Holographic and QFT Complexity with angular momentum

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June 9^{th}-11^{th}, 2021

Coming soon:
Alice Bernamonti, Francesco Bigazzi, DB, Lapo Faggi, Federico Galli

New Frontiers in Theoretical Physics - Cortona Young 2021

The AdS/CFT correspondence and holographic states

[1997: Maldacena] [1998: Witten] [1998: Gubser, Klebanov, Polyakov]

Bulk AdS

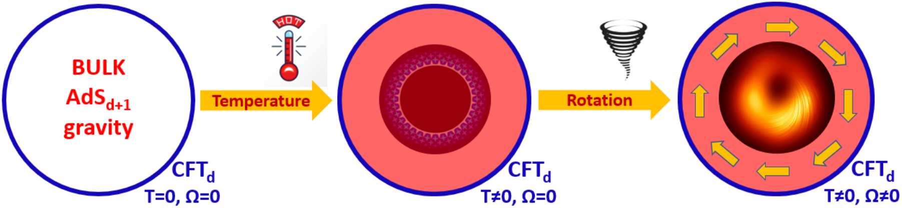
- Anti-de Sitter gravity $\Lambda < 0$
- $d+1$ spacetime dimensions
- weak coupling (= easy)

1. empty AdS spacetime
2. AdS black hole
3. AdS rotating black hole

Boundary CFT

- Conformal Field Theory
- d spacetime dimensions
- strong coupling (= hard)

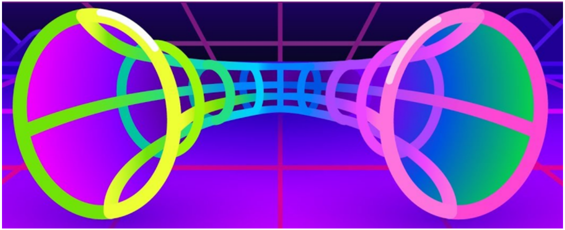
1. vacuum state
2. thermal density matrix ...
3. ... with angular momentum



The wormhole elongation puzzle

Eternal black holes dual to **thermofield double states** *[2001: Maldacena]*

Eternal AdS (rotating) black holes have a **wormhole** behind the horizon and are dual to (rotating) thermofield double (**TFD**) states



Wormhole elongation puzzle *[2014: Susskind]*

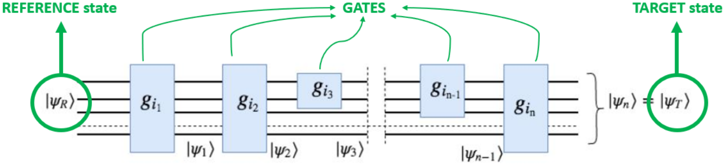
*How can it be that the bulk **wormhole grows for a very long time**? Is there a quantity in the boundary theory that captures this behavior?*

Candidate answer: **Computational Complexity**

Complexity: from discrete gates to continuous paths

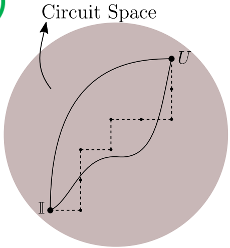
Basic idea of circuit complexity

- Q: Given a *reference* state and a set of (elementary) unitary operators called *gates*, how **hard** is it to prepare a particular *target* state?
- A: Compute **circuit complexity**, the *minimum number of gates* that build up the *unitary circuit U* such that $|\psi_T\rangle = U\,|\psi_R\rangle$



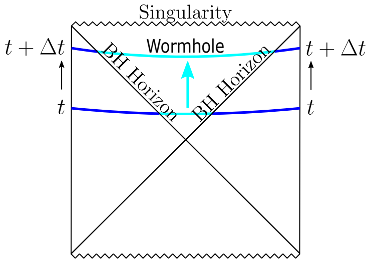
Geometrization of circuit complexity *[2005: Nielsen]*

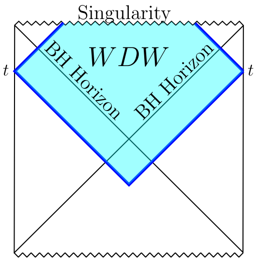
Complexity is the **length of the shortest geodesic** connecting reference and target **in circuit space**



Holographic complexity proposals: CV and CA

Example: non-rotating TFD state dual to non-rotating eternal black hole





Complexity = Volume

[2014: Stanford, Susskind]

$$\mathcal{C}_V \propto \mathit{Volume}_{MAX}$$

Complexity = Action

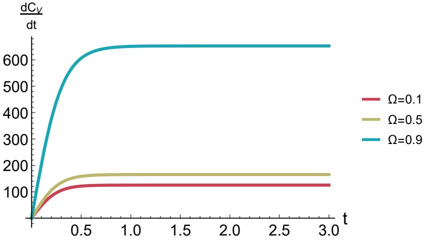
[2016: Brown, Roberts, Susskind, Swingle, Zhao]

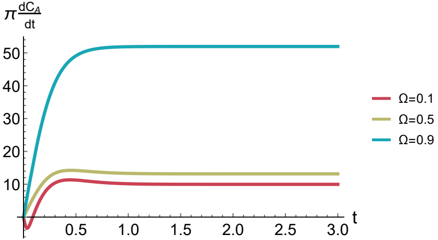
$$\mathcal{C}_A \propto \mathit{Action}_{WDW}$$

- Volume and Action manifest a constant growth rate at late times
- Same behavior for complexity of "fast-scramblers", which are quantum circuits that rapidly spread information

CV and CA growth rate in rotating black holes

- Using techniques of *[2017: Carmi, Chapman, Marrochio, Myers, Sugishita]*, previously employed in non-rotating black holes, we found in the rotating BTZ:





- Both for CV and CA, **late time growth rate is constant** (as expected from *[2016: Cai, Ruan, Wang, Yang, Peng]*):

$$\lim_{t\to\infty} \frac{d\mathcal{C}}{dt} \propto (M - \Omega J)_{outer} - (M - \Omega J)_{inner}$$

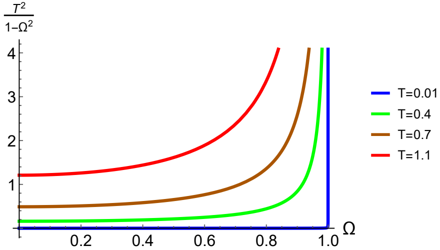
- Same formula holds for Kerr-AdS$_{3+1}$ (using full CA prescription!)

Late time growth rate in rotating BTZ

- For rotating BTZ we can rewrite:

$$\lim_{t\to\infty} \frac{d\mathcal{C}^{BTZ}}{dt} \propto \frac{T^2}{1-\Omega^2}$$

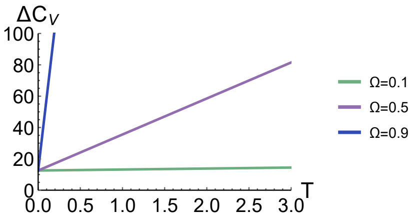
- At fixed $T \neq 0$, it **diverges** when the boundary rotates at the **speed of light** ($\Omega \to 1$):

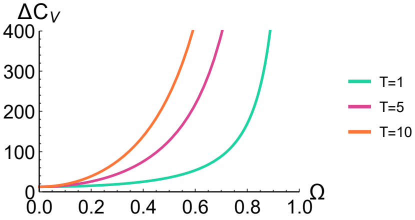


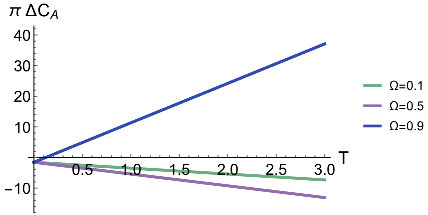
- The **extremal** BTZ ($T \to 0$, $\Omega \to 1$, $\frac{T}{1-\Omega^2} = const.$) has **vanishing late time growth rate**, like electrically charged extremal black holes studied in *[2017: Carmi, Chapman, Marrochio, Myers, Sugishita]*

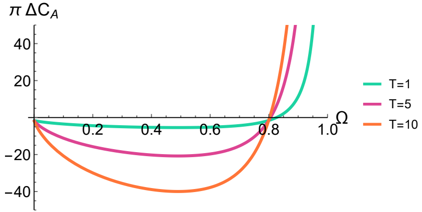
Holographic complexity of formation for rotating TFD

UV-finite measure of holographic complexity: $\Delta\mathcal{C} = \mathcal{C}(\mathrm{BTZ}) - 2\mathcal{C}(\mathrm{AdS})$



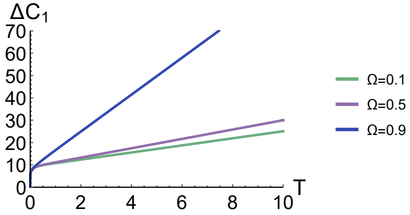


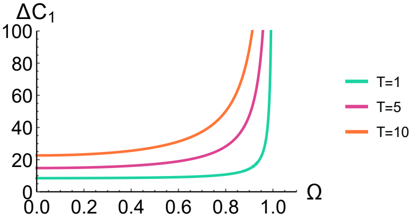




Circuit complexity of rotating TFD in free scalar QFT

- Free scalar QFT in 1+1 dimensions (spatial direction is a circle)
- Build rotating TFD and map into non-rotating TFD
- Adopt circuit setup of *[2018: Chapman, Eisert, Hackl, Heller, Jefferson, Marrochio, Myers]* and compute **minimal length geodesic** following Nielsen approach
- Complexity of formation similar to BTZ but not the same:





- Complexity oscillates in time and does not saturate at late times!
- Free QFT circuits are **not** "fast-scramblers" like holographic circuits!