# Thermal transport, geometry, and anomalies
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**Abstract**

The aim of this course is to introduce students to the ideas and techniques that, originating in so-called "high-energy" physics, have become standard tools in statistical physics and condensed matter physics in the 21st century. In particular, we will revisit effective actions, introduce quantum anomalies and anomaly-related transport, and the relationship between thermal transport and general relativity. The course is motivated by some of the new developments in condensed matter physics in the 21st century that have led to a new grand unification of high- and low-energy physics.

## Prerequisites

- Mechanics: Lagrangean and Hamiltonian formalism, canonical transformations, Poisson brackets. [1]
- Classical fields: special relativity, Lorentz transformations. The electromagnetic field. [2]
- Quantum Mechanics: Path integral formulation. [3]
- Statistical Physics: Partition function, Green's functions. [4]
- Condensed matter: The Landau Fermi liquid. [4]

[1] Landau series Vol. 1. Mechanics.

[2] Landau series Vol. 2. The Classical Theory of Fields.

[3] A. Altland and B.D. Simons, Condensed Matter Field Theory, Cambridge University Press (Cambridge, 2010).

[4] - A. A. Abrikosov, L. P. Gorkov, I. E. Dzyaloshinki, Richard A. Silverman, Methods of Quantum Field Theory in Statistical Physics, Dover Pub. (1975).

**Tentative program**

## 1. Introduction and generalities.

- Why QFT? From classical mechanics to QFT.
- Classical fields and special relativity. Gauge fields.
- The action. Path integral formalism. Constructing an action. Symmetries.
- Effective actions. (1).
- The Lorentz group: Scalars, vectors, tensors, spinors.
- Fermions in D=2,3,4. Helicity and chirality.

## 2. QFT and CM

- The $XX^{th}$ century revolution in CM: graphene.
- The Dirac equation. The vacuum: the Dirac sea.
- The Landau Fermi liquid. The vacuum: the Fermi sea.
- Renormalization in QFT: The renormalization group.
- Effective actions (2).
- Landau Fermi liquid as a fixed point of RG. Luttinger liquid in (1+1). The case of graphene in (2+1).

## 3. Symmetries and anomalies

- Global and local gauge invariance. Gauge fields.
- The Noether's theorems.
- QFT anomalies.
- Chiral anomaly and mixed chiral-gravitational anomaly.
- Anomaly induced transport .
- Applications to condensed matter.

## 4. Geometry and thermal transport

- The energy-momentum tensor. Components.
- Noether construction (flat space) versus metric stress tensor (curved space).
- Kubo formula for the thermal conductivity. Luttinger trick.
- Applications to condensed matter.

## Bibliography[1,2]

### Introduction and background


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- A. A. Abrikosov, L. P. Gorkov, I. E. Dzyaloshinki, Richard A. Silverman, *Methods of Quantum Field Theory in Statistical Physics*, Dover Pub. (1975)
- D.J. Amit, *Field theory, the renormalization group, and critical phenomena,* World Scientific (Singapore, 1984).
- C. Itzykson and J.M. Drouffe, *Statistical field theory, Vol. I,* Cambridge University Press (Cambridge, 1989).
- M. Le Bellac, *Quantum and statistical field theory*, Oxford University Press (Oxford, 1991).
- J. Zinn-Justin, *Quantum field theory and critical phenomena*, Clarendon Press (Oxford, 1993).
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- E. Fradkin, Field Theoretic Aspects of Condensed Matter Physics: An Overview, arXiv:2301.13234v2
- D. Tong, Lectures on Quantum Field Theory, https://www.damtp.cam.ac.uk/user/tong/qft.html
- B. Shatiapalan, Quantum Field Theory (Lecture Notes) https://www.imsc.res.in/~bala/qftI.pdf


### Specific topics


- J. Polchinski, Effective Field Theory and the Fermi Surface, arXiv1993.
- J.M. Luttinger, Theory of thermal transport coefficients, Phys. Rev. 135 (1964) A1505–A1514.
- K. Landsteiner, Anomaly related transport of Weyl fermions for Weyl semi-metals, arXiv:1306.4932v3.
- M. N. Chernodub, Y. Ferreiros, A. G. Grushin, K. Landsteiner, M.A.H. Vozmediano, Thermal transport, geometry, and anomalies, arXiv:2110.05471.


1. This is a personal choice by no means aimed to be complete. Choose the book-review whose language fits you better. Original articles, no matter how old, have always to be checked.
2. For easier access I chose the arXiv version for the reviews available there. I've included also some web pages.