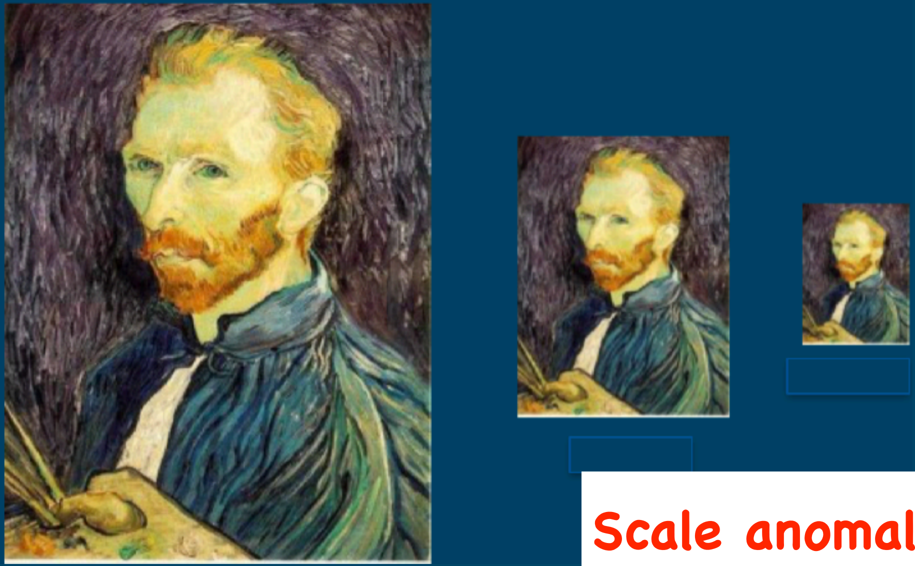
(HEP) QED Conformal anomaly

Scale invariance: $x \to \lambda x$

Noether: $j^{\mu} = T^{\mu\nu} x_{\nu}$ $\quad T^{\mu}_{\mu} = 0$

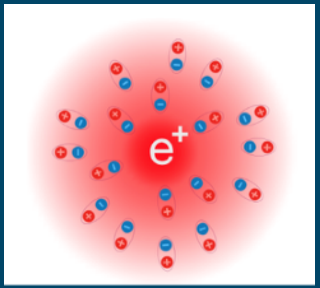


Massless Dirac action is scale invariant

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} i \not{D} \psi ,$$

Scale anomaly: renormalization of the charge introduces a scale:

$$\beta_e = \frac{\mathrm{d}e(E)}{\mathrm{d}\ln E} ,$$



$$\langle T^{\alpha}_{\ \alpha}(x) \rangle = \frac{\beta_e(e)}{2e} F^{\mu\nu}(x) F_{\mu\nu}(x) ,$$

$$S^{EM}_{eff} = \int dx \, \tau(x) T^{\mu}_{\mu}(x)$$

HEP Anomaly induced transport

$$g_{\mu\nu}(x) = \boxed{e^{2\tau(x)}}\eta_{\mu\nu}, \longrightarrow S_{eff} = \int d^4x\,\tau(x)T^{\mu}_{\mu}(x) \longrightarrow J^{\mu}(x) = -\frac{2\beta_e(e)}{e}F^{\mu\nu}(x)\partial_{\nu}\tau(x)\,.$$

B ↙ ↘ E

Scale magnetic effect

$$\vec{J}(x) = -\frac{2\beta_e(e)}{e}\vec{B}(x)\times\vec{\nabla}\tau(x)\,.$$

Inhomogeneous scale factor:
Generation of an electric current
perpendicular to B and grad τ

Scale electric effect

$$\vec{J}_{\mathrm{SEE}} = \sigma(x)\vec{E}(x)\,,$$

$$\sigma(t,\vec{x}) = -\frac{2\beta_e(e)}{e}\frac{\partial\tau(t,\vec{x})}{\partial t}\,.$$

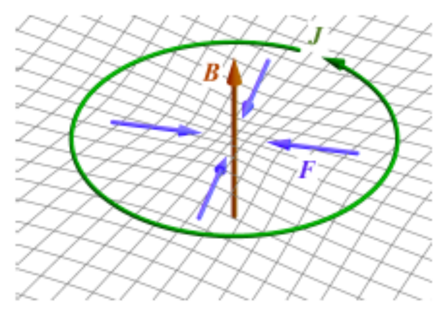
Time varying scale factor:
Ohm law with anomalous conductivity

Very hard to test in the cosmological context

M Chernodub, PRL’2016

3D Condensed matter

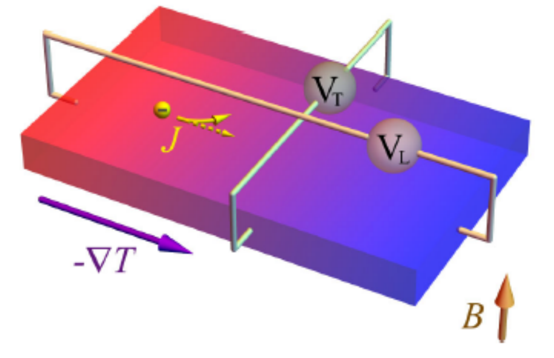
Scale magnetic effect*



$$\boldsymbol{J} = -\frac{2\beta(e)}{e}\boldsymbol{\nabla}\tau(x) \times \boldsymbol{B}(x).$$

$$\nabla\phi \leftrightarrow \frac{\nabla T}{T} \quad {}^{**}$$

Nernst effect



$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T\hbar}\boldsymbol{B} \times \boldsymbol{\nabla}T\,.$$

Experimental signatures

The thermoelectric coefficient α should.

- Extrapolate to a non zero value as T->0 and μ=0.
- Linear in B.
- Proportional to v_F (larger in cleaner samples).

$$\boldsymbol{J} = \sigma\boldsymbol{E} + \alpha\boldsymbol{\nabla}T,$$

$$\alpha \sim v_F B/T$$

* M. Chernodub, **PRL117**, 141601 (2016)
** J. M. Luttinger, PR135, 1505 (1964)

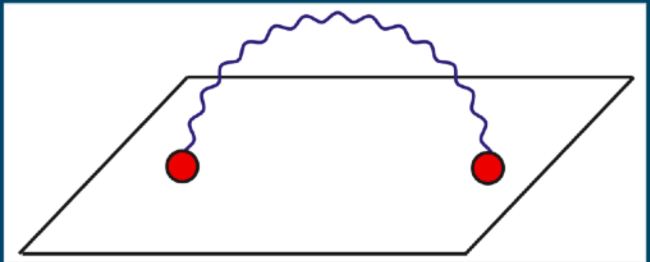
Anomaly based: Chernodub, Cortijo, MV, **PRL120**, 206601 (2018)
Kubo calculation: Arjona, Chernodub , MV, PRB**99** , 235123 (2019)
Effect of tilt: Ballestad, Cortijo, MV, Qaiumzadeh, PRB**107** , 014410 (2023)

2D?: Graphene vs QED(2+1)

1. **QED(2+1) is not scale invariant (coupling dimension-full).**
   The theory is superrenormalizable.
   Why?: photon propagator $\sim 1/k^2$

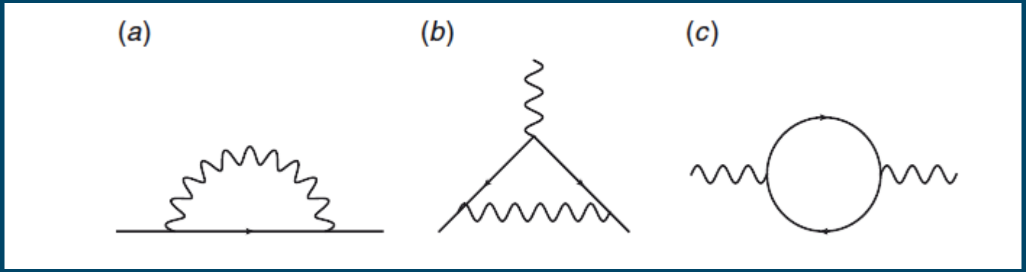
$$L = \int d^2 r dt \bar{\Psi}(\mathbf{r}, t)\gamma^\mu(\partial_\mu - ieA_\mu)\Psi(\mathbf{r}, t)$$

2. Graphene: brane reduced QED
   Photon propagator $\sim 1/k$. The model is renormalizable
   And scale invariant!



$$L_{int} = \frac{e^2}{4\pi}\int d^3k \bar{\Psi}(\gamma^0 A_0 + v_F \gamma^i A_i)\Psi$$

RG Analysis. Results



Diverges

Finite

$$\beta_v = -\frac{e^2}{16\pi v_F}$$

$$\beta_e = 0$$

1. The electric charge is not renormalized -> **no QED scale anomaly**
2. The Fermi velocity grows to the infrared -> **?**

J. González, F. Guinea, MAHV
Nucl. Phys. **B424**, 593 (94)
Phys. Rev. Lett.**77**, 3589 (96)
Phys. Rev. **B59**, R2474 (99)

A reasonable question: will v_F (E) generate a scale anomaly?

(With observable consequences?)

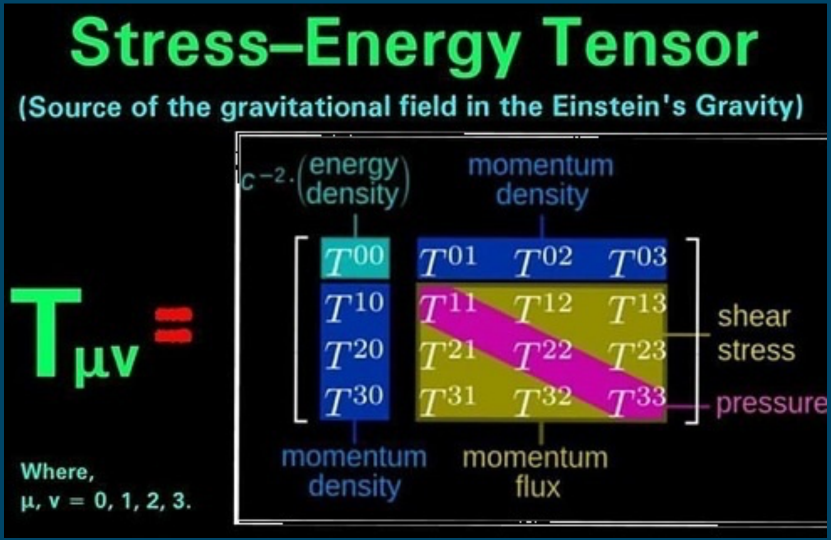
1. The answer to the first Q is (obviously) yes.

2. Observables?

Effective action associated to a scale transformation <->
trace of the stress tensor

$$\boxed{T^\mu_\mu = \beta_v < \bar{\Psi} i\vec{\gamma}\,\vec{\nabla}\,\Psi > \, = -\frac{\beta_v}{v_F} < \epsilon >}$$

It affects the EOS:
<u>thermodynamics and hydro effects</u>



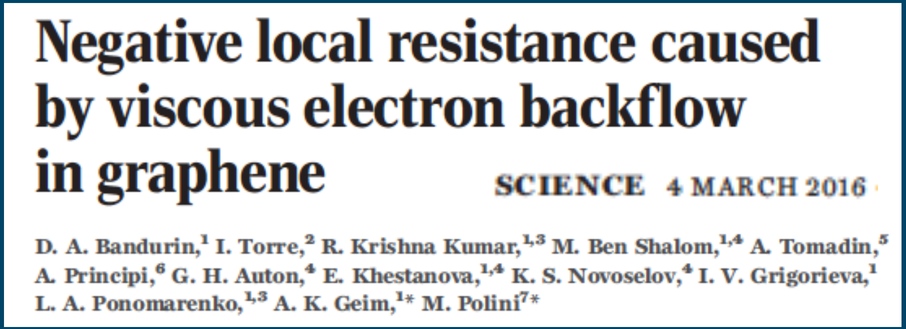
Hydro aspects of Dirac matter

$\tau_{ee} >> \tau_{any}$ Fermi liquids in ultrapure crystals

JHEP March 24, 2015

Negative magnetoresistivity in chiral fluids and holography

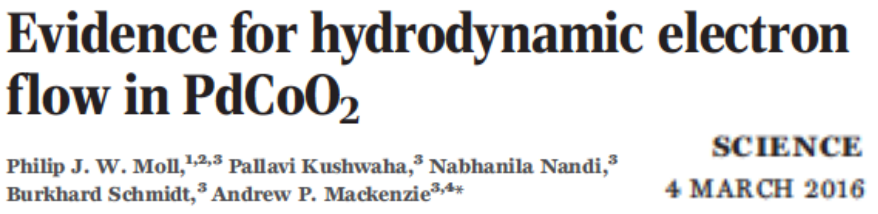
Karl Landsteiner, Yan Liu and Ya-Wen Sun



Negative local resistance caused by viscous electron backflow in graphene

SCIENCE 4 MARCH 2016

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1]* M. Polini[7]*



Evidence for hydrodynamic electron flow in $PdCoO_2$

SCIENCE 4 MARCH 2016

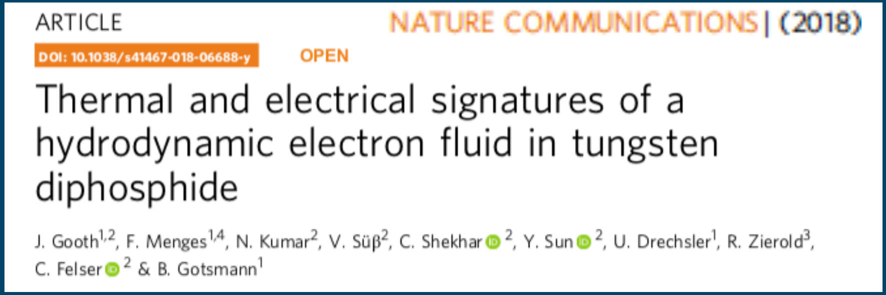
Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3] Burkhard Schmidt,[3] Andrew P. Mackenzie[3,4]*

ELECTRON TRANSPORT

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

SCIENCE 4 MARCH 2016

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2]* Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5]*

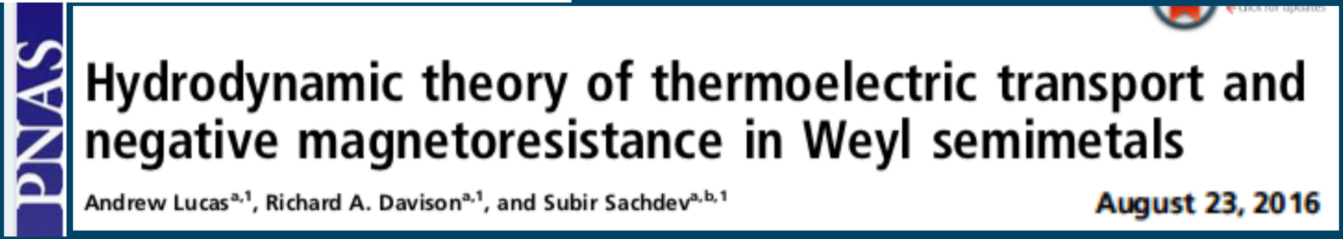


ARTICLE NATURE COMMUNICATIONS | (2018)

DOI: 10.1038/s41467-018-06688-y OPEN

Thermal and electrical signatures of a hydrodynamic electron fluid in tungsten diphosphide

J. Gooth[1,2], F. Menges[1,4], N. Kumar[2], V. Süß[2], C. Shekhar[2], Y. Sun[2], U. Drechsler[1], R. Zierold[3], C. Felser[2] & B. Gotsmann[1]



PNAS

Hydrodynamic theory of thermoelectric transport and negative magnetoresistance in Weyl semimetals

Andrew Lucas[a,1], Richard A. Davison[a,1], and Subir Sachdev[a,b,1]

August 23, 2016

Summary and conclusion

Quantum field theory

Condensed matter

Anomalies → New transport phenomena

New anomalies
New QFTs ← Lorentz breaking terms

Exp-confirmation
of theories ← Experimental accessibility

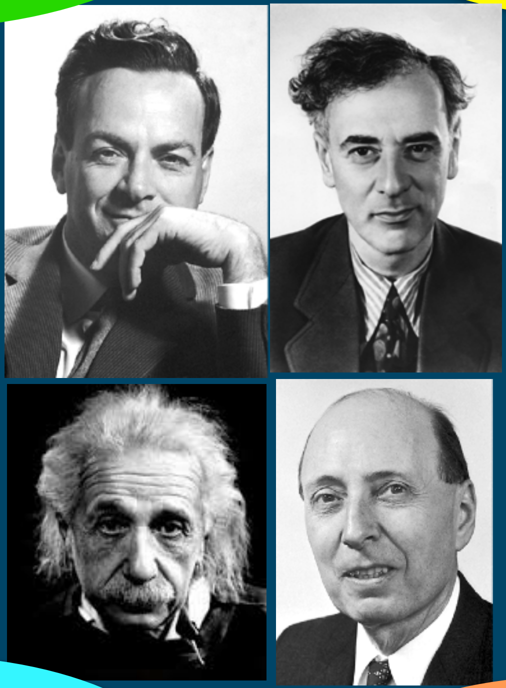
DIRAC MATTER

Quantum field theory

Condensed matter

Plasma physics

Statistical physics



Hydrodynamics

Elasticity

Relativity and cosmology

String theory (holography)

Condensed matter merges HEP (again)

The adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing.

Key expressions

General for massless QED

- The action $S_0 = \int dt\, d^n\vec{x}\, \bar{\Psi}\, i\, (\gamma^t \partial_t + v_F \vec{\gamma} \cdot \vec{\nabla})\Psi$
- The stress tensor $T^{\mu\nu} = \frac{i}{2}\bar{\Psi}\left(\gamma^\mu \nabla^\nu + \gamma^\nu \nabla^\mu\right)\Psi - \eta^{\mu\nu}\bar{\Psi}\, i\, (\gamma^t \partial_t + v_F \vec{\gamma} \cdot \vec{\nabla})\Psi$
- The energy density $\boxed{\epsilon \equiv T^{00} = -\, v_F\, \bar{\Psi} i \vec{\gamma} \cdot \vec{\nabla}\Psi}$
- The trace $T^\alpha_{\ \alpha} = -\, 2\bar{\psi} i \gamma^\mu \nabla_\mu \psi \equiv 0$ (traceless using eq. of motion)

The (new) conformal anomaly

Under a scale transformation $g_{\mu\nu} = e^{2\tau}\eta_{\mu\nu}$, $\delta g_{\mu\nu} = 2\tau\eta_{\mu\nu}$

$$S \rightarrow S_\tau = S + \tau \int dt d^n\vec{x}\, T^\mu_\mu(x) + O(\tau^2) \qquad \boxed{v_F \rightarrow v_F + \tau\beta_v}$$

$$\frac{\partial S_\psi}{\partial \tau} = < \int dt\, d^2\vec{x}\, T^\mu_{\ \mu} > \qquad \boxed{T^\mu_\mu = \beta_v < \bar{\Psi} i \vec{\gamma} \vec{\nabla} \Psi > = -\frac{\beta_v}{v_F} < \epsilon >}$$

arXiv:2505.13712

Graphene: more recent hydro exp.

Visualizing Poiseuille flow of hydrodynamic electrons Sulpizio,Geim et al Nature | Vol 576 | 5 December 2019

Scanning carbon nanotube single-electron transistor to image the Hall voltage of electronic flow through channels of high-mobility graphene.

Article

Imaging viscous flow of the Dirac fluid in graphene Nature | Vol 583 | 23 July 2020 | **537**

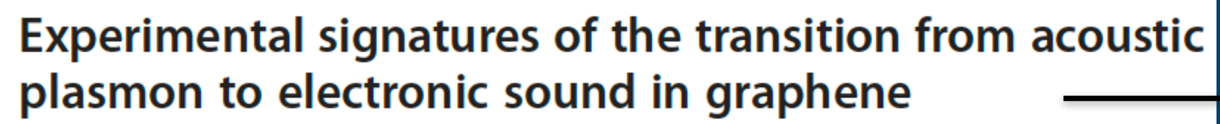
Probing viscous electronic transport via magnetic field imaging

Kim, Jacobi et al

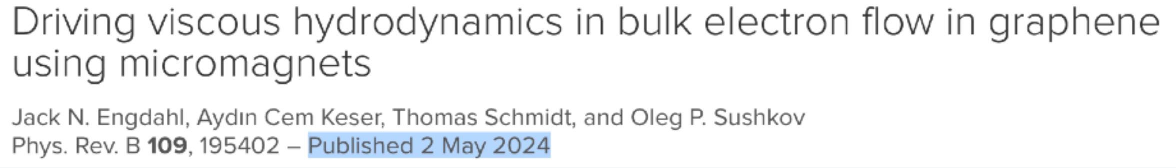
Observation of hydrodynamic plasmons and energy waves in graphene

THz absorption spectra. Zhao et al.

Nature | Vol 614 | 23 February 2023

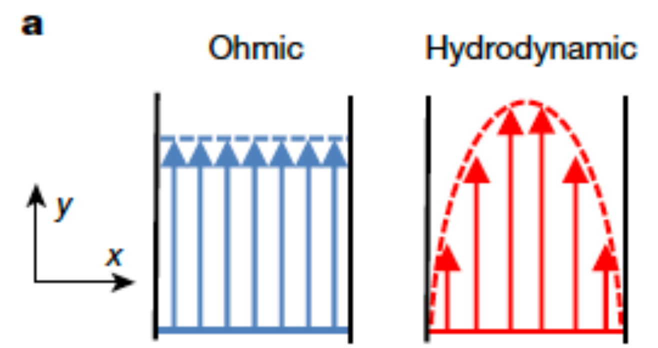


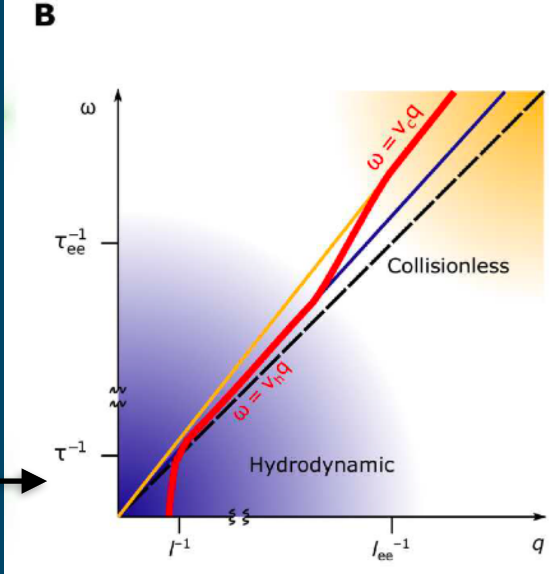
Experimental signatures of the transition from acoustic plasmon to electronic sound in graphene



Driving viscous hydrodynamics in bulk electron flow in graphene using micromagnets

Jack N. Engdahl, Aydın Cem Keser, Thomas Schmidt, and Oleg P. Sushkov
Phys. Rev. B **109**, 195402 – Published 2 May 2024





Barcons Ruiz *et al.*, *Sci. Adv.* **9**, eadi0415 (2023)

Consequences of conformal anomaly

Thermodynamics

- Conformal equation of state:

$$< T^\mu_\mu > = E - 2P = 0\ , \ \ E = 2P$$

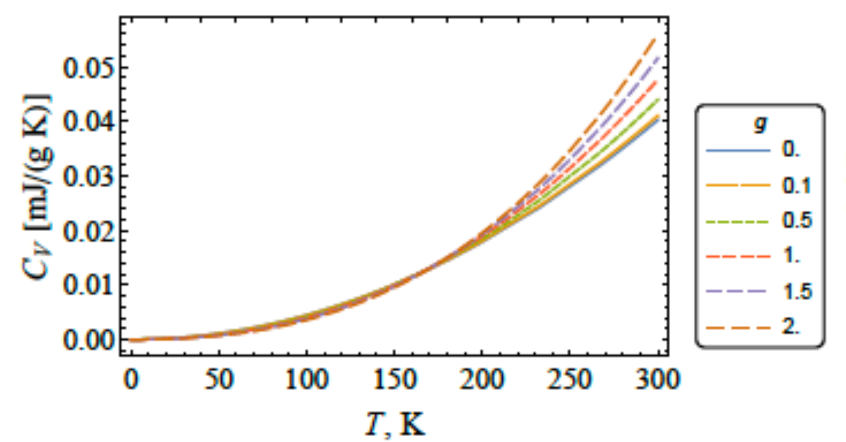
- Conformal anomaly:

$$< T^\mu_\mu > = -\frac{\beta_v}{v_F} < E >$$

- Modified EOS:

$$\boxed{\left(1 + \frac{\beta_v}{v_F}\right) E = 2P}$$

Specific heat*



arXiv:2505.13712

* First noticed by Vafek PRL07.

Measurable

Consequences of conformal anomaly
Hydrodynamics

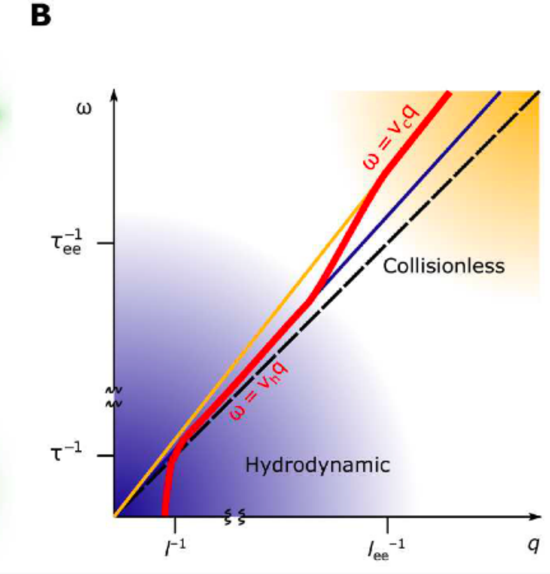
- **Sound wave velocity**

Measurable?

EOS of relativistic fluid $(c \to v_F)$

$$\frac{\partial P}{\partial E} = \frac{v_s^2}{v_F^2} \qquad \left(1 + \frac{\beta_v}{v_F}\right) E = 2P$$

$$v_s(T) = \frac{v_F(T)}{\sqrt{2}} \left(1 - \frac{\beta_v}{v_F(T)}\right)^{-1/2}$$



Barcons Ruiz *et al.*, *Sci. Adv.* **9**, eadi0415 (2023)

arXiv:2505.13712

Measurable?

Evidence for second sound in graphene via THz pulses.
Crosson et al., Nat Phys'23