EW SMEFT – Useful formulas

Contents

1 SMEFT Lagrangian in Warsaw basis after all redefinitions

The following results are given in the Warsaw basis, assuming a flavor symmetry $U(3)^5$ and ignoring all Yukawas and fermion mixings. They are the final results after applying all field and parameter redefinitions. The "small deltas" $\delta e/e, \delta g/g \dots$ represent the shifts coming from input parameter scheme definitions. Their expressions in termis of Wilson coefficients is input scheme dependent. Explicit expressions are given below for the $\{\alpha, m_Z, G_F\}$ and $\{m_W, m_Z, G_F\}$ sets.

1.1 Fermion - gauge interactions

$$\mathcal{L}_{Vff} = -g_V V_\mu \bar{\psi}\gamma^\mu (g^L_{\psi V} P_L + g^R_{\psi V} P_R)\psi \tag{1}$$

where we defined

$$g_Z = \frac{g}{c_\theta} \qquad g_\gamma = e \qquad g_W = \frac{g}{\sqrt{2}} \tag{2}$$

For the Z couplings:

$$g^L_{eZ} = -\frac{1}{2} + s_\theta^2\left[1 + \frac{\delta s_\theta^2}{s_\theta^2}\right] - \frac{\bar{C}^{(3)}_{Hl} + C^{(1)}_{Hl}}{2} \qquad g^R_{eZ} = s_\theta^2\left[1 + \frac{\delta s_\theta^2}{s_\theta^2}\right] - \frac{\bar{C}_{He}}{2} \tag{3}$$

$$g^L_{\nu Z} = \frac{1}{2} + \frac{\bar{C}^{(3)}_{Hl} - C^{(1)}_{Hl}}{2} \qquad g^R_{\nu Z} = 0 \tag{4}$$

$$g^L_{uZ} = \frac{1}{2} - \frac{2s_\theta^2}{3}\left[1 + \frac{\delta s_\theta^2}{s_\theta^2}\right] + \frac{\bar{C}^{(3)}_{Hq} - C^{(1)}_{Hq}}{2} \qquad g^R_{uZ} = -\frac{2}{3}s_\theta^2\left[1 + \frac{\delta s_\theta^2}{s_\theta^2}\right] - \frac{\bar{C}_{Hu}}{2} \tag{5}$$

$$g^L_{dZ} = -\frac{1}{2} + \frac{s_\theta^2}{3}\left[1 + \frac{\delta s_\theta^2}{s_\theta^2}\right] - \frac{\bar{C}^{(3)}_{Hq} + C^{(1)}_{Hq}}{2} \qquad g^R_{dZ} = \frac{1}{3}s_\theta^2\left[1 + \frac{\delta s_\theta^2}{s_\theta^2}\right] - \frac{\bar{C}_{Hd}}{2} \tag{6}$$

while for the γ and W:

$$g^L_{e\gamma} = g^R_{e\gamma} = -\left[1 + \frac{\delta e}{e}\right] \qquad g^L_{e\nu W} = 1 + \frac{\delta g}{g} + \bar{C}^{(3)}_{Hl} \tag{7}$$

$$g^L_{\nu\gamma} = g^R_{\nu\gamma} = 0 \qquad g^R_{e\nu W} = 0 \tag{8}$$

$$g^L_{u\gamma} = g^R_{u\gamma} = \frac{2}{3}\left[1 + \frac{\delta e}{e}\right] \qquad g^L_{udW} = 1 + \frac{\delta g}{g} + \bar{C}^{(3)}_{Hq} \tag{9}$$

$$g^L_{d\gamma} = g^R_{d\gamma} = -\frac{1}{3}\left[1 + \frac{\delta e}{e}\right] \qquad g^R_{udW} = 0 \tag{10}$$

1.2 TGC

$$\mathcal{L}_{TGC} = -ig_{VWW}\left[g_1^V\left(W^+_{\mu\nu}W^{-\mu}V^\nu - W^-_{\mu\nu}W^{+\mu}V^\nu\right) + \kappa_V V^{\mu\nu}W^+_\mu W^-_\nu + \frac{\lambda_V}{m_W^2}V^\mu_\nu W^{+\nu}_\rho W^{-\rho}_\mu\right] \tag{11}$$

Where

$$g_{ZWW} = g c_\theta \qquad g_{\gamma WW} = e \tag{12}$$

$$g_1^Z = 1 - s_\theta^2 \frac{\delta g'}{g'} + (1+s_\theta^2)\frac{\delta g}{g} + s_\theta^2 t_\theta \bar{C}_{HWB} \qquad g_1^\gamma = 1 + s_\theta^2 \frac{\delta g}{g} + c_\theta^2 \frac{\delta g'}{g'} - \frac{s_{2\theta}}{2}\bar{C}_{HWB} \tag{13}$$

$$\kappa_Z = 1 - s_\theta^2 \frac{\delta g'}{g'} + (1+s_\theta^2)\frac{\delta g}{g} - \frac{s_{2\theta}}{2}\bar{C}_{HWB} \qquad \kappa_\gamma = 1 + s_\theta^2 \frac{\delta g}{g} + c_\theta^2 \frac{\delta g'}{g'} + \frac{c_\theta^2}{t_\theta}\bar{C}_{HWB} \tag{14}$$

$$\lambda_Z = \frac{3g}{2}\bar{C}_W = \frac{6m_W^2}{g}\frac{C_W}{\Lambda^2} \qquad \lambda_\gamma = \lambda_Z \tag{15}$$

And gauge invariance requires

$$(\kappa_Z - g_1^Z) = -t_\theta^2(\kappa_\gamma - g_1^\gamma) \tag{16}$$

1.3 QGC

$$\mathcal{L}_{QGC} = \frac{g^2}{2}\Big[g_{WW}\left(W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu} - (W_\mu^+ W^{-\mu})^2\right) + g_{VV'}\left(W^{+\mu}W^{-\nu}(V_\mu V'_\nu + V_\nu V'_\mu) - 2W_\nu^+ W^+ - \nu V_\mu V\prime\mu\right)\Big] \tag{17}$$

$$g_{WW} = 1 + 2\frac{\delta g}{g} \tag{18}$$

$$g_{ZZ} = c_\theta^2\left[1 - 2s_\theta^2\frac{\delta g'}{g'} + 2(1+s_\theta^2)\frac{\delta g}{g} + 2s_\theta^2 t_\theta \bar{C}_{HWB}\right] \tag{19}$$

$$g_{Z\gamma} = s_{2\theta}\left[1 + c_{2\theta}\frac{\delta g'}{g'} + (1+2s_\theta^2)\frac{\delta g}{g} - c_{2\theta}t_\theta\bar{C}_{HWB}\right] \tag{20}$$

$$g_{\gamma\gamma} = s_\theta^2\left[1 + 2c_\theta^2\frac{\delta g'}{g'} + 2s_\theta^2\frac{\delta g}{g} - s_{2\theta}\bar{C}_{HWB}\right] \tag{21}$$

They are related to TGC corrections via

$$\frac{\delta g_{ZZ}}{g_{ZZ}} = 2(g_1^Z - 1) \tag{22}$$

$$\frac{\delta g_{Z\gamma}}{g_{Z\gamma}} = (g_1^Z - 1) + (g_1^\gamma - 1) \tag{23}$$

$$\frac{\delta g_{\gamma\gamma}}{g_{\gamma\gamma}} = 2(g_1^\gamma - 1) \tag{24}$$

1.4 HVV

$$\mathcal{L}_{HVV} = \frac{m_Z^2}{v}\,hZ_\mu Z^\mu\,[1+\Delta_{HZZ}] + \frac{2m_W^2}{v}\,hW_\mu^+ W^{-\mu}\,[1+\Delta_{HWW}] + \frac{\Delta_{HZZ}^{(p)}}{v}hZ_{\mu\nu}Z^{\mu\nu} + 2\frac{\Delta_{HWW}^{(p)}}{v}hW_{\mu\nu}^+ W^{-\mu\nu} + \Delta_{HAA}^{(p)}\frac{h}{v}A_{\mu\nu}A^{\mu\nu} + \Delta_{HAZ}^{(p)}\frac{h}{v}A_{\mu\nu}Z^{\mu\nu} + \Delta_{HGG}^{(p)}\frac{h}{v}G_{\mu\nu}^a G^{a\mu\nu} \tag{25}$$

with

$$
\begin{aligned}
\Delta_{HZZ} &= \Delta\kappa_H + \frac{\delta v_T}{v_T} + 2s_\theta^2\frac{\delta g'}{g'} + 2c_\theta^2\frac{\delta g}{g} + s_{2\theta}\bar{C}_{HWB} + \bar{C}_{HD} && (26)\\
\Delta_{HWW} &= \Delta\kappa_H + \frac{\delta v_T}{v_T} + 2\frac{\delta g}{g} && (27)\\
\Delta^{(p)}_{HWW} &= \bar{C}_{HW} && (28)\\
\Delta^{(p)}_{HZZ} &= c_\theta^2\bar{C}_{HW} + s_\theta^2\bar{C}_{HB} + s_\theta c_\theta\bar{C}_{HWB} && (29)\\
\Delta^{(p)}_{HAA} &= s_\theta^2\bar{C}_{HW} + c_\theta^2\bar{C}_{HB} - s_\theta c_\theta\bar{C}_{HWB} && (30)\\
\Delta^{(p)}_{HAZ} &= s_{2\theta}\left(\bar{C}_{HW} - \bar{C}_{HB}\right) - c_{2\theta}\bar{C}_{HWB} && (31)\\
\Delta^{(p)}_{HGG} &= \bar{C}_{HG} && (32)
\end{aligned}
$$

1.5 Higgs potential

$$
\begin{aligned}
V(H) = {} & h^2\,\lambda v_T^2\left[1 + 2\Delta\kappa_H - \frac{3}{2\lambda}\,\bar{C}_H + \frac{\delta\lambda}{\lambda} + 2\frac{\delta v_T}{v_T}\right] + h^3\,\lambda v_T\left[1 + 3\Delta\kappa_H - \frac{5}{2\lambda}\,\bar{C}_H + \frac{\delta\lambda}{\lambda} + \frac{\delta v_T}{v_T}\right]\\
& + h^4\,\frac{\lambda}{4}\left[1 + 4\Delta\kappa_H - \frac{15}{2\lambda}\,\bar{C}_H + \frac{\delta\lambda}{\lambda}\right] - \frac{3}{4}\frac{h^5}{v_T}\,\bar{C}_H - \frac{1}{8}\frac{h^6}{v_T^2}\,\bar{C}_H\,. && (33)
\end{aligned}
$$

2 Useful definitions

$$
\begin{aligned}
\bar{C}_i &= \frac{v^2}{\Lambda^2}C_i && (34)\\
\theta &= \arctan\left[\frac{g'}{g} + \frac{1}{2}\frac{gg'}{g^2 + (g')^2}\bar{C}_{HWB}\right] && (35)
\end{aligned}
$$

Generic covariant derivative (NC only):

$$
\begin{aligned}
D_\mu = {} & \partial_\mu + iQ\frac{g_1 g_W}{\sqrt{g_1^2 + g_W^2}}A_\mu\left[1 - \bar{C}_{HWB}\frac{g_1 g_W}{g_W^2 + g_1^2}\right] && (36)\\
& + i\sqrt{g_1^2 + g_W^2}\,Z_\mu\,\left[T_3 - \frac{g_1^2}{g_1^2 + g_W^2}\,Q + \bar{C}_{HWB}\frac{g_1 g_W}{g_1^2 + g_W^2}\left(T_3 - \frac{g_W^2}{g_1^2 + g_W^2}\,Q\right)\right]\\
& + \dots\\
= {} & \partial_\mu + iQ\,g_W\,s_\theta\,A_\mu\left[1 - \frac{1}{2}\frac{c_\theta}{s_\theta}\bar{C}_{HWB}\right] + i\frac{g_W}{c_\theta}\,Z_\mu\,\left(T_3 - Qs_\theta^2\right)\left[1 + \frac{1}{2}\frac{s_\theta}{c_\theta}\bar{C}_{HWB}\right] && (37)\\
& + \dots
\end{aligned}
$$

Parameter shifts:

$$
\begin{aligned}
\frac{\delta s_\theta^2}{s_\theta^2} &= 2c_\theta^2\left(\frac{\delta g'}{g'} - \frac{\delta g}{g}\right) + \frac{s_{4\theta}}{4s_\theta^2}\bar{C}_{HWB} && (38)\\
\frac{\delta e}{e} &= c_\theta^2\frac{\delta g'}{g'} + s_\theta^2\frac{\delta g}{g} + \frac{\Delta\alpha}{2} && (39)
\end{aligned}
$$

2.1 Input shifts

In the $\{\alpha, m_Z, G_F, m_h\}$ scheme:

$$\frac{\delta g}{g} = \frac{1}{2c_{2\theta}}\left[-c_\theta^2\left(\Delta m_Z^2 + \Delta G_F\right) + s_\theta^2 \Delta\alpha\right] \qquad \frac{\delta v_T}{v_T} = \frac{\Delta G_F}{2} \tag{40}$$

$$\frac{\delta g'}{g'} = \frac{1}{2c_{2\theta}}\left[s_\theta^2\left(\Delta m_Z^2 + \Delta G_F\right) - c_\theta^2 \Delta\alpha\right] \qquad \frac{\delta\lambda}{\lambda} = -\Delta G_F - \Delta m_h^2 \tag{41}$$

In the $\{m_W, m_Z, G_F, m_h\}$ scheme:

$$\frac{\delta g}{g} = -\frac{1}{2}\left[\Delta G_F + \frac{\Delta m_Z^2}{s_\theta^2}\right] \qquad \frac{\delta v_T}{v_T} = \frac{\Delta G_F}{2} \tag{42}$$

$$\frac{\delta g'}{g'} = -\frac{\Delta G_F}{2} \qquad \frac{\delta\lambda}{\lambda} = -\Delta G_F - \Delta m_h^2 \tag{43}$$

with

$$\Delta\kappa_H = \bar{C}_{H\Box} - \frac{\bar{C}_{HD}}{4} \qquad \Delta m_h^2 = 2\Delta\kappa_H - \frac{3}{2\lambda}\bar{C}_H \tag{44}$$

$$\Delta m_Z^2 = \frac{2gg'}{g^2 + (g')^2}\bar{C}_{HWB} + \frac{\bar{C}_{HD}}{2} \qquad \Delta\alpha = -\frac{2gg'}{g^2 + (g')^2}\bar{C}_{HWB} \tag{45}$$

$$\Delta m_W^2 = 0 \qquad \Delta G_F = 2\bar{C}_{Hl}^{(3)} - \bar{C}'_{ll} \tag{46}$$