$\mathcal{N} = 2$ Conformal quivers as interpolating theories

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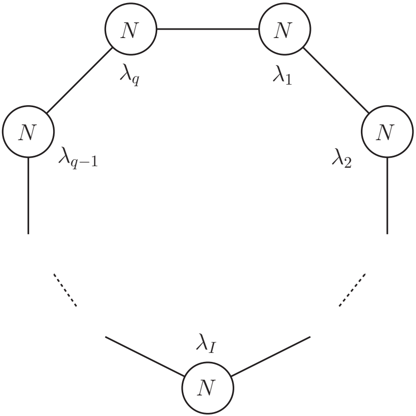
Based on [2012.15792] - [2105.00257] with M. Preti



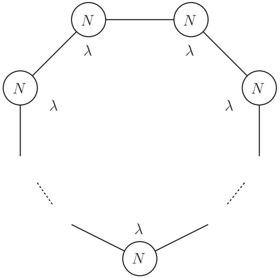
- (N) $= \left(A_\mu, \lambda^a, \bar{\lambda}^a, \varphi, \bar{\varphi}\right)_I$

  Adj of $SU(N)_I$

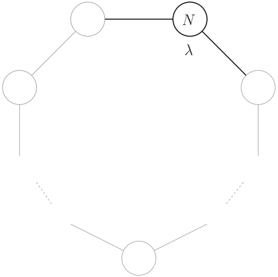
- —— $= (q, \bar{q}, \psi, \bar{\psi})$

  $\left(\square, \bar{\square}\right)$ of $SU(N)_I \times SU(N)_{I+1}$

Interpolation between $\mathcal{N} = 4$ and $\mathcal{N} = 2$ SCQCD

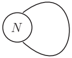


$$\lambda_I = \lambda \qquad \forall I$$

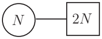


$$\lambda_1 = \lambda \,, \qquad \lambda_{I \neq 1} = 0$$

$\mathbb{Z}_q$ orbifold of $\mathcal{N} = 4$ SYM



$\mathcal{N} = 2$ SCQCD



Observables in A_{q-1} theories

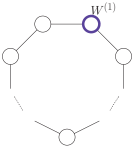
- For each node we can define:

$$\text{Wilson loop}: \quad W_I = \frac{1}{N} \text{tr}\, \mathcal{P} \exp \left\{ \sqrt{\frac{\lambda_I}{N}} \oint_C d\tau \left[\mathrm{i}\, A_\mu^I \, \dot{x}^\mu + \frac{1}{\sqrt{2}} (\varphi_I + \bar{\varphi}_I) \right] \right\}$$

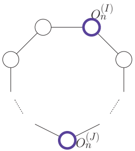
$$\text{Chiral operator}: \quad O_{\vec{n}}^{(I)}(x) = \text{tr}\, \varphi_I^{n_1} \, \text{tr}\, \varphi_I^{n_2} \ldots , \qquad \sum_i n_i = n$$

- We study several classes of observables[1]

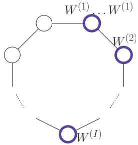
$\left\langle W^{(I)} \right\rangle$



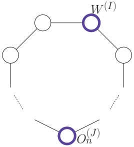
$\left\langle O_{\vec{n}}^{(I)} \bar{O}_{\vec{n}}^{(J)} \right\rangle$



$\left\langle W^{(I)} \ldots W^{(J)} \right\rangle$



$\left\langle W^{(I)} O_{\vec{n}}^{(J)} \right\rangle$



[1] Results collected in two Mathematica datasets available on Arxiv.

Tools: supersymmetric localization

The path integral is mapped to a sphere S_4 and reduced to a finite dimensional integral[2]

$$Z_{\mathbb{R}^4} = \int [\mathcal{D}\Phi] \longrightarrow \mathcal{Z}_{S^4} = \int da$$

a is a $N \times N$ matrix.
A_{q-1} partition function is reduced to a q-multimatrix model:

$$\mathcal{Z}_{q-1} = \int \prod_{I=1}^{q} da_I \, e^{-\frac{8\pi^2 N}{\lambda_I} \operatorname{tr} a_I^2 - \mathcal{S}_{\text{int}}(a_I)}$$

where

$$\mathcal{S}_{\text{int}}(a_I) = \sum_{m=2}^{\infty} \sum_{k=0}^{2m} \frac{(-1)^{m+k}}{m} \zeta_{2m-1} \binom{2m}{k} \left(\operatorname{tr} a_I^{2m-k} - \operatorname{tr} a_{I+1}^{2m-k}\right)\left(\operatorname{tr} a_I^{k} - \operatorname{tr} a_{I+1}^{k}\right)$$

Expanding S_{int} in perturbation theory, any observables can be computed in terms of q factorized Gaussian models, using recursive methods [3].

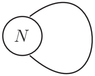
$$\langle f(a_I) \rangle_q = \frac{1}{\mathcal{Z}_{q-1}} \prod_{I=1}^{q} \left\langle e^{-\mathcal{S}_{\text{int}}(a_I)} \, f(a_I) \right\rangle_{\text{Gaussian}}$$

[2] [Pestun, 2007]

[3] [Billò, Fucito, Lerda, Morales, Stanev, Wen, FG, Gregori, 2017-2020]

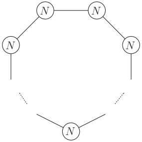
Interpolating results for Wilson loops

- $\mathcal{N} = 4$



$$\langle W \rangle_0 = w(\lambda) = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

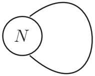
- Quiver



$$\begin{aligned}\left\langle W^{(1)} \right\rangle_q = \; & w(\lambda_1) - \frac{3\zeta_3 \lambda_1^2 (2\lambda_1 - \lambda_2 - \lambda_q)\partial_1 w_1}{128\pi^4} + \frac{5\zeta_5 \lambda_1^2}{1024\pi^6}\Big[4\lambda_1(2\lambda_1 - \lambda_2 - \lambda_q)(2\lambda_1 \partial_1^3 w_1 + 3\partial_1^2 w_1) \\ & + (4\lambda_1^2 - (\lambda_2 + \lambda_q)\lambda_1 - \lambda_2^2 - \lambda_q^2)\partial_1 w_1\Big] + \frac{9\zeta_3^2 \lambda_1^2}{32768\pi^8}\Big[(2\lambda_1 - \lambda_2 - \lambda_q)^2 \lambda_1^2 \partial_1^2 w_1 + 2(8\lambda_1^3 \\ & - 6(\lambda_2 + \lambda_q)\lambda_1^2 + 2(\lambda_2^2 + \lambda_2\lambda_q + \lambda_q^2)\lambda_1 - 2\lambda_2^3 + (\lambda_{q-1} - 2\lambda_q)\lambda_q^2 + \lambda_2^2\lambda_3)\partial_1 w_1\Big] + \ldots\end{aligned}$$

Interpolating results for Wilson loops

- $\mathcal{N} = 4$



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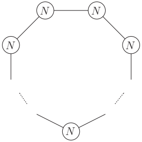
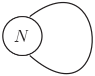


$$\left\langle W^{(1)} \right\rangle_q = w(\lambda_1) - \frac{3\zeta_3 \lambda_1^2 (2\lambda_1 - \lambda_2 - \lambda_q)\partial_1 w_1}{128\pi^4} + \frac{5\zeta_5 \lambda_1^2}{1024\pi^6}\Big[4\lambda_1(2\lambda_1 - \lambda_2 - \lambda_q)(2\lambda_1 \partial_1^3 w_1 + 3\partial_1^2 w_1) + (4\lambda_1^2 - (\lambda_2 + \lambda_q)\lambda_1 - \lambda_2^2 - \lambda_q^2)\partial_1 w_1\Big] + \frac{9\zeta_3^2 \lambda_1^2}{32768\pi^8}\Big[(2\lambda_1 - \lambda_2 - \lambda_q)^2 \lambda_1^2 \partial_1^2 w_1 + 2(8\lambda_1^3 - 6(\lambda_2 + \lambda_q)\lambda_1^2 + 2(\lambda_2^2 + \lambda_2\lambda_q + \lambda_q^2)\lambda_1 - 2\lambda_2^3 + (\lambda_{q-1} - 2\lambda_q)\lambda_q^2 + \lambda_2^2\lambda_3)\partial_1 w_1\Big] + \ldots$$

$$\lambda_1 = \lambda\,,\ \lambda_{I\neq 1} = 0 \quad \downarrow$$

$$\langle W \rangle_{\rm SCQCD} = w(\lambda) - \frac{3\zeta_3 \lambda^3}{(8\pi^2)^2}\partial_\lambda w + \frac{10\zeta_5 \lambda^4}{(8\pi^2)^3}\Big[4\lambda\partial_\lambda^3 w + 6\partial_\lambda^2 w + \partial_\lambda w\Big] + \frac{9\zeta_3^2 \lambda^5}{2(8\pi^2)^4}\Big[\lambda\partial_\lambda^2 w + 4\partial_\lambda w\Big] - \frac{7\zeta_7 \lambda^5}{4(8\pi^2)^4}\Big[256\lambda(\lambda\partial_\lambda^5 w + 5\partial_\lambda^4 w) + 16(7\lambda + 60)\partial_\lambda^3 w + 168\partial_\lambda^2 w + 21\partial_\lambda w\Big] + \ldots$$

- $\mathcal{N} = 4$



$$\langle W \rangle_0 = w(\lambda) = \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$$

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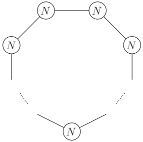


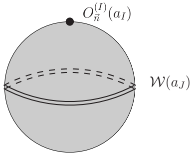
$$\left\langle W^{(1)} \right\rangle_q = w(\lambda_1) - \frac{3\zeta_3 \lambda_1^2 (2\lambda_1 - \lambda_2 - \lambda_q)\partial_1 w_1}{128\pi^4} + \frac{5\zeta_5 \lambda_1^2}{1024\pi^6}\Big[4\lambda_1(2\lambda_1 - \lambda_2 - \lambda_q)(2\lambda_1 \partial_1^3 w_1 + 3\partial_1^2 w_1)$$

$$+ (4\lambda_1^2 - (\lambda_2 + \lambda_q)\lambda_1 - \lambda_2^2 - \lambda_q^2)\partial_1 w_1\Big] + \frac{9\zeta_3^2 \lambda_1^2}{32768\pi^8}\Big[(2\lambda_1 - \lambda_2 - \lambda_q)^2 \lambda_1^2 \partial_1^2 w_1 + 2(8\lambda_1^3$$

$$- 6(\lambda_2 + \lambda_q)\lambda_1^2 + 2(\lambda_2^2 + \lambda_2\lambda_q + \lambda_q^2)\lambda_1 - 2\lambda_2^3 + (\lambda_{q-1} - 2\lambda_q)\lambda_q^2 + \lambda_2^2\lambda_3)\partial_1 w_1\Big] + \ldots$$

$$\lambda_I = \lambda\,, \quad \forall I \quad \downarrow$$

$$\left\langle W^{(1)} \right\rangle_q \xrightarrow{\lambda_I = \lambda} w(\lambda)$$



In the context of one-point functions of chiral operators

we define twisted and untwisted combinations :

$$U_{\vec{n}} = \frac{1}{\sqrt{q}} \sum_{I=1}^{q} O^{(I)}_{\vec{n}} \,, \qquad T^{(I)}_{\vec{n}} = \frac{1}{\sqrt{2}} \left(O^{(I)}_{\vec{n}} - O^{(I+1)}_{\vec{n}} \right) .$$

which transform properly under the $\mathbb{Z}_q$ action. The untwisted one-point functions read:

$$\left\langle U_{\vec{n}} \, W^{(1)} \right\rangle_q \xrightarrow{\lambda_I = \lambda} \mathcal{A}_n(\lambda)$$

where $\mathcal{A}_n(\lambda) = \frac{n}{2^{n/2}} \lambda^{\frac{n}{2}-1} I_n(\sqrt{\lambda})$ is the $\mathcal{N} = 4$ result.

$$\left\langle T^{(1)}_n W^{(1)} \right\rangle_q \xrightarrow{\lambda_I = \lambda} \mathcal{A}_n(\lambda) \left(1 - \frac{\lambda^n}{(8\pi^2)^n} \frac{3}{2^n} \binom{2n}{n} \zeta_{2n-1} \right) + \ldots$$

differ from $\mathcal{N} = 4$ result by a codified transcendentality expansion.