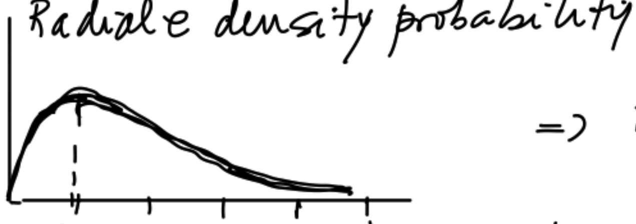
Ionization of a bound e

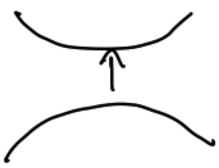


$R_{Bohr} = (\alpha m_e)^{-1} \simeq (\text{keV})^{-1}$

$\Rightarrow k \sim \alpha m_e \simeq 3.7\,\text{keV}$

$v_i = \alpha \simeq 10^{-2} > v_\chi$

Excitation in semiconductors



← conduction band

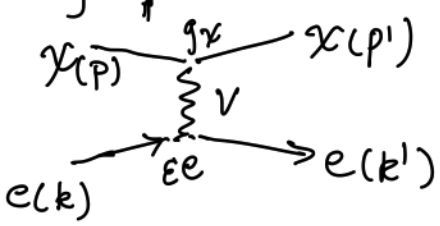
← valence band

band gap $\simeq$ eV

Example of calculation of scattering rate:

ionization of ~~H~~ χ (Tongyan Lin, TASI, 1904.07915)

E.g. for a DP V



$q^\mu = (p'^\mu - p^\mu)$

but for NR particles $E \ll |\vec{p}|$

so $p_\mu p^\mu = -|\vec{p}|^2$, also $q^\mu q_\mu = -|\vec{q}|^2$

$\Rightarrow$ spin averaged scattering amplitude square for a free e is

$$|\mathcal{M}_{free}|^2 = \frac{16\, g_\chi^2 (\varepsilon e)^2 m_\chi^2 m_e^2}{(|\vec{q}|^2 + m_V^2)^2}$$

For a bound electron

$$\langle \sigma v \rangle_j = \int \frac{d^3p'}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3}\, \overbrace{2\pi\, \delta(\Delta E_e - \omega)}^{\text{only E is conserved - not momentum}}\, \frac{|\mathcal{M}_{free}|^2}{16\, m_\chi^2 m_e^2} \left| f_{j \to \vec{k}'}(\vec{q}) \right|^2$$

labels initial state

free outgoing labelled by $\vec{k}'$

$f_{j\to\vec{k}'}$ accounts for overlap of i,f wavefunctions

$$f_{j\to\vec{k}'}(\vec{q}) = \sqrt{V} \int d^3\vec{r}\, \psi_j(\vec{r})\, \psi^*_{\vec{k}'}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}$$

wavefunction normalization

plane wave for free DM particle

If free e: $\psi_j = \psi_{\vec{k}} = e^{i\vec{k}\cdot\vec{r}} / \sqrt{V}$

If free e : $|f_{j\to \vec{k}'}(\vec{q})|^2 = \sqrt{V}\int d^3\vec{r}\, \frac{e^{i\vec{k}\cdot\vec{r} - i\vec{k}'\vec{r}}}{V}\, e^{i\vec{q}\vec{r}}$

Using $\frac{(2\pi)^3\,\delta^3(0)}{V} \to 1$ $\quad = (2\pi)^3\,\delta^3(\vec{k} - \vec{k}' + \vec{q})$

Taking $q_0 \simeq (\alpha m_e)^2$ as a refference value we define

$$\mu_{\chi e}^2 \frac{|\mathcal{M}_{free}|^2}{16\, m_\chi^2 m_e^2} = \underbrace{\mu_{\chi e}^2 \frac{g_\chi^2 (\varepsilon e)^2}{[(\alpha m_e)^2 + m_V^2]^2}}_{\equiv\ \bar{\sigma}_e} \underbrace{\frac{[(\alpha m_e)^2 + m_V^2]^2}{[|\vec{q}|^2 + m_V^2]^2}}_{\times\ F_{DM}^2(q)}$$

After some manipulations

$$\sigma \simeq \bar{\sigma}_e \int^{E_R^{max}} \frac{dE_R}{E_R^{max}} F_{DM}^2(q)$$

which $\Rightarrow$ $\sigma \simeq \bar{\sigma}_e$ when $F_{DM}^2(q) = 1$

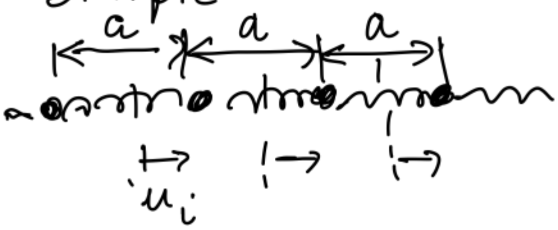
With a complete calculation $|f^{ion}(k', q)|^2$ peaks at $k' \simeq q \simeq \alpha m_e \simeq \frac{1}{R_{Bohr}}$

$q_0 = \alpha m_e$

$$|F_{DM}| = \frac{(\alpha m_e)^2 + m_V^2}{|\vec{q}|^2 + m_V^2} = \begin{cases} m_V \gg q_0, q, & F_{DM} = 1 \\ m_V \ll q_0, q, & F_{DM} = \frac{q_0^2}{|\vec{q}|^2} \end{cases}$$

Phonons : (Tongyan Lin TASI 1904.07915)

Simple 1-D model : lightly coupled springs



displacements from equilibrium u_i, spring constant K

$$H = \sum_i \frac{M}{2}\dot{u}_i^2 + \frac{1}{2}K(u_{i+1}-u_i)^2 + \underbrace{\cdots}_{\text{neglect higher order}}$$

Divide and multiply by the separation "a", then take $a \to 0$ to get in the continuum limit with related constants, $K_{eff} = aK$

$$H = \int dx \left[\frac{M}{2}\dot{u}^2 + \frac{K_{eff}}{2}(\nabla u)^2\right]$$

writing the corresponding $\mathcal{L}$

$$\mathcal{L} = \int dx \left[\frac{M\dot{u}^2}{2} - \frac{K_{eff}}{2}(\nabla u)^2\right]$$ corresponds to

"free particles" with $\omega_q = \sqrt{\frac{K_{eff}}{M}}\,|\vec{q}| \equiv C_{sound}|\vec{q}|$

These are called "acoustic phonons"

($\omega_q \xrightarrow[|\vec{q}| \to 0]{} 0$ is characteristic of Goldstone bosons)

The field $u(x)$ can be quantized (written in terms of quantum creation and destruction operators)

In 3-D, one has $\vec{u}(x)$, $\vec{r}_j = \vec{r}_j^{\,0} + \vec{u}_j$, etc. One deals with it as if it corresponded to particles