THE TOPOLOGICAL LINE OF ABJ(M) THEORY

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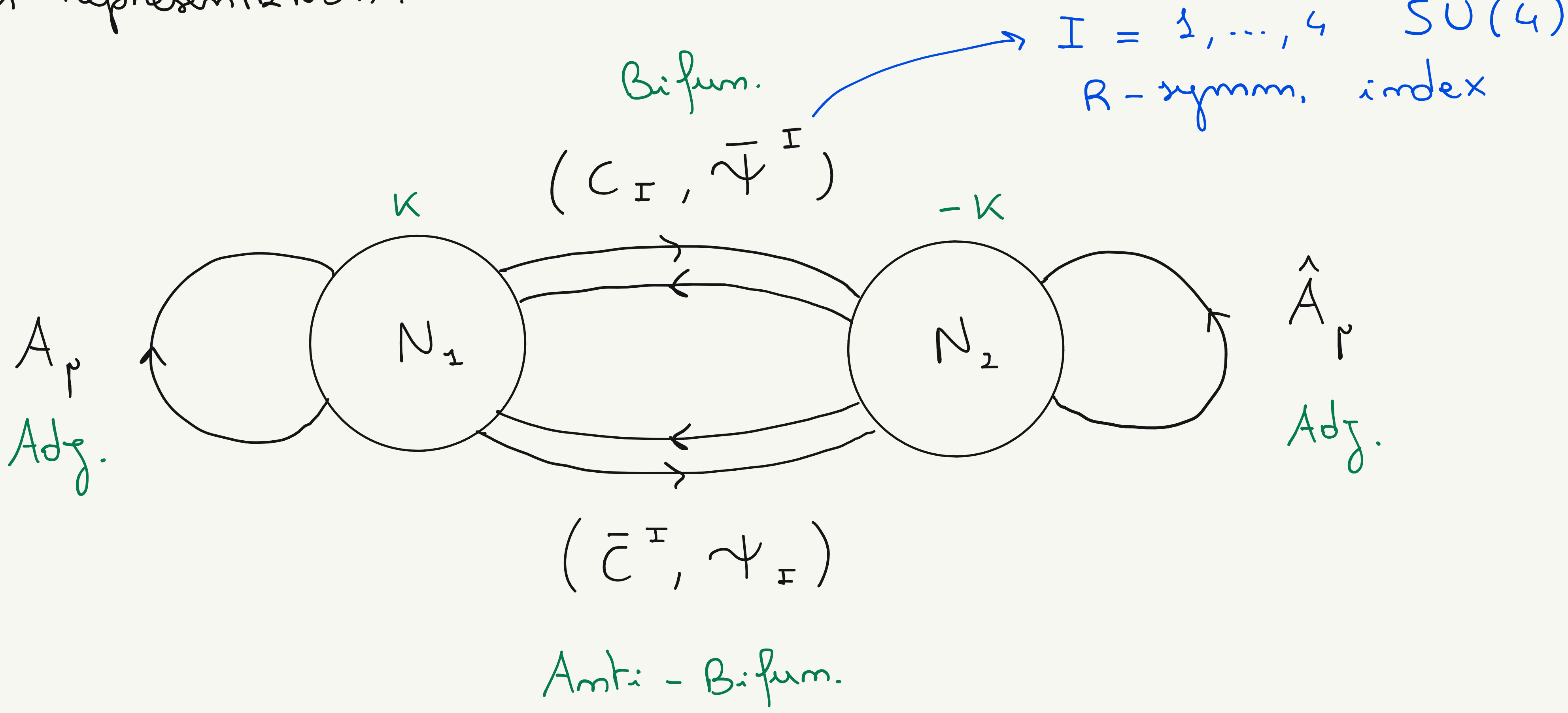
What is ABJ(M)

ABJ(M) is a 3d superconformal CS - Matter theory with $\mathcal{N} = 6$ susy [A, B, J, M, '08].

Gauge group: $G = U(N_1)_k \times U(N_2)_{-k}$

Couplings: N_1, N_2, k

Quiver representation:



Action: $S_{ABJ(M)} = S_{CS} + \hat{S}_{CS} + S_{matter} + S^B_{int} + S^F_{int}$

Why do we study it

Because we can explicitly test a duality...

ABJ(M) $\xleftrightarrow[\text{Correspondence}]{\text{AdS/CFT}}$ M-Theory on

CFT_3 $\qquad\qquad\qquad$ $AdS_4 \times S^7/\mathbb{Z}_k$

So what can we compute? Observables!

LOCAL
ABJ(M) fields $\rightarrow$

Extract useful informations
(OPE coeff., Central charge, ...)
Perturbative and Non-perturbative

NON-LOCAL
Wilson loops / Lines $\rightarrow$

Is there a simpler way to extract relevant informations?

$\Longrightarrow$ Restrict to the Topological sector!

The Topological Sector

A topological sector is a sector of the theory where

- correlators are independent of sp.-time coordinates to all order in perturbation theory

$$\partial_i \langle O(x_1) \cdots O(x_n) \rangle = 0 \qquad i = 1, \cdots, n$$

- Operators are protected

$$\Delta = \Delta_0 + \cancel{\gamma(g)} \quad (= 0)$$

Its existence has been proven for all 4d $\mathcal{N} \geq 2$ SCFTs [Rastelli, ..., '15] and for 3d $\mathcal{N} \geq 4$ SCFTs [Pufu, ..., '14]

But ⟶ An explicit construction for 3d $\mathcal{N} = 6$ theories was still lacking!

Moreover...

The Conjecture

For 3d $\mathcal{N} \geq 4$ SCFTs on S^3 it is believed that (proved for $\mathcal{N} = 4, 8$)

$$Z_{3d}[m] = Z_{1d}[m]$$

$Z_{3d}[m]$ → Mass-deformed 3d theory

$$m^A \int d^3x \sqrt{g} \left(\alpha^{ab} J^A_{ab} + \bar{\alpha}^{\dot{a}\dot{b}} K^A_{\dot{a}\dot{b}} \right)$$

$Z_{1d}[m]$ → Mass-deformed 1d theory

$$-4\pi r^2 m^A \int_{-\pi}^{\pi} d\varphi \, J^A(\varphi)$$

Therefore:

$$\left\langle \int d\varphi \, J^{A_1}(\varphi) \cdots \int d\varphi \, J^{A_m}(\varphi) \right\rangle = \frac{1}{(4\pi r^2)^m} \frac{1}{Z_{3d}} \frac{d^m Z_{3d}}{dm^{A_1} \cdots dm^{A_m}} \Bigg|_{m=0}$$

⟹ We check the validity of this relation for ABJ(M) in the weak coupling limit ($N_{1,2}$ generic, large K)

Topological Sector for ABJ(M)

We restrict to the $x^\mu = (0,0,s)$ line and implement the so called Topological Twist [Witten, '88]. What we observe is

ABJ(M)		1d line
$so(3,2) \oplus su(6)$	Restriction $\longrightarrow$	$so(1,2) \oplus su(3) \oplus u(1)$
$\subset osp(6\|4)$		$\subset su(1,1\|3)$
12 Poincaré supercharges		6 Poincaré supercharges
		Quantum numbers $[\Delta, m, j_1, j_2]$

The recipe for the twist:

- Extract an $sl(2|2) \subset su(1,1|3)$.
- Take $sl(2)_{sp.-t} \oplus_{diag} sl(2)_{R-sym.} = \widehat{sl}(2) = \langle \hat{P}, \hat{K}, \hat{D} \rangle$.
- Select a $\mathbb{Q} = Q + S$ such that $\hat{P}, \hat{K}, \hat{D}$ are $\mathbb{Q}$-exact.

The Magic of the Twist

It is known that:

$$\partial_i \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle = 0 \quad \text{when } \mathcal{O} \text{ is chiral} \quad ([Q, \mathcal{O}] = 0)$$

But

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle = \text{const.} \xrightarrow[\text{under } R\text{-symm.}]{\mathcal{O} \text{ is changed}} \text{const.} = 0$$

$\Longrightarrow$ The twist allow us to recover a non-vanishing result if now $\mathcal{O}$ is $\mathbb{Q}$-exact.

What we need to do therefore is to study:

~~Neutral $\mathcal{O}$ in the Cohomology of Q~~ $\longrightarrow$ Neutral $\mathcal{O}$ in the Cohomology of $\mathbb{Q}$

Empty!   Non-Empty!

Perturbation theory

In our work we found that there are two candidates which satisfy the previous property: $C_2 = Y_1$ and $\bar{C}^4 = \bar{Y}^3$.

At this point we need to:

- Construct a gauge-inv. combination $\mathcal{O}(0) = \mathrm{Tr}\left[(Y_1 \bar{Y}^3)(0)\right]$
- Construct $\mathcal{O}(s)$ from $\mathcal{O}(0)$ with the Twisted translation $\hat{P}$
- Compute perturbatively the n-point functions

$$\langle \mathcal{O}(s_1) \cdots \mathcal{O}(s_n) \rangle$$

- Compare the results with the Matrix Model ones.

Results

By computing all the suitable derivatives of the ABJ(M) mass-deformed Matrix Model we find

$$\langle \mathcal{O}(\varphi_1)\,\mathcal{O}(0)\rangle^{(2)} = -\frac{N_1 N_2}{(4\pi)^2}\left(1-\frac{\pi^2}{6k^2}\left(N_1^2+N_2^2-2\right)\right) = \frac{1}{\pi^2}\frac{1}{(2\pi)^2}\frac{1}{Z}\frac{d^2 Z}{dm^2}\bigg|_{m=0}^{(2)}$$

$$\langle \mathcal{O}(\varphi_1)\,\mathcal{O}(\varphi_2)\,\mathcal{O}(0)\rangle^{(1)} = 0 = \frac{1}{\pi^3}\frac{1}{(2\pi)^3}\frac{d^3 \log Z}{dm^3}\bigg|_{m=0}^{(\text{all})}$$

$$\langle \mathcal{O}(\varphi_1)\,\mathcal{O}(\varphi_2)\,\mathcal{O}(\varphi_3)\,\mathcal{O}(\varphi_4)\rangle^{(1)} = 2\,\frac{N_1 N_2}{(4\pi)^4} = \frac{1}{\pi^4}\frac{1}{(2\pi)^4}\frac{d^4 \log Z}{dm^4}\bigg|_{m=0}^{(1)}$$

Additional Results: 4-pt function and the central charge at 2-loops.

$$\frac{1}{\pi^4}\frac{1}{(2\pi)^4}\frac{d^4 \log Z}{dm^4}\bigg|_{m=0}^{(2)} = 2\,\frac{N_1 N_2}{(4\pi)^4} - \frac{N_1 N_2\left(N_1^2+N_2^2-2\right)}{192\,\pi^2 k^2}$$

$$c_T^{(2)} = -\frac{64}{\pi^2}\frac{d^2 \log Z}{dm^2}\bigg|_{m=0}^{(2)} = 16\, N_1 N_2\left(1-\frac{\pi^2}{6k^2}\left(N_1^2+N_2^2-2\right)\right)$$

Conclusions

- We showed how to construct the 1d topological sector for the ABJ(M) theory and how to extract ABJ(M) data from it by computing correlators of suitable topological operators $\mathcal{O}$.

- We presented a perturbative highly non-trivial evidence that the conjecture holds also for 3d $\mathcal{N} = 6$ SCFTs.

Future directions

- Promote the 1d topological sector to a 1d topological defect by considering the $1/2$ - BPS fermionic Wilson line.
- Applying bootstrap techniques to study the topological sector and extract OPE coefficients.
- Localization of the full 1d topological sector in ABJ(M) with generic $\mathcal{O}^m$ operators.