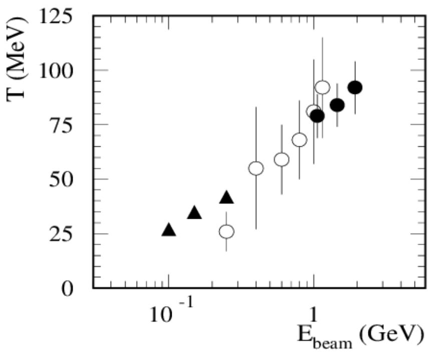
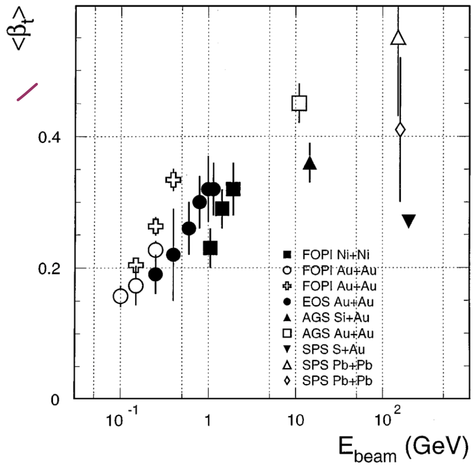
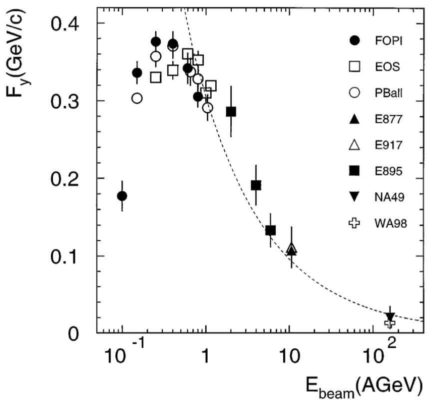
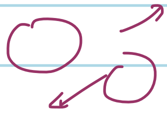
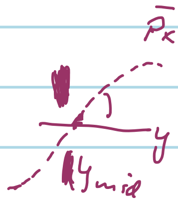


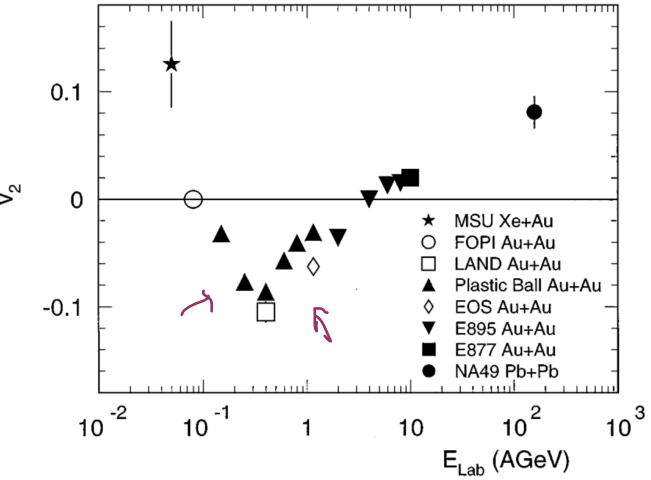




$F_y = \frac{d\bar{p}_x}{dy}$







$v_2 = \langle \cos 2\varphi \rangle$

MIDRAPIDITY

$v_2 < 0$ OUT OF REACTION PLANE

$v_2 > 0$ IN THE REACTION PLANE

SPATIAL INFO FROM CORRELATIONS

ENTROPY FROM CLUSTER YIELDS

NUCLEAR TRANSPORT THEORY

STATISTICAL THEORY USING PHASE SPACE VBLES $\vec{r}$ & $\vec{p}$ FOR NUCLEONS & OTHER PTCLES

? HISTORY OF REACTIONS?

A-BODY SCHRODINGER EQ SOLUTION

t-DEP ?

CAN BE SOLVED FOR FEW BODIES

LEARN ABOUT PROGRESS OF A REACTION FROM INITIAL STATE TILL FREEZE-OUT

TRANSPORT THEORY SHOULD COPE
W/PAST STAGES
STATISTICAL THEORY ADDITION FOR SLOW

MICROSCOPIC DERIVATIONS USED TO JUSTIFY STRUCTURE & FOR INSPIRATIONS
+ PHENOMENOLOGY EXPLOITING

$E = E\{f\}$ ENERGY FUNCTIONAL
f - PHASE-SPACE DENSITIES FOR PARTICLES

f TIME-DEPENDENT HARTREE-FOCK
USED F/ LOW ENERGY REACTIONS
GOOD UP TO TENS OF MeV/nucl
WHEN EFFECTS OF SHORT-RANGE CORRELATIONS SUPPRESSED

$$i\frac{\partial}{\partial t}\varphi_\alpha(\bar{r}t) = \left(-\frac{\nabla^2}{2m} + U(\bar{r}t)\right)\varphi_\alpha(\bar{r}t)$$

$$U(\bar{r}t) \equiv U(n(\bar{r}t))$$

$$n(\bar{r}t) = \sum_\alpha |\varphi_\alpha|^2$$

ONE-BODY DENSITY MTX $\propto e^{-ip\bar{r}_1'}$

$$\rho(\bar{r}_1,\bar{r}_1' t) = \sum_\alpha \varphi_\alpha(\bar{r}_1 t)\,\varphi^*_\alpha(\bar{r}_1' t)$$

$e^{ip\bar{r}_1}$ EXPECTATION OF MOMENTUM OPERATOR

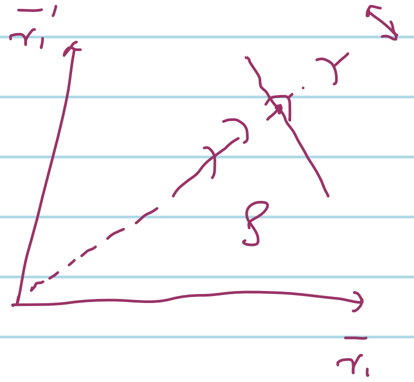
$$n(\bar{r}t) = \rho(\bar{r},\bar{r}\,t)$$

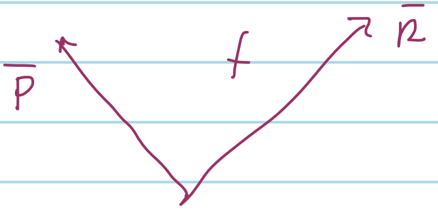
$$\langle\bar{p}\rangle = \int d^3r_1\, \frac{1}{2i}\left(\vec{\nabla}_1 - \vec{\nabla}_1'\right)\rho(\bar{r}_1,\bar{r}_1' t)\Big|_{\bar{r}_1'=\bar{r}_1}$$

WIGNER DISTRIBUTION

$$f(\bar R, \bar p\, t) = \int d(\bar r_1 - \bar r_1')\, e^{-i\bar p(\bar r_1 - \bar r_1')}\, \rho(\bar r_1 \bar r_1'\, t)$$

$\leftrightarrow \frac{\hbar}{p_{ch}} \sim \frac{\hbar}{p_F}$ AT FIXED $\bar R = \frac{1}{2}(\bar r_1 + \bar r_1')$





EQ OF MOTION

$$\frac{\partial}{\partial t}\rho = \frac{1}{i}\left[-\frac{\nabla_1^2}{2m} + \frac{\nabla_1'^2}{2m} + U(\bar r_1 t) - U(\bar r_1' t)\right]\rho(\bar r_1 \bar r_1' t)$$

EQ OF MOTION FOR f

$$f(\bar R\, \bar p\, t) = \int d\bar r\, e^{-i\bar p \bar r}\, \rho\left(\bar R + \frac{\bar r}{2}, \bar R - \frac{\bar r}{2}, t\right)$$

$\bar r = \bar r_1 - \bar r_1'$ $\quad \bar R = \frac{1}{2}(\bar r_1 + \bar r_1')$

$$\frac{\partial f}{\partial t} = ?$$

$$\frac{\partial}{\partial r_1} = \frac{\partial \bar R}{\partial \bar r_1}\frac{\partial}{\partial \bar R} + \frac{\partial \bar r}{\partial \bar r_1}\frac{\partial}{\partial \bar r}$$

$$= \frac{1}{2}\frac{\partial}{\partial \bar R} + \frac{\partial}{\partial \bar r} \qquad \frac{\partial}{\partial \bar r_1'} = \frac{1}{2}\frac{\partial}{\partial \bar R} - \frac{\partial}{\partial \bar r}$$

$$-\nabla_1^2 + \nabla_1'^2 = -\left(\frac{1}{2}\frac{\partial}{\partial \bar R} + \frac{\partial}{\partial \bar r}\right)^2 + \left(\frac{1}{2}\frac{\partial}{\partial \bar R} - \frac{\partial}{\partial \bar r}\right)^2 = -2 \cdot 2\,\frac{1}{2}\frac{\partial}{\partial \bar R}\frac{\partial}{\partial \bar r}$$

$$= -2\frac{\partial}{\partial \bar R}\frac{\partial}{\partial \bar r}$$

$$\int d\bar r\, e^{-i\bar p \bar r}\, \frac{1}{2m}\left(-2\frac{\partial}{\partial \bar R}\frac{\partial}{\partial \bar r}\right)\rho\left(\bar R + \frac{\bar r}{2}, \bar R - \frac{\bar r}{2}, t\right)$$

PARTIAL INTEGRATION

$$\frac{1}{i}\frac{1}{m}(-i\bar{p})\frac{\partial}{\partial \bar{R}}\int d\bar{r}\, e^{-i\bar{p}\bar{r}}\rho = -\frac{\bar{p}}{m}\frac{\partial f}{\partial \bar{R}}$$

$$\frac{\partial}{\partial t} f = -\frac{\bar{p}}{m}\frac{\partial f}{\partial \bar{R}} + \dots$$

$$\bar{r} \sim \frac{1}{p_F}$$

$$\frac{1}{i}\int e^{-i\bar{p}\bar{r}}\Big(U(\bar{R}+\tfrac{\bar{r}}{2}) - U(\bar{R}-\tfrac{\bar{r}}{2})\Big)\rho(R+\tfrac{\bar{r}}{2}, \bar{R}-\tfrac{\bar{r}}{2}, t)$$

$$\simeq \bar{r}\frac{\partial U}{\partial \bar{R}}\Big|_{\bar{r}=0}$$

$$\frac{1}{i}\int e^{-i\bar{p}\bar{r}}\frac{\partial U}{\partial \bar{R}}\bar{r}\rho = \frac{1}{i}\frac{\partial U}{\partial \bar{R}}\int \underbrace{e^{-i\bar{p}\bar{r}}\bar{r}}_{i\frac{\partial}{\partial \bar{p}}e^{-i\bar{p}\bar{r}}}\rho\, d\bar{r}$$

$$= \frac{\partial U}{\partial R}\frac{\partial}{\partial \bar{p}}\int e^{-i\bar{p}\bar{r}}\rho\, d\bar{r} = \frac{\partial U}{\partial \bar{R}}\frac{\partial f}{\partial \bar{p}}$$

$$\frac{\partial f}{\partial t} = -\frac{\bar{p}}{m}\frac{\partial f}{\partial \bar{R}} + \frac{\partial U}{\partial \bar{R}}\frac{\partial f}{\partial \bar{p}}$$

$$\frac{\partial f}{\partial t} + \frac{\bar{p}}{m}\frac{\partial f}{\partial \bar{R}} - \frac{\partial U}{\partial \bar{R}}\frac{\partial f}{\partial \bar{p}} = 0$$

VLASOV EQ
1-PARTICLE LIOUVILLE EQ

f CAN BE USED TO CALCULATE 1-BODY OBSERVABLES

$$n(\bar{R},t) = \int d^3\bar{p}\; f(\bar{R},\bar{p}\,t)$$

$$\langle \bar{p} \rangle = \int d^3 r\, d^3 p\; \bar{p}\, f(\bar{R}\,\bar{p}\,t)$$

$$\varepsilon = \frac{p^2}{2m} + U(\bar{R}\, t) \qquad \frac{\bar{p}}{m} = \frac{\partial \varepsilon}{\partial \bar{p}} \qquad -\frac{\partial U}{\partial \bar{R}} = -\frac{\partial \varepsilon}{\partial \bar{R}}$$

$$\frac{\partial f}{\partial t} + \frac{\partial \varepsilon}{\partial \bar{p}}\frac{\partial f}{\partial \bar{R}} - \frac{\partial \varepsilon}{\partial \bar{R}}\frac{\partial f}{\partial \bar{p}} = 0$$

POISSON BRACKET FORM

ε – 1-PTCLE HAMILTONIAN $\qquad \varepsilon = \frac{\delta E}{\delta f}$

PHASE SPACE DENSITY CONSERVED ALONG CLASSICAL TRAJECTORIES

$$\frac{d\bar{R}}{dt} = \frac{\partial \varepsilon}{\partial \bar{p}} = \frac{\bar{p}}{m} \qquad \frac{d\bar{p}}{dt} = -\frac{\partial \varepsilon}{\partial \bar{R}} = -\frac{\partial U}{\partial \bar{R}}$$

$$f(\bar{R}(t), \bar{p}(t), t)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial \bar{R}}\frac{\partial \bar{R}}{\partial t} + \frac{\partial f}{\partial \bar{p}}\frac{\partial \bar{p}}{\partial t} + \frac{\partial f}{\partial t}$$

$$= \frac{\partial f}{\partial \bar{R}}\frac{\partial \varepsilon}{\partial \bar{p}} - \frac{\partial f}{\partial \bar{p}}\frac{\partial \varepsilon}{\partial \bar{R}} + \frac{\partial f}{\partial t} = 0$$

$$f = \begin{cases} 1 & p < p_F \\ 0 & p > p_F \end{cases} \qquad f = e^{\left(\mu - \frac{p^2}{2m}\right)/T}$$

VALUES STUCK FOREVER!

VLASOV EQ CANNOT DESCRIBE THERMALIZATION!

$$\rho(r_1\, r_1'\, t) = \int d^3r_2 \ldots d^3r_A\, \Psi_A(\bar{r}_1, r_2 \ldots \bar{r}_A\, t)\, \Psi_A^*(\bar{r}_1'\, r_2 \ldots \bar{r}_A\, t)$$

↑ HERMITIAN POSITIVE DEFINITE OPERATOR

$$\rho(\bar{r}_1, \bar{r}_1, t) = \sum_\alpha n_\alpha(t)\, \varphi_\alpha(\bar{r}_1\, t)\, \varphi_\alpha^*(\bar{r}_1'\, t)$$

TEST-PARTICLE METHOD

$$f(\bar r\, \bar p\, t) = \frac{1}{N} \sum_{i=1}^{NA} \delta(\bar r - \bar r_i(t))\, \delta(\bar p - \bar p_i(t))$$

$$\frac{\partial \bar r_i}{\partial t} = \left.\frac{\partial \mathcal{E}}{\partial \bar p}\right|_{r_i p_i} \qquad \frac{\partial \bar p_i}{\partial t} = -\left.\frac{\partial \mathcal{E}}{\partial \bar r}\right|_{r_i p_i}$$

$$\frac{\partial f}{\partial t} = \frac{1}{N} \Sigma \left\{ -\delta' \delta \frac{\partial \bar r_i}{\partial t} - \delta \delta' \frac{\partial \bar p_i}{\partial t} \right\}$$

$$= \frac{1}{N} \Sigma \left\{ -\delta' \delta \frac{\partial \mathcal{E}}{\partial \bar p_i} + \delta \delta' \frac{\partial \mathcal{E}}{\partial \bar r_i} \right\}$$

$$= -\frac{\partial \mathcal{E}}{\partial \bar p} \frac{\partial f}{\partial \bar r} + \frac{\partial \mathcal{E}}{\partial \bar r} \frac{\partial f}{\partial \bar p}$$ VLASOV EQ

$$\rho_2(\bar r_1, \bar r_2\, r_1', \bar r_2'\, t) = \int d\bar r_3\, d\bar r_A\, \psi(\bar r, \bar r_2, \bar r_3 \ldots t)\, \psi^*(\bar r_1', \bar r_2'\, \bar r_3 \ldots t)$$

$$\rho_2(\bar r_1, \bar r_2\, \bar r_1'\, \bar r_2'\, t) \simeq \rho_1(\bar r_1\, \bar r_1' t)\, \rho_1(\bar r_2\, \bar r_2' t) \Rightarrow \text{TDHF}$$

$\rho_2 = \rho_1 \rho_1 + \delta \rho_2 \leftarrow$ CORRELATIONS

$\rho_3 = \rho_1 \rho_1 \rho_1 + \Sigma\, \delta \rho_2\, \rho_1 \Rightarrow$ SOLUTION FOR ρ_2 ASSUMING THAT $\rho_2 \simeq \rho_1 \rho_1$ AS $t \to -\infty$

$\Rightarrow$ WIGNER REPRESENTATION

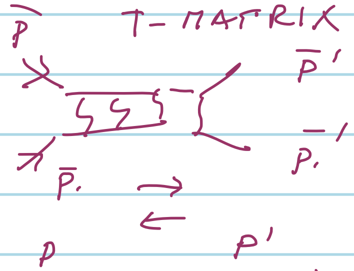
$$\frac{\partial f}{\partial t} + \frac{\partial \mathcal{E}}{\partial \bar p} \frac{\partial f}{\partial \bar r} - \frac{\partial \mathcal{E}}{\partial \bar r} \frac{\partial f}{\partial \bar p} = I$$

$\leftarrow$ COLLISION INTEGRAL

RATE OF CHANGES W/TIME

RATHER THAN BEING CONSTANT ALONG CLASSICAL TRAJECTORIES, f CHANGES DUE TO SHORT-RANGE CORRELATIONS COLLISIONS

UNDER APPROXIMATION OF SLOW CHANGES IN SPACE AND TIME, I DEPENDS ONLY ON LOCAL VALUES OF f AND ACQUIRES FORM OF FERMI GOLDEN RULE FOR RATES



PARTICLE AT $(\bar{r}, \bar{p})$

$$I(\bar{r}, p) = \int d\bar{p}_1\, d\bar{p}'\, d\bar{p}_1'\, \delta(\bar{p}+\bar{p}_1-\bar{p}'-\bar{p}_1')\, \delta(\varepsilon+\varepsilon_1-\varepsilon'-\varepsilon_1')$$

FEEDING

$$|T(p\,p_1\,p'\,p_1')|^2 \Big[f(\bar{p}'\,\bar{r}\,t)\, f(\bar{p}_1'\,\bar{r}\,t)\,(1-f)(1-f_1) - (1-f')(1-f_1')\, f f_1 \Big]$$

DEPLETION

$$\varepsilon+\varepsilon_1 = \frac{P^2}{2M} + \frac{q^2}{2\mu} \qquad \varepsilon'+\varepsilon_1' = \frac{P'^2}{2M} + \frac{q'^2}{2\mu}$$

$$\int d^3q'\, |T|^2\, \delta\!\left(\frac{q^2}{2\mu} - \frac{q'^2}{2\mu}\right)$$

$$\overset{\shortparallel}{\int} d\Omega\, q'^2\, dq'\, |T|^2\, \delta(\quad)$$

$$\int d\Omega\, \frac{\mu}{q}\, q^2\, |T|^2 = \int d\Omega\, \frac{d\sigma}{d\Omega}\, v \qquad v = \frac{q}{\mu}$$

$$\frac{d\sigma}{d\Omega} = \mu^2\, |T|^2$$

FEEDING DEPLETION

$$I = \int dp_1 \int d\Omega\, v\, \frac{d\sigma}{d\Omega} \Big\{ f' f_1' (1-f)(1-f_1) - (1-f')(1-f_1') f f_1 \Big\}$$

TEST-PARTICLE METHOD WORKS HERE TOO

PARTICLES MOVE ALONG CLASSICAL TRAJECTORIES AND COLLIDE WITH EACH OTHER

IN EQUILIBRIUM $\frac{df}{dt} = 0$ $\quad I = 0$

$$f' f_1' (1 \mp f)(1 \mp f_1) = (1 \mp f')(1 \mp f_1') f f_1$$

BOSON OPTION

IN EQUILIBRIUM $I = 0$

$$\frac{f'}{1 \mp f'} \frac{f_1'}{1 \mp f_1'} = \frac{f}{1 \mp f} \frac{f_1}{1 \mp f_1}$$

$$\log \frac{f'}{1 \mp f'} + \log \frac{f_1'}{1 \mp f_1'} = \log \frac{f}{1 \mp f} + \log \frac{f_1}{1 \mp f_1}$$

CONSERVATION
PARTICLE
ENERGY
MOMENTUM

$$\log \frac{f}{1 \mp f} = \beta\mu - \beta\varepsilon + \beta\bar{u}\,\bar{p}$$

↑ CONSTANT

$$\frac{f}{1 \mp f} = e^{\beta(\mu - \varepsilon + \bar{u}\bar{p})} \hat{=} e^{\beta \cdots}$$

$$f = e^{\beta \cdots}(1 \mp f) = e^{\beta \cdots} \mp e^{\beta \cdots} f$$

$$(1 \pm e^{\beta \cdots}) f = e^{\beta \cdots}$$

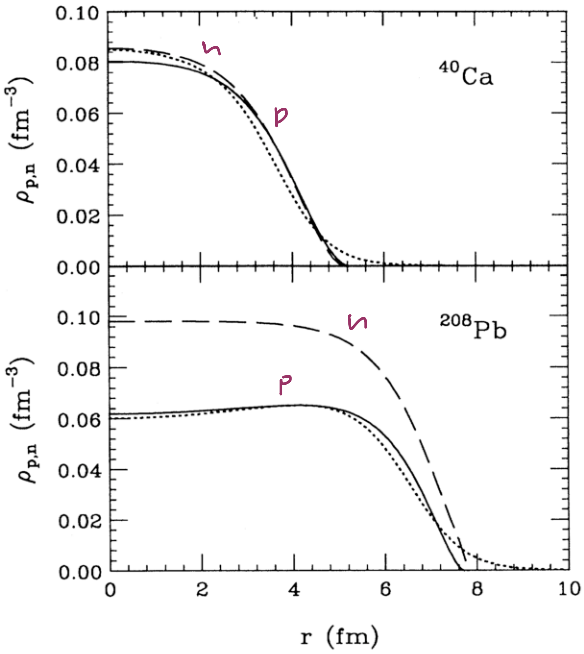
$$f = \frac{e^{\beta \cdots}}{1 \pm e^{\beta \cdots}} = \frac{1}{e^{-\beta \cdots} \pm 1} = \frac{1}{e^{-\beta\mu + \beta\varepsilon - \beta\bar{u}\bar{p}} \pm 1}$$

FERMI DIRAC FOR +
BOSE-EINSTEIN FOR −
DISTRIBUTION

$$\beta = \frac{1}{T}$$

$\beta\mu \to -\infty$ LIMIT OF BOLTZMANN STATISTICS

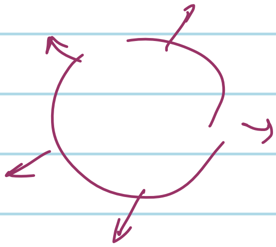
$T \to 0$ $\quad f \to 0, 1$ $\quad$ VLASOV THOMAS FERMI



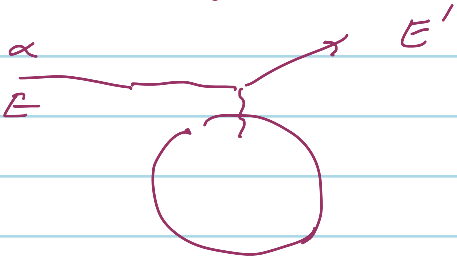
$\rho_n + \rho_p \simeq 0.16\, fm^{-3}$

.... ρ FROM ELECTRON SCATTERING

? COLLECTIVE OSCILLATIONS



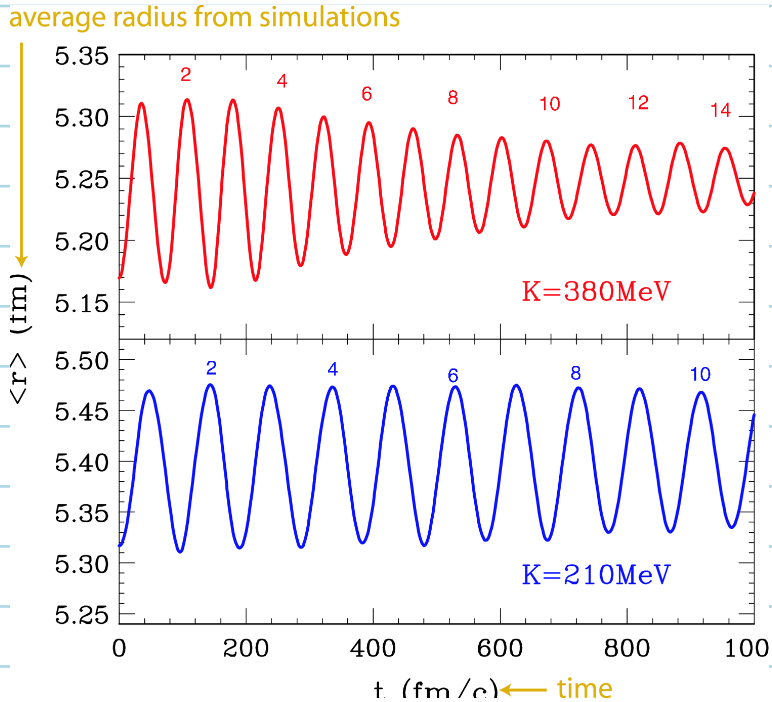
? DATA ON MONOPOLE RESONANCES?



$E' \simeq E - \hbar\Omega$

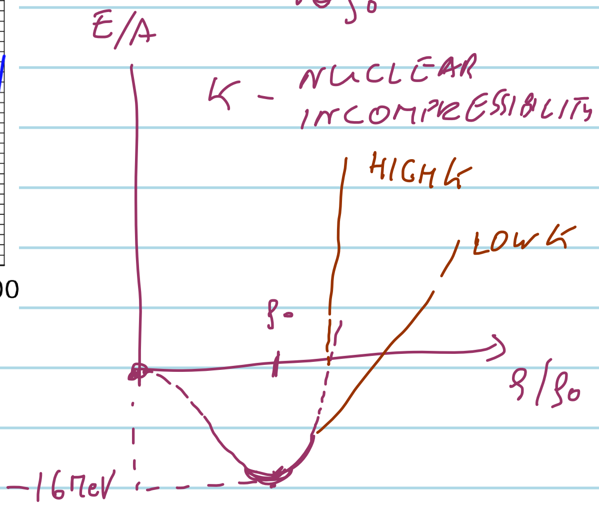
Ω - FREQUENCY OF OSCILLATIONS

$$\Omega \sim \frac{c_s}{R} \propto \frac{\sqrt{K}}{R}$$

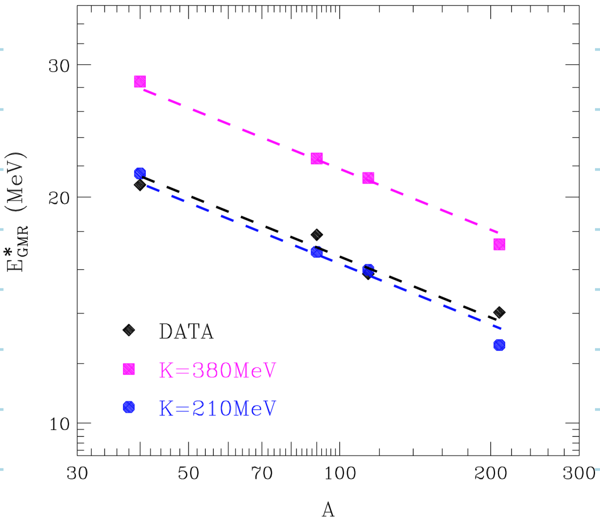


$$\frac{E}{A} = -16\,\text{MeV} + \frac{K}{18\rho_0^2}(\rho-\rho_0)^2$$

K – NUCLEAR INCOMPRESSIBILITY

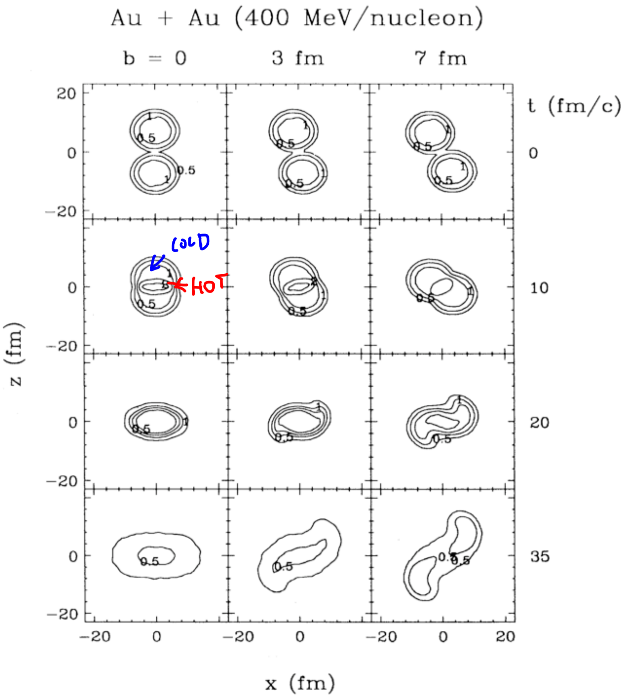


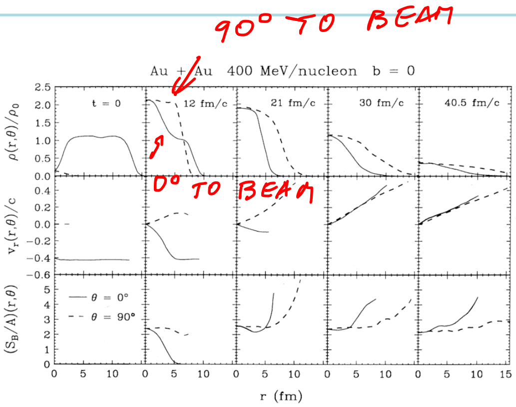
$\rho_0 = 0.16\ \text{fm}^{-3}$ ← DENSITY AT CENTERS OF NUCLEI



$K \sim 230$ MeV

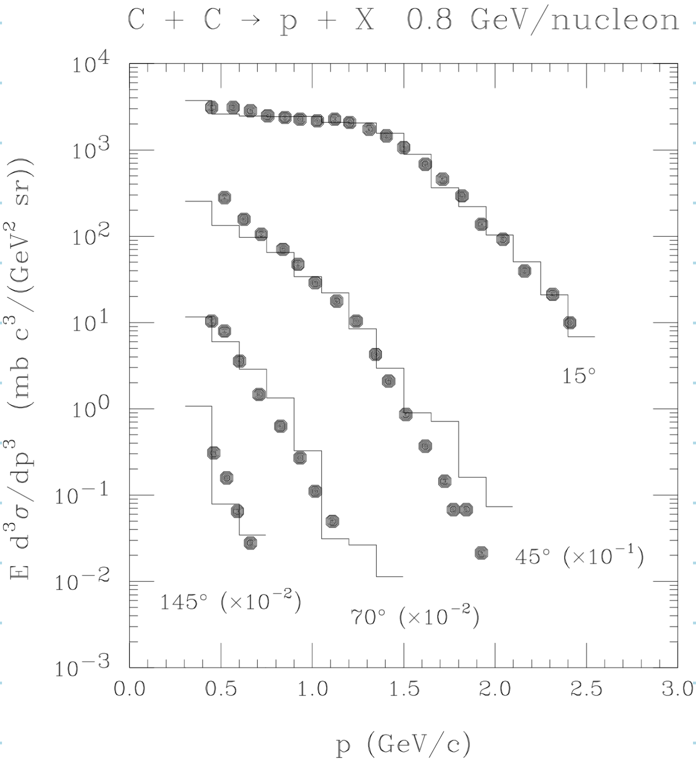
? HIGHER DENSITY THAN NORMAL ?





SPECTATORS MOVE
AT ORIGINAL
PROJ/TARGET VELOCITY

EXPANDING PARTICIPANT
REGION SHADOWED BY SPECTATORS



POINTS DATA

HISTOGRAMS CALCULATIONS

? RADIAL FLOW
EQ OF STATE
TEST ?

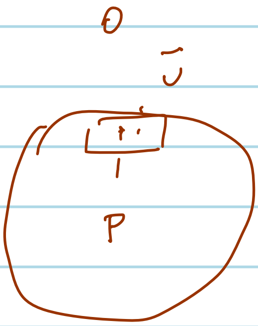
SAME RADIAL
FLOW NO MATTER
WHAT ASSUMPTIONS

EULER EQ IN HYDRO LIMIT

$$m_N n \frac{\partial \vec{v}}{\partial t} = -\nabla p$$ ← PRESSURE

MASS DENSITY

RATE OF CHANGE OF COLLECTIVE VELOCITY



FRAME $\vec{v} = 0$

$v \propto p t$

HIGH p ⇔ SHORT t

PRESSURE HIGH ⇒ DYNAMICS EXPLOSIVE
t IS SHORT

$v \propto p_t$ - THE SAME IN BOTH CASES   LOW p - DYNAMICS SLUGGISH T LONG

YOU MEASURE v TO FIND p YOU NEED TO FIX t

HIGH P → FAST EXPANSION → STRONG SPECTATOR SHADOWS

LOW   SLO EXPANSION → WEAK SHADOWS

REACTION-PLANE OBSERVABLES F_y v_2