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Curvature and its correlation with some properties of the neutron star

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References

- S. K. Biswal, H. C. Das, Ankit Kumar *et al.* **arXiv:2012.13673**.
- H. C. Das, Ankit Kumar, Bharat Kumar *et al.* **JCAP01(2021)007**.

Introduction

- According to Einstein's theory of general relativity, massive objects warp the space-time around them, and the effect a warp has on objects is called gravity. The amount of wrap is measured as strength of gravity.
- The curvature of space-time directly depends on the mass of the objects.

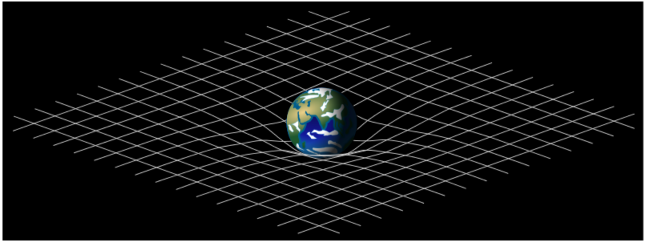

Figure: Curvature in space-time caused by an object.

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- The curvature depends on the metric, which defines the geometry of the manifold.
- The measurement of the curvature is the connection between the vectors in the tangent spaces in the manifold, which is called affine connection and also it has a direct connection with metric elements.
- There is a unique connection which can be found from the metric is defines by the Christoffel symbol $\Gamma^{\lambda}_{\mu\nu}$ defined as

$$\Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} [\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu}]$$

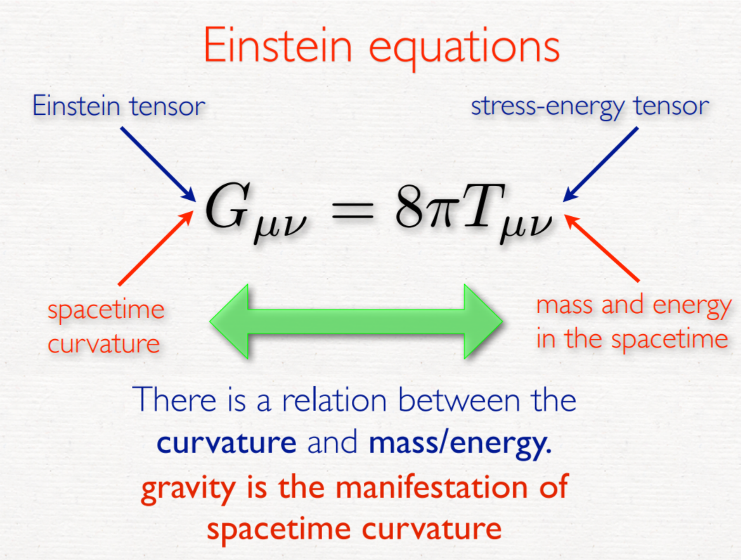
- All informations are contained in the Riemann tensor and it is defined as

$$\mathcal{R}^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

Different tensors and its significance

- Ricci tensor ($\mathcal{R}_{\mu\nu}$), Ricci scalar ($\mathcal{R}$), Weyl tensor ($\mathcal{W}_{\mu\nu}$) etc. to measure the space-time curvature.
- The Ricci scalar and the Ricci tensor contain all the informations about the Riemann tensor leaving only the trace-less part.
- Weyl tensor can be formed by removing all the contraction terms of the Riemann tensor.
- Physically, the Ricci scalar and the Ricci tensor measure the volumetric change of the body in presence of the tidal effect.
- Weyl tensor gives information about the shape distortion of the body. However, the Riemann tensor measures both the distortion of shape and the volumetric change of the body in presence of the tidal force.

Einstein Tensor ($G_{\mu\nu}$)



Magnitude of curvatures

- A measure of the space-time curvature is contained in the ratio **M/R**, which is also called as compactness.
- The larger this ratio, the larger the gravity/curvature.
- For example, our Earth,
  $C_e \simeq \frac{M_e}{R_e} = \frac{5.97 \times 10^{24}\ \ kg}{6372\ \ km} \simeq 3 \times 10^{-9}$.
- In our neighbourhoods, the largest curvature due to the Sun.
  $C_\odot \simeq \frac{M_\odot}{R_\odot} \simeq \frac{1.98 \times 10^{30}\ \ kg}{6.95 \times 10^{5}\ \ km} \simeq 2 \times 10^{-6}$.
- For White dwarfs, $C_{WD} \simeq \frac{M_{WD}}{R_{WD}} \simeq \frac{1 M_\odot}{10^4\ \ km} \simeq 3 \times 10^{-4}$.
- For Neutron star, $C_{NS} \simeq \frac{M_{NS}}{R_{NS}} \simeq \frac{2 M_\odot}{10\ \ km} \simeq 0.2$.
- For Black hole, $C_{BH} \simeq 0.5$

Ref. CSP Book by J. Schaffner-Bielich

Types of curvatures, (PRD 89,063003 (2014))

- Ricci Scalar,

$$\mathcal{R} = \kappa(\mathcal{E} - 3P). \tag{1}$$

where $\kappa = 8\pi$. P, $\mathcal{E}$ is the pressure, energy density respectively.

- Full contraction of Ricci Tensor,

$$\mathcal{J} = \sqrt{\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}} = \kappa[\mathcal{E}^2 + 3P^2]^{1/2}. \tag{2}$$

- Full contraction of Riemann Tensor (Kretschmann scalar),

$$\mathcal{K} = \sqrt{\mathcal{R}^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}} = \kappa\left[3\mathcal{E}^2 + 3P^2 + 2P\mathcal{E} - \frac{128\mathcal{E}m}{r^3} + \frac{48m^2}{r^6}\right]^{1/2}. \tag{3}$$

- Full contraction of Weyl Tensor,

$$\mathcal{W} = \sqrt{\mathcal{C}^{\mu\nu\rho\sigma}\mathcal{C}_{\mu\nu\rho\sigma}} = \left[\frac{4}{3}\left(\frac{6m}{r^3} - \kappa\mathcal{E}\right)^2\right]^{1/2}. \tag{4}$$

Curvature and Compactness with radius

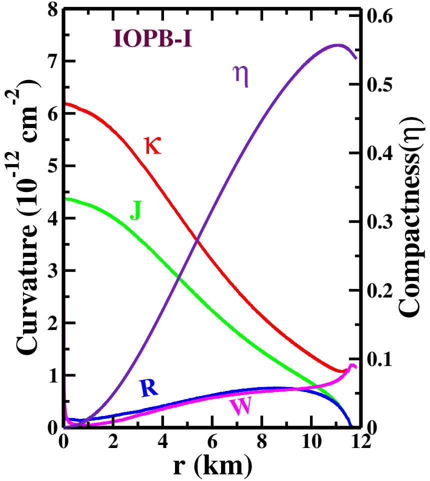


Figure: The radial variation of curvatures and compactness for IOPB-I parameter set.

Surface curvature ($\mathcal{K}(R)/\mathcal{K}_\odot$) with mass

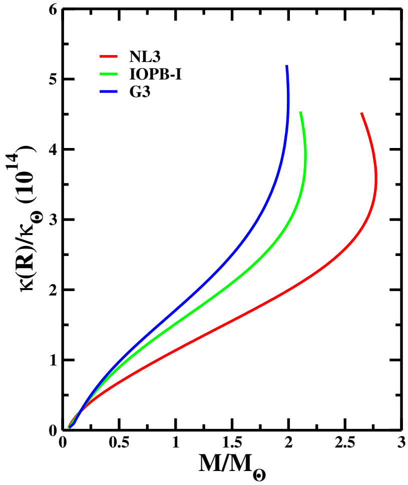


Figure: Mass variation of surface curvatures for 3-parameter set.

Properties of different curvatures

- Ricci curvature gives the matter content of the universe by means of the Einstein field equation.
- The components of Ricci curvatures are zero for outside the star as in equations (1) and (2). At the surface both $\mathcal{E}$ and P is zero. It implies $\mathcal{R}(r)$ and $\mathcal{J}(r)$ are zero.
- Similarly case for Weyl tensor, which is vanish outside of the uniform star $(m = \frac{4}{3}\pi r^3 \mathcal{E})$.
- But the Riemann tensor has non-vanishing components in the vacuum. So that Riemann tensor is more suitable for the curvature measurements of the space-time.

Some facts about the NS

- The compactness of a spherical objects, which measures the strength of its gravity is defined for a neutron stars is :
  $\eta_{NS} = \frac{2GM}{c^2R} \simeq 0.3(\frac{M}{M_\odot})(\frac{R}{10\ km})^{-1}$.

- The space time curvature is yet another measure of the strength of gravity. The curvature at the surface of a typical neutron star is
  $\mathcal{K}_{NS} = \frac{4\sqrt{3}GM}{c^2R^3} = 1 \times 10^{-12}\ \text{cm}^{-2}\ (\frac{M}{M_\odot})(\frac{R}{10\ km})^{-3}$.

- The compactness ratio $\eta_{NS}/\eta_\odot \sim 10^5$ and the curvature ratio, $\mathcal{K}_{\mathcal{NS}}/\mathcal{K}_\odot \sim 10^{14}$.

Ref. Turk J Phys 40, (2016) 127-138.

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- These two quantity's compactness and curvatures assert the relativistic GR is indispensable for the the description of neutron stars.
- Their strong field gravity suggests that they can also be used to constrain alternative or modified models of gravity.
- The high density behaviour at the core of the NS, which helps to calculate the EOS of dense matter.
- Neutron stars are the best natural testing grounds of the grand unification theories of fundamental forces.

Motivation

- The quantities $\mathcal{K}$ and $\mathcal{W}$ are significant to measure the curvatures both inside and outside the NS.
- Since those quantity's are depends on the EOS, and some observables like mass and radius of the NS, hence one can numerically calculate their values for the NS with different formalism.
- We have taken 30 RMF, 15 SHF and 6 DD-RMF models and calculate the EOS, $M - R$ relation, curvatures etc.
- Are there any correlations exist between the curvatures and properties of the NS?

Correlation between surface curvature and compactness

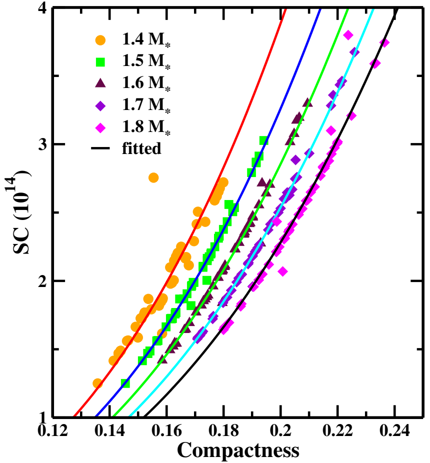

Figure: Surface curvature with compactness for different masses of the NS for 36 RMF and 15 SHF EOSs.

Correlation between surface curvature and radius

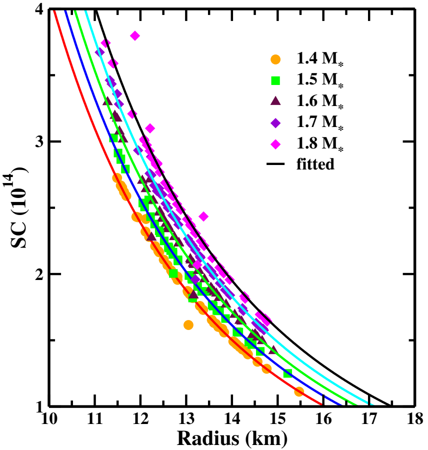

Figure: Surface curvature with radius for different masses of the NS for 36 RMF and 15 SHF EOSs.

Pearson formula

The correlation co-efficient

$$\xi = \frac{\Sigma_{xy}}{\sqrt{\Sigma_{xx}\Sigma_{yy}}}, \tag{5}$$

where

$$\Sigma_{xy} = \frac{1}{N}\sum_{i=0}^{N} x_i y_i - \frac{1}{N^2}\left(\sum_{i=0}^{N} x_i\right)\left(\sum_{i=0}^{N} y_i\right). \tag{6}$$

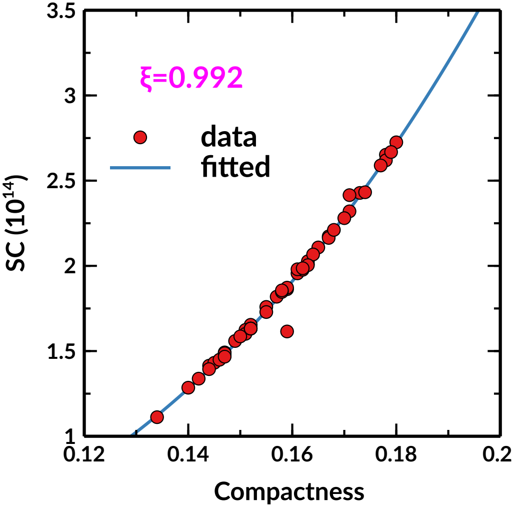

Figure: Left: Surface curvature with compactness for canonical NS. Right: The correlation co-efficients with different masses of the NS.

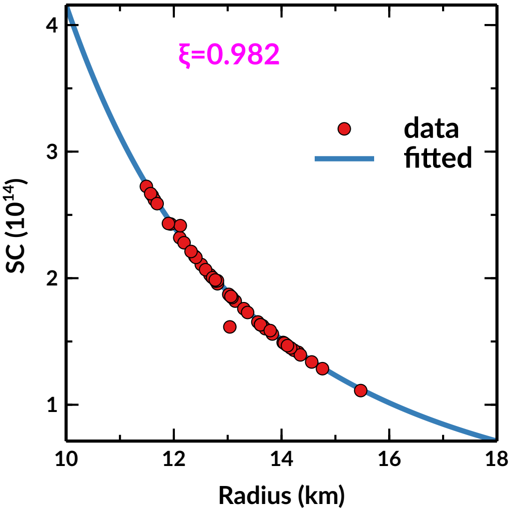

Figure: Left: Surface curvature with radius for canonical NS. Right: Correlation co-efficients with different masses of the NS.

Correlation heat map

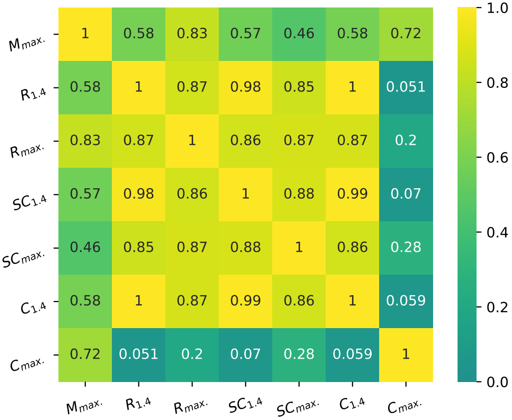

Figure: The correlation matrix heat map shows the correlations between each pairs and the value of ξ are written in the box.

Conclusion

- We take 36 RMF, 15 SHF EOSs as input to the TOV equations and calculate mass, radius, curvatures.
- The $\mathcal{K}$ and $\mathcal{W}$ are prominent quantity to measure the curvature inside and outside the NS.
- We find a strong correlations between the $\mathrm{SC}_{1.4}$-$C_{1.4}$ (99%) and $\mathrm{SC}_{1.4}$-$R_{1.4}$ (98%).
- More than 85% correlation find for $\mathrm{SC}_{max.}$-$C_{1.4}$ and $\mathrm{SC}_{max.}$-$R_{1.4}$.

Thank You