Anomalous Spin-Rotation coupling and the breaking of Einstein Equivalence Principle in a thermalized medium

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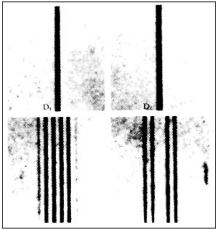
Based on [MB & Kharzeev, 2102.01676, to appear in PRD]

Magnetic Moment

Particle with mass m charge e and spin $\mathbf{S}$

$$\boldsymbol{\mu}_{\mathrm{B}} = -g_B \frac{e}{2m} \mathbf{S}$$

Zeeman effect $\quad H = H_0 - \boldsymbol{\mu}_{\mathrm{B}} \cdot \mathbf{B}$



Larmor frequency $\quad \omega = g_{\mathrm{B}} \dfrac{eB}{2mc}$

Magnetic Moment g-factor: $\quad g_{\mathrm{B}}$

Spin-Rotation Coupling or Gravitomagnetic Moment g_Ω

$$H = H_0 - g_\Omega \mathbf{S} \cdot \mathbf{\Omega} \qquad \mathbf{\Omega}\text{: Rotation}$$

Dirac equation

$$\left[\mathrm{i}\gamma^\mu (D_\mu + \Gamma_\mu) - m\right] \psi = 0$$

$$D_\mu = \partial_\mu - \mathrm{i}eA_\mu, \qquad \Gamma_\mu = \frac{1}{8}\omega_{\mu ij}[\gamma^i, \gamma^j]$$

2° order Dirac equation

$$\left[\partial_t^2 - \nabla^2 - e\mathbf{B}(\mathbf{L} + 2\mathbf{S}) - \mathbf{\Omega}(\mathbf{L} + \mathbf{S}) + m^2\right] \psi = 0$$

$e\mathbf{B} \cdot \mathbf{S}$: Magnetic moment $g_B = 2$

$\mathbf{\Omega} \cdot \mathbf{S}$: Gravitomagnetic Moment $g_\Omega = 1$

Evidence for Spin-Rotation Coupling

Experiment: [B. Venema et al, PRL (1992)]
Interpretation: [B. Mashhoon, Physics Letters A (1995)]

$$^{199}\mathrm{Hg}(I = \tfrac{1}{2}) \qquad ^{201}\mathrm{Hg}(I = \tfrac{3}{2})$$

$$H^i_{\mathrm{int}} = -g^i_{\mathrm{B}} \mu_{\mathrm{N}} \mathbf{B} \cdot \mathbf{S} - g_\Omega \hbar \boldsymbol{\omega}_{\mathrm{Earth}} \cdot \mathbf{S}$$

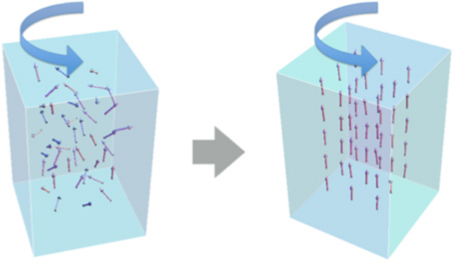
Zeeman splitting: parallel ΔE_+, anti-parallel ΔE_-

$$\frac{\Delta E^{201}_+}{\Delta E^{199}_+} - \frac{\Delta E^{201}_-}{\Delta E^{199}_-} \simeq 2 \left(1 - \frac{g^{201}_{\mathrm{B}}}{g^{199}_{\mathrm{B}}}\right) \frac{1}{g^{199}_{\mathrm{B}}} g_\Omega \frac{\hbar \omega_{\mathrm{Earth}}}{\mu_{\mathrm{N}} B}$$

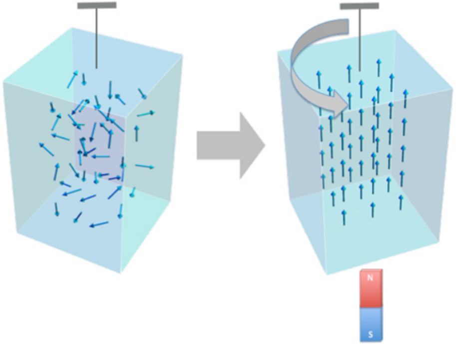
$$\text{Data} \Rightarrow |g_\Omega - 1| < 0.03$$

see also [Obukhov, Silenko, Teryaev, Int.J. of Modern Physics (2016)]

Barnett effect:



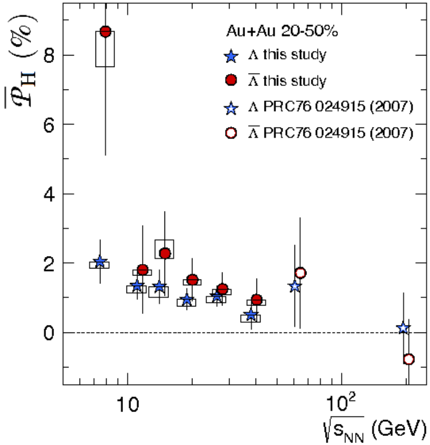
Einstein-de-Haas Effect:

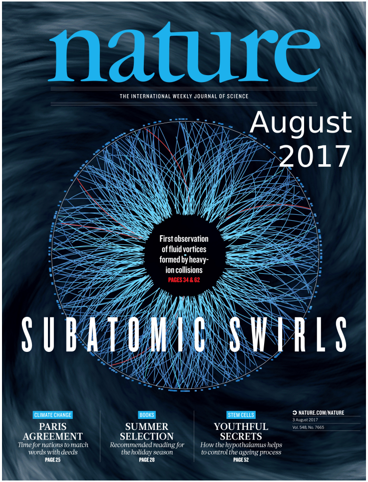


[M. Matsuo, J.Ieda, S. Maekawa, Frontiers in Physics (2015)]

Global polarization (Lambda Polarization)

[STAR Collaboration, Global Lambda hyperon polarization in nuclear collisions, Nature 548 6265, 2071]





Axial Vortical Effect (AVE)

[Vilenkin, PRD 20 (1979); Landsteiner, Megias, Melgar, and Pena-Benitez JHEP 09 (2011); Gao, Liang, Pu, Wang, and Wang, PRL 109 (2012)]

Spin $\frac{1}{2}$ Dirac particle ar thermal equilibrium in a rotating medium

$$\text{Axial current: } \widehat{j}^\mu_{\mathrm{A}} = \Psi\gamma^\mu\gamma^5\Psi$$

$$\langle \widehat{j}^\mu_{\mathrm{A}} \rangle = W^{\mathrm{A}} \frac{\omega^\mu}{T}$$

$$W^{\mathrm{A}}_{\text{Non-Int}} = \frac{1}{2\pi^2\beta} \int \mathrm{d}k \, \frac{\varepsilon_k^2 + k^2}{\varepsilon_k} n_{\mathrm{F}}(\varepsilon_k)$$

[MB, Grossi, Becattini, JHEP 10 (2017); MB, LNP “Strongly Interacting Matter under Rotation” (2021)]

$$\text{Massless field: } W^{\mathrm{A}}_{\text{Non-Int}} = \frac{T^3}{6}$$

[review: Kharzeev, Liao, Voloshin, and Wang, Prog. Part. Nucl. Phys. 88 (2016)]

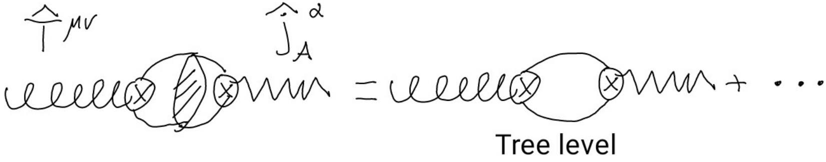
Axial Vortical Effect (AVE) Conductivity W^{A}

Kubo formula:

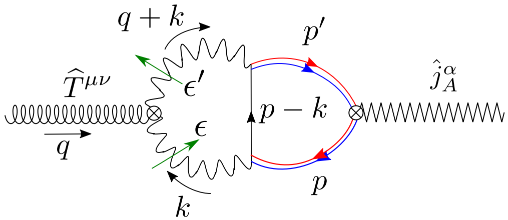
$$W^{\mathrm{A}} = 2 \int_0^\beta \frac{\mathrm{d}\tau}{\beta} \int \mathrm{d}^3 x \, x^1 \langle \widehat{T}^{02}(-\mathrm{i}\tau, \mathbf{x}) \, \widehat{j}^3_{\mathrm{A}}(0) \rangle_{\mathrm{T,c}}$$

Axial Vortical Effect (AVE) Conductivity W^{A}

Diagrams

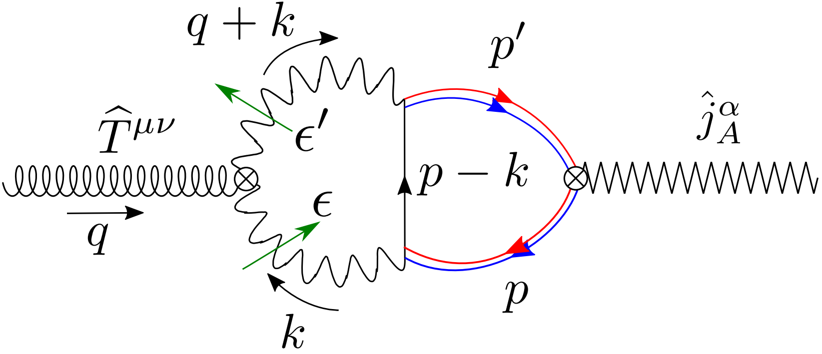


First loop QED



$$W^{\mathrm{A}}_{QED,m=0} = \left(\frac{1}{6} + \frac{e^2}{24\pi^2}\right) T^3$$

[D.-F. Hou, H. Liu, and H.-c. Ren, PRD 86 (2012); S. Golkar and D. T. Son, JHEP 02 (2015)]



The thermal bath provides the rotated photons

Main Results

[MB & Kharzeev, 2102.01676, to appear in PRD]

1. Gravitomagnetic moment of a Dirac massive particle at **thermal** equilibrium is anomalous; AGM for QED:

$$g_\Omega - 1 = \begin{cases} -\dfrac{1}{6}\dfrac{e^2T^2}{m^2} & T \ll m \\ -\dfrac{5}{36}\dfrac{e^2T^2}{m^2} & T \gg m,\ m > eT \end{cases}$$

2. AGM causes radiative corrections to the axial vortical effect

$$W^{\mathrm{A}} = \frac{1}{2\pi^2\beta}\int \mathrm{d}k \left[g_\Omega(\varepsilon_k)\frac{\varepsilon_k^2 + k^2}{\varepsilon_k} + g'_\Omega(\varepsilon_k)\left(k^2 + \frac{k^4}{3\varepsilon_k^2}\right)\right] n_{\mathrm{F}}(\varepsilon_k - \mu)$$

$$\simeq \left(1 - \frac{1}{6}\frac{e^2T^2}{m^2}\right)\frac{(mT)^{3/2}}{\sqrt{2}\pi^{3/2}}\mathrm{e}^{-(m-\mu)/T}$$

Breaking of Einstein Equivalence Principle

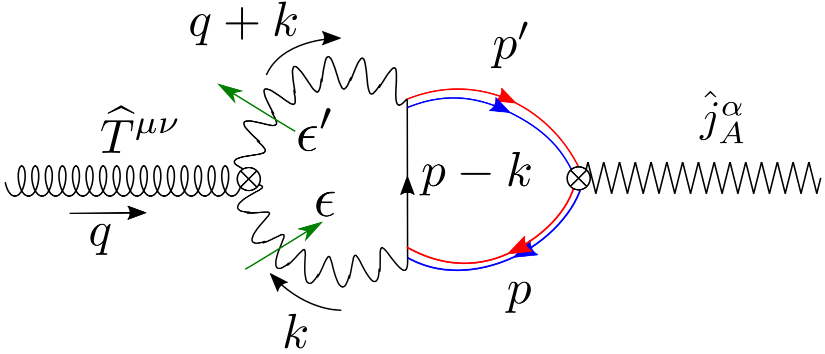
The Einstein Equivalence Principle (EEP) forbids the appearance of an anomalous spin-rotation coupling $g_\Omega \equiv 1$.

[Kobzarev and Okun, Zh. Eksperim. i Teor. Fiz. (1962); Cho and Dass, PRD 14 (1976); de Oliveira and Tiomno, Nuovo Cim. 24 (1962); Teryaev, Front. Phys. (2016)]

EEP premises do not hold in the presence of a medium
$\Rightarrow$ Breaking of EEP is possible at finite temperature

$$\frac{m_{\text{Inertial}}}{m_{\text{Gravitational}}} = 1 + \frac{e^2}{3}\frac{T^2}{m^2}$$

[Donoghue, Holstein, and Robinett, PRD 30 (1984) and Gen. Rel. Grav. 17, 207 (1985); Mitra, Nieves, and Pal, PRD 64 (2001)]



The thermal bath provides the rotated photons

Anomalous Magnetic Moment

$$\mathcal{L} = J^\mu A_\mu$$

$$\langle p', s' | J^\mu(0) | p, s \rangle = \bar{u}(p', s') \left\{ \frac{P^\mu}{2m} + \frac{\mathrm{i}\sigma^{\mu\nu} q_\nu}{2m} \left[1 + F_2(0)\right] \right\} u(p, s) + \mathcal{O}(q^2)$$

g_B is anomalous

$$g_B = 2(1 + F_2(0)) = 2\left(1 + \frac{\alpha}{2\pi} - \frac{1}{18}\frac{e^2 T^2}{m^2}\right), \quad T \ll m$$

[Schwinger (1948), Fujimoto and Jae (1982), Peressutti and Skagerstam (1982)]

Anomalous Gravitomagnetic Moment

$$\widehat{H}_{\rm int} = \frac{1}{2}\int \mathrm{d}^3x\, \widehat{T}_{\mu\nu} h^{\mu\nu}_{\rm Ext} \qquad g^{\mu\nu}_{\rm Ext} = \eta^{\mu\nu} + h^{\mu\nu}_{\rm Ext}$$

$$\begin{aligned}\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle =& \bar{u}(p', s')\Big\{ I_{P\gamma}(P, q)\,(P_\mu\gamma_\nu + P_\nu\gamma_\mu) + I_{u\gamma}(P, q)\,(u_\mu\gamma_\nu + u_\nu\gamma_\mu) \\ & + I_{Pl}(P, q)\hat{\not{l}}\left(P_\mu\hat{l}_\nu + P_\nu\hat{l}_\mu\right) + I_{ul}(P, q)\hat{\not{l}}\left(u_\mu\hat{l}_\nu + u_\nu\hat{l}_\mu\right) + \cdots\Big\} u(p, s) + \mathcal{O}(q^2).\end{aligned}$$

Scattering theory

$$\mathcal{A} = -\mathrm{i}(2\pi)\delta(p\cdot u - p'\cdot u)\frac{1}{2}\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle h^{\mu\nu}_{\rm Ext}(p' - p, \mathbf{\Omega})$$

Spin-rotation coupling

$$V = -g_\Omega \mathbf{S}\cdot\mathbf{\Omega} \;\Rightarrow\; \mathcal{A} = -\mathrm{i}(2\pi)\delta(p_0 - p'_0)\left[-g_\Omega\, \xi'^\dagger\frac{\boldsymbol{\sigma}}{2}\xi\cdot\mathbf{\Omega}\right]$$

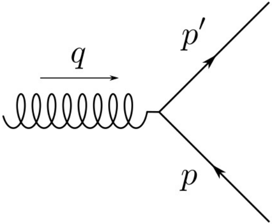
Anomalous Gravitomagnetic Moment

$$P = p' + p,\; q = p' - p, \quad \omega_P = P \cdot u,\; P_s = \sqrt{\omega_P^2 - P^2}$$

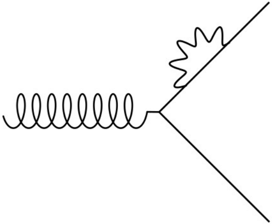
$$l^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho q_\sigma, \quad \hat{l}^\mu = \frac{l^\mu}{\sqrt{-l^2}}$$

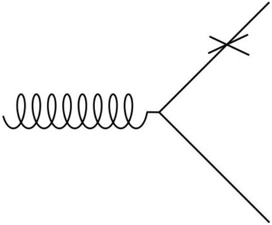
$$\begin{aligned}\langle p', s'|\widehat{T}_{\mu\nu}(0)|p, s\rangle =& \bar{u}(p', s')\Big\{ I_{P\gamma}(P,q)\left(P_\mu\gamma_\nu + P_\nu\gamma_\mu\right) + I_{u\gamma}(P,q)\left(u_\mu\gamma_\nu + u_\nu\gamma_\mu\right) \\ &+ I_{Pl}(P,q)\hat{\not{l}}\left(P_\mu\hat{l}_\nu + P_\nu\hat{l}_\mu\right) + I_{ul}(P,q)\hat{\not{l}}\left(u_\mu\hat{l}_\nu + u_\nu\hat{l}_\mu\right) + \cdots\Big\} u(p,s) + \mathcal{O}(q^2).\end{aligned}$$

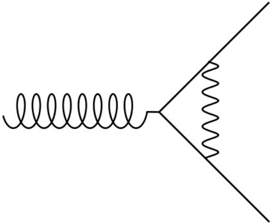
$$g_\Omega = \lim_{\substack{q\to 0 \\ P_s\to 0}} 4\left(I_{P\gamma}(P,q) + \frac{I_{u\gamma}(P,q)}{\omega_P} - I_{Pl}(P,q) - \frac{I_{ul}(P,q)}{\omega_P} \right)$$

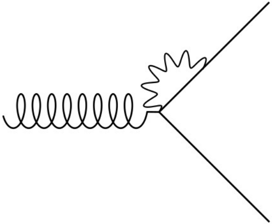


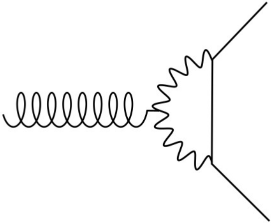
(a)Tree Level


(b)Self Energy


(c)Counter Term


(d)Electromagnetic Vertex


(e)Contact


(f)Photon Polarization

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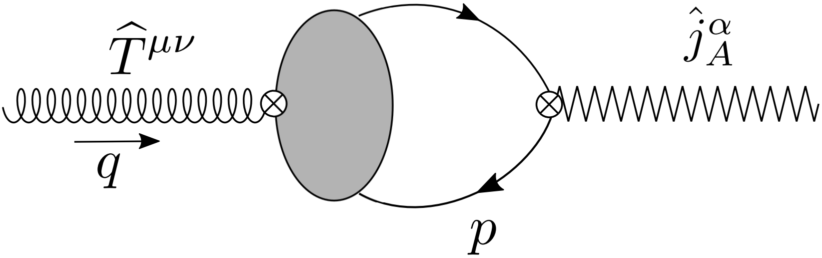

Quantum operators

[Weinberg, QFT Vol I]: Any operator can be written as sums of multi-particle states. For additive operators:

$$\widehat{T}^{\mu\nu}(x) = \frac{1}{2\pi^2}\sum_{\tau,\tau'}\int \frac{\mathrm{d}^3 q'}{2\varepsilon_{q'}}\int \frac{\mathrm{d}^3 q}{2\varepsilon_q}\widehat{a}^\dagger_{\tau'}(q')\widehat{a}_\tau(q)\mathrm{e}^{\mathrm{i}(q-q')\cdot x}\langle q',\tau'|\widehat{T}^{\mu\nu}(0)|q,\tau\rangle$$

$$+ \ \text{anti-particles} + \text{photons}$$

For axial current

$$\widehat{j}^{\mu}_{\mathrm{A}}(0) = \frac{1}{(2\pi)^3}\sum_{\sigma,\sigma'}\int \frac{\mathrm{d}^3 k}{2\varepsilon_k}\frac{\mathrm{d}^3 k'}{2\varepsilon_{k'}}\bar{u}_\sigma(k)_{A'}\left(\gamma^\mu\gamma^5\right)_{A'B'}u_{\sigma'}(k')\widehat{a}^\dagger_\sigma(k)_{B'}\widehat{a}_{\sigma'}(k')$$

$$+ \ \text{anti-particles}$$

Thermal averages: $\langle\widehat{a}^\dagger_{\tau'}(q')\widehat{a}_{\sigma'}(k')\rangle_T = \delta_{\tau'\sigma'}2\varepsilon_{q'}\delta^3(\mathbf{k}'-\mathbf{q}')n_{\mathrm{F}}(k')$

Axial Vortical Effect (AVE) Conductivity W^{A}

$T \ll m$

$$W^{\mathrm{A}} \simeq \frac{1}{2\pi^2\beta} \int \mathrm{d}k \left[g_\Omega(\varepsilon_k) \frac{\varepsilon_k^2 + k^2}{\varepsilon_k} + g'_\Omega(\varepsilon_k) \left(k^2 + \frac{k^4}{3\varepsilon_k^2} \right) \right] n_{\mathrm{F}}(\varepsilon_k - \mu)$$

$$g_\Omega(\varepsilon_k) = 4 \left(I_{P\gamma}(\varepsilon_k) + \frac{I_{u\gamma}(\varepsilon_k)}{\varepsilon_k} - I_{Pl}(\varepsilon_k) - \frac{I_{ul}(\varepsilon_k)}{\varepsilon_k} \right)$$

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