Characterizing Entanglement in Quantum Many-Body System



$|\Psi\rangle$

e.g. ground state

Reduced density matrix

$$\rho_A = \mathrm{Tr}_B\left[\,|\Psi\rangle\langle\Psi|\,\right]$$

Two systems are bipartite entangled iff $|\Psi\rangle \neq |\Psi\rangle_A \otimes |\Psi\rangle_B$

$$\mathrm{Tr}_A[\rho_A^2] < 1$$

purity

$$S_A^{\mathrm{VN}} = -\,\mathrm{Tr}_A[\rho_A \log \rho_A]$$

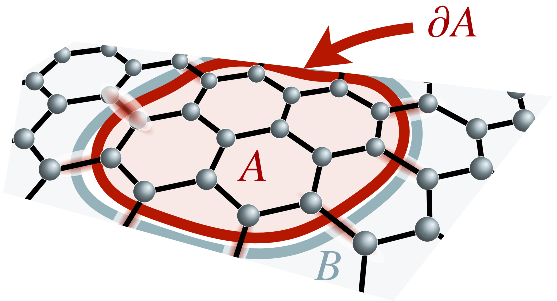
Von Neumann
entanglement entropy

How to measure?

e.g. via randomized measurements

A Elben, ST Flammia, H-Y Huang, R Kueng, J Preskill, B Vermersch & PZ, arXiv:2203.11374

Characterizing Entanglement in Quantum Many-Body System



$|\Psi\rangle$

Reduced density matrix

$$\rho_A = \mathrm{Tr}_B \left[|\Psi\rangle\langle\Psi| \right]$$

e.g. ground state

Two systems are bipartite entangled iff $|\Psi\rangle \neq |\Psi\rangle_A \otimes |\Psi\rangle_B$

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi} e^{-\xi_\alpha/2} |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$$

$\chi = 1$ product state

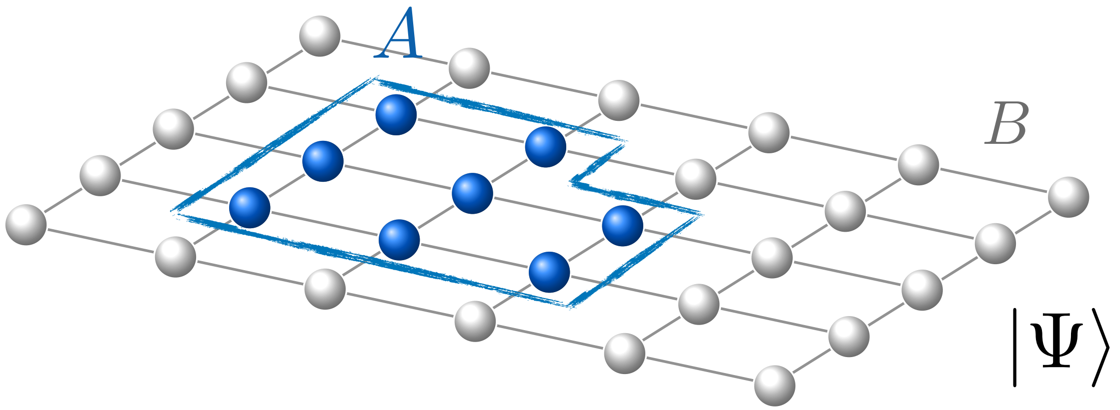
$\chi > 1$ entangled state

ρ_A has rank χ

Schmidt decomposition

Reviews: L. Amico, R. Fazio, A. Osterloh and V. Vedral, RMP (2008); J. Eisert, M. Cramer and M. B. Plenio, RMP (2010); P. Calabrese, J. Cardy and B. Doyon, JPA (2009); I. Peschel and V. Eisler, JPA (2009); O. Gühne, and G. Tóth, Phys. Rep. (2009); J. I Cirac, D Pérez-García, N Schuch, and F Verstraete, RMP (2021)

Characterizing Entanglement in Quantum Many-Body System



Reduced density matrix

$$\rho_A = \mathrm{Tr}_B\left[\,|\Psi\rangle\langle\Psi|\,\right] = e^{-\tilde{H}_A}$$

mixed state: Gibbs ensemble with EH

entanglement Hamiltonian (EH)

$$= \sum_{\alpha=1}^{\chi} e^{-\xi_\alpha} |\Phi_\alpha^A\rangle\langle\Phi_\alpha^A|$$

entanglement spectrum

Why interesting?

- Entanglement measures
- fingerprint of topological order (Li-Haldane)
- detection of quantum phase transitions

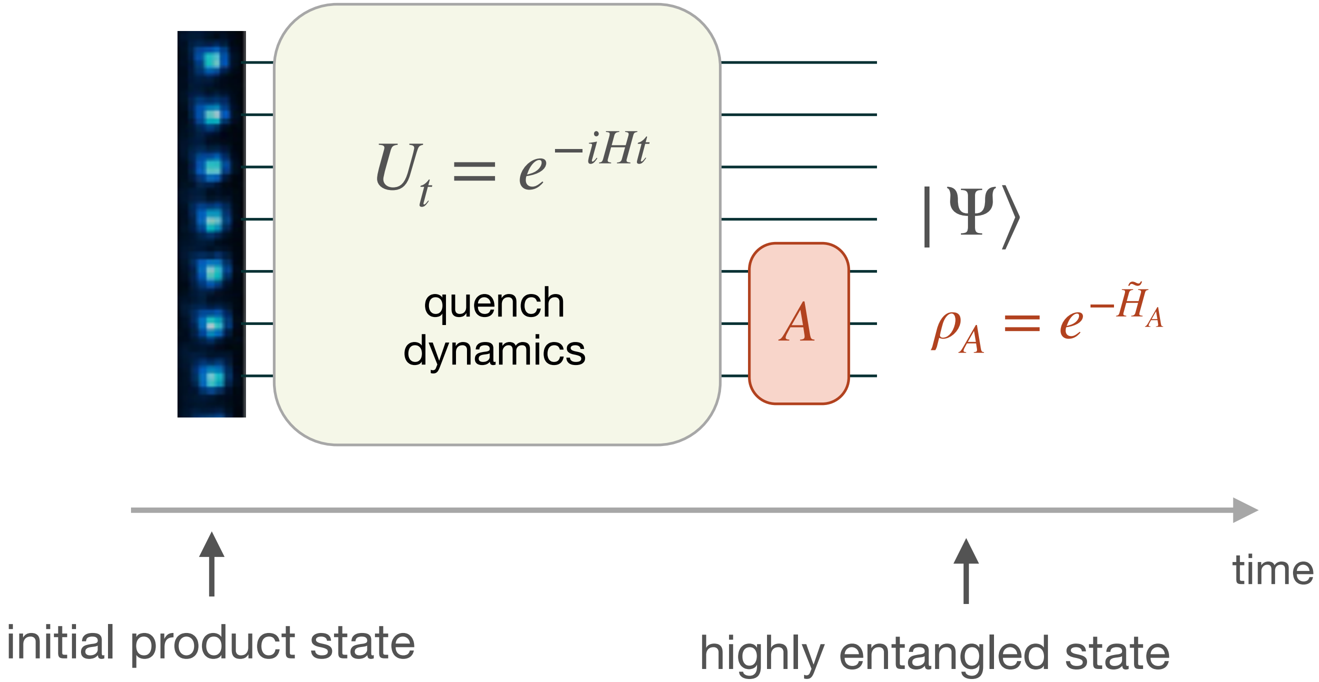
… low-lying entanglement spectrum can be used as a "fingerprint" to identify topological order. [PRL 2008]

D. Haldane

Can we `learn' operator structure of Entanglement Hamiltonian? (sample-)efficient?

Example: Entanglement Spectrum & Quench Dynamics

Quench dynamics with analog quantum simulator



$U_t = e^{-iHt}$

quench dynamics

$|\Psi\rangle$

A

$\rho_A = e^{-\tilde{H}_A}$

time

initial product state

highly entangled state

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi} e^{-\xi_\alpha/2} |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$$

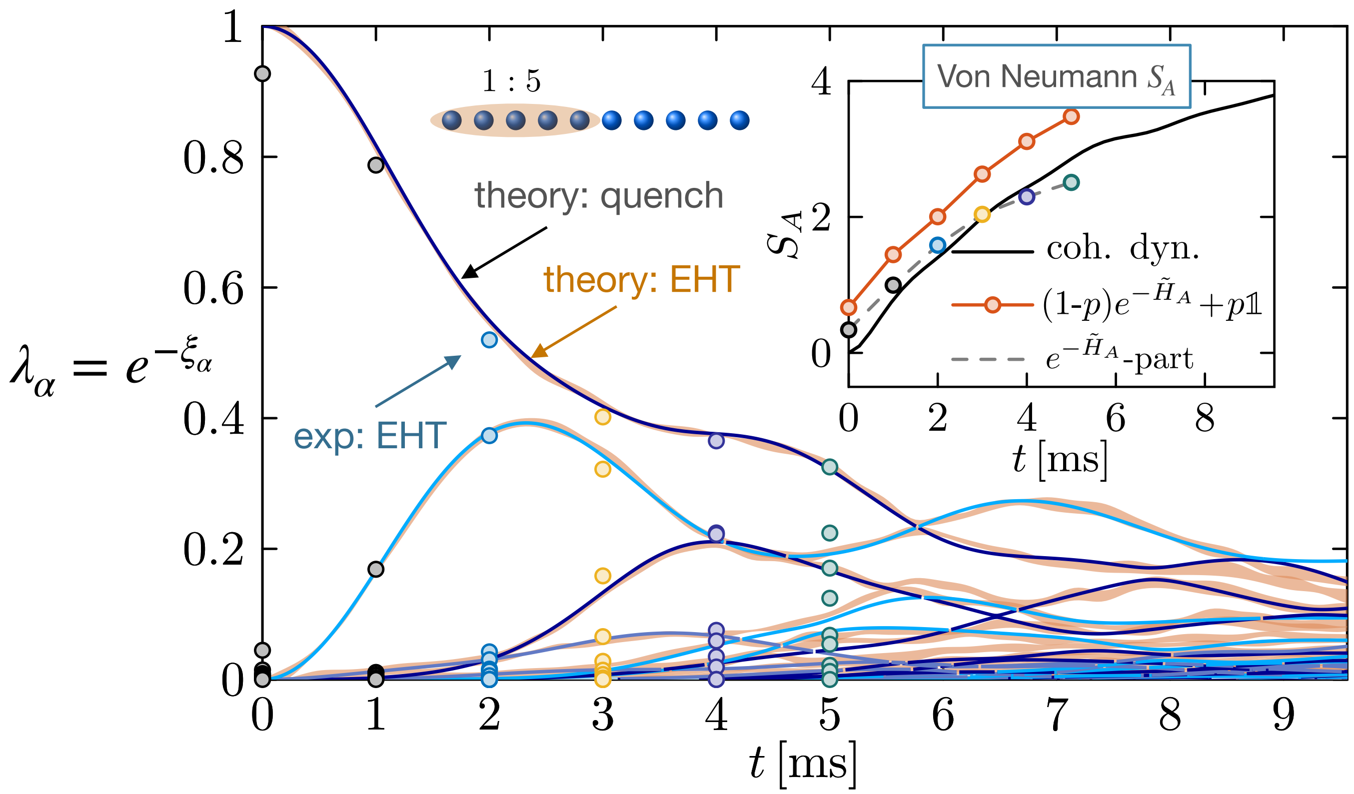
Schmidt values as function of time

product → entangled state

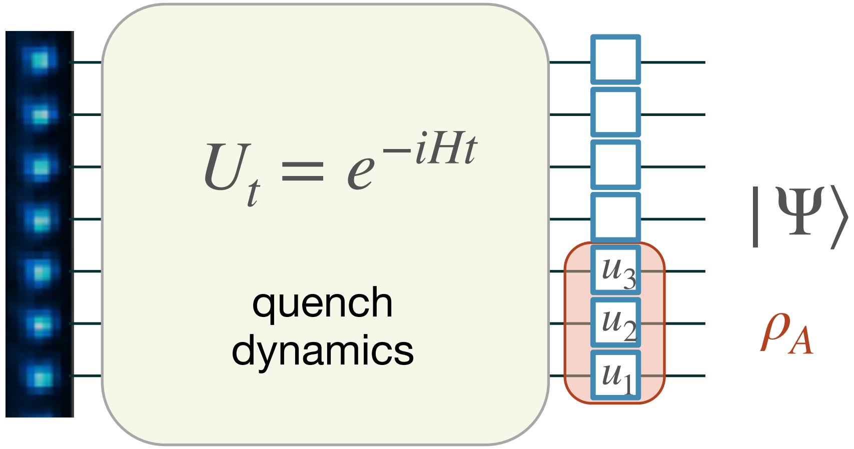
C Kokail, R van Bijnen, A Elben, B Vermersch, & P.Z, Nature Physics (2019); with experimental data from T. Brydges et al., Science (2019)

Example: Entanglement Spectrum & Quench Dynamics Th+Exp

Sub-system [1:5] of 10 ions [similar data for 20 ions and subsystem [8:14]]



$$H = \sum_{i<j} \left(J_{ij}\sigma_i^+\sigma_j^- + \text{ h.c.}\right) + B \sum_i \sigma_i^z$$



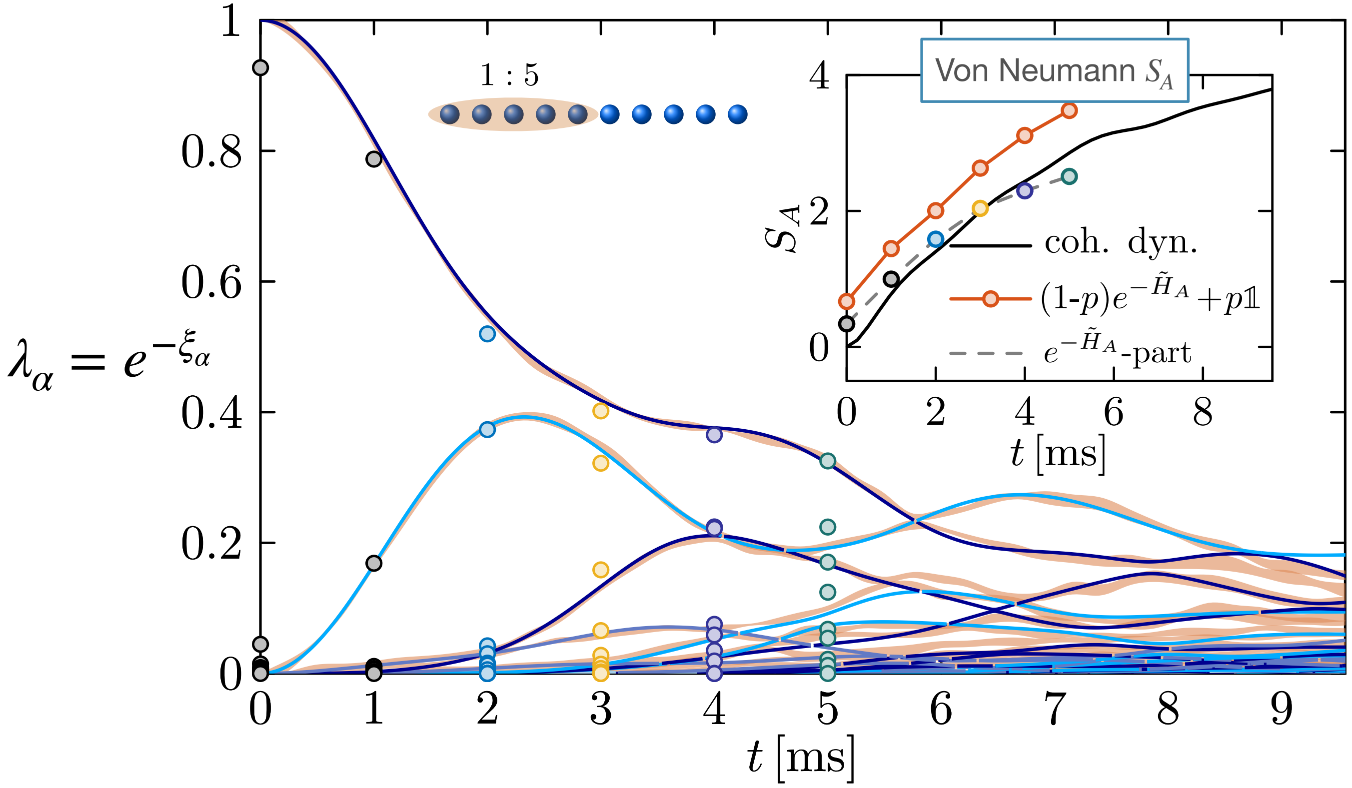
$$|\Psi\rangle = \sum_{\alpha=1}^{\chi} e^{-\xi_\alpha/2} |\Phi_\alpha^A\rangle \otimes |\Phi_\alpha^B\rangle$$

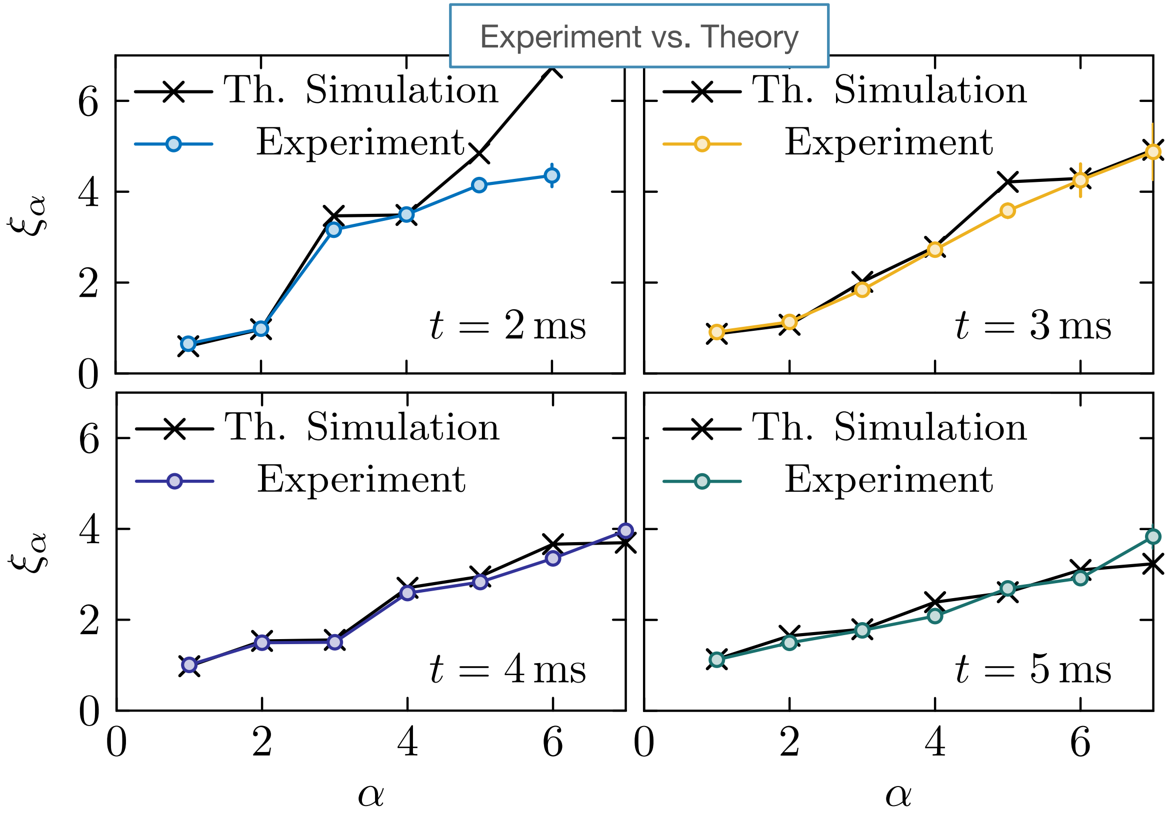
We wish to `see' the Schmidt values $\lambda_\alpha = e^{-\xi_\alpha}$ in quench dynamics in time

C Kokail, R van Bijnen, A Elben, B Vermersch, & P.Z, Nature Physics (2019); with experimental data from T. Brydges et al., Science (2019)

Example: Entanglement Spectrum & Quench Dynamics Th+Exp

Sub-system [1:5] of 10 ions





$$H = \sum_{i<j} \left(J_{ij}\sigma_i^+\sigma_j^- + \text{ h.c.}\right) + B\sum_i \sigma_i^z \quad \text{for } B \gg J$$

Q.: How to extract the ES (and EH) from experimental data?

C Kokail, R van Bijnen, A Elben, B Vermersch, & P.Z, Nature Physics (2019); with experimental data from T. Brydges et al., Science (2019)

Outline

Introduction & Background Material

- Programmable Quantum Simulators - Atomic Platforms
- Characterizing Entanglement in Many-Body Systems

How to measure Entanglement

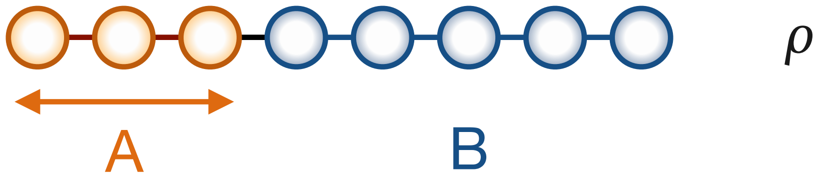
- Renyi Entanglement Entropy
- …

- quantum state tomography
- copies - quantum protocol
- randomized measurements & classical shadows

The randomized measurement toolbox,
A Elben, ST Flammia, HY Huang, R Kueng, J Preskill, B Vermersch & PZ,
Nature Review Physics (2022)

Measuring Renyi Entanglement Entropy

task: measure 2nd order Renyi entanglement entropy



$\mathrm{Tr}_A\, \rho_A^2$ Renyi entropy n=2

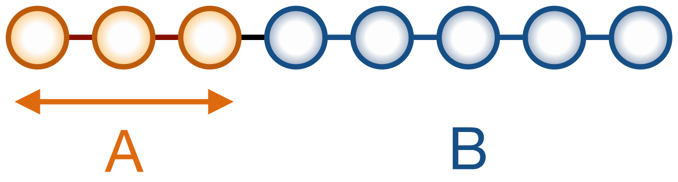
~ purity of subsystem

nonlinear functional of density matrix

but expectation values are always linear: $\langle \hat{A} \rangle = \mathrm{Tr}[\hat{A}\rho]$:-(

Measuring Renyi Entanglement Entropy

Protocol 0: Quantum State Tomography



$\mathrm{Tr}_A\, \rho_A^2$ Renyi entropy n=2

~ purity of subsystem

↑

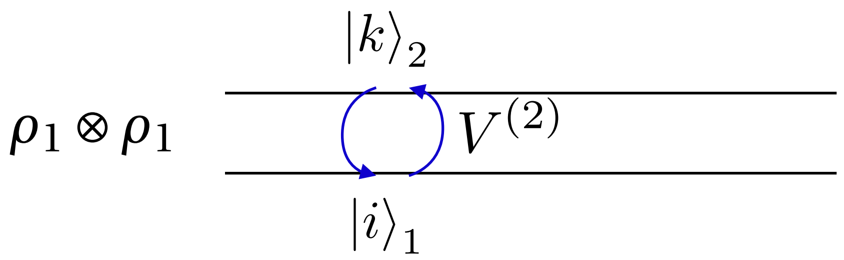
measurement data → ρ_A ✓expensive* $\sim \mathrm{rank}(\rho_A)\; 2^{N_A}$ (scales exponentially)

* except very small (sub)systems,
or we know something about quantum state

Measuring Renyi Entanglement Entropy

Protocol 1: Copies of the quantum system [quantum protocol]

Example n=2:



swap operator

$$V^{(2)} |i\rangle_1 \otimes |k\rangle_2 = |k\rangle_1 \otimes |i\rangle_2$$

$$\mathrm{tr}\{V^{(2)} \rho_1 \otimes \rho_2\} = \mathrm{tr}\left\{ V^{(2)} \sum_{ijkl} \rho^{(1)}_{ij} \rho^{(2)}_{kl} |i\rangle \langle j| \otimes |k\rangle \langle \ell| \right\}$$

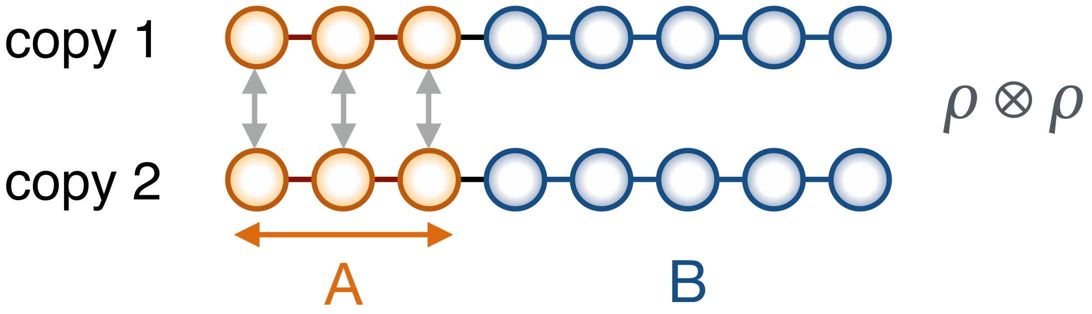
expectation value

$$= \mathrm{tr}\{\rho_1 \rho_2\}$$

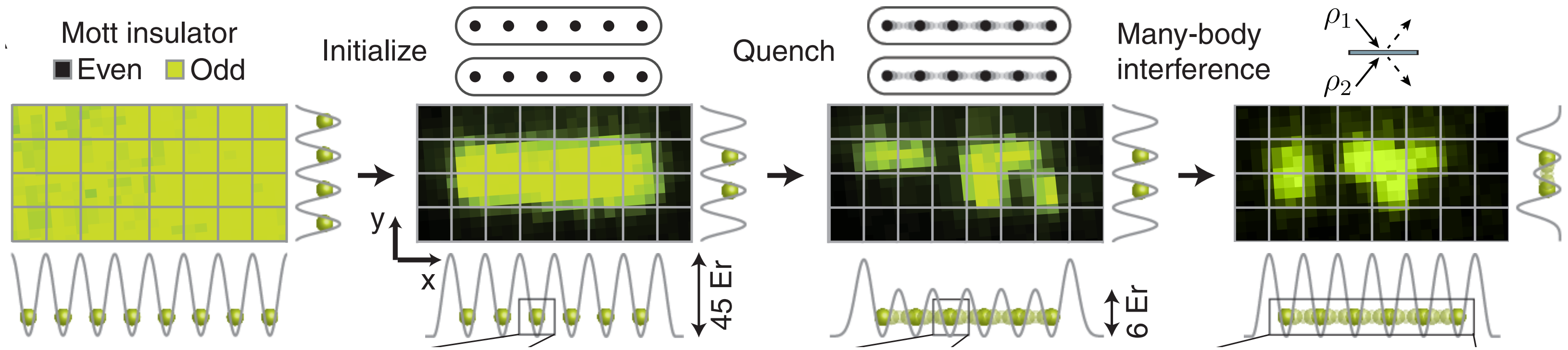
theory: AJ Daley, H Pichler, J Schachenmayer, PZ, PRL (2012); C Moura Alves & D Jaksch, PRL (2004); A. K. Ekert et al.PRL (2002).

Measuring Renyi Entanglement Entropy

Protocol 1: Copies of the quantum system [quantum protocol]



Controlled few-atom systems & quantum gas microscope



see Appendix with details of protocol

experiment: R Islam *et al.*, Nature (2015); AM Kaufmann *et al.*, Science (2016) [Greiner Group]

experiment: R Islam *et al.*, Nature (2015); AM Kaufmann *et al.*, Science (2016) [Greiner Group]

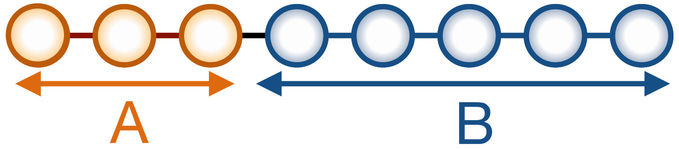
Measuring Renyi Entanglement Entropy


A Elben


B Vermersch

Protocol 2: Single copy of quantum system

single system



$\rho \otimes \rho$?

virtual copy*
(replica trick)

how?

$\mathrm{Tr}_A\, \rho_A^2$ …

from Statistical Correlations
in Random Measurements

signal is in the noise

---

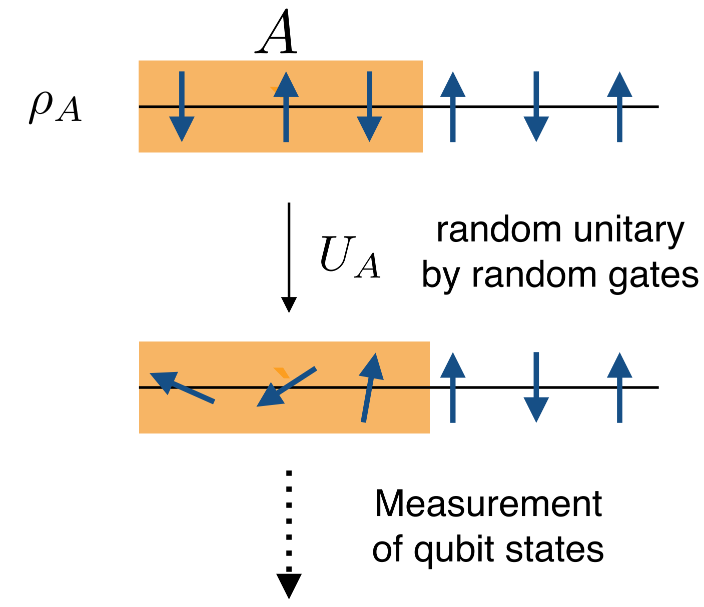
* in contrast to real copies, virtual copies are legal in quantum mechanics

Statistical Correlations in Random Measurements

Protocol for a chain of qubits:

Random measurement

$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[\boldsymbol{U_A \rho_A U_A^\dagger} \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$



$$(\,0\,,\,1\,,\,0\,) = \mathbf{s}_A$$

Average over the Circular Unitary Ensemble (CUE)

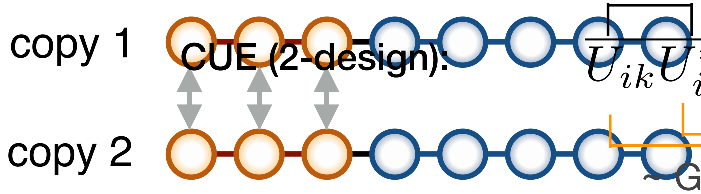
$$\overline{P_U(\mathbf{s}_A)} = \frac{1}{N_{\mathcal{H}_A}}$$

Hilbertspace dimension of A

$$\overline{P_U(\mathbf{s}_A)^2} = \frac{1 + \mathrm{Tr}\left[\rho_A^2\right]}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$

Virtual copies:

$$\overline{P_U(\mathbf{s}_A)^2} = \overline{\mathrm{Tr}_{1\oplus 2}\left[\ldots U_A \rho_A U_A^\dagger \otimes U_A \rho_A U_A^\dagger\right]} = \frac{\mathrm{Tr}_{1\oplus 2}\left[(1+S)\rho_A \otimes \rho_A\right]}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$



CUE (2-design):

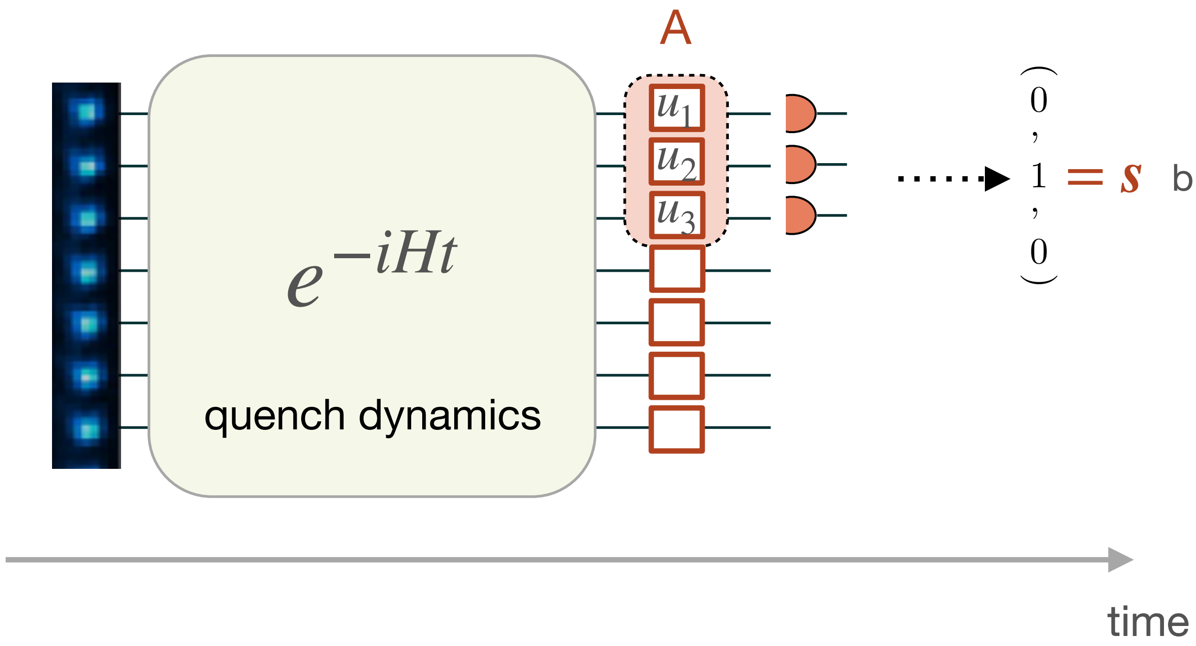
$$\overline{U_{ik}U_{il}^* U_{im} U_{in}^*} = \frac{\delta_{kl}\delta_{mn} + \delta_{kn}\delta_{ml}}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$

~Gaussian

S van Enk, C Beenakker (PRL 2012)

Randomized Measurements: Local Random Unitaries

Measurement post-processing



$\begin{pmatrix}0\\,\\1\\,\\0\end{pmatrix} = s$ bit string

$$P_U(\mathbf{s}) = \mathrm{Tr}\left[U\rho_A U^\dagger |\mathbf{s}\rangle\langle\mathbf{s}|\right]$$

$$U = \bigotimes_{i\in A} u_i \qquad u_i \in \mathrm{CUE}(d)$$

evenly distributed on Bloch sphere

purity - Renyi entropy

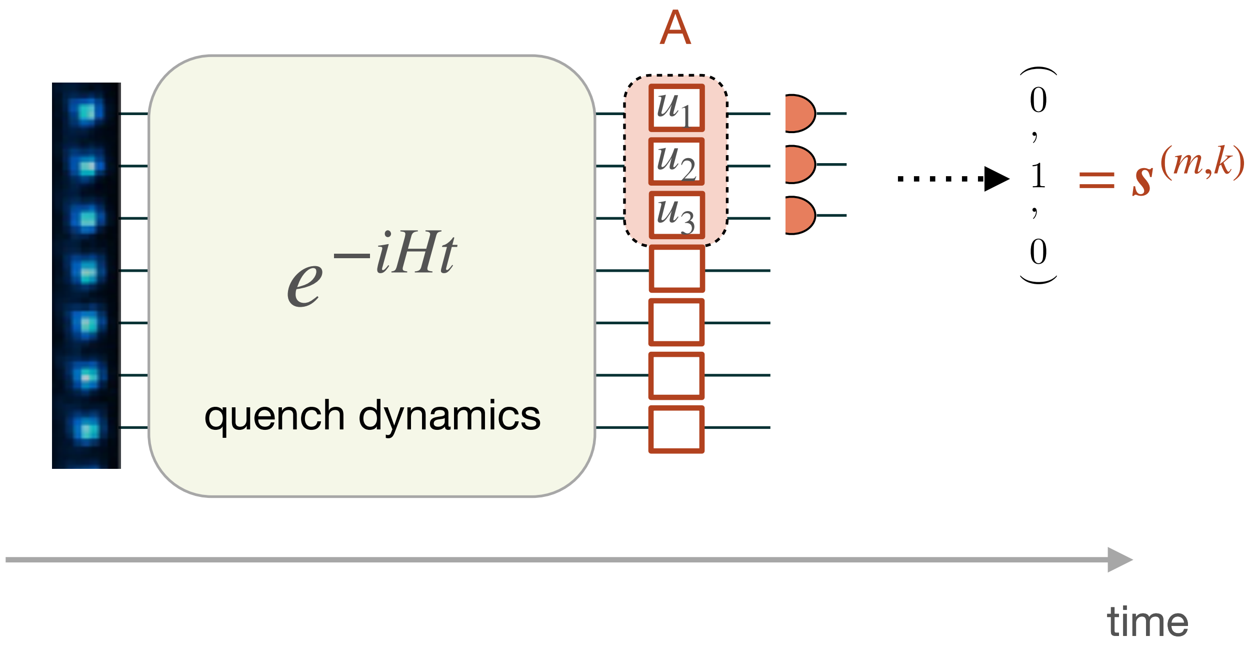
$\mathrm{Tr}\,\rho_A^2 = \mathbb{E}_{U\sim\mathrm{CUE}}[\hat{P}_2]$ with $\hat{P}_2 = 2^{|A|}\sum_{s,s'}(-2)^{\underset{\uparrow \text{ Hamming distance}}{-D[s,s']}}\underbrace{P_U(\mathbf{s})P_U(\mathbf{s}')}_{\text{cross-correlation}}$

Random *single spin* rotations are sufficient!

T Brydges, A Elben et al., Science (2019)
A Elben, B Vermersch, et al., PRA (2019)

Randomized Measurements: Local Random Unitaries

Measurement post-processing



$$P_U(\mathbf{s}) = \mathrm{Tr}\left[U\rho_A U^\dagger |\mathbf{s}\rangle\langle\mathbf{s}|\right]$$

$$U = \bigotimes_{i\in A} u_i \qquad u_i \in \mathrm{CUE}(d)$$

evenly distributed on Bloch sphere

purity - Renyi entropy

$$\mathrm{Tr}\,\rho_A^2 = \mathbb{E}_{U\sim\mathrm{CUE}}[\hat{P}_2] \quad \text{with} \quad \hat{P}_2 = \frac{1}{MK(K-1)}\sum_{m=1}^{M}\sum_{k\neq k'=1}^{K}(-2)^{-D[s^{(m,k)},s^{(m,k')}]}$$

$k, k' = 1,\ldots,N_U \equiv K$ random unitaries

$m = 1,\ldots,N_M \equiv M$ measurements

$N_U \times N_M$ # exp runs

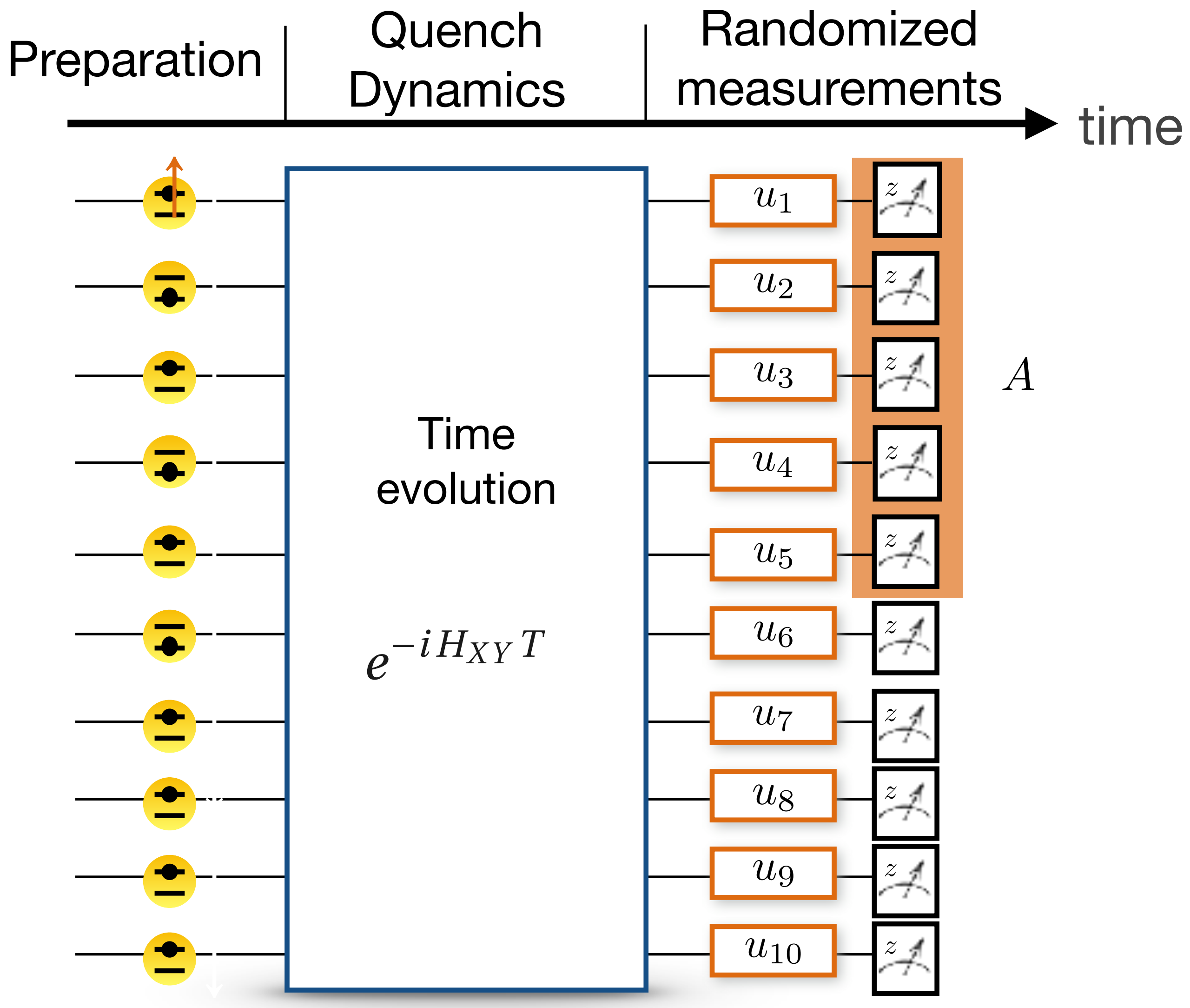
Features:

- local operations & measurements
- scaling with #unitaries and #measurements?

T Brydges, A Elben et al., Science (2019)

Proof → A Elben, B Vermersch, et al., PRA (2019)

Example 1: Experiment —Entanglement in Quench Dynamics



Preparation

Quench Dynamics

Randomized measurements

time

Time evolution

$e^{-iH_{XY}T}$

u_1 u_2 u_3 u_4 u_5 u_6 u_7 u_8 u_9 u_{10}

A

Hamiltonian

$$H_{XY} = \hbar \sum_{i<j} \boxed{J_{ij}} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+)$$

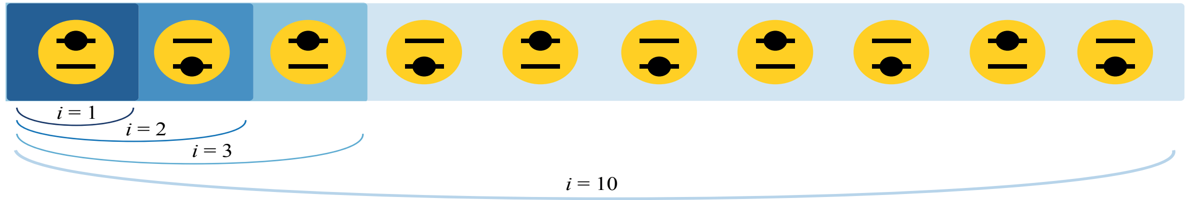
long range interaction

$$+ \hbar \sum_j (B + \boxed{b_j}) \sigma_j^z$$

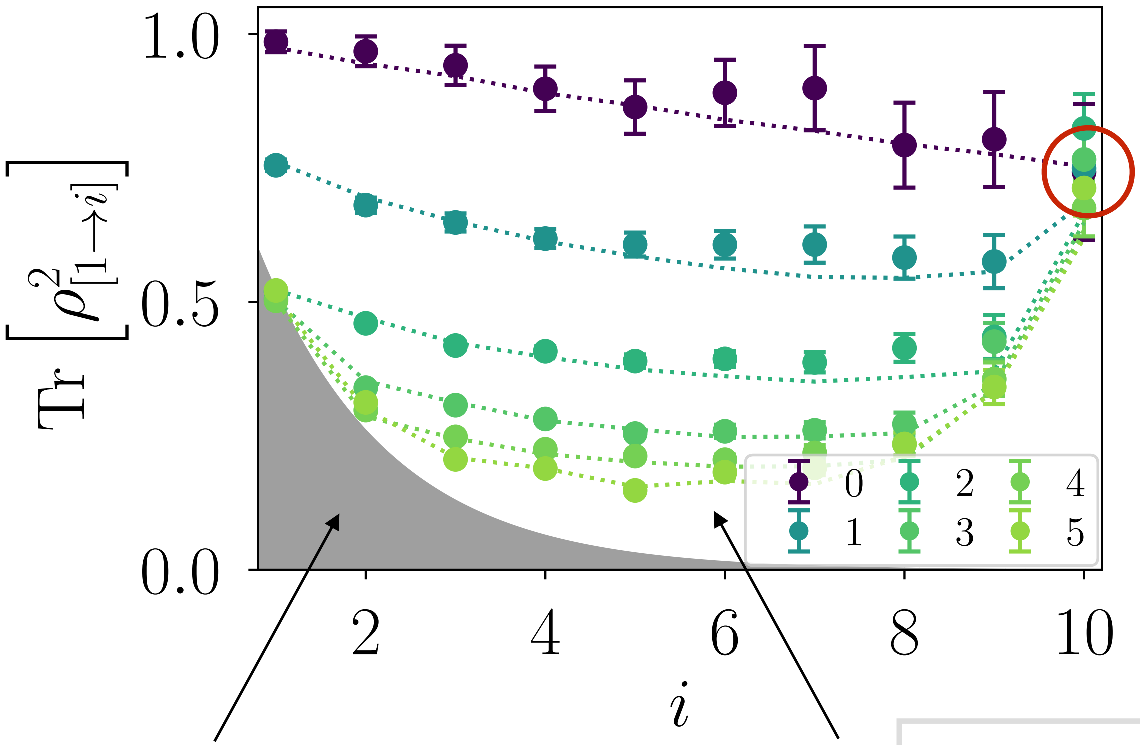
local disorder potentials

long range interaction local disorder potentials

10 Ions [no disorder]

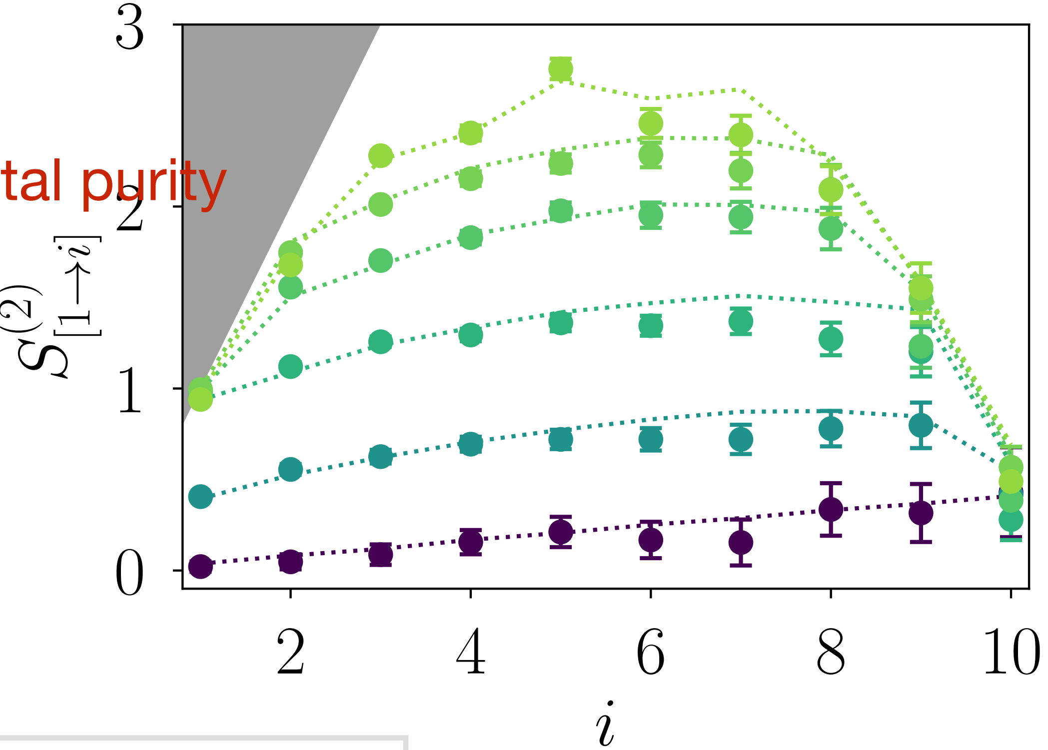


purity $\mathrm{Tr}[\rho_A^2]$



=0.76 total purity

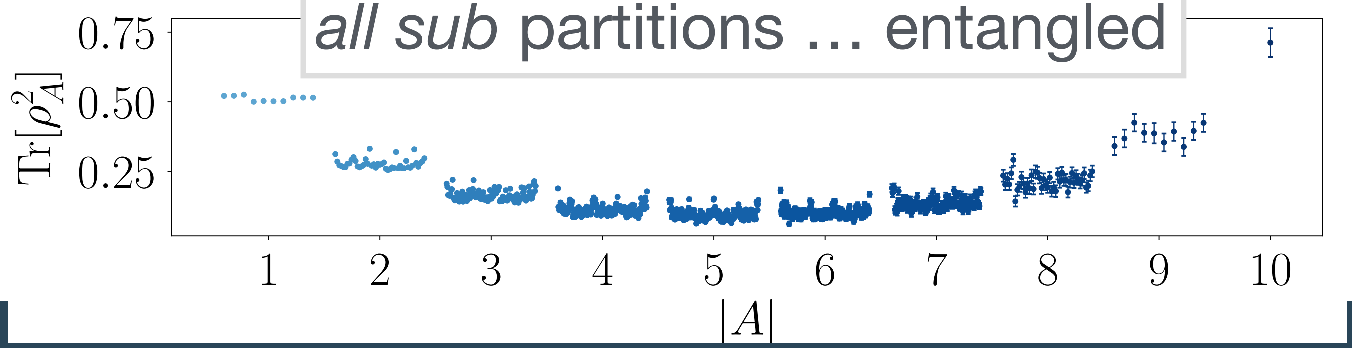
Renyi entropy



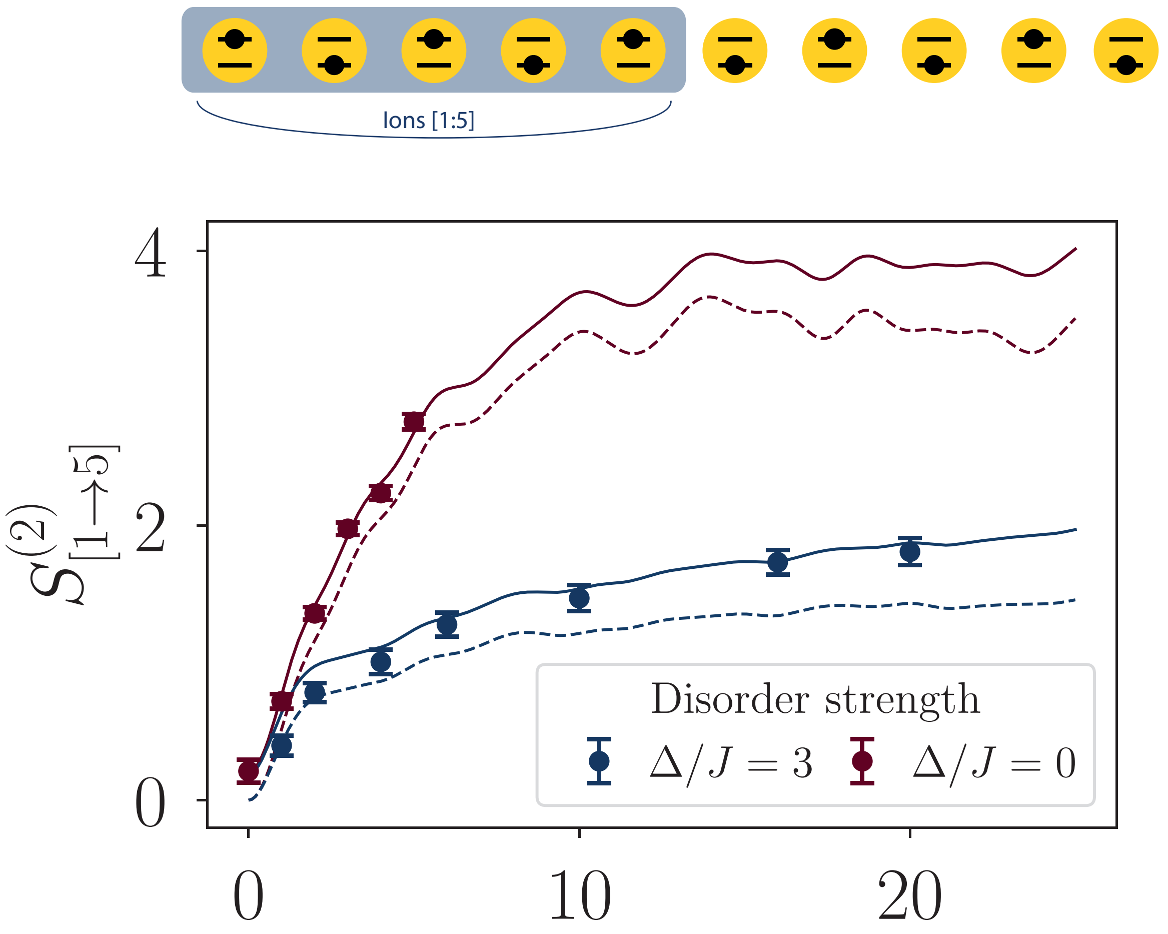
ms

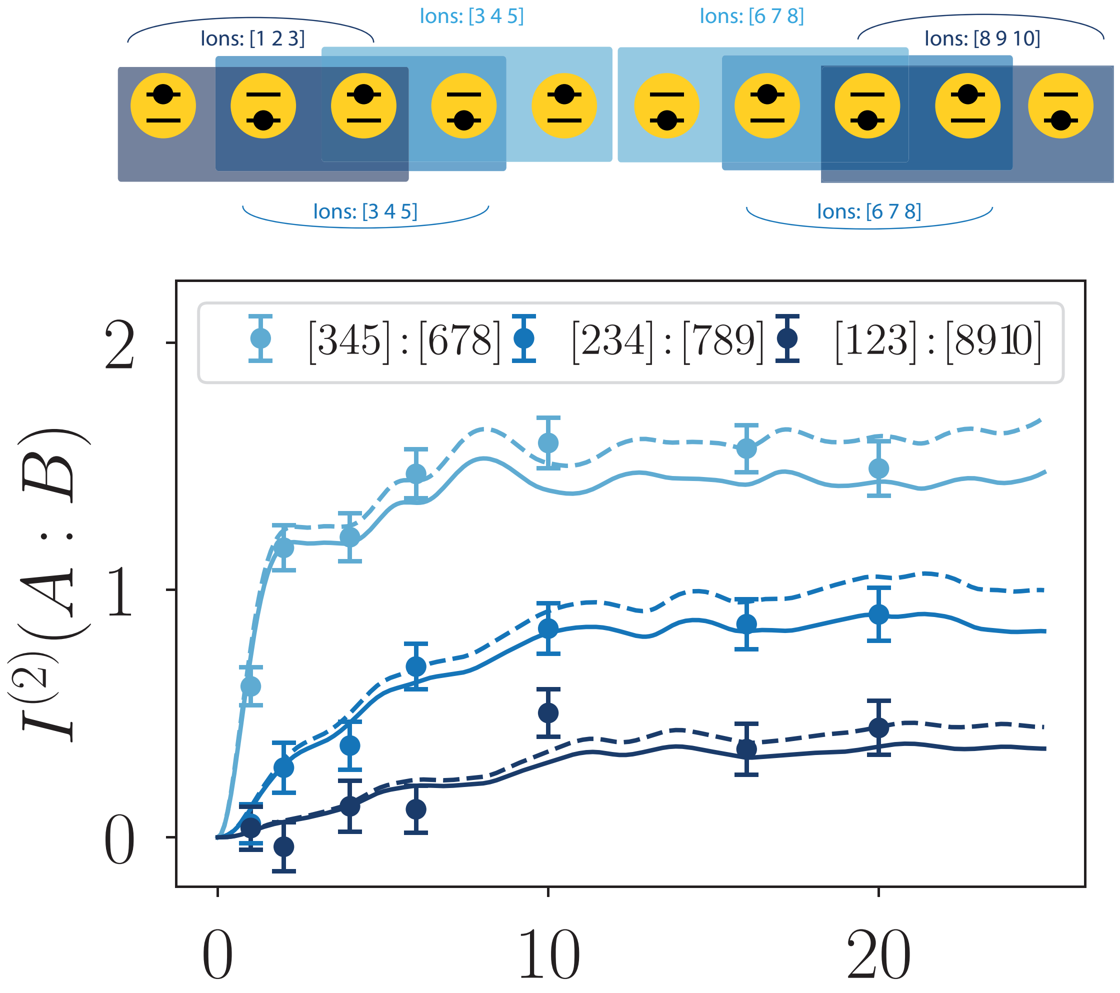
maximally mixed

all sub partitions … entangled



10 Ions [disorder]





$$H_{XY} = \hbar \sum_{i<j} \boxed{J_{ij}} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + \hbar \sum_j (B + \boxed{b_j}) \sigma_j^z$$

long range interaction

local disorder potentials

Example 2:

A Elben
→ Caltech

B Vermersch
→ Grenoble

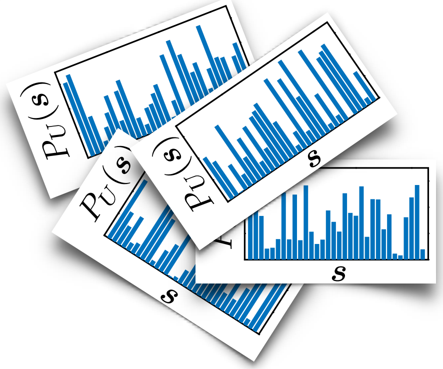
T Brydges

MK Joshi

Editors' Suggestion Featured in Physics PHYSICAL REVIEW LETTERS **124,** 010504 (2020) published 6 January 2020

Cross-Platform Verification of Intermediate Scale Quantum Devices

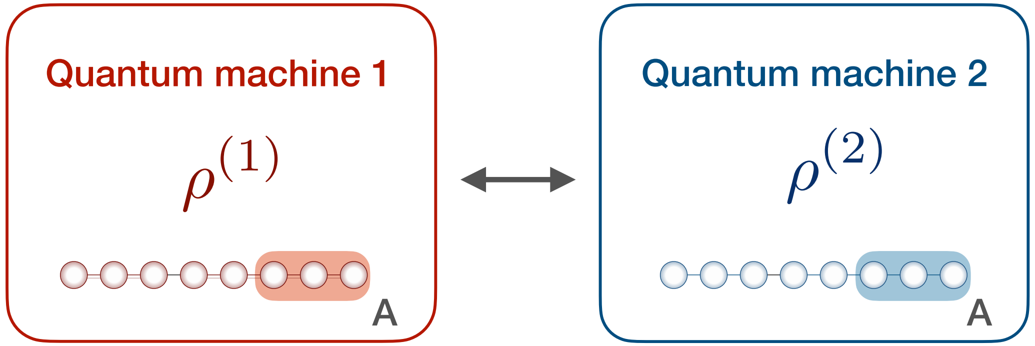
Andreas Elben, Benoît Vermersch, Rick van Bijnen, Christian Kokail, Tiff Brydges, Christine Maier,
Manoj K. Joshi, Rainer Blatt, Christian F. Roos, and Peter Zoller



data from Brydges et al.

$$\mathcal{F}(\rho_A^{(1)}, \rho_A^{(2)}) \sim \mathrm{Tr}\left[\rho_A^{(1)}\rho_A^{(2)}\right]$$

How to measure?



Quantum machine 1

$\rho^{(1)}$

A

Quantum machine 2

$\rho^{(2)}$

A

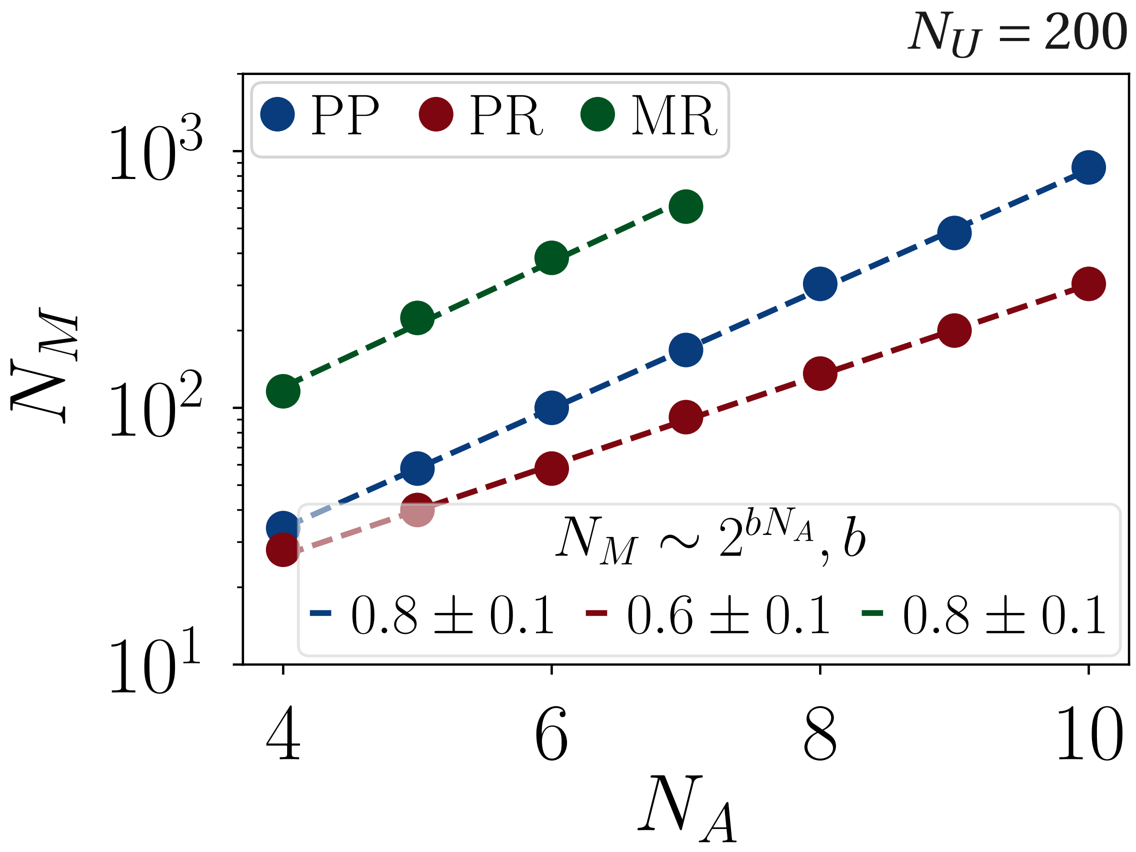
J. Carrasco, A. Elben, C. Kokail, B. Kraus, and P. Zoller, *Theoretical and Experimental Perspectives of Quantum Verification*, PRX Quantum (2021).

J. Eisert, D. Hangleiter, N. Walk, I. Roth, D. Markham, R. Parekh, U. Chabaud, and E. Kashefi, *Quantum certification and benchmarking*, Nature Reviews Physics (2020).

Exp.: D. Zhu et al. (C Monroe), Nat Comm (2022)

Scaling of the required number of measurements [numerical results]

Minimal number of required measurements N_M to estimate $(\mathscr{F}_{\text{max}}(\rho_A, \rho_A))_e$ for error $\epsilon = 0.05$ vs. number qubits N_A for $N_U = 100$.



PP: pure product state
PR: pure Haar random state
MR: mixed random states

Results:

- Scaling statistical error

$$|[\mathscr{F}_{\text{max}}(\rho_A, \rho_A)]_e - 1| \sim 1/(N_M \sqrt{N_U})$$

for $N_M \lesssim D_A = 2^{N_A}$ and $N_U \gg 1$,

- Scaling experimental runs

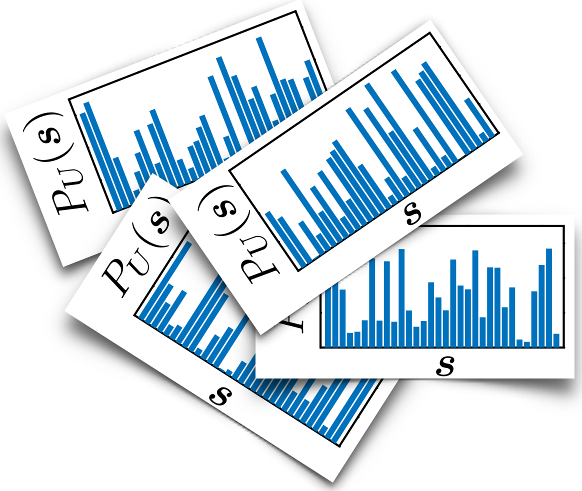
$$N_U N_M \sim 2^{bN_A}$$

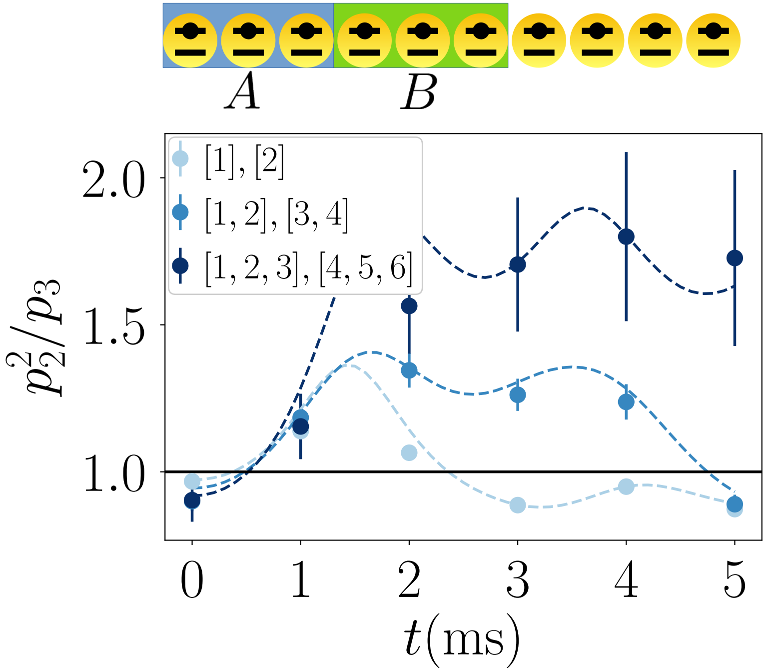
with $b \lesssim 1$ vs. full tomography $b \geq 2$

Example 3:

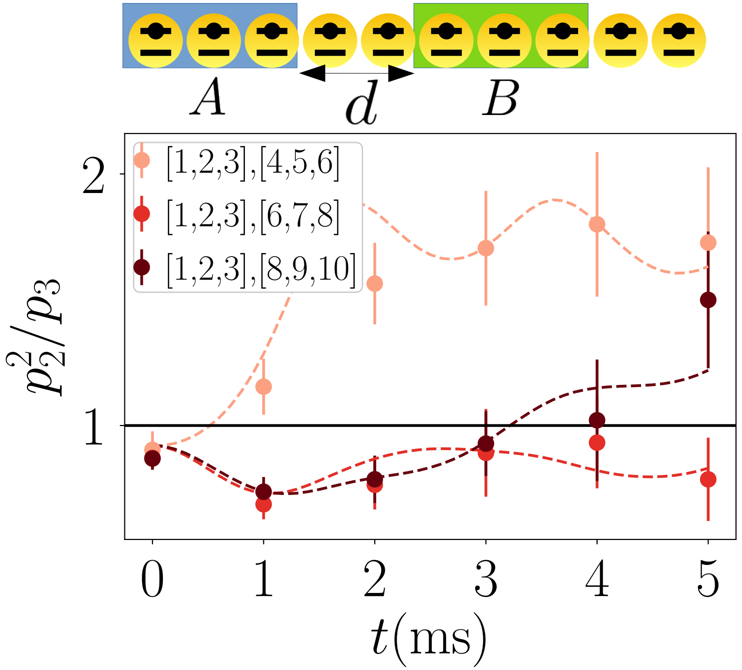
Mixed State Entanglement

Measure first,
ask questions later

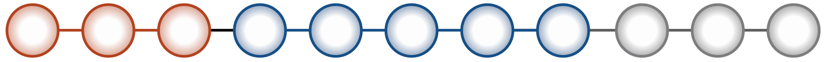




connected subpartitions



disconnected subpartitions



A and B bipartite entangled iff

$$\rho_{AB} \neq \sum_i p_i \, \rho_A^{(i)} \otimes \rho_B^{(i)}$$

p3-PPT condition

$$p_3 < p_2^2$$

where

$$p_n = \mathrm{Tr}[(\rho_{AB}^{T_A})^n]$$

PT-moments

➡ sufficient condition

A Elben, R Kueng, HY Huang, R van Bijnen, C Kokail, M Dalmonte, P Calabrese, B Kraus, J Preskill, PZ & B Vermersch, PRL (2020)

Randomized Measurements

Exp: T. Brydges, A. Elben et al., Science (2019),
Probing Renyi Entanglement Entropy via Randomized Measurements

Theory: A Elben, B Vermersch, CF Roos, and PZ, PRA (2019),
Randomized Measurements: A Toolbox …


A Elben
→ Caltech


B Vermersch
→ Grenoble


C Kokail


R van Bijnen

Classical Shadows

H.-Y. Huang, R. Kueng, and J. Preskill, Nat. Phys. 16, 1050 (2020)
Predicting Many Properties of a Quantum System from Very Few Measurements


Review article Nature Reviews Physics 1 (2022).

The randomized measurement toolbox

Andreas Elben[1,2,3,4], **Steven T. Flammia**[1,5], **Hsin-Yuan Huang**[1,6], **Richard Kueng**[7], **John Preskill**[1,2,5,6], **Benoît Vermersch**[3,4,8] & **Peter Zoller**[3,4] ✉

Measure first, ask questions later

Randomized Measurements

Exp: T. Brydges, A. Elben et al., Science (2019),
Probing Renyi Entanglement Entropy via Randomized Measurements

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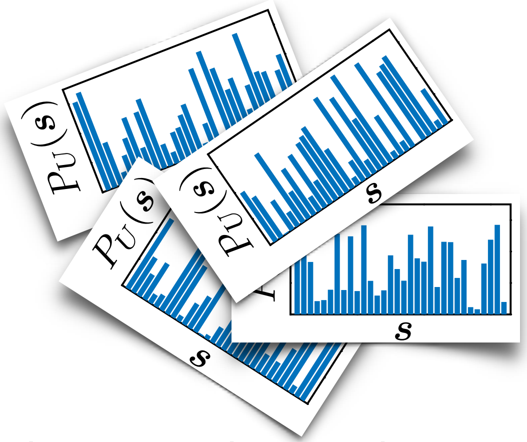


data reused many times …

Classical Shadows

H.-Y. Huang, R. Kueng, and J. Preskill, Nat. Phys. 16, 1050 (2020)
Predicting Many Properties of a Quantum System from Very Few Measurements

Rigorous error bounds:

$$M \propto \log(L) 4^w/\epsilon^2$$

independent randomized measurements suffice to …

Review article Nature Reviews Physics 1 (2022).

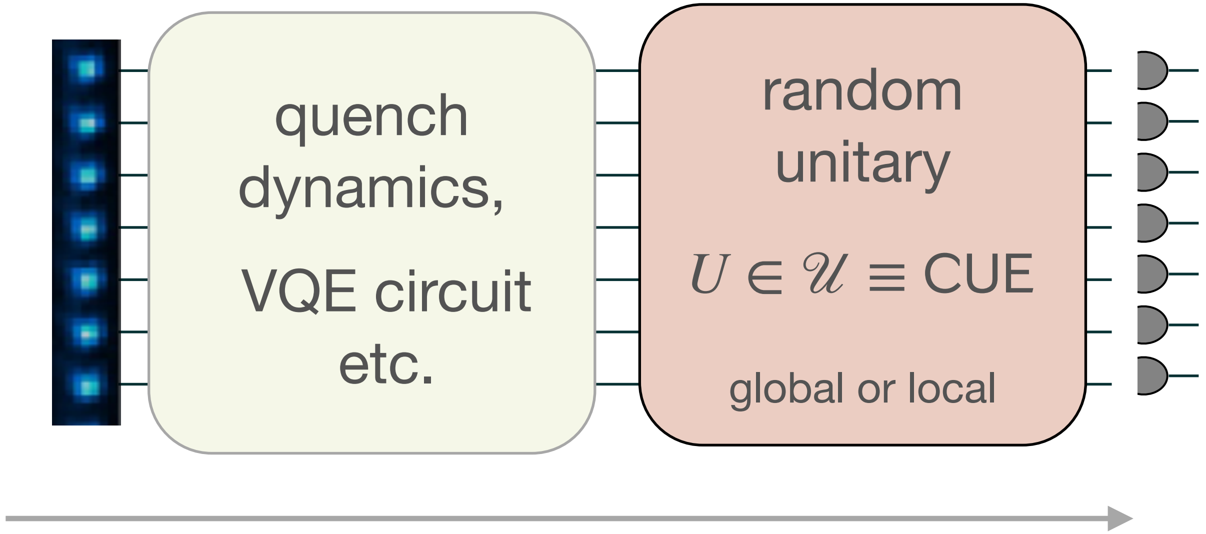
The randomized measurement toolbox

Andreas Elben [1,2,3,4], **Steven T. Flammia** [1,5], **Hsin-Yuan Huang** [1,6], **Richard Kueng** [7], **John Preskill** [1,2,5,6], **Benoît Vermersch** [3,4,8] & **Peter Zoller** [3,4] ✉

Measure first, ask questions later

Randomized Measurements

Measurement post-processing



quench dynamics,

VQE circuit etc.

random unitary

$U \in \mathscr{U} \equiv \mathrm{CUE}$

global or local

time

Repeat

Probability for bitstring s

$$P_U(\mathbf{s})$$

is a random variable depending on U

(Cross-) Correlation of probabilities

‘Noise’ or ensemble average (e.g. CUE)

$$\mathbb{E}_{U\sim\mathscr{U}}[P_U(\mathbf{s})P_U(\mathbf{s}')]$$

experiment `day 1, lab 1’

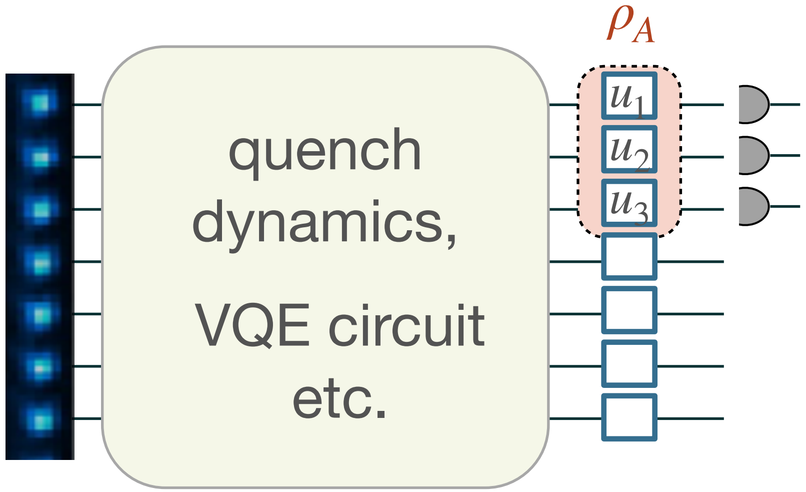
experiment `day 2 lab 2’

2-design, …

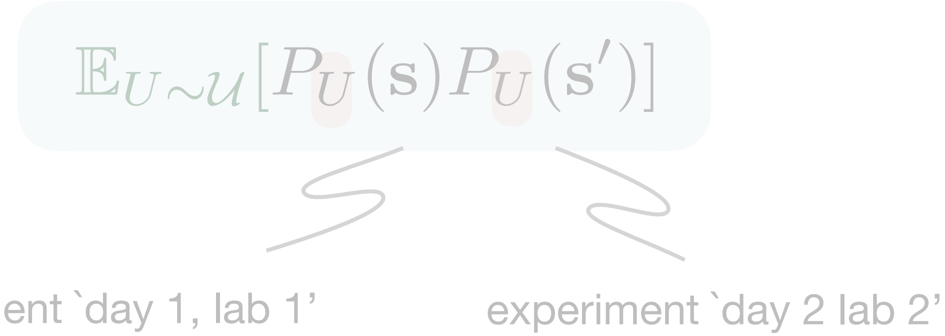
… hybrid classical-quantum protocols

Randomized Measurements

Measurement post-processing



(Cross-) Correlation of probabilities



- OTOCS

  theory - B. Vermersch et al., Phys. Rev. X **9**, 021061 (2019).
  exp - M. K. Joshi et al., Phys. Rev. Lett. **124**, 240505 (2020).

- topological invariants

  theory - A. Elben et al., Science Advances **6**, eaaz3666 (2020).

- Partially transposed density matrix

  theory [+ exp] - A. Elben et al., Phys. Rev. Lett. 125, 200501 (2020).
  theory [+ exp] - A. Neven et al., Npj Quantum Inf. **7**, (2021).

- Entanglement Hamiltonian Tomography

  theory - C Kokail et al., Nat. Phys. **17**, 936 (2021).
  theory + exp - MK Joshi, C Kokail, R van Bijnen et al., arXiv 2023

- Spectral form factor & quantum chaos

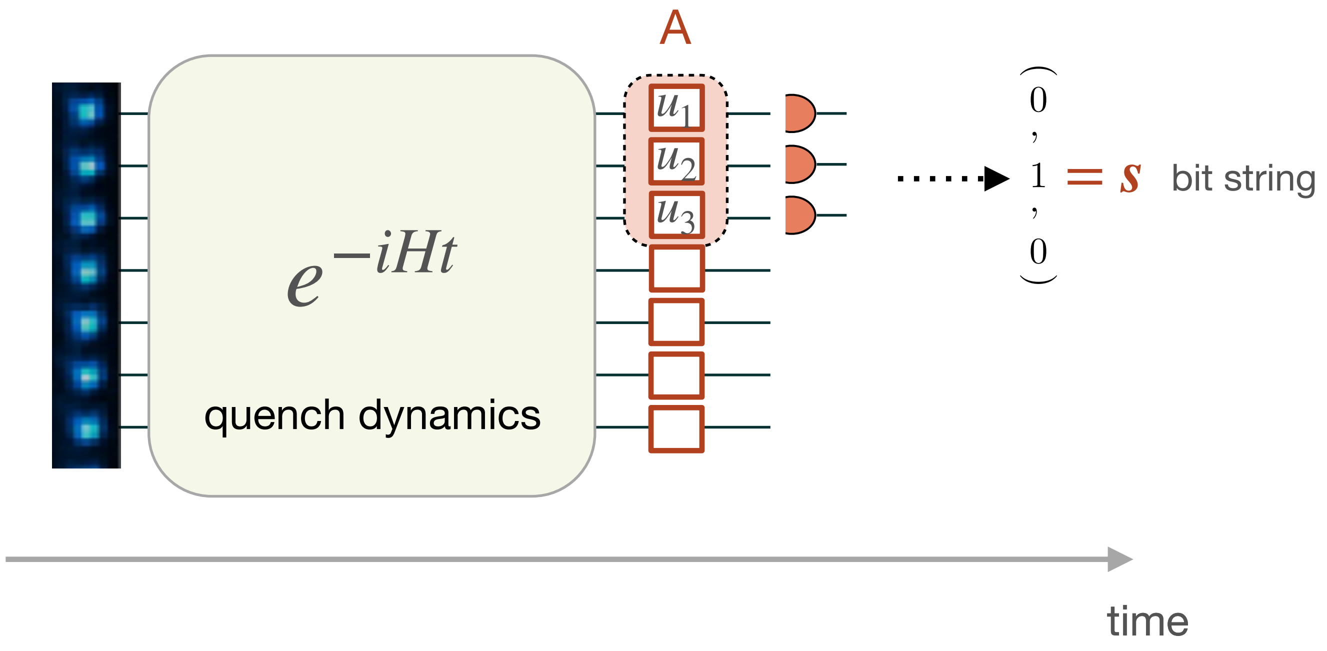
  theory - L. K. Joshi et al., Phys. Rev. X. **12**, (2022).
  exp - L. K. Joshi et al. with Monroe group, unpublished

- observation of quantum Mpemba effect

  theory + exp - MK Joshi et al. unpublished

Randomized Measurements: Local Random Unitaries

Measurement post-processing



$$P_U(\mathbf{s}) = \mathrm{Tr}\left[U\rho_A U^\dagger |\mathbf{s}\rangle\langle\mathbf{s}|\right]$$

$$U = \bigotimes_{i\in A} u_i$$

$u_i \in \mathrm{CUE}(d)$

evenly distributed
on Bloch sphere

purity - Renyi entropy

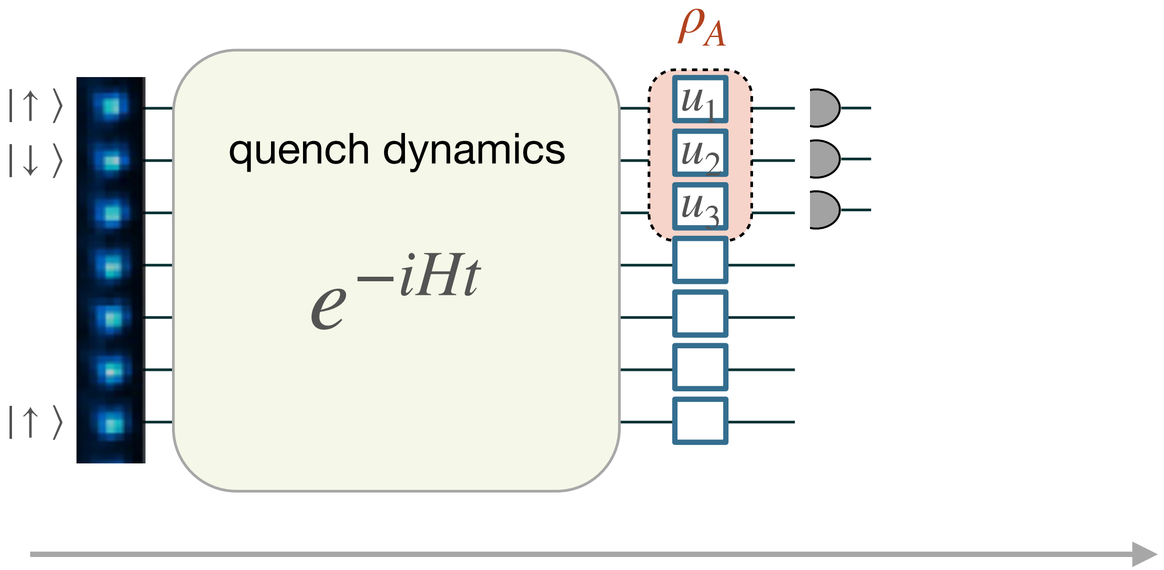
$\mathrm{Tr}\,\rho_A^2 = \mathbb{E}_{U\sim\mathrm{CUE}}[\hat{P}_2]$ with $\hat{P}_2 = 2^{|A|}\sum_{s,s'}\underset{\substack{\uparrow\\ \text{Hamming distance}}}{(-2)^{-D[s,s']}}\underbrace{P_U(\mathbf{s})P_U(\mathbf{s}')}_{\text{cross-correlation}}$

Random *single spin* rotations are sufficient!

T Brydges, A Elben et al., Science (2019)
A Elben, B Vermersch, et al., PRA (2019)

Randomized Measurements: Tomography

Quench dynamics with analog quantum simulator



time

product state → entangled state

time

Goal: learn operator structure of Entanglement Hamiltonian

$$\rho_A = e^{-\tilde{H}_A}$$

… learn efficiently only if we know something about EH

Randomized Tomography

$$\rho_A = \mathbb{E}_{U\sim\mathrm{CUE}}[\hat{\rho}_A] \quad \longleftarrow \quad \hat{\rho}_A = \sum_{\mathbf{s},\mathbf{s}'} \sum_{U} P_U(\mathbf{s})(-2)^{-D[\mathbf{s},\mathbf{s}']} U|\mathbf{s}'\rangle\langle\mathbf{s}'|U^\dagger$$

exponentially expensive

tomographically complete

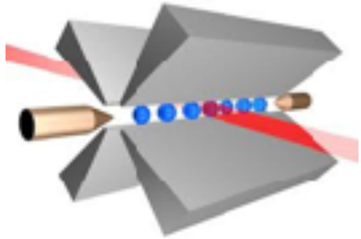
C Kokail

R van Bijnen

M Joshi

Exploring Large-Scale Entanglement in Quantum Simulation

Trapped ions



UIBK & IQOQI

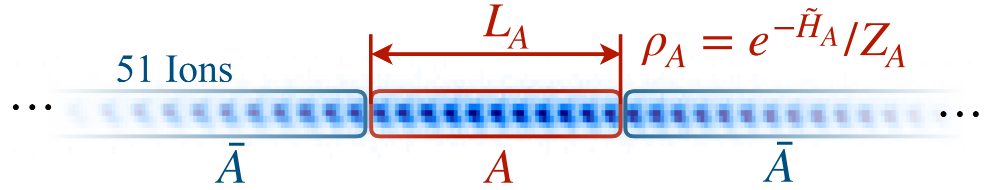
Theory: C Kokail, R van Bijnen, TV Zache, and P.Z.

Experiment: ML Joshi, F Kranzl, R Blatt, CF Roos

Nature online Nov 23, 2023

Early collaborations: M Dalmonte (→ ICTP), B Vermersch (→ Grenoble)

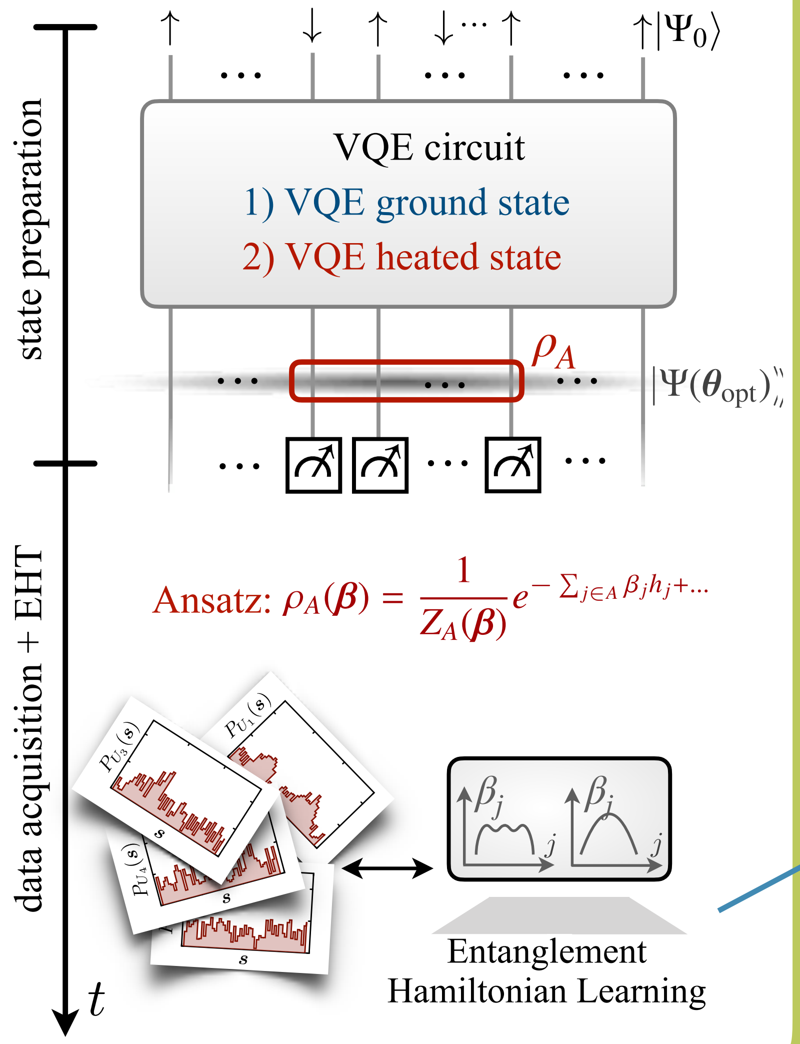
Heisenberg model



XXZ chain

$$H = J\sum_j \left(S^x_j S^x_{j+1} + S^y_j S^y_{j+1} + \Delta S^z_j S^z_{j+1}\right) = \sum_j h_j$$

State preparation & analysis



VQE circuit

1) VQE ground state

2) VQE heated state

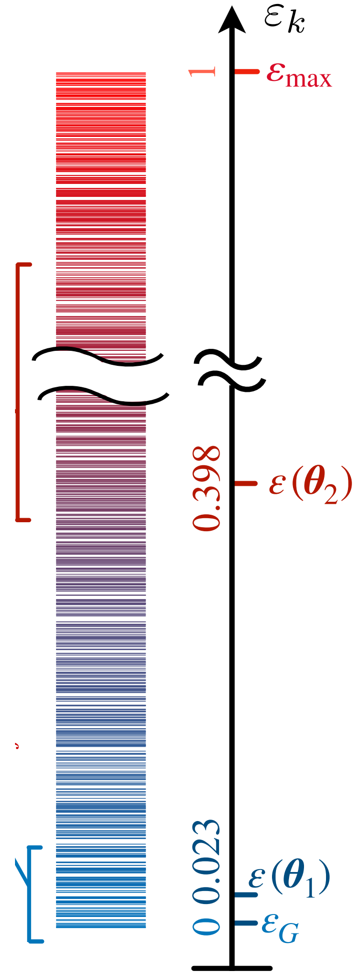
Ansatz: $\rho_A(\boldsymbol{\beta}) = \frac{1}{Z_A(\boldsymbol{\beta})} e^{-\sum_{j\in A}\beta_j h_j + \ldots}$

Entanglement Hamiltonian Learning

Entanglement properties

phase diagram

energy spectrum



volume law entanglement → heated state

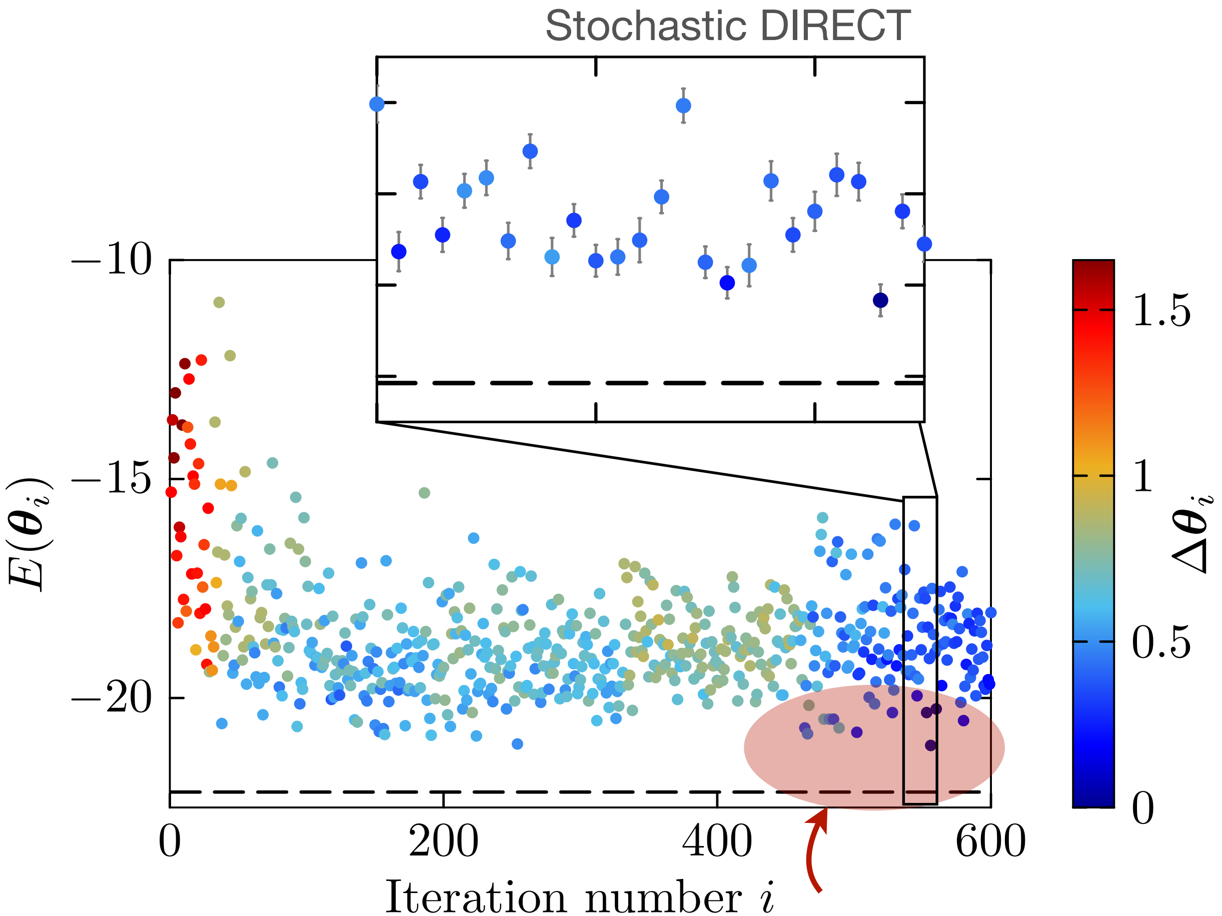
sample-efficient **tomography of ρ_A**
for *subsystems* > 20 lattice sites

area law entanglement → ground state

Experimental Energy Optimization Trajectory for Ground State (VQE)

Theory: C Kokail, R van Bijnen et al., unpublished
Experiment: M Joshi et al., unpublished

N = 51 ions



to be improved

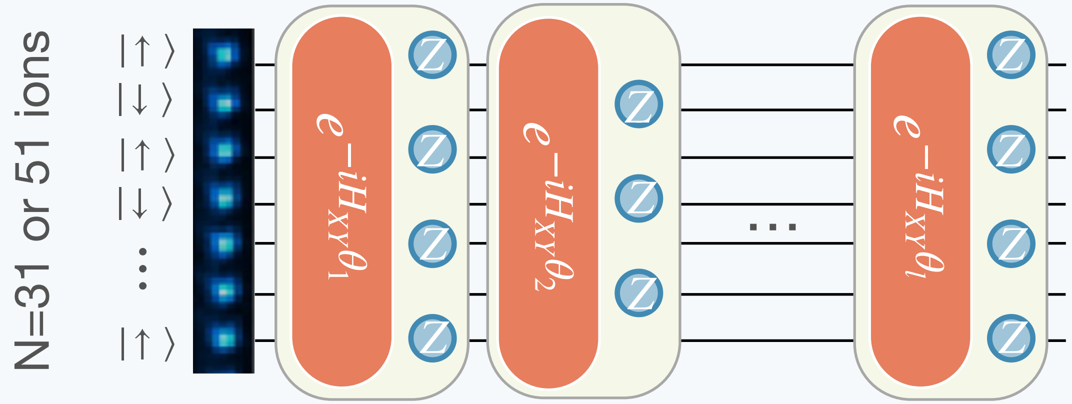
~ low temperature state T ~ few J

Heisenberg Model (spin-½)

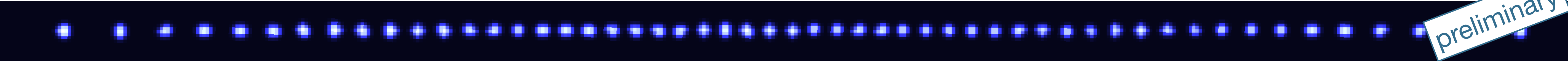
$$\hat{H} = J\sum_{i=1}^{N-1}\left(\hat{S}_i^x\hat{S}_{i+1}^x+\hat{S}_i^y\hat{S}_{i+1}^y\right)+\Delta\sum_{i=1}^{N-1}\hat{S}_i^z\hat{S}_{i+1}^z+h\sum_{i=1}^{N}\hat{S}_i^z$$

$J=1 \quad \Delta=1 \quad h=0.5$

VQE Circuit with Trapped Ion Resources

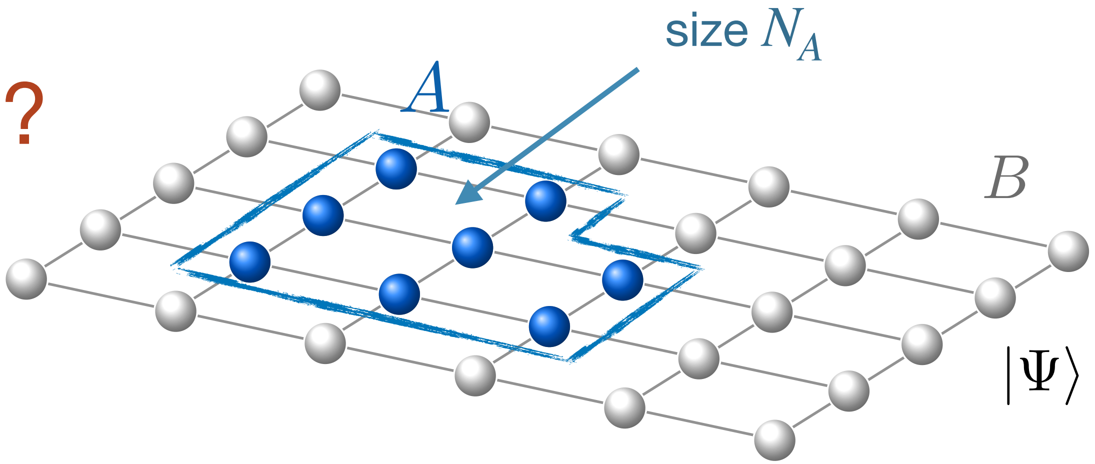


5 layers and 10 parameters



preliminary

Learning the Entanglement Hamiltonian?



Protocol 0: Quantum state tomography

measurement data ⟶ $\rho_A = \exp\left(-\tilde{H}_A\right)$

✓ expensive *

exponential in subsystem size N_A

?

* except: for small system sizes, or if we know something about the quantum state

Do we know something about the *structure of* $\tilde{H}_A$ to make tomography *efficient*?

A. Anshu et al., *Sample-Efficient Learning of Interacting Quantum Systems*, Nat. Phys. **17**, 931 (2021).

Entanglement Hamiltonian in QFT: *Bisognano-Wichmann Theorem*

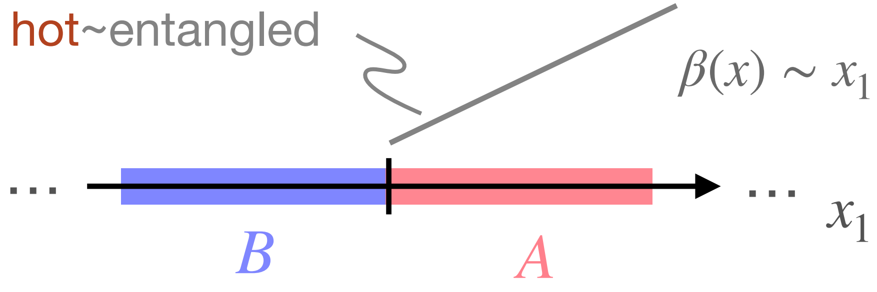
Relativistic Quantum Field Theory

Lorentz invariance

$$H = \int_{A \cup B} d^d x \, \mathscr{H}(x)$$

vacuum state $|\Omega\rangle$ →

Entanglement Hamiltonian



$$\rho_A \equiv \mathrm{Tr}_B |\Omega\rangle\langle\Omega| = \exp\left[- \int_A d^d x \, \beta(x) \mathscr{H}(x) \right]$$

$\tilde{H}_A$

Gibbs state with *local* temperature $\beta(x) \sim x_1$

EH $\tilde{H}_A$ as *deformed* system Hamiltonian

Bisognano and Wichmann, J. Math. Phys. (1976) Casini, Huerta & Myers, Journal of HEP (2011)

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

Entanglement Hamiltonian in QFT: *Bisognano-Wichmann Theorem*

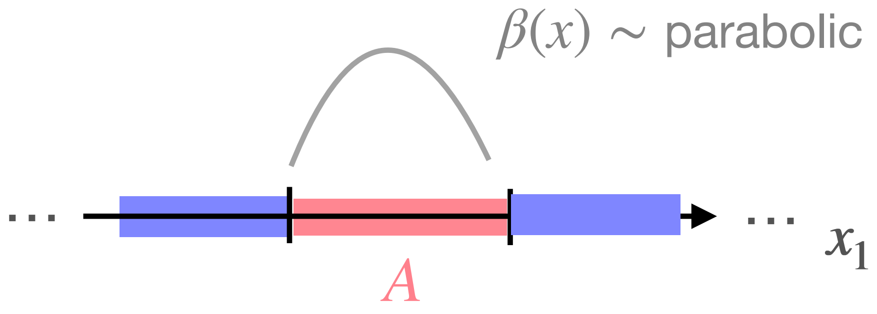
Conformal Field Theory

scale invariance

$$H = \int_{A\cup B} d^dx\,\mathscr{H}(x)$$

vacuum state $|\Omega\rangle$ $\longrightarrow$

Entanglement Hamiltonian



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Gibbs state with *local* temperature $\beta(x) \sim x_1$

Entanglement Hamiltonian as *deformed* system Hamiltonian

Casini, Huerta & Myers, Journal of HEP (2011)

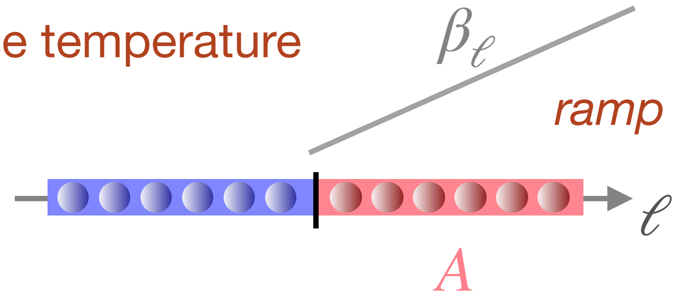
1+1 CFT: Hislop, Longo, Cardy, Calabrese, Tonni, Wen, Ryu, Ludwig, … (ground state, thermal & quench)

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

Lattice Bisognano-Wichmann & beyond

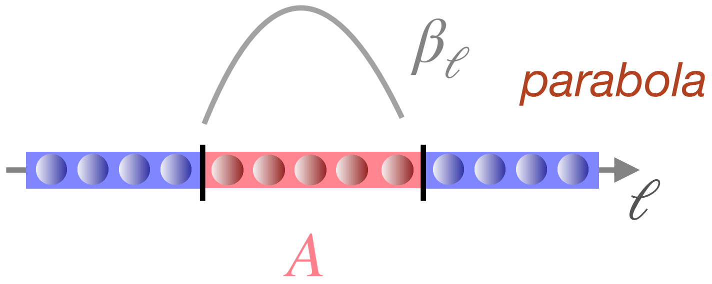
ground state of many-body lattice model

Gibbs state with local inverse temperature



$$\rho_A = e^{-\tilde{H}_A}$$

entanglement Hamiltonian



BW recipe

$$\hat{H} = \sum_{\ell} \hat{h}_\ell$$

k-local Hamiltonian

$$\downarrow$$

$$\tilde{H}_A = \sum_{\ell \in A} \beta_\ell \, \hat{h}_\ell + \dots$$

EH as *local deformation* of system Hamiltonian

1. Validity of BW-like EH, non-local corrections often sub-leading

Analytical results: non-interacting, non-critical chains

Numerical evidence: lattice models, quench dynamics

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

Lattice Bisognano-Wichmann & beyond

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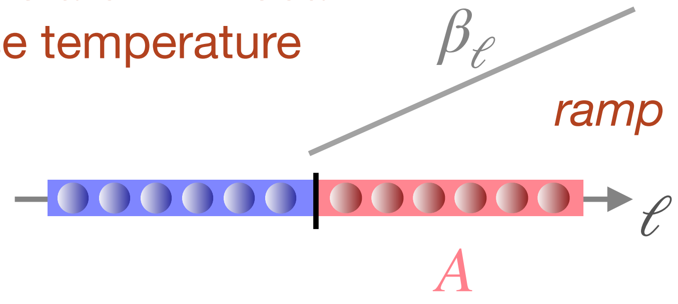

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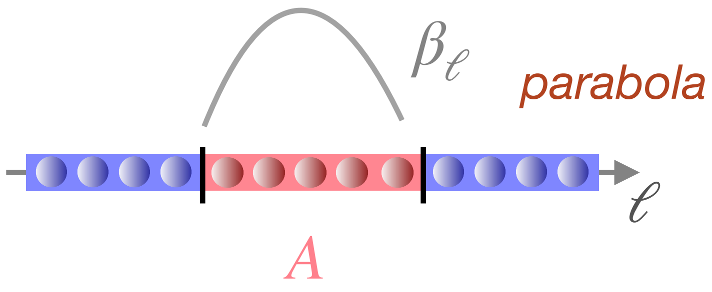


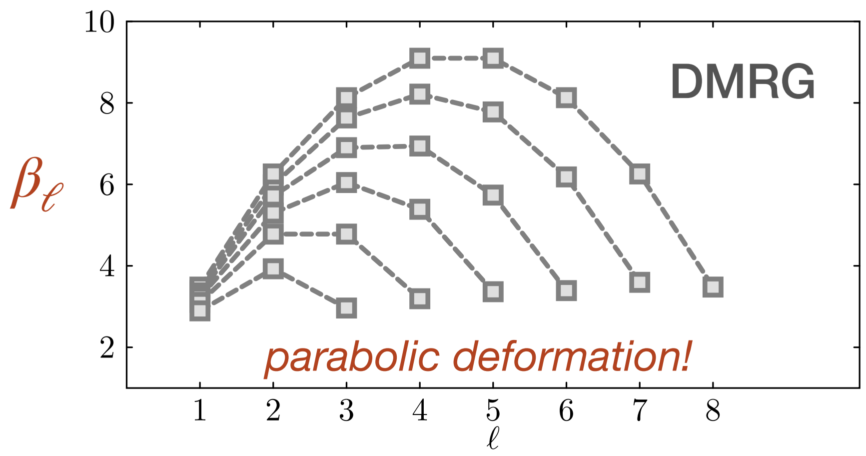
BW recipe

$$\hat{H} = \sum_\ell \hat{h}_\ell$$ k-local Hamiltonian

$$\downarrow$$

$$\tilde{H}_A = \sum_{\ell \in A} \beta_\ell \hat{h}_\ell + \dots$$ EH as *local deformation* of system Hamiltonian

Numerical example: Heisenberg model 1D



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Lattice Bisognano-Wichmann & beyond

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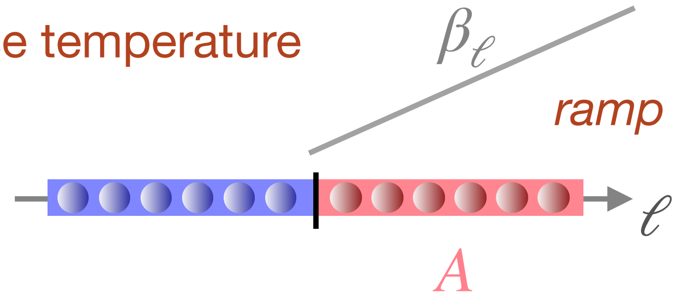
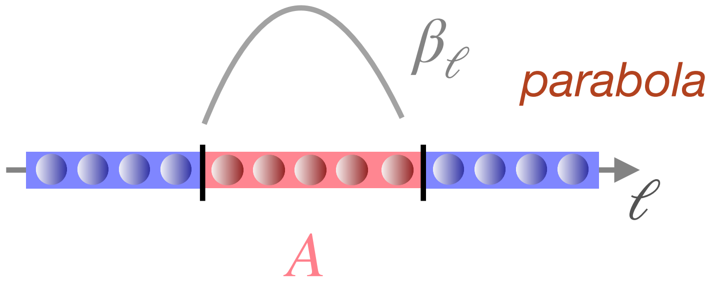


$$\rho_A = e^{-\tilde{H}_A}$$

entanglement Hamiltonian



BW recipe

$$\hat{H} = \sum_\ell \hat{h}_\ell$$ k-local Hamiltonian

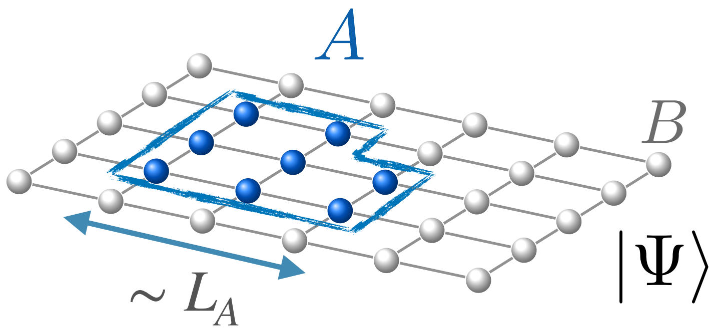
$$\downarrow$$

$$\tilde{H}_A = \sum_{\ell \in A} \beta_\ell \hat{h}_\ell + \dots$$ EH as *local deformation* of system Hamiltonian

2. suggests an efficient ansatz to \`learn' Entanglement Hamiltonian

Review: M Dalmonte, V Eisler, M Falconi, B Vermersch, *Entanglement Hamiltonians - from field theory to lattice models & experiments*, Ann Phys 2022,534, 2200064

Entanglement in Many-Body Quantum Systems



Many-Body Problem

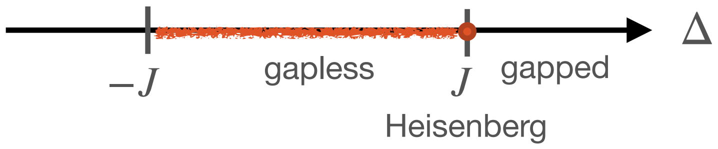
Hamiltonian

$$\hat{H} = \sum_j \hat{h}_j$$

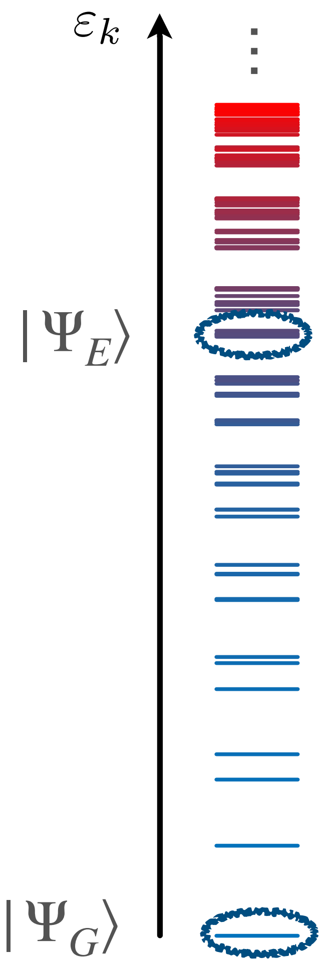
k-local

Example: XXZ / Heisenberg model (1D)

$$\hat{H}_T = J \sum_{i=1}^{N-1} \left(\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y \right) + \Delta \sum_{i=1}^{N-1} \hat{S}_i^z \hat{S}_{i+1}^z$$



energy spectrum

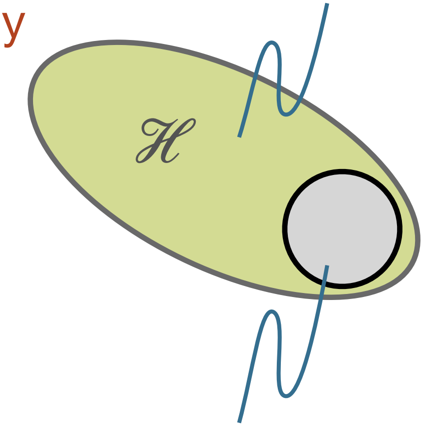


excited state

$$S_A \propto V = L_A^d$$

thermal entropy

volume law entanglement

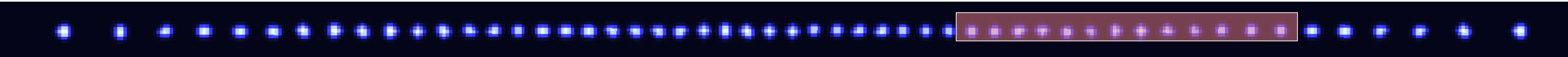


area law entanglement

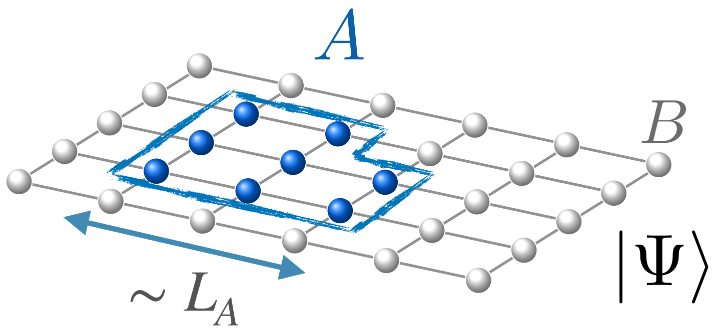
ground state

$$S_A \propto L_A^{d-1}$$

$\sim c \log L_A$ CFT $d = 1$



Area Law vs. Volume Law Entanglement

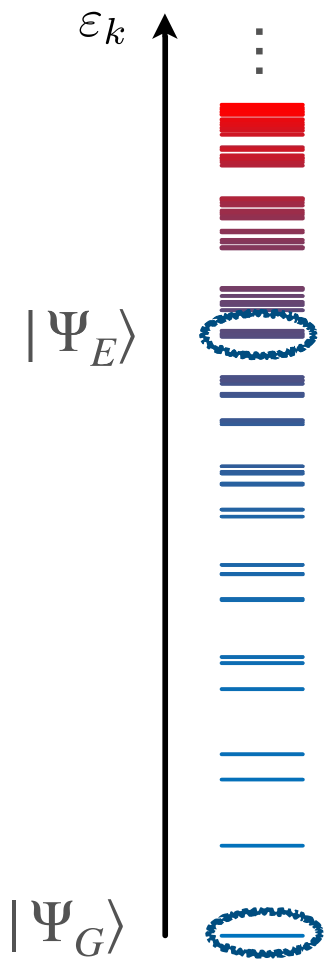


Many-Body Problem

Hamiltonian

$$\hat{H} = \sum_j \hat{h}_j$$ k-local

energy spectrum



q-simulators can represent volume law states → quantum advantage

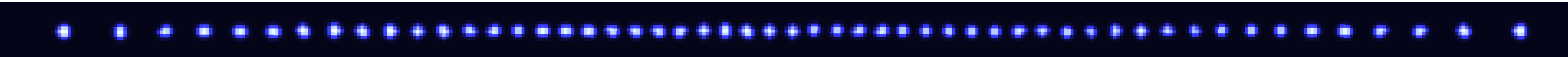
excited state $S_A \propto V = L_A^d$ volume law entanglement

thermal entropy

classical simulations with tensor networks

ground state $S_A \propto L_A^{d-1}$ area law entanglement

$\sim c \log L_A$ CFT $d = 1$



Entanglement Hamiltonian & Eigenstate Thermalization Hypothesis

Eigenstate Thermalization Hypothesis (ETH):

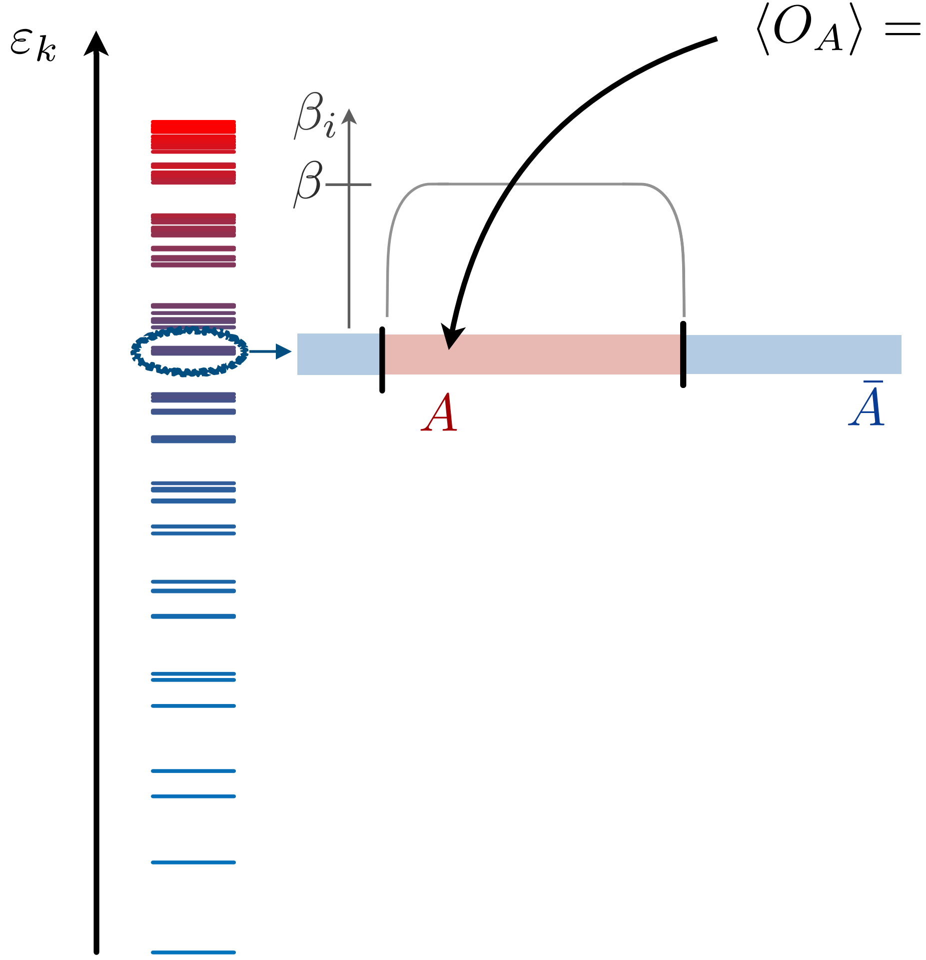
local Hamiltonian: $\hat{H} = \sum_i \hat{h}_i$

reduced density matrix of a subsystem :A

$$\rho_A \approx \frac{1}{Z} e^{-\beta \hat{H}_A}$$

$$\approx \frac{1}{Z} e^{-\sum_i \beta_i \hat{h}_i}$$

local temperature profile



$$\langle \hat{O}_A \rangle = \frac{\mathrm{Tr}\left(\hat{O}_A e^{-\beta \hat{H}}\right)}{\mathrm{Tr}\left(e^{-\beta \hat{H}}\right)}$$

Garrison et.al. PRX 8, 021026 (2018)

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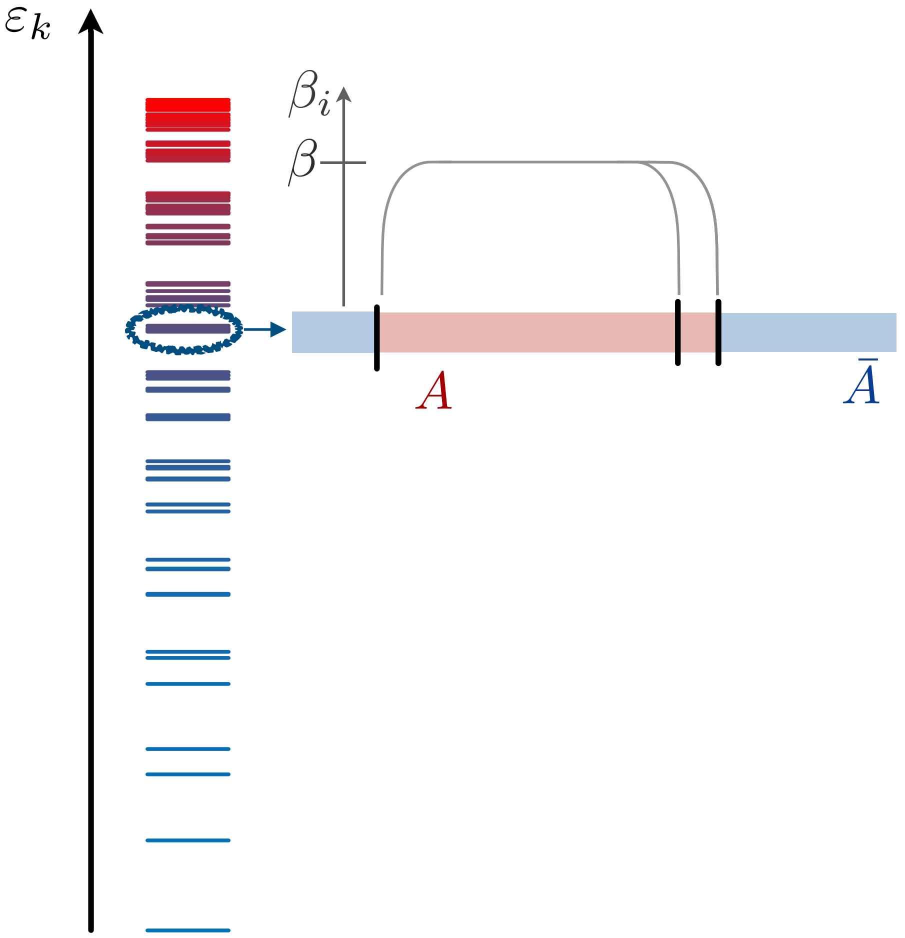

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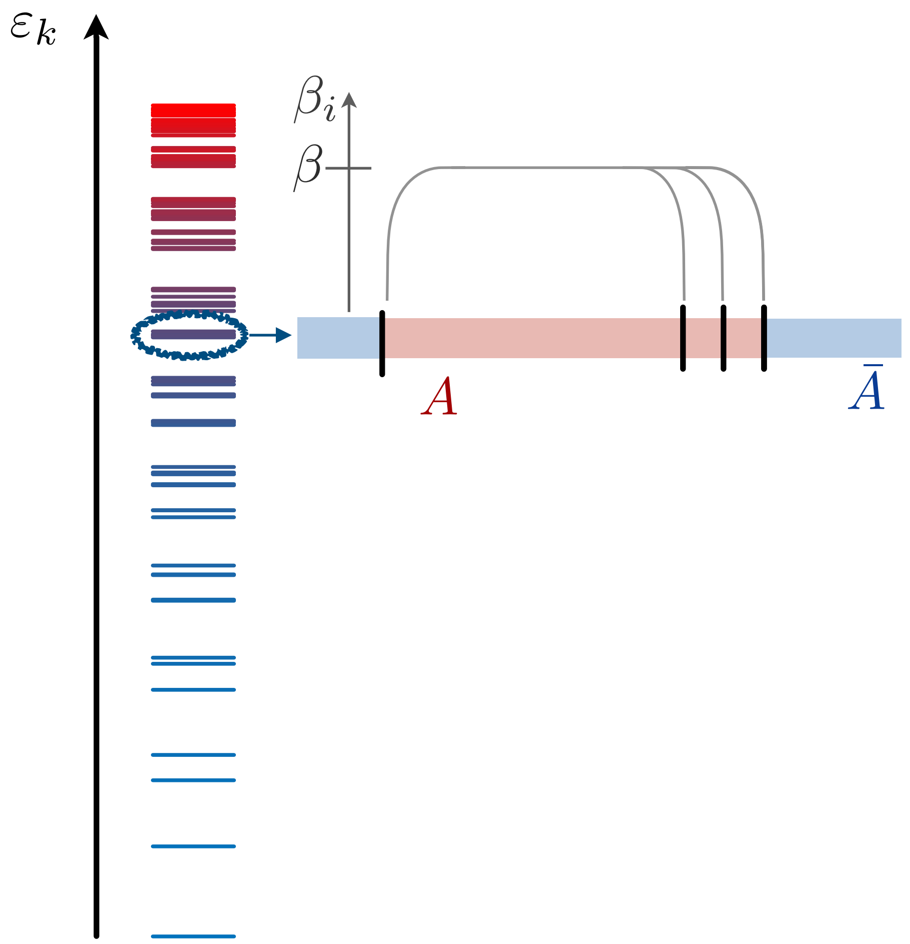

Garrison et.al. PRX 8, 021026 (2018)

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$$S_A^{\mathrm{VN}} =\sim \langle \hat{\mathcal{H}}_A \rangle$$

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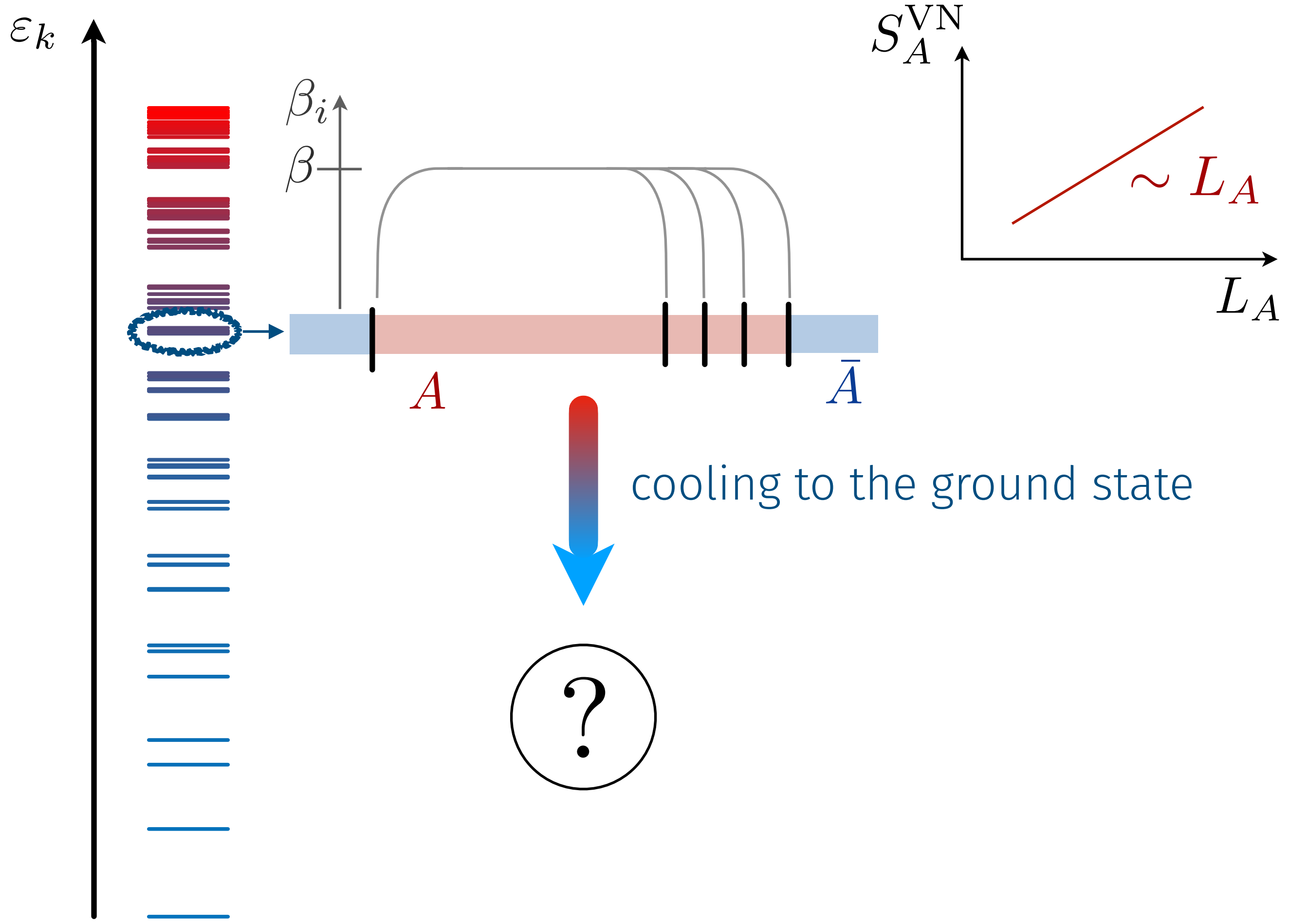


cooling to the ground state

Garrison et.al. PRX 8, 021026 (2018)

Entanglement Hamiltonian & Eigenstate Thermalization Hypothesis

EH from CFT for ground state:
(at the critical point)

$$\rho_A = \frac{1}{Z}\exp\left[-\int_A dx\,\beta(x)\mathcal{H}(x)\right]$$

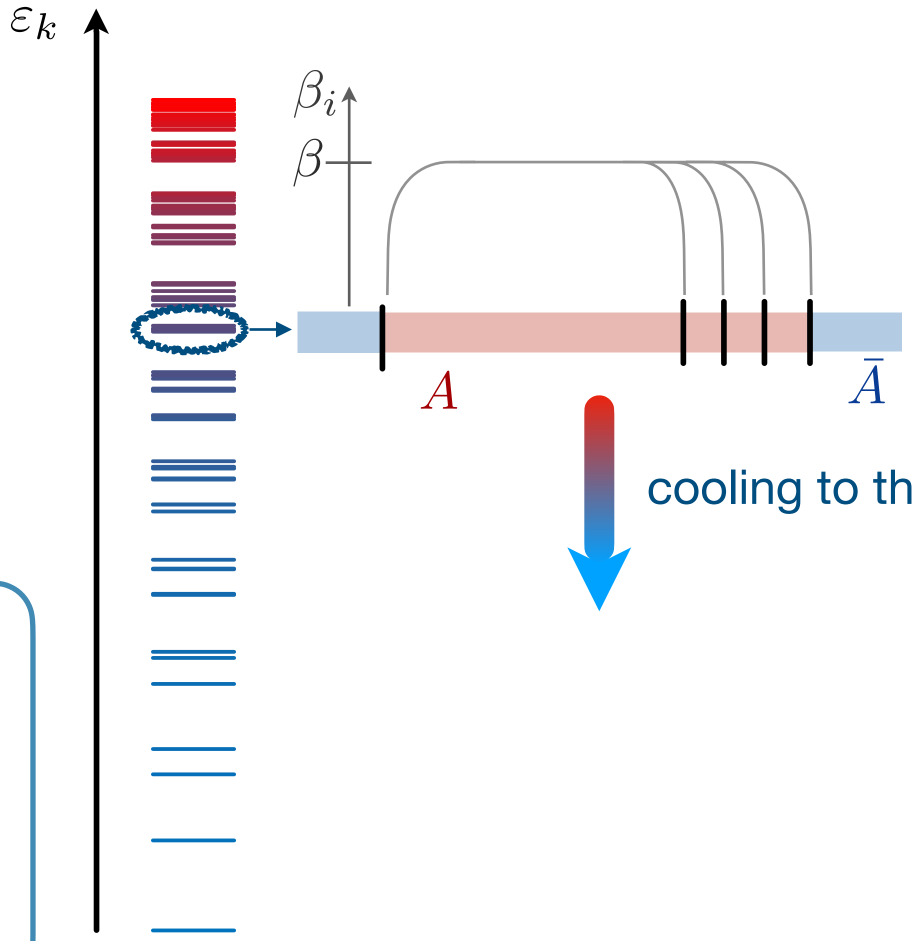
with

$$\beta(x) \propto \frac{L_A^2 - 4x^2}{L_A}$$

Parametrization as Gibbs state with *local inverse temperature*

$$\rho_A \sim \mathrm{e}^{-\sum_{i\in A}\beta_i \hat{h}_i}$$

local temperature



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Casini, Huerta & Myers, JHEP, 2011(5), 1-41

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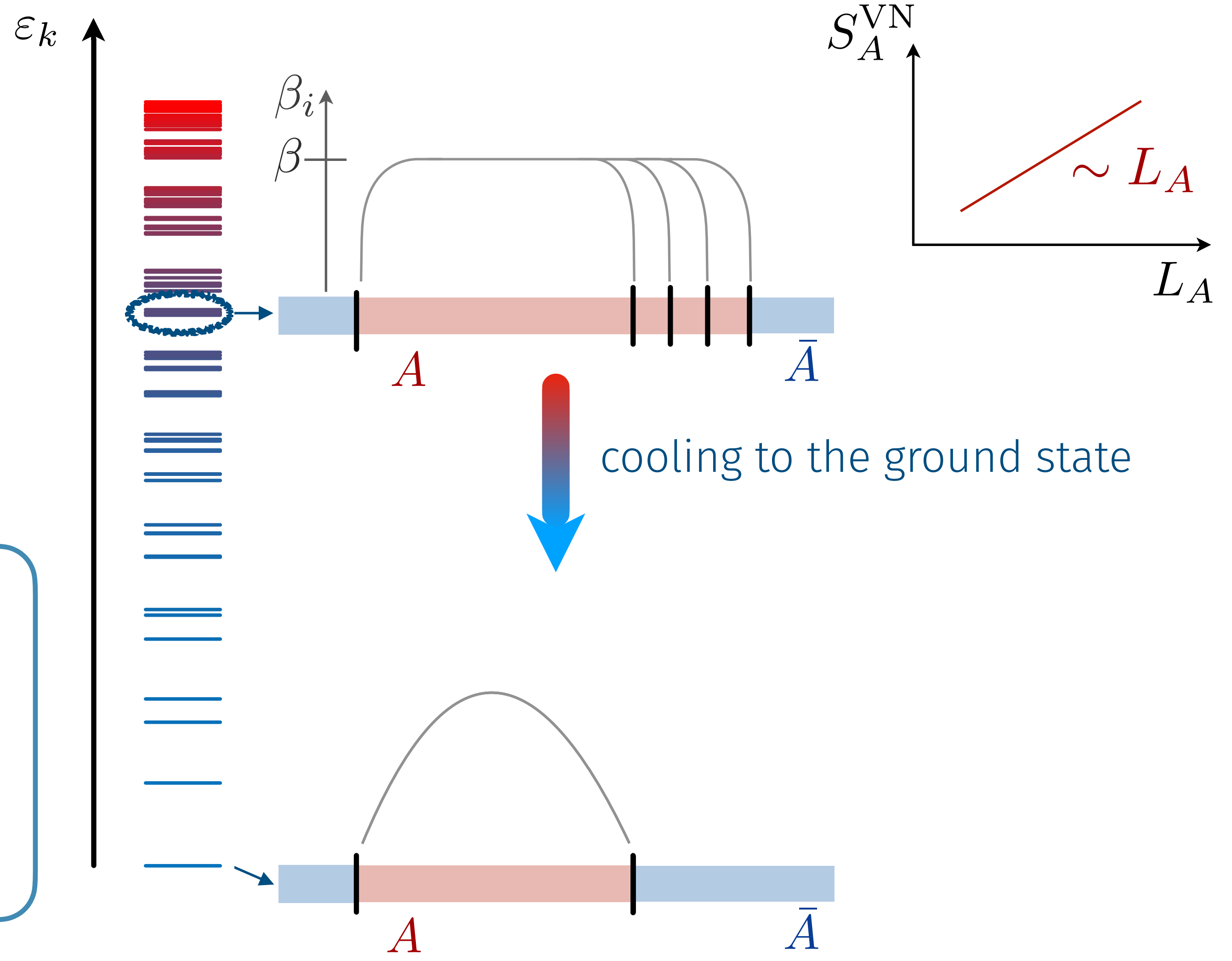

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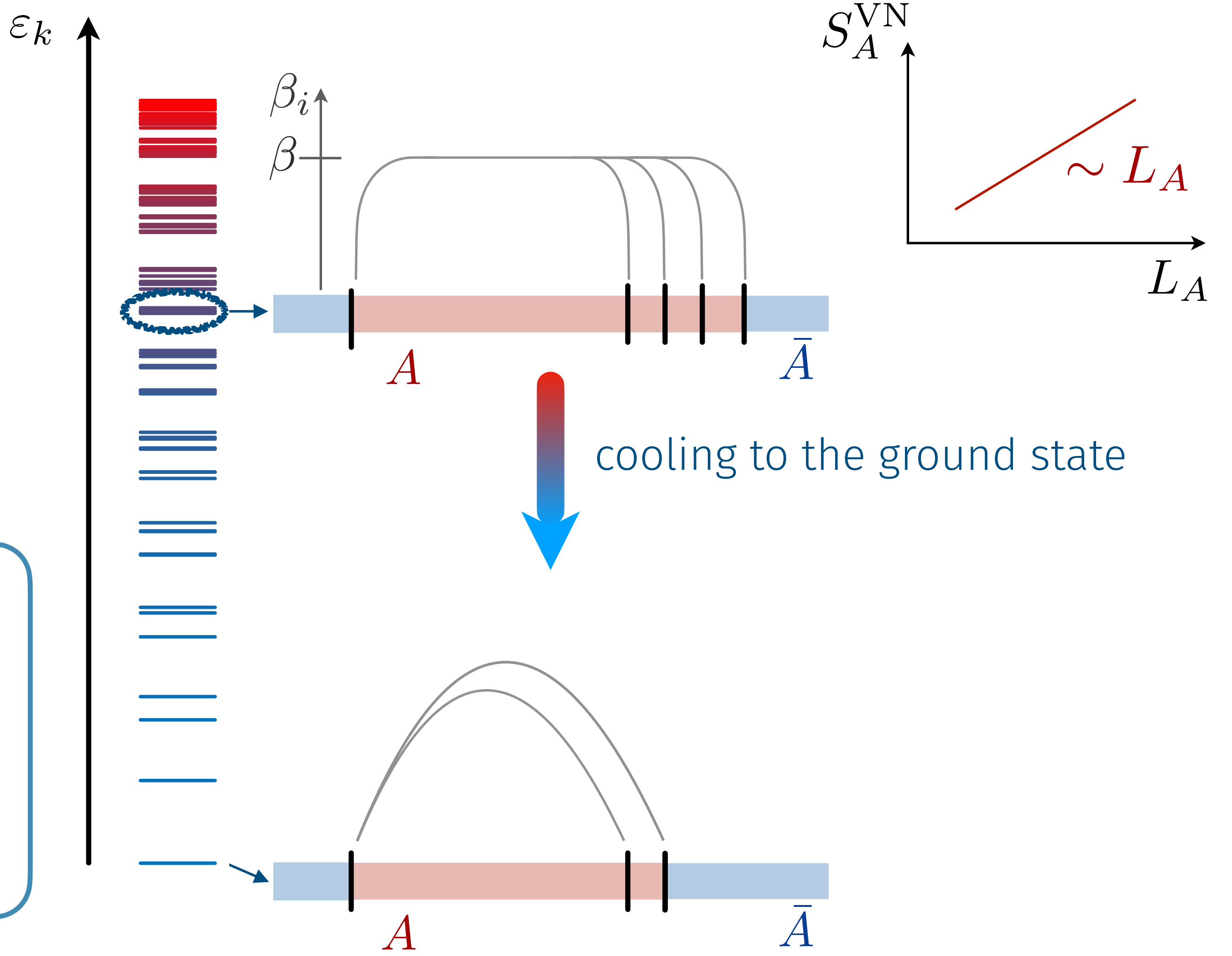

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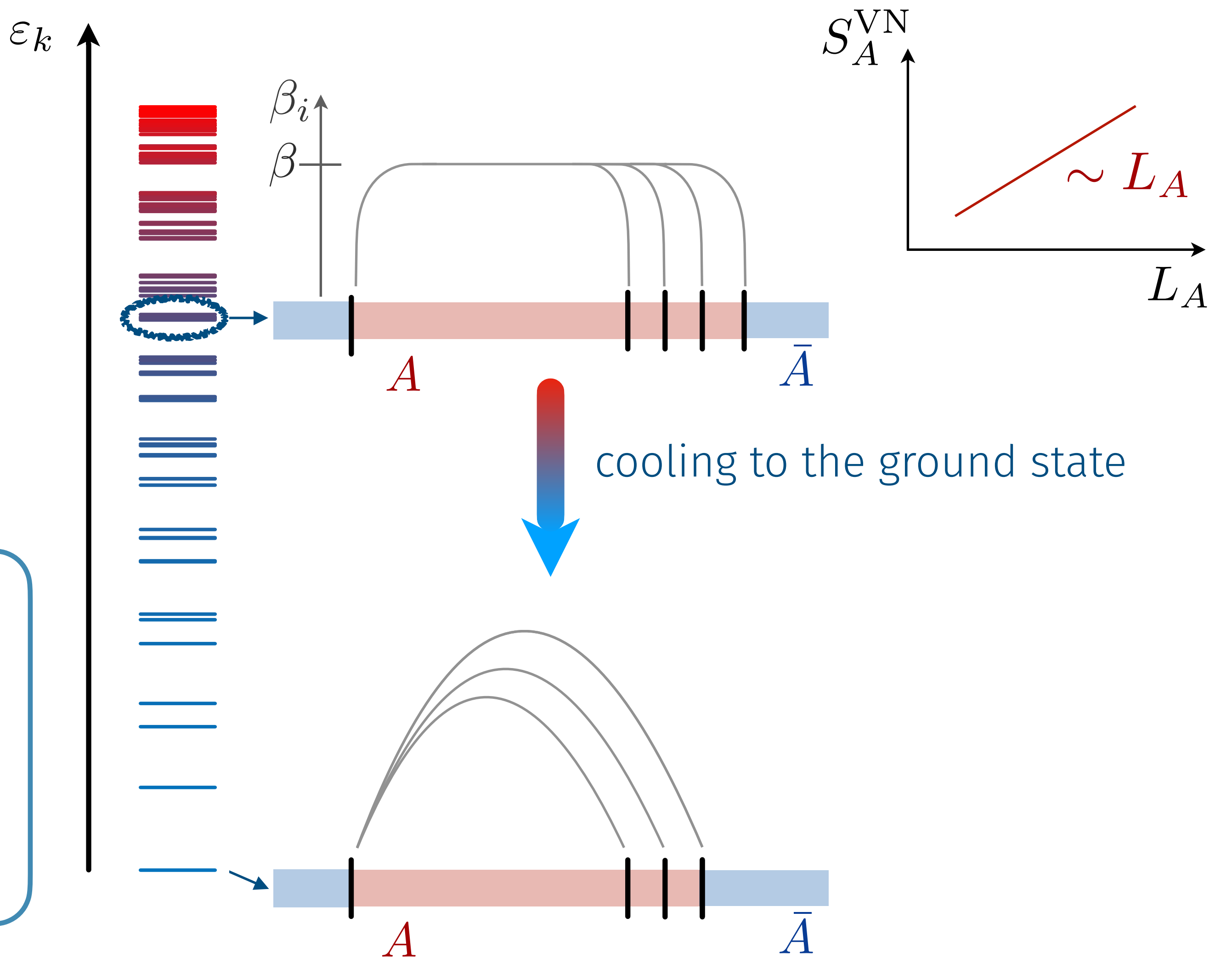

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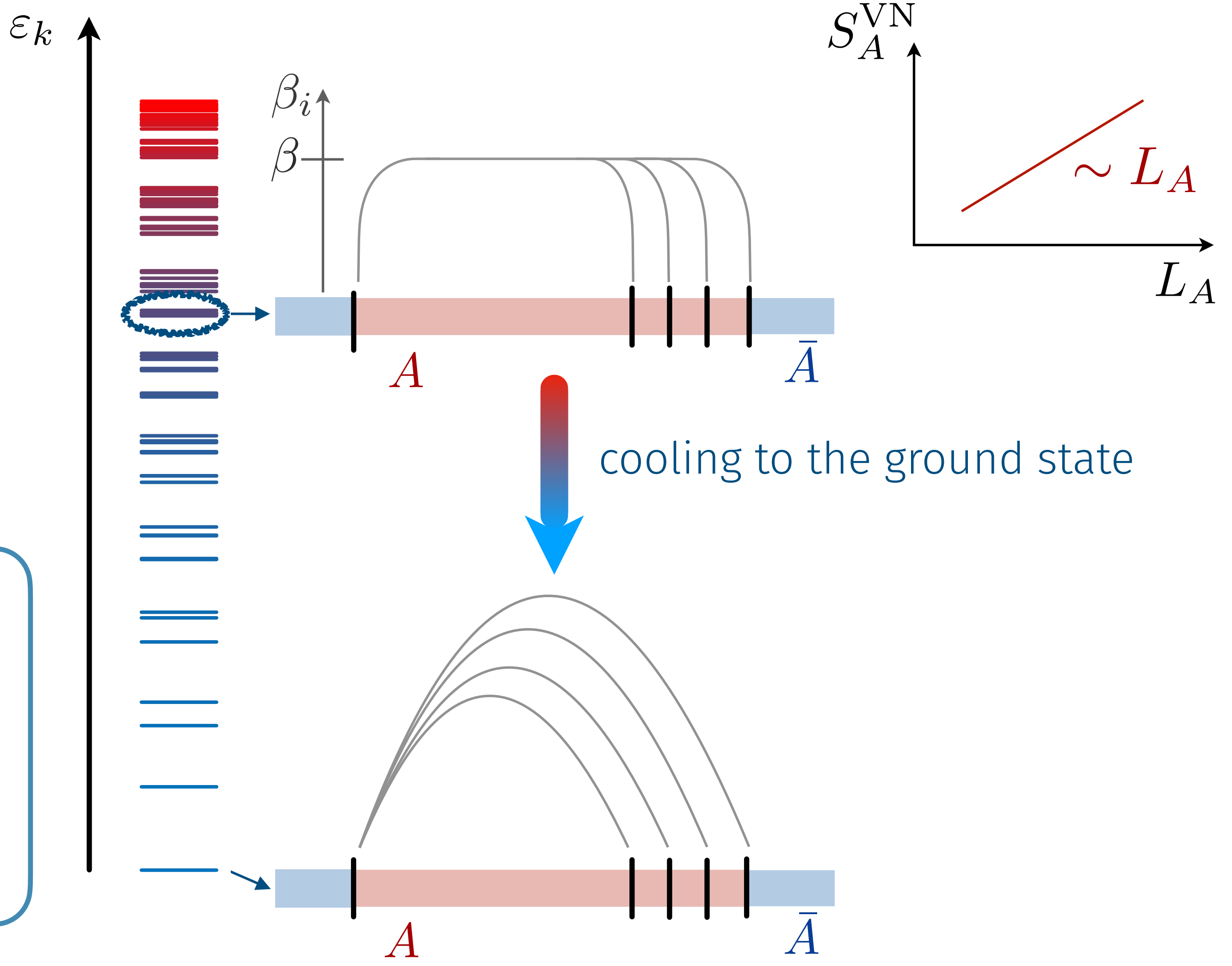

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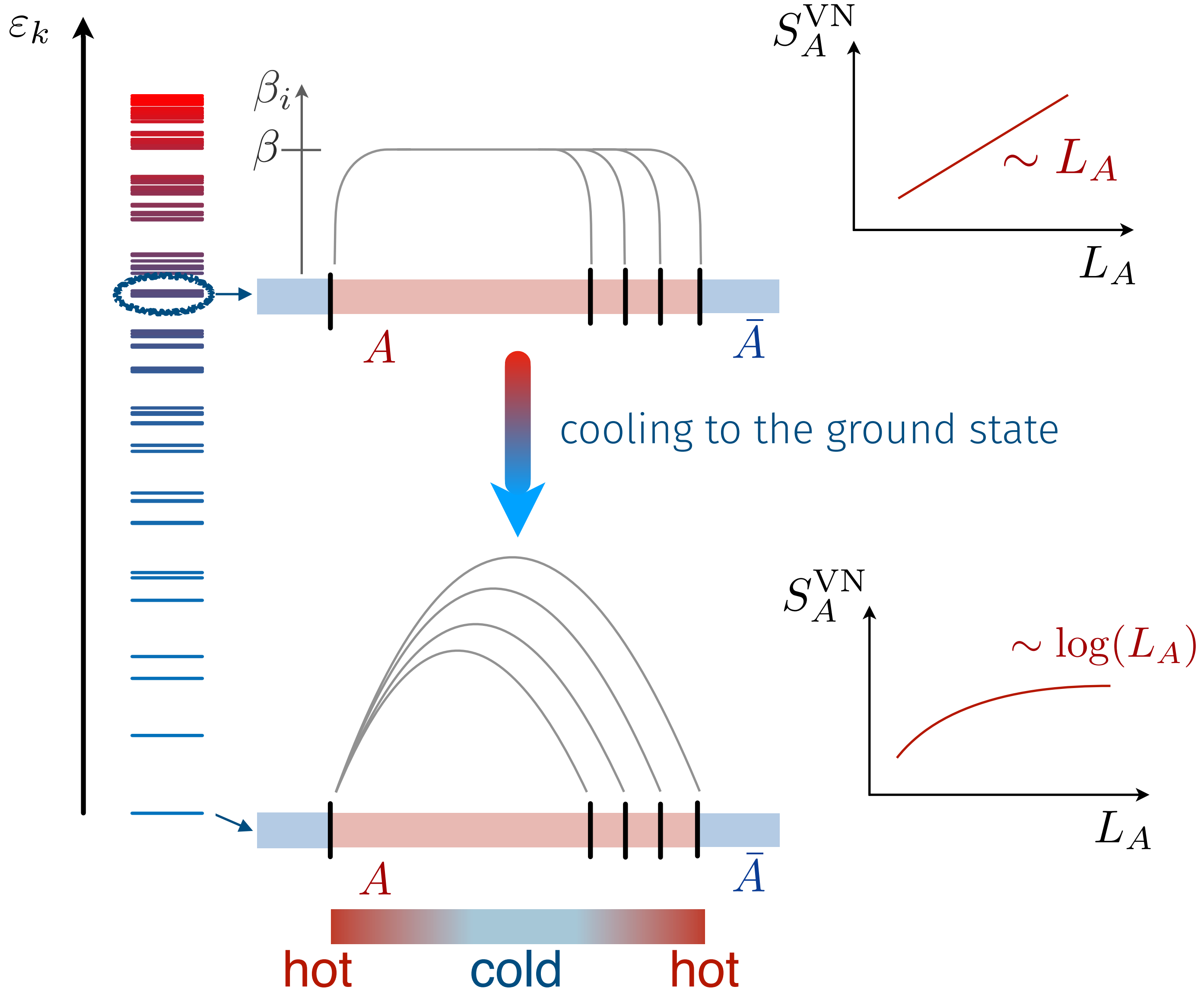

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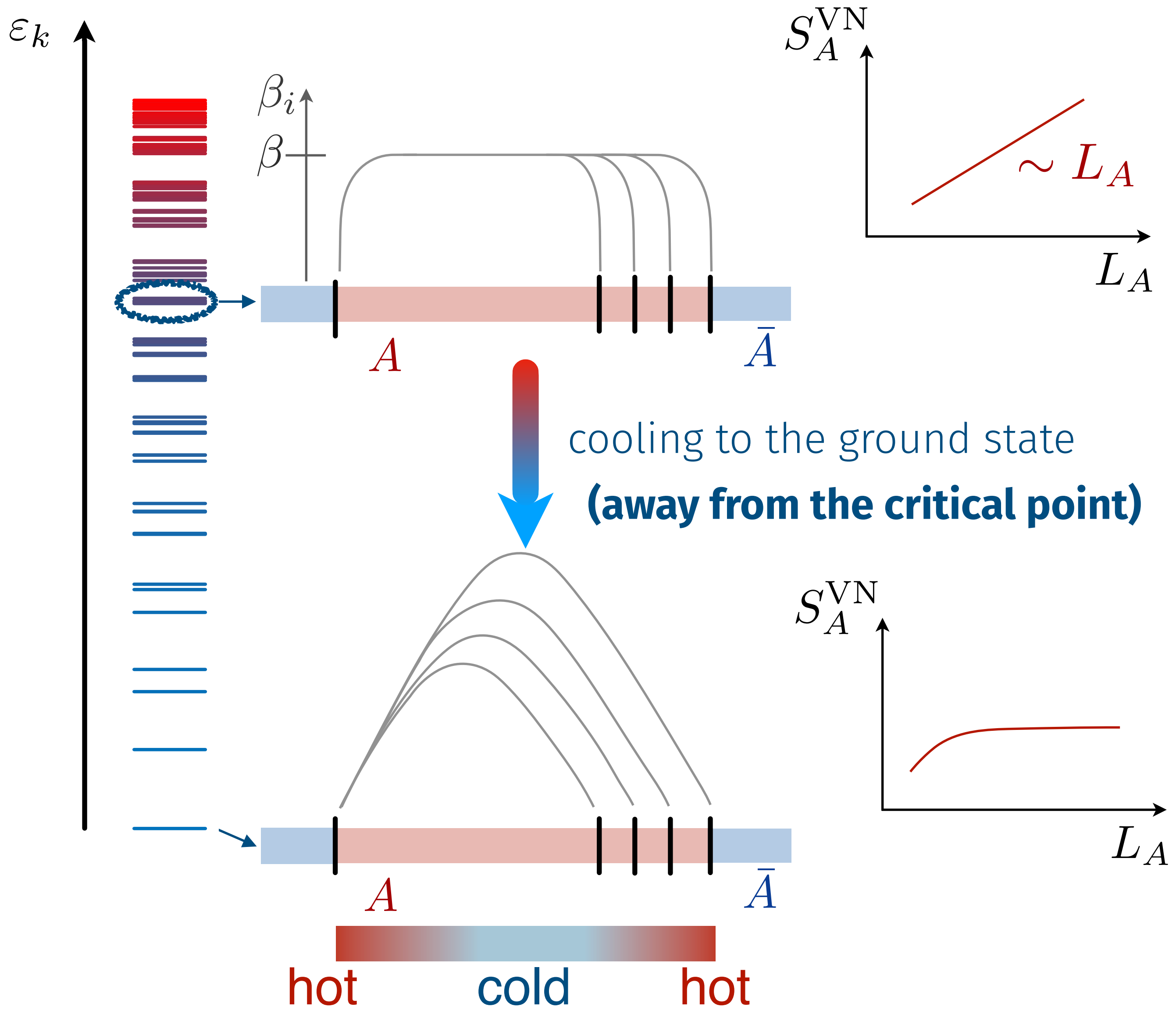


cooling to the ground state

(away from the critical point)

Viktor Eisler *et al J. Stat. Mech.* (2020) 103102

Casini, Huerta & Myers, JHEP, 2011(5), 1-41

Entanglement Hamiltonian & Eigenstate Thermalization Hypothesis

Example: XXZ model (51 sites)

$$\hat{H} = \frac{1}{2}\sum_i \left(\hat{S}_i^+\hat{S}_{i+1}^- + \mathrm{H.c}\right) + \Delta\sum_i \hat{S}_i^z\hat{S}_{i+1}^z$$

Ansatz for the Entanglement Hamiltonian:

ferromagnet c=1 CFT antiferromagnet

$\Delta = -1$ $\Delta = 1$

$$\hat{\mathcal{H}}_A = \sum_\ell \beta_\ell \hat{h}_\ell$$

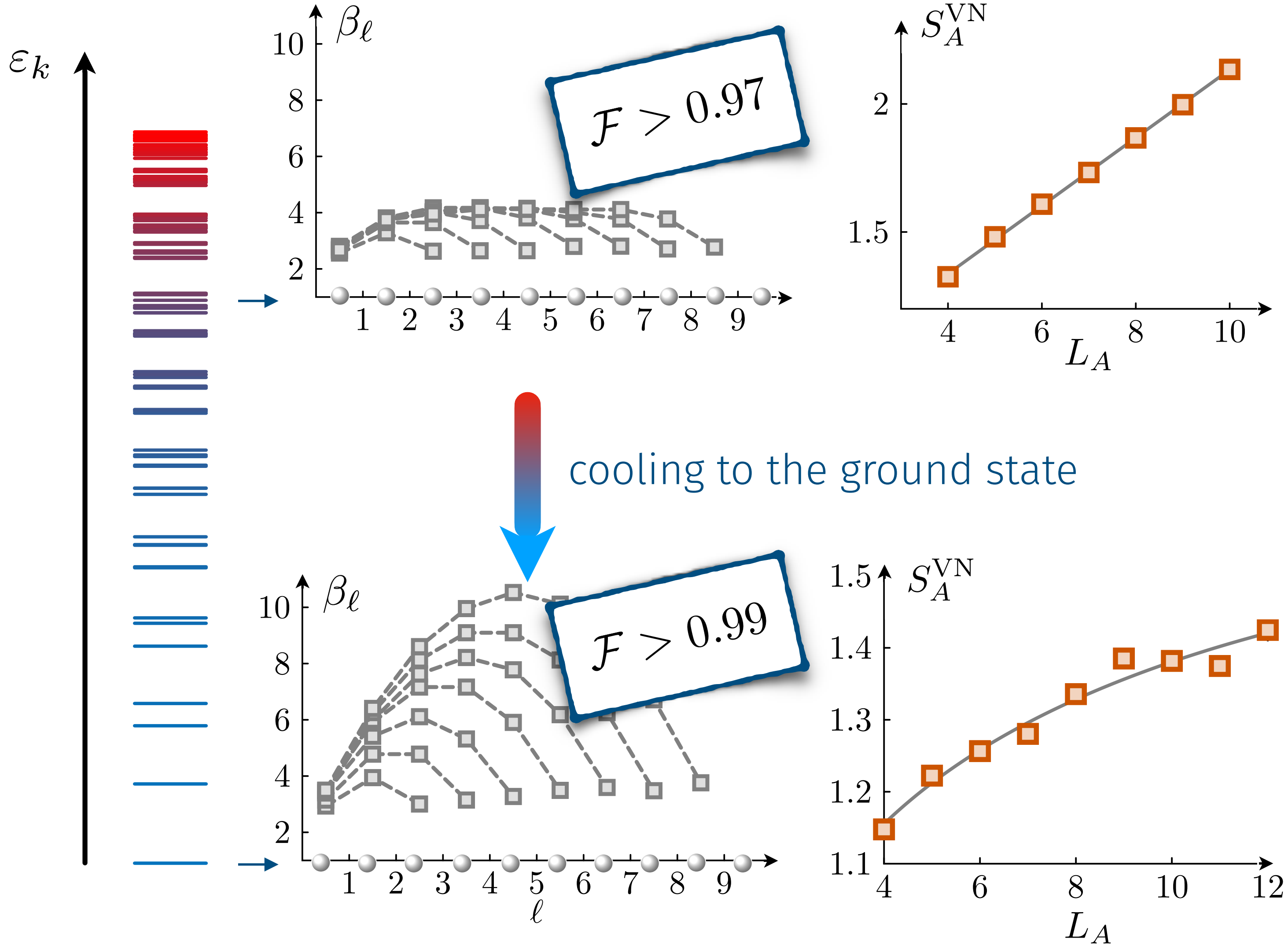
$$\hat{h}_\ell = \frac{1}{2}\left(\hat{S}_\ell^+\hat{S}_{\ell+1}^- + \mathrm{H.c.}\right) + \Delta\hat{S}_\ell^z\hat{S}_{\ell+1}^z$$

Qi & Ranard Quantum 3, 159 (2019)

E. Bairey et.al. Phys. Rev. Lett. (2019)

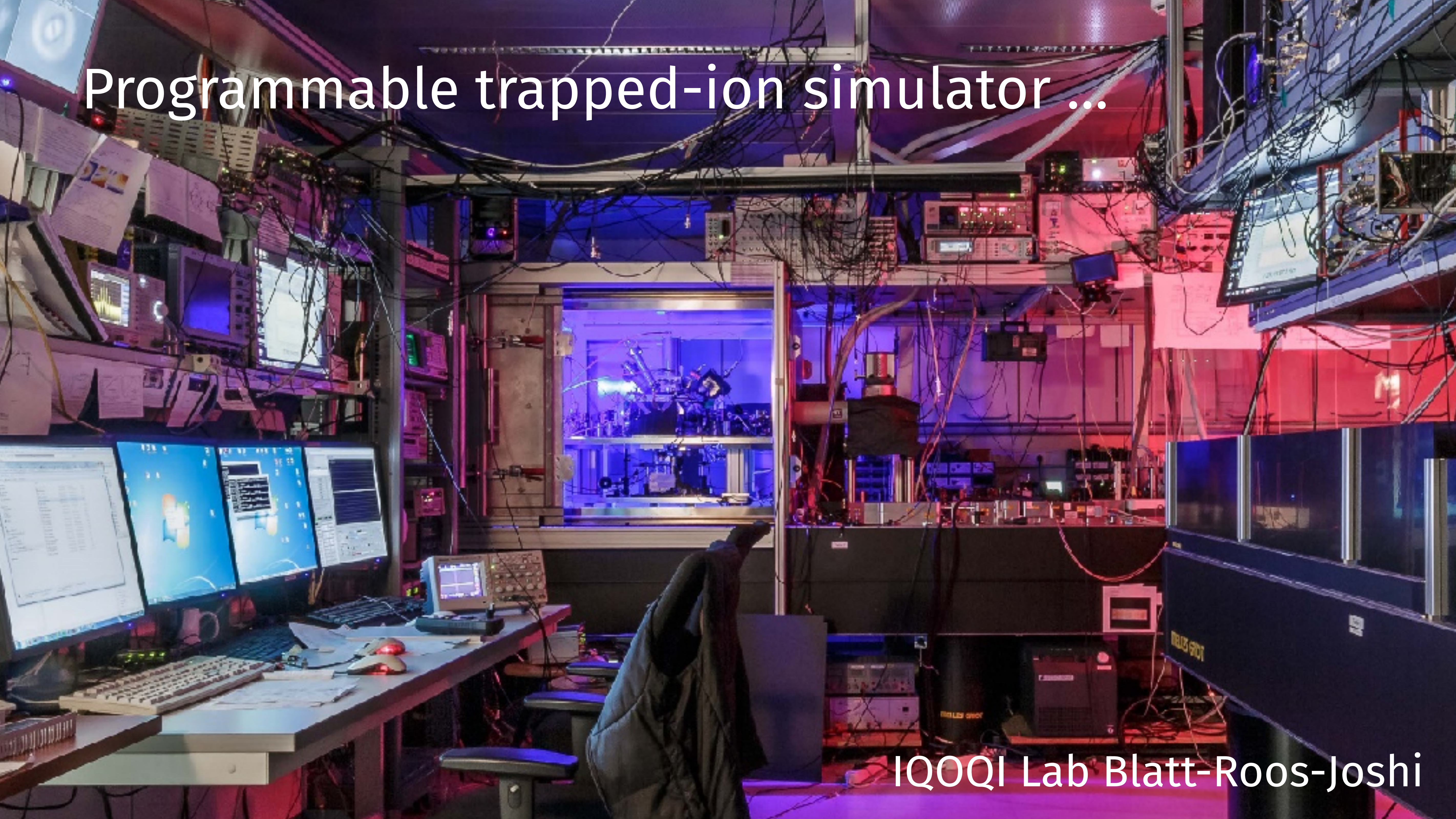
W. Zhu et.al. Phys. Rev. B 99, 235109

C. Kokail et.al. Nat.Phys. 2021



cooling to the ground state

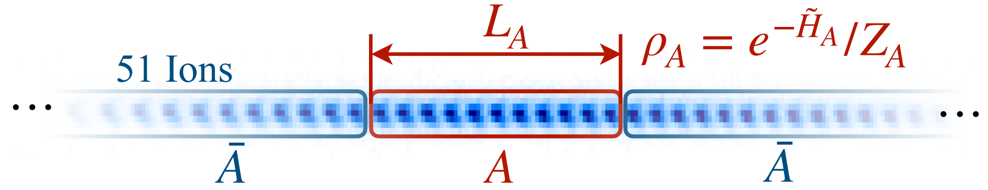
Programmable trapped-ion simulator …



IQOQI Lab Blatt-Roos-Joshi

Results: Theory and Experiment

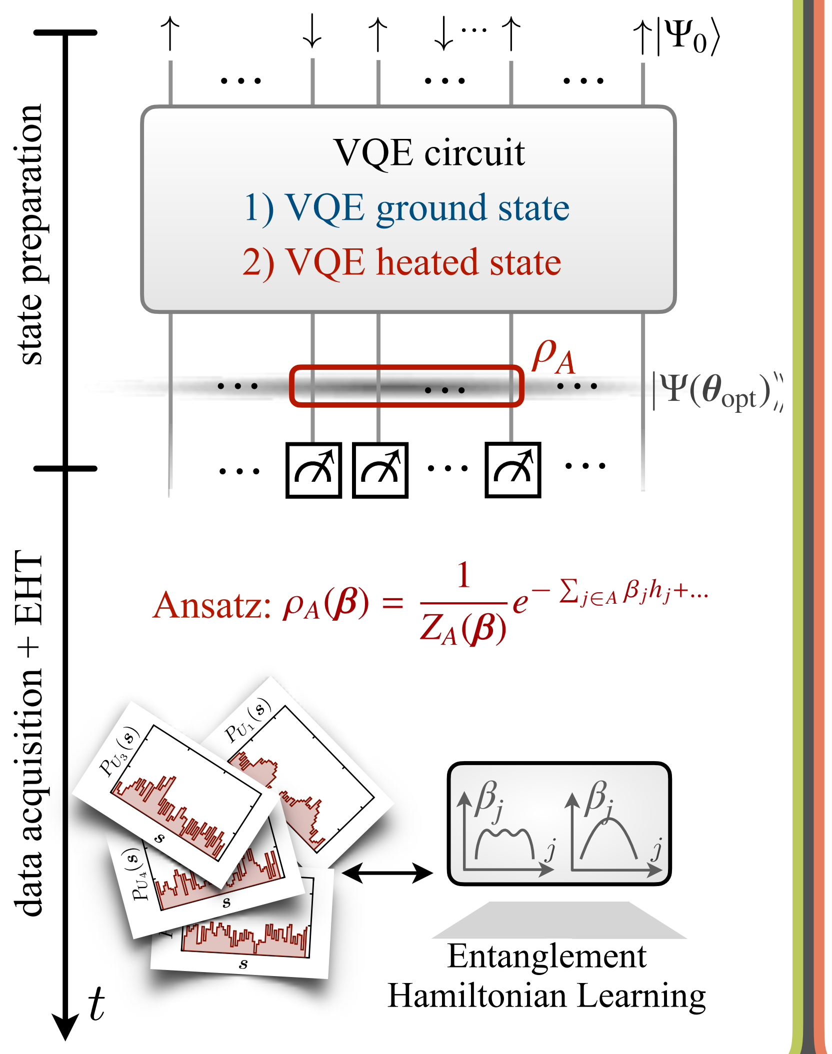
Heisenberg model



XXZ chain

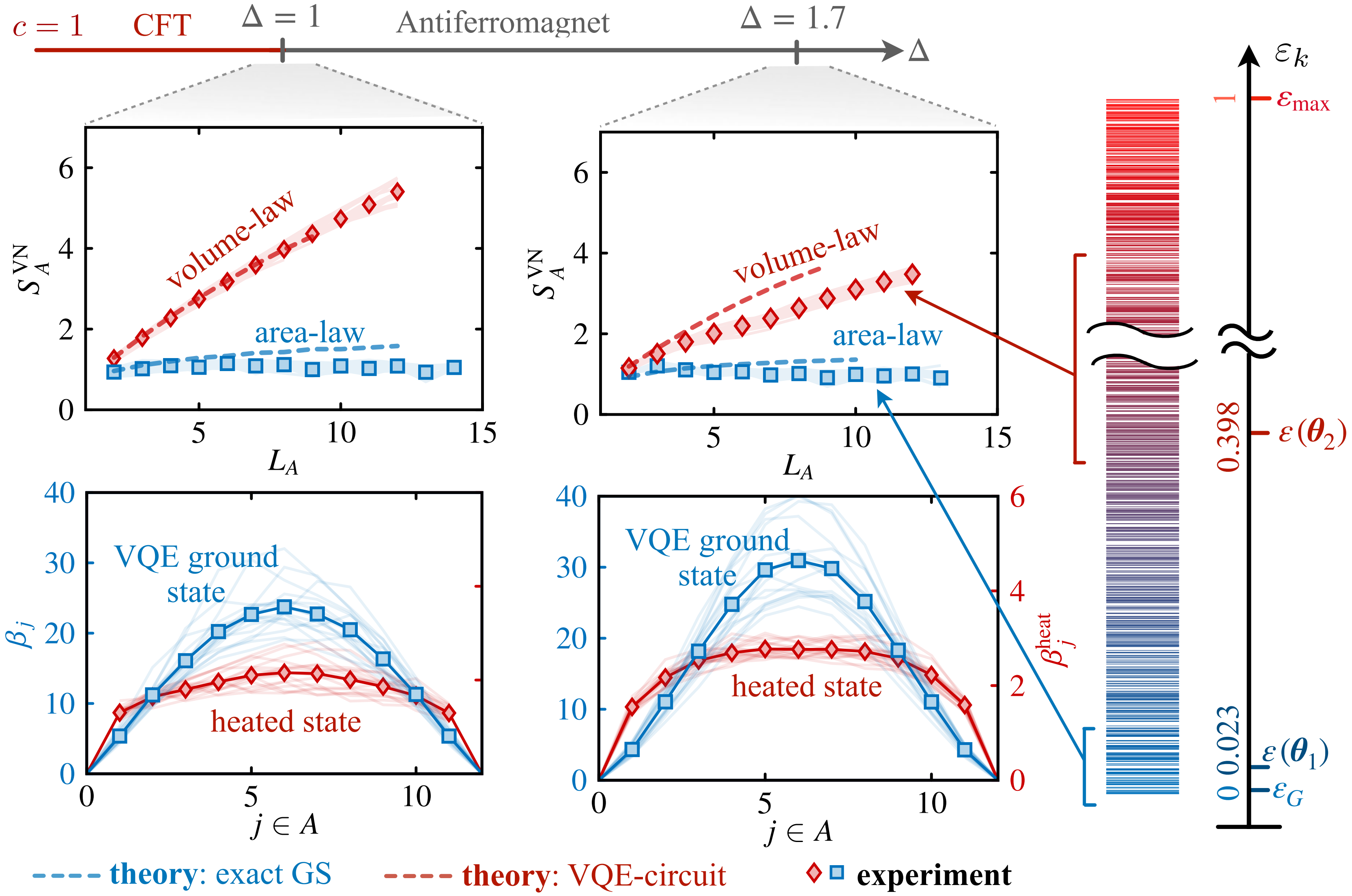
$$H = J\sum_j \left(S^x_j S^x_{j+1} + S^y_j S^y_{j+1} + \Delta S^z_j S^z_{j+1}\right) = \sum_j h_j$$

State preparation & analysis



Ansatz: $\rho_A(\boldsymbol{\beta}) = \dfrac{1}{Z_A(\boldsymbol{\beta})} e^{-\sum_{j\in A}\beta_j h_j + \dots}$

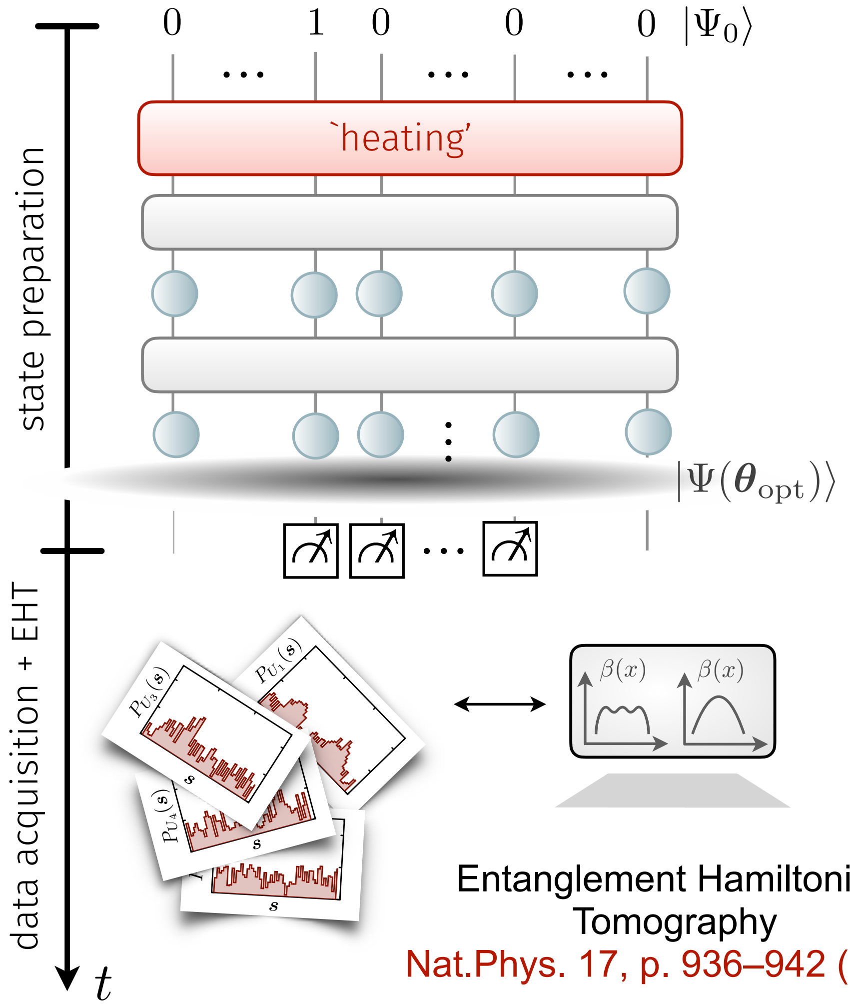
Entanglement properties



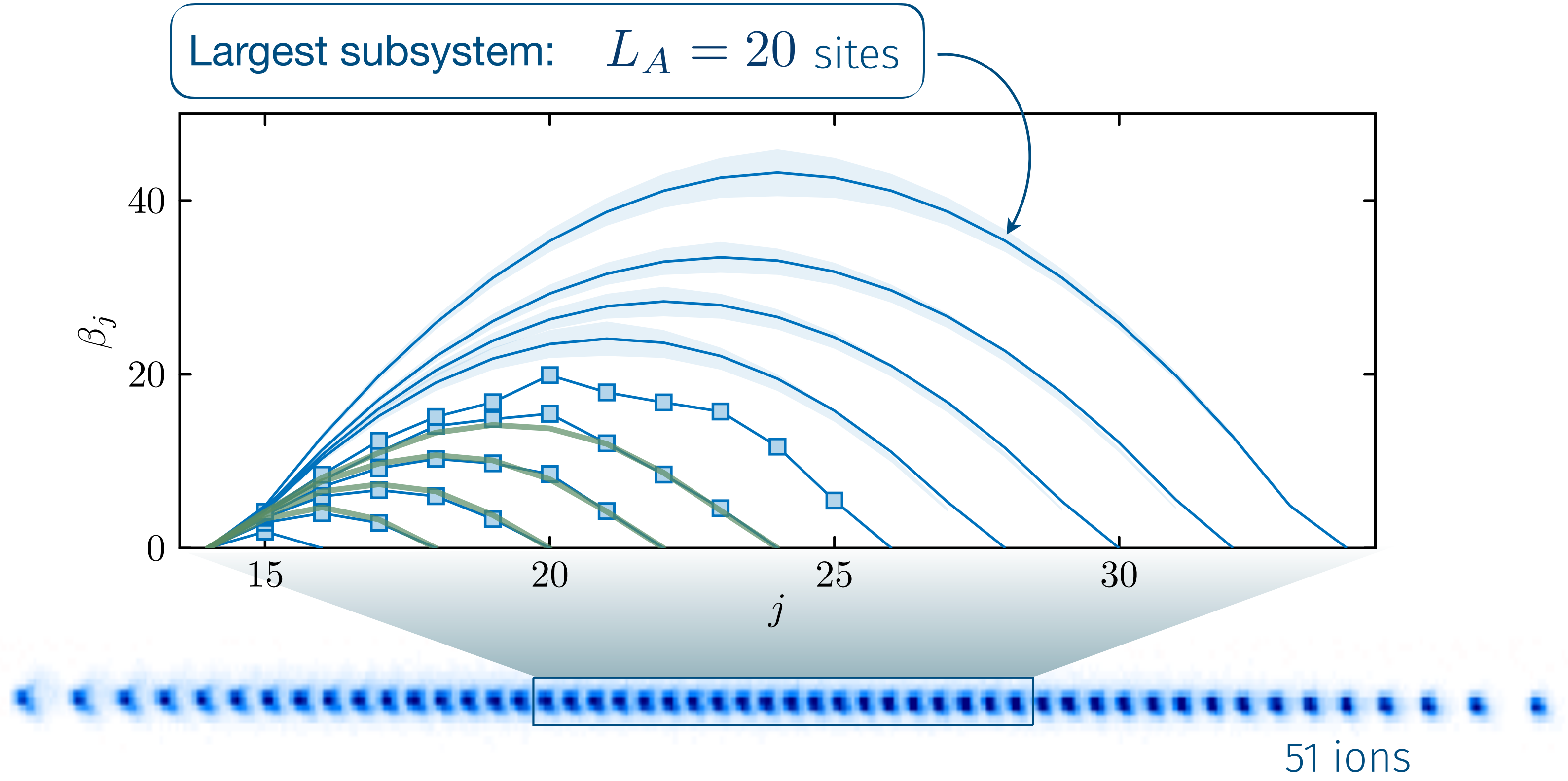
Scaling

variational prep. of the XXZ GS: $$\hat{H} = J\sum_{j=1}^{L}\left(\hat{S}_j^x\hat{S}_{j+1}^x + \hat{S}_j^y\hat{S}_{j+1}^y + \Delta\hat{S}_j^z\hat{S}_{j+1}^z\right) = \sum_{j=1}^{L}\hat{h}_j \qquad L = 51$$

51 particles

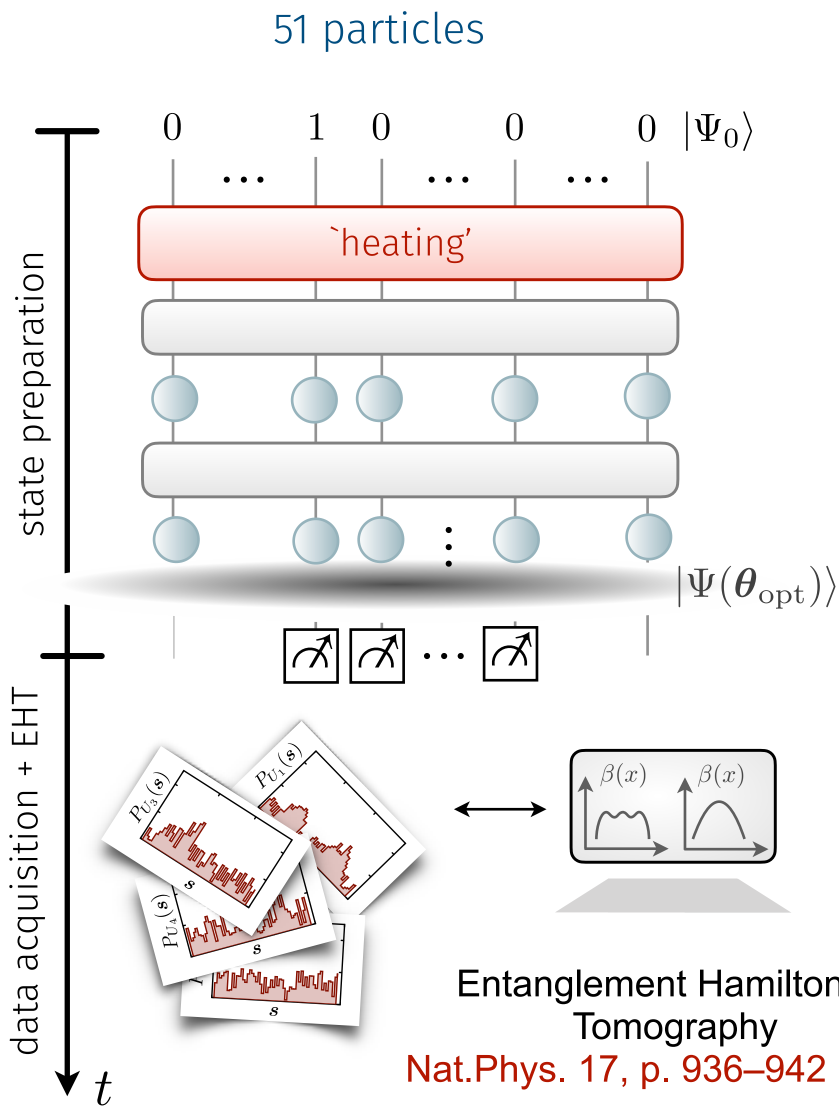


Entanglement Hamiltonian
Tomography
Nat.Phys. 17, p. 936–942 (2021)

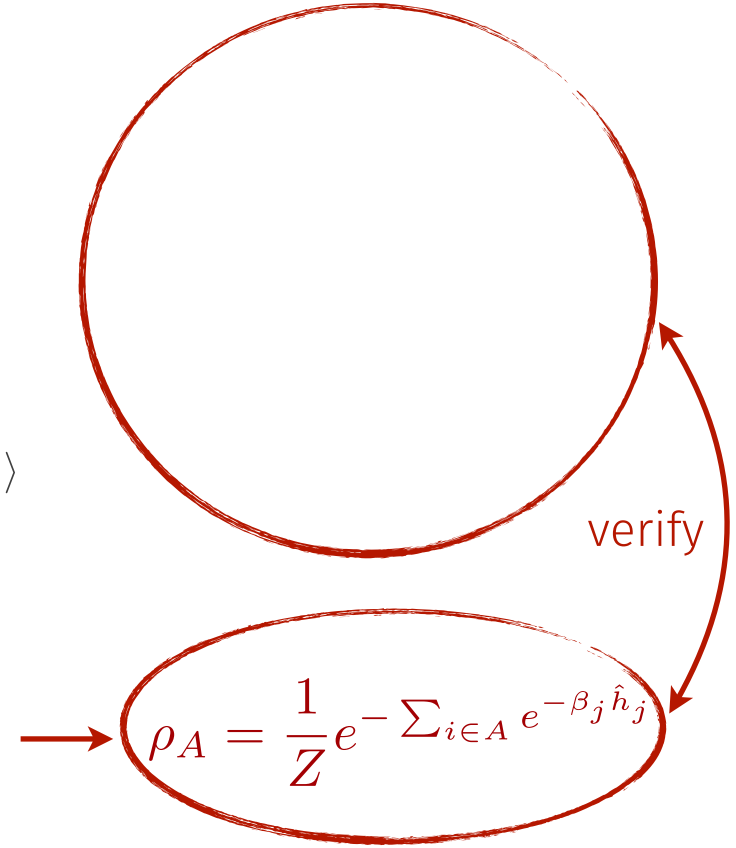


Verification of Entanglement Hamiltonian Tomography

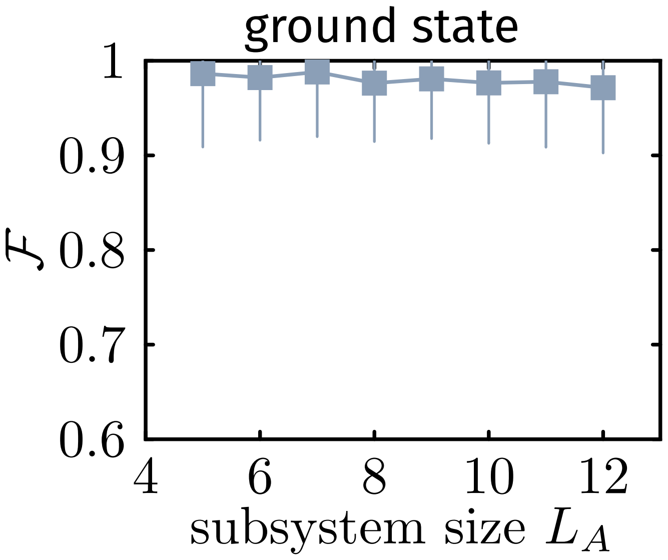
variational prep. of the XXZ GS: $$\hat{H} = J\sum_{j=1}^{L}\left(\hat{S}_j^x\hat{S}_{j+1}^x + \hat{S}_j^y\hat{S}_{j+1}^y + \Delta\hat{S}_j^z\hat{S}_{j+1}^z\right) = \sum_{j=1}^{L}\hat{h}_j \qquad L = 51$$

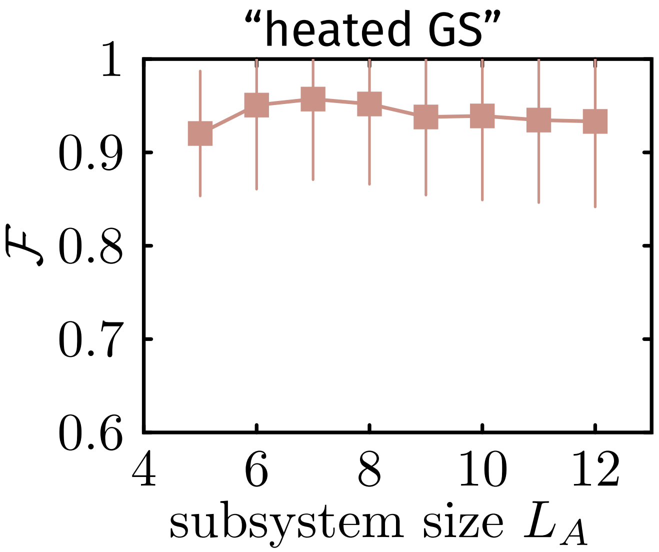


Entanglement Hamiltonian Tomography
Nat.Phys. 17, p. 936–942 (2021)



Cross-Platform Verification
Phys. Rev. Lett. 124, 010504





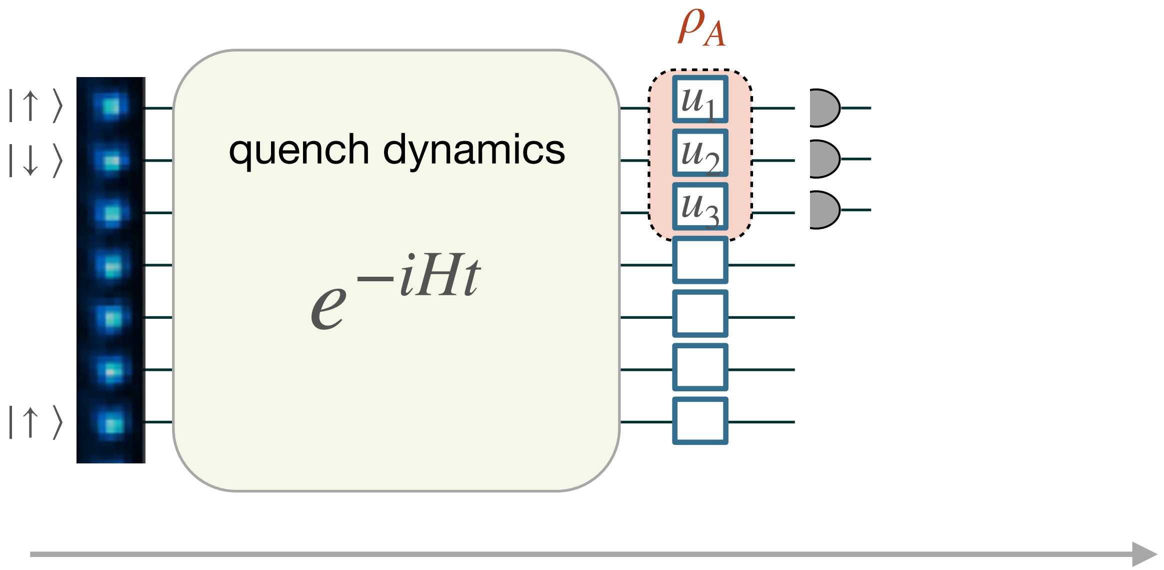
Details of *Entanglement Hamiltonian Learning*

C. Kokail, R. van Bijnen, A. Elben, B. Vermersch, & PZ, *Entanglement Hamiltonian Tomography in Quantum Simulation*, Nat. Phys. (2021).

A. Anshu, S. Arunachalam, T. Kuwahara, and M. Soleimanifar, *Sample-Efficient Learning of Interacting Quantum Systems*, Nat. Phys. (2021).

Randomized Measurements: Tomography

Quench dynamics with analog quantum simulator



time

product state → entangled state

time

Randomized Tomography

$$\rho_A = \mathbb{E}_{U\sim\mathrm{CUE}}[\hat{\rho}_A]$$

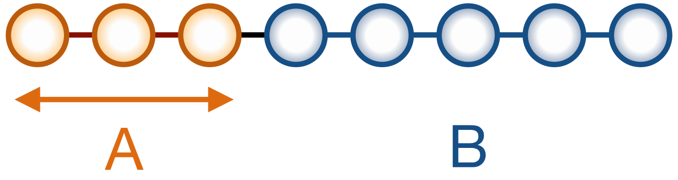
exponentially expensive

$$\hat{\rho}_A = \sum_{\mathbf{s},\mathbf{s}'} \sum_{U} P_U(\mathbf{s})(-2)^{-D[\mathbf{s},\mathbf{s}']} U|\mathbf{s}'\rangle\langle\mathbf{s}'|U^\dagger$$

tomographically complete

Measuring (Large-Scale) Entanglement

Protocol 0: Quantum State Tomography



data ⟶ ρ_A ✓expensive* $\sim \text{rank}(\rho_A)\, 2^{N_A}$ (scales exponentially)

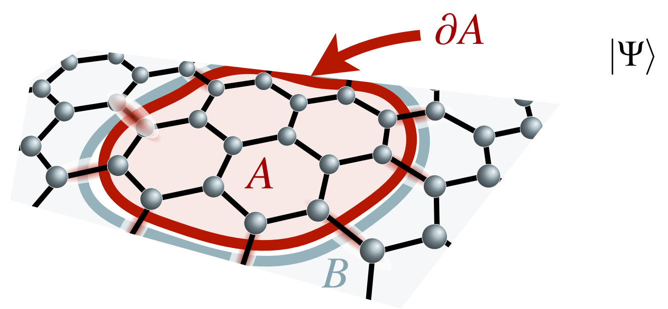
sample-efficient entanglement Hamiltonian tomography

* tomography can be made \`more efficient' if we *know* something about the quantum state: MPS, low rank, neural network, …

Classical Shadows

* or, we are only interested in certain functionals of ρ_A, e.g. expectation values $\langle O \rangle = \text{Tr}\, O_A \rho_A$

Sample-Efficient Learning of the Entanglement Hamiltonian (EH)

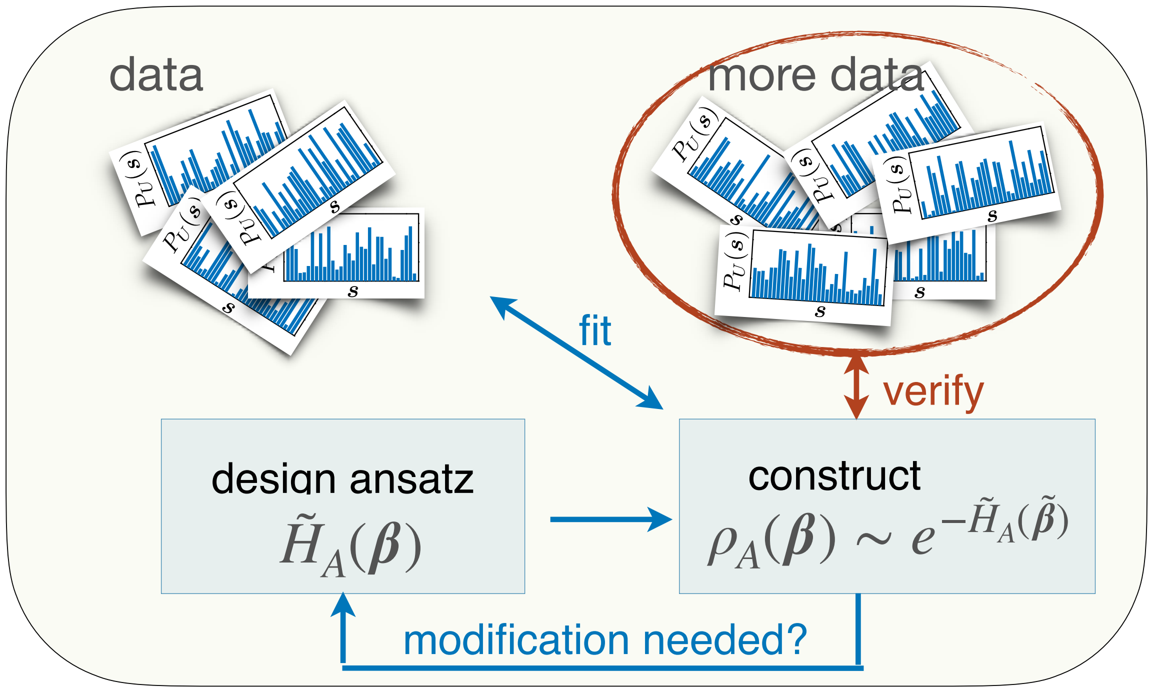


Protocol: *efficient* parametrization of EH

data $\longrightarrow$ $\boxed{\rho_A = e^{-\tilde{H}_A(\boldsymbol{\beta})}}$ with $\tilde{H}_A(\boldsymbol{\beta}) = \sum_{i \in A} \beta_i \hat{h}_\ell + \ldots$ polynomial # β_i ?

Gibbs state

simple operator structure



data

more data

fit

verify

design ansatz $\tilde{H}_A(\boldsymbol{\beta})$

construct $\rho_A(\boldsymbol{\beta}) \sim e^{-\tilde{H}_A(\tilde{\boldsymbol{\beta}})}$

modification needed?

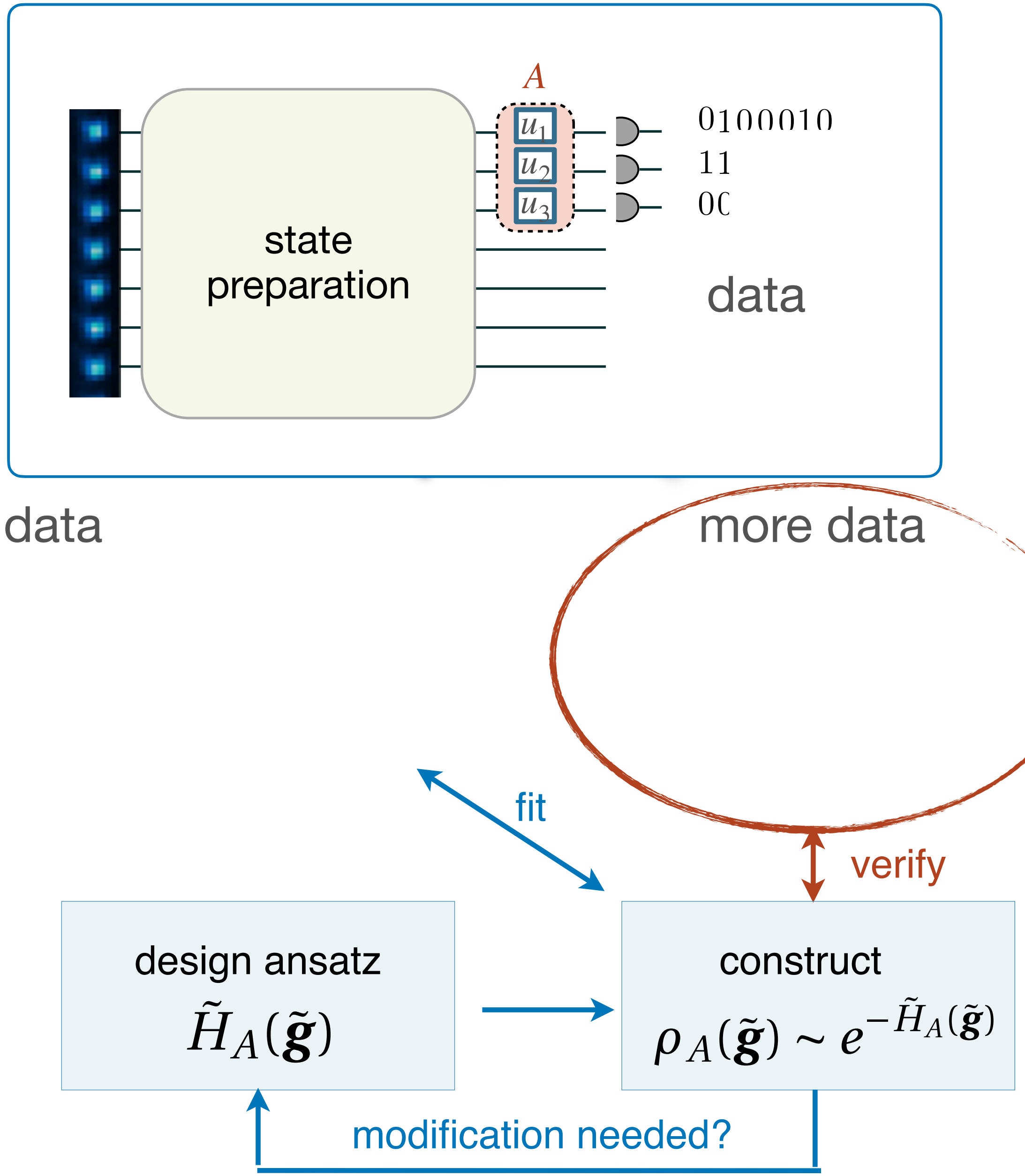
measurement protocol

- sample complexity
- time efficiency

C. Kokail, R. van Bijnen, A. Elben, B. Vermersch, & PZ, *Entanglement Hamiltonian Tomography in Quantum Simulation*, Nat. Phys. (2021).

A. Anshu, S. Arunachalam, T. Kuwahara, and M. Soleimanifar, *Sample-Efficient Learning of Interacting Quantum Systems*, Nat. Phys. (2021).

Learning the Entanglement Hamiltonian



state preparation

data

data

more data

fit

verify

design ansatz

$\tilde{H}_A(\tilde{\mathbf{g}})$

construct

$\rho_A(\tilde{\mathbf{g}}) \sim e^{-\tilde{H}_A(\tilde{\mathbf{g}})}$

modification needed?

Learning Protocol

- **Measure** experimental frequencies:

$$P_U(\mathbf{s}) = \mathrm{Tr}\left(U\rho_A U^\dagger |\mathbf{s}\rangle\langle\mathbf{s}|\right)$$

- **Ansatz** for Entanglement Hamiltonian:

$\tilde{H}_A(\tilde{\mathbf{g}})$

- e.g. deformation of system Hamiltonian plus corrections

- **Fit** optimal parameters $\tilde{\mathbf{g}}$ by minimizing the distance to the frequencies

$$\chi^2(\tilde{\mathbf{g}}) = \sum_{U,\mathbf{s}}\left[\mathrm{Tr}\left(U|\mathbf{s}\rangle\langle\mathbf{s}|U^\dagger \frac{e^{-\tilde{H}_A(\tilde{\mathbf{g}})}}{Z(\tilde{\mathbf{g}})}\right) - P_U(\mathbf{s})\right]^2$$

- **Verify** by measuring Hilbert-Schmidt fidelities

$$\mathcal{F} \sim \mathrm{Tr}\left[\rho_A^{\text{data}}\rho_A^{\text{more data}}\right] = \ldots$$

A. Elben et. al. PRL (2020)

EHT for the Ground state of a long-range Ising chain (Theory)

System Hamiltonian

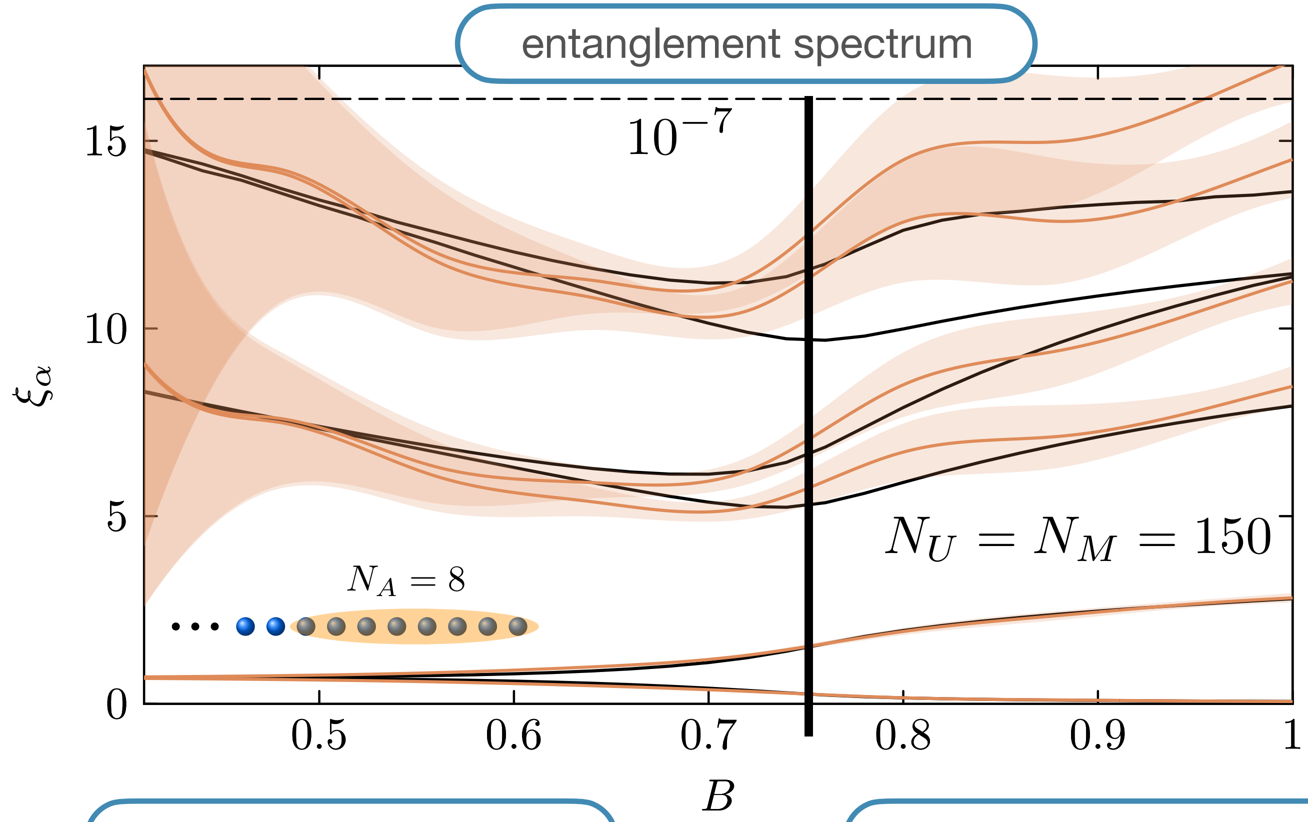
$$H = \sum_{i,j>i} \frac{J_0}{|i-j|^\alpha} \sigma_i^x \sigma_j^x + B \sum_i \sigma_i^z$$

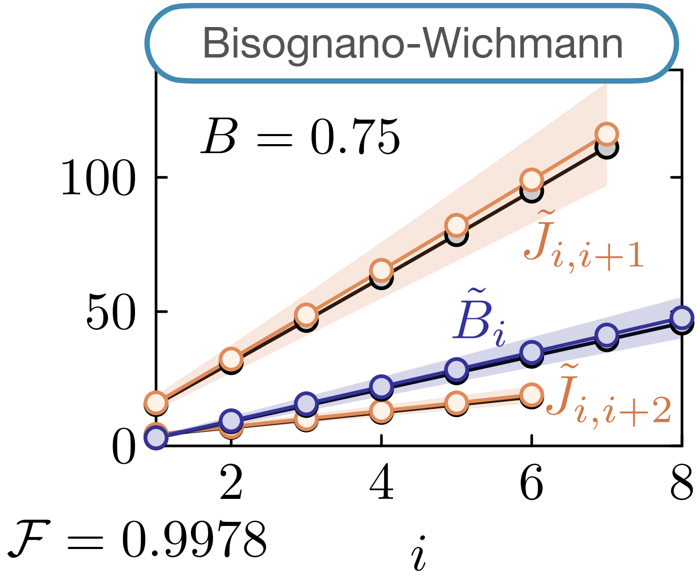
$$\alpha = 2.5$$

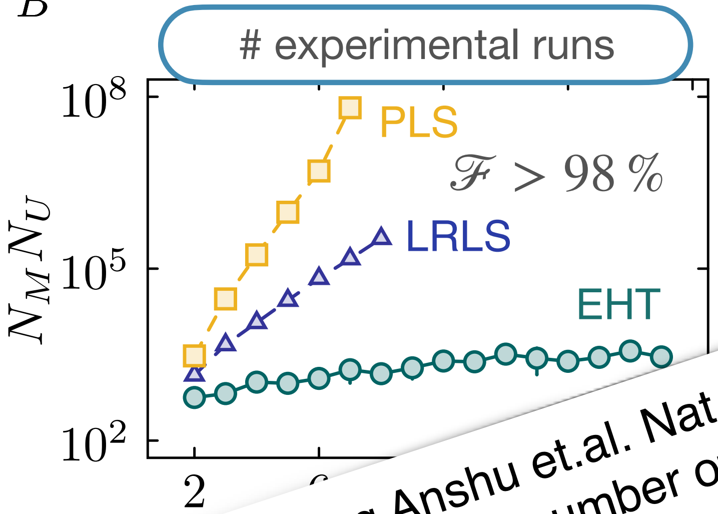
Ansatz for Entanglement Hamiltonian

We choose a simple deformation of the system Hamiltonian

$$\tilde{H}_A = \sum_{i,j\in A} \tilde{J}_{ij} \sigma_i^x \sigma_j^x + \sum_{i\in A} \tilde{B}_i \sigma_i^z$$


entanglement spectrum


Bisognano-Wichmann


experimental runs

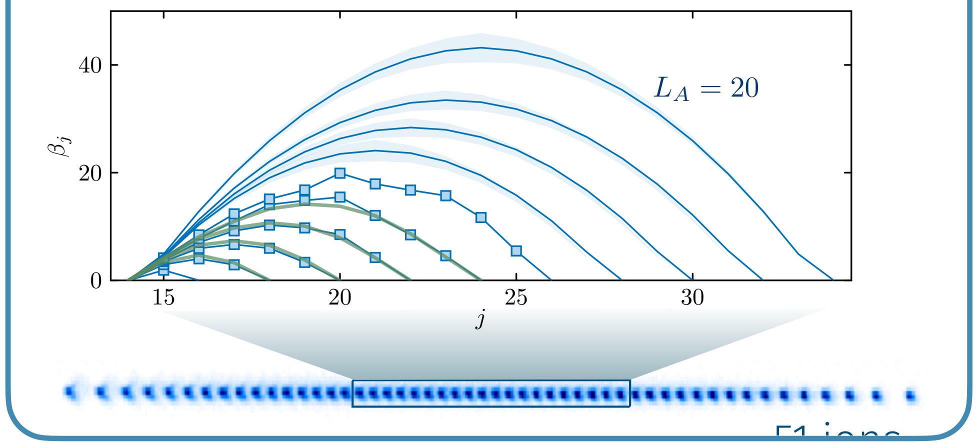
Anurag Anshu et.al. Nat.Phys 20
(polynomial number of sample

N=51 ions

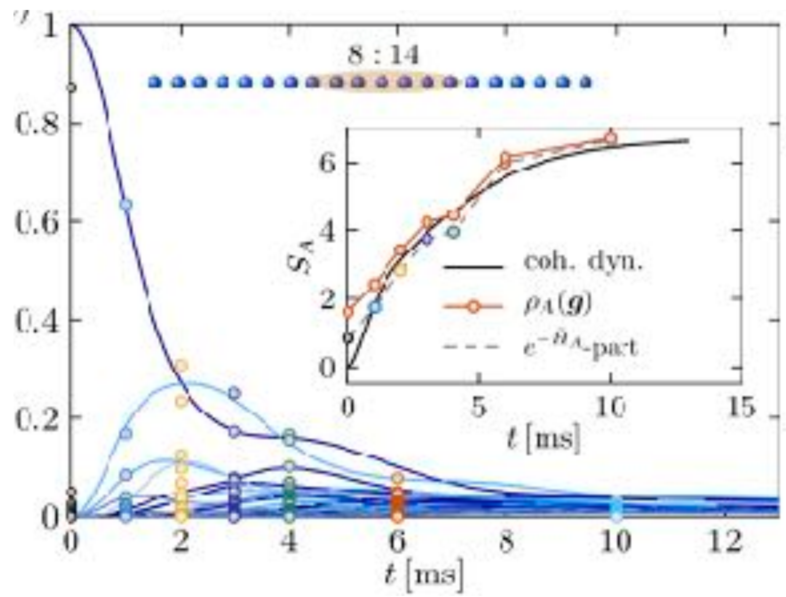
Conclusions & Outlook

- Programmable Quantum Simulators with Atoms
- Hybrid Classical-Quantum / Variational Algorithms
- Randomized Measurements Toolbox
- Programmable Quantum Sensors

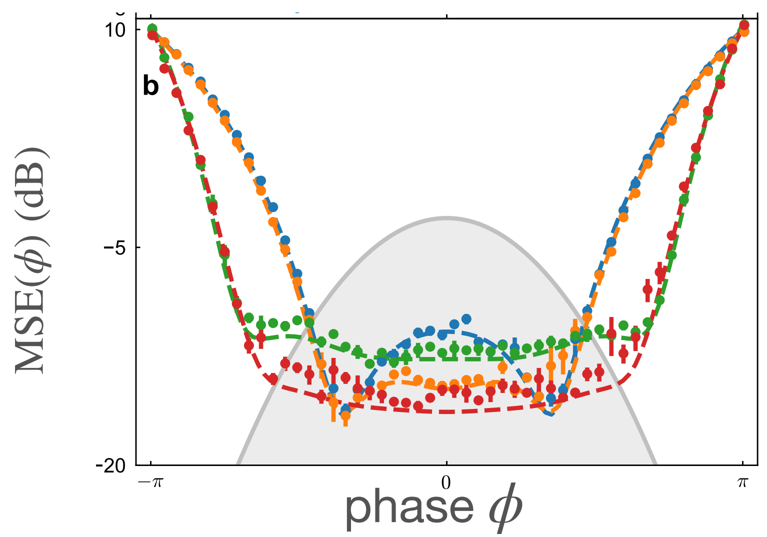
Observation of Bisognano-Wichmann

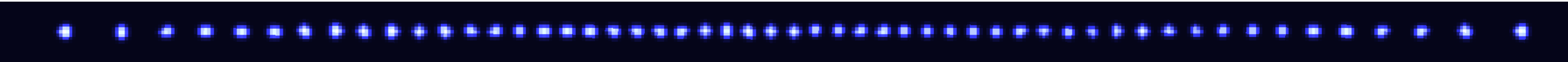


Entanglement spectrum: quench dynamics



Variational Optimal Ramsey





The Team

Theory:


C. Kokail


R. van Bijnen


T Zache


P. Zoller

Experiment:


M. Joshi


F. Kranzl


C. Roos


R. Blatt