Axion Electrodynamics

→ Most exploitable coupling

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 - J^\mu A_\mu - \frac{g_{a\gamma}}{4} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$

see e.g. 1612.07057

$$-\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

$$\vec{E} = -\nabla A_0 - \dot{\vec{A}} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$g_{a\gamma} = -\frac{\alpha}{2\pi f_a}C_{a\gamma}\,, \quad C_{a\gamma} = \frac{E}{N} - 1.92$$

Many ways to solve
- Classical
- QM mixing
- QFT
- Thermal Feild Theory.
- . . .

tools not topic

Euler-Lagrange

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)}\right) - \frac{\partial \mathcal{L}}{\partial A_\nu} = 0$$

$$\Downarrow$$

$$\partial_\mu F^{\mu\nu} = J^\nu \underbrace{- g_{a\gamma} \tilde{F}^{\mu\nu} \partial_\mu a}_{\text{axion "current"}}$$

Inhomogeneus Maxwell

$$\vec{\nabla}\cdot\vec{E} = \rho - g_{a\gamma}\vec{B}\cdot\vec{\nabla}a$$

$$\vec{\nabla}\times\vec{B} - \dot{\vec{E}} = \vec{J} + g_{a\gamma}(\vec{B}\dot{a} - \vec{E}\times\vec{\nabla}a)$$

Homogeneus: Bianchi identity

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$\Downarrow$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} + \dot{\vec{B}} = 0$$

+ axion E-L

$$\ddot{a} - \nabla^2 a + m_a^2 a = g_{a\gamma}\vec{E}\cdot\vec{B}$$

What about a medium?

split "free" and "bound" charges

$$\rho = \rho_f + \rho_b \,, \qquad \vec{J} = \vec{J}_f + \vec{J}_b$$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} \qquad \vec{J}_f = \vec{\nabla}\times\vec{M} + \vec{P}$$

gives

$$\vec{D} \equiv \vec{E} + \vec{P} \qquad \vec{H} = \vec{B} - \vec{M}$$

$$\Downarrow$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f - g_{a\gamma}\vec{B}\cdot\vec{\nabla}a$$
$$\vec{\nabla}\times\vec{H} - \dot{\vec{D}} = \vec{J}_f + g_{a\gamma}\left(\vec{B}\dot{a} - \vec{E}\times\vec{\nabla}a\right)$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} + \dot{\vec{B}} = 0$$
$$\ddot{a} - \nabla^2 a + m_a^2 a = g_{a\gamma}\vec{E}\cdot\vec{B}$$

Two main limits

non-rel (haloscopes)

rel (astro, LSW, helio...)

non-rel

$\nabla a \simeq 0$

Usually (not always) apply some external B-feild B_e

neglect back reaction on the axion

$$\vec{\nabla} \times \vec{H} - \dot{\vec{D}} = g_{a\gamma} \vec{B_e} \dot{a}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{E_a} = \frac{g_{a\gamma} \vec{B_e}}{\varepsilon} a$$

1912.11467

$$\int P d\nu \leq \pi \frac{g_{a\gamma}^2 \rho_a V B^2}{}$$

$\frac{S}{N} \propto P \propto Q^2$

scan $\propto \dot{Q}$

sees what photon sees

in some infinite medium

Never neglect boundary conditions!

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example: Cavitity (1803.01243)

E-field is a sum over modes

$$E(\vec{x}) = \sum_i E_i e_i(\vec{x}) \quad \text{with}$$

$$\int dV e_i e_j = V \delta_{ij}$$

Helmholtz $\nabla^2 e_i = \omega_i^2 e_i$ (no axion, losses)

$$\nabla^2 \vec{E} + \omega^2 \vec{E} + m_a^2 g_{a\gamma} \vec{B_e} a + i\Gamma E = 0$$

project over modes

$$\omega_i^2 E_i + \omega^2 E_i + i\Gamma E_i = -m_a^2 g_{a\gamma} B_e G$$

$$G = \frac{1}{B_e V} \int dV \vec{B_e} \cdot \vec{e_i}$$

$$|E_i(m_a=\omega_i)| = \frac{m_a g_{a\gamma} B_e a}{\Gamma} G$$

$$P = \Gamma U$$

$$\int E^2 dV$$

$$P = \frac{Q}{m_a} g_{a\gamma}^2 B_e^2 G^2 V \rho_a \,, \quad Q = \frac{\omega}{\Gamma}$$

<u>non-rel</u>

$\nabla a \neq 0$

Typically make a wave equation

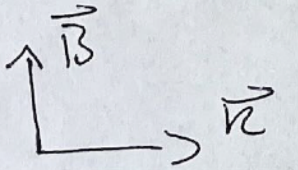
$\mu = 1, \ \vec{B} = \vec{B}_e$

$$\omega^2 a + \nabla^2 a = m_a^2 a - g_{a\gamma} \vec{E}\cdot\vec{B}_e$$

$$-\nabla^2 \vec{E} + \nabla(\nabla\cdot\vec{E}) = \omega^2 \vec{D} + \omega^2 g_{a\gamma} a \vec{B}_e$$

This nicely leads to a mixing picture

<u>Simple case</u>



transverse photon mixing

$$\begin{pmatrix} \omega^2 - k^2 - m_a^2 & \omega g_{a\gamma} B_e \\ \omega g_{a\gamma} B_e & \varepsilon\omega^2 - k^2 \end{pmatrix} \begin{pmatrix} a \\ iA_t \end{pmatrix} = 0$$

propogating states (eigenmodes)

axion-like:

$$k^2 = \omega^2 - m_a^2 + \mathcal{O}(g_{a\gamma}^2)$$

$$\begin{pmatrix} a \\ iA_t \end{pmatrix} = \begin{pmatrix} 1 \\ \dfrac{-g_{a\gamma} B_e \omega}{\omega^2(n^2-1) - m_a^2} \end{pmatrix} + \mathcal{O}(g_{a\gamma}^2)$$

photon-like:

$$k^2 = \varepsilon\omega^2 + \mathcal{O}(g_{a\gamma}^2)$$

$$\begin{pmatrix} a \\ iA_t \end{pmatrix} = \begin{pmatrix} \dfrac{g_{a\gamma} B_e \omega}{\omega^2(n^2-1) - m_a^2} \\ 1 \end{pmatrix} + \mathcal{O}(g_{a\gamma}^2)$$

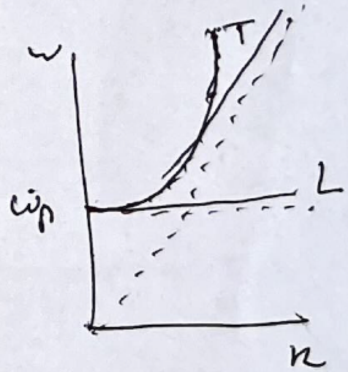
Note: resonance when

$$(n^2-1)\omega^2 = m_a^2$$

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} \quad \text{(plasma)}$$

$$\Rightarrow m_a = \omega_p$$

(L modes when $\omega_a = \omega_p$)



conversion requires

i) dispersion relation matching or

or

ii) breaking translation

example resonant conversion in astro-plasma 7

(Raffelt PRD 1988)

2107.07399

Slowly varying B or ω_p

$$\frac{\partial_z^2 E_t}{\partial x^2} - \omega^2 g_{a\gamma} B_e + (\omega^2 - \omega_p^2) E_t = 0$$

$$E_t(x) \equiv \hat{E}_t(x) e^{i(\omega t - kx)}$$

$$a \equiv \tilde{a}(x) e^{i(\omega t - kx)}$$

WKB approximation

$$k \tilde{E}_t \gg \frac{\partial E_y}{\partial x}$$

linearizes to

$$2ik \frac{\partial E_t}{\partial x} \simeq (m_a^2 - \omega_p^2) E_t - \omega^2 g_{a\gamma} B_e \tilde{a}$$

Can integrate with stationary phase approximation

$$\tilde{E}_t(x_0 + L) = \omega^2 \sqrt{\frac{\pi}{2k\omega_p |\omega_p'|}} g_{a\gamma} \tilde{a} \vec{B}_e$$

$$P_{a\to\gamma} = \frac{\text{Energy in } \gamma}{\text{Energy in } a}$$

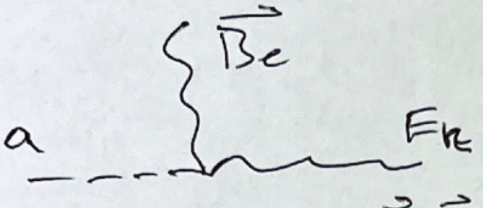
$$= \underbrace{\frac{\pi \omega_p}{2k|\omega_p'|}}_{\sim L^2 \left(\frac{1}{2}\frac{\omega^2}{k} L\right)^2} g_{a\gamma}^2 B_e^2$$

What about QFT?

e.g. 1707.00701
1812.05487

$$\Gamma_{a\to\gamma} = 2\pi \sum_k |M|^2 \delta(\omega_a - \omega_k)$$

Need to be careful with normalizations and states



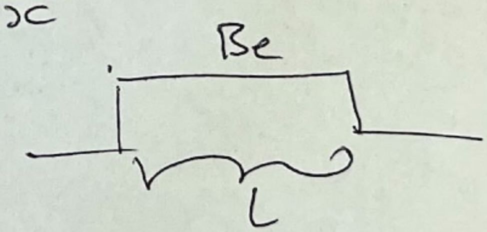
$$M = \frac{g_{a\gamma}}{2\omega V} \underbrace{\int d^3x\, e^{i\vec{p}\cdot\vec{x}}\, \vec{B}_e \cdot \vec{E}_k^*}_{\text{overlap integral}}$$

p_{axion}

<u>example</u> Helioscope

$$E_k = i\omega_k e^{i\vec{k}\cdot\vec{x}}$$, aligned with B

$$M = i\frac{g_{a\gamma} B_e}{2V}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}$$



$$q = k - p$$

$$= \frac{g_{a\gamma} B_e}{2L}\frac{2\sin(qL/2)}{q}$$

$$\sum_k \delta(\omega_a - \omega_k) \to \frac{L}{2\pi}\int dk\, \delta(\omega_a - \omega_k)$$

$$\Gamma_{a\to\gamma} = \frac{1}{L} P_{a\to\gamma}$$

9)a

$$P_{a\to\gamma} = \sum_{q=\pm q} \left(g_{a\gamma} B_e \frac{\sin(q L/2)}{q} \right)^2$$

↑ forward + backscatter

if $q_+ L << 1$

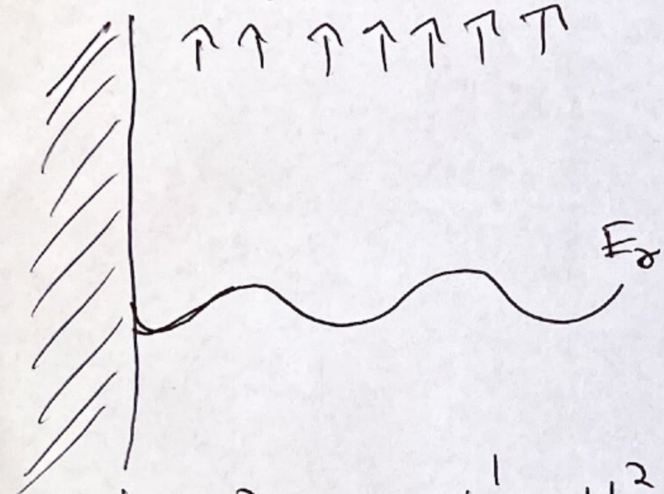
$$P = \frac{g_{a\gamma}^2 B_e^2 L^2}{4}$$

9b)

Dish antenna 3 ways

1st) background axion feild

$E_a^1 = 0$ $\quad E_a^2 = g_{a\gamma} \vec{B}_e a$



$E_{\parallel}^1 = E_{\parallel}^2$, $H_{\parallel}^1 = H_{\parallel}^2$

at $x = 0$

$E_\gamma = -E_a$

So $E_\gamma = -E_a e^{i(kx - \omega t)}$

Classical vs Quantum

For axion-photon conversion almost everything is classical

Two requirements for non-classical behavior: non-classical state and non-classical question

$$H \propto a_{axion}\, a_\gamma^\dagger + a_{axion}^\dagger\, a^\gamma$$

total rate $a \to \gamma$

$$\Gamma_{a\to\gamma} - \Gamma_{\gamma\to a} \propto \left|\langle N_a-1, N_\gamma+1 \,|\, a_a a_\gamma^\dagger \,|\, N_a, N_\gamma \rangle\right|^2 - \left|\langle N_a+1, N_\gamma-1 \,|\, a_a^\dagger a_\gamma \,|\, N_a, N_\gamma \rangle\right|$$

$$\propto N_a(N_\gamma+1) - (N_a+1)N_\gamma = N_a - N_\gamma$$

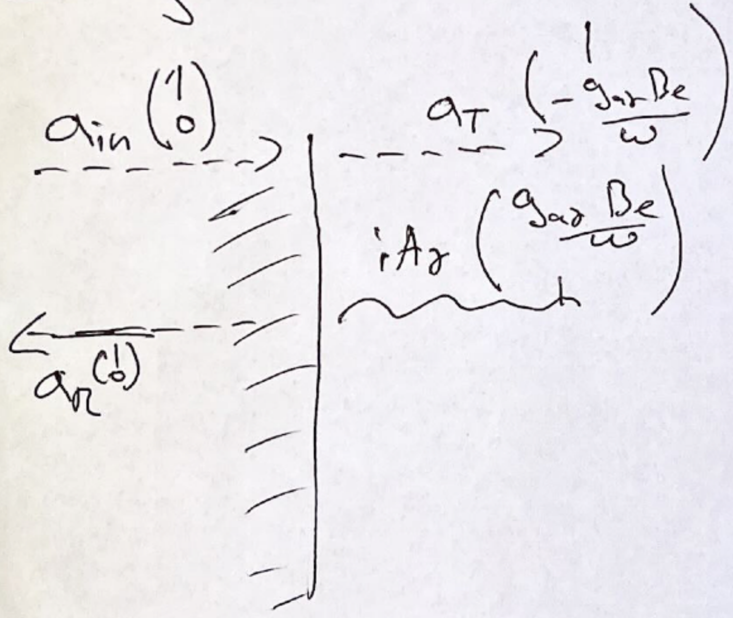
Poynting Outside Be

$$\cancel{S = \tfrac{1}{2}(E_\gamma \times H_\gamma)}$$

$$\bar{S} = \frac{1}{2} Re(E^a) \times Re(H^a)$$
$$= \frac{1}{2} E_a^2$$

$$\Rightarrow \frac{P_\gamma}{A} = \rho_a \left(\frac{g_{a\gamma} Be}{m_a}\right)^2$$

2) mixing



lowest order in $g_{a\gamma}$

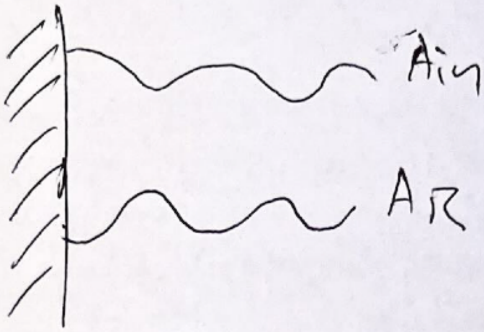
$|a_{in}| = a$ $\quad |a_T| \simeq a$, $|a_R| \simeq 0$

$$|A_\gamma| = \frac{-1}{i} \frac{g_{a\gamma} Be}{\omega} a$$

$$\Rightarrow \frac{P_\gamma}{A} = \rho_a \left(\frac{g_{a\gamma} Be}{m_a}\right)^2$$

3) QFT

what is a photon wavefunction?



$$A_R = -A_{in}$$

$$\Rightarrow E_R = i\omega\, 2\sin(kx)$$

$$\mathcal{M} = \frac{g_{a\gamma}}{2\omega L}\int_{-\infty}^{\infty} dx\, \vec{B}_e \cdot \vec{E}_R^{\,*}$$

$$\int_0^{\infty} 2\sin(kx) = \frac{2}{k} = \frac{2}{\omega}$$

$$\frac{\Gamma}{A} = \frac{g_{a\gamma}^2 B_e^2}{V m_a^2}$$

N axions in V

$$\Phi = \frac{\rho_a}{m_a}\left(\frac{g_{a\gamma} B_e}{m_a}\right)^2$$

$$\frac{P}{A} = \rho_a \frac{g_{a\gamma}^2 B_e^2}{m_a^2}$$