Stochastic sampling effects and heterogeneities in control of epidemic processes

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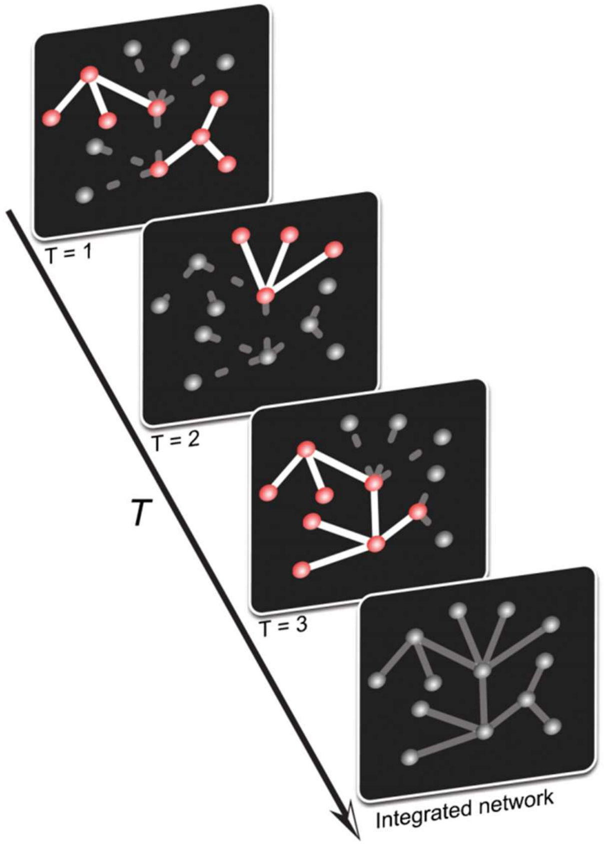
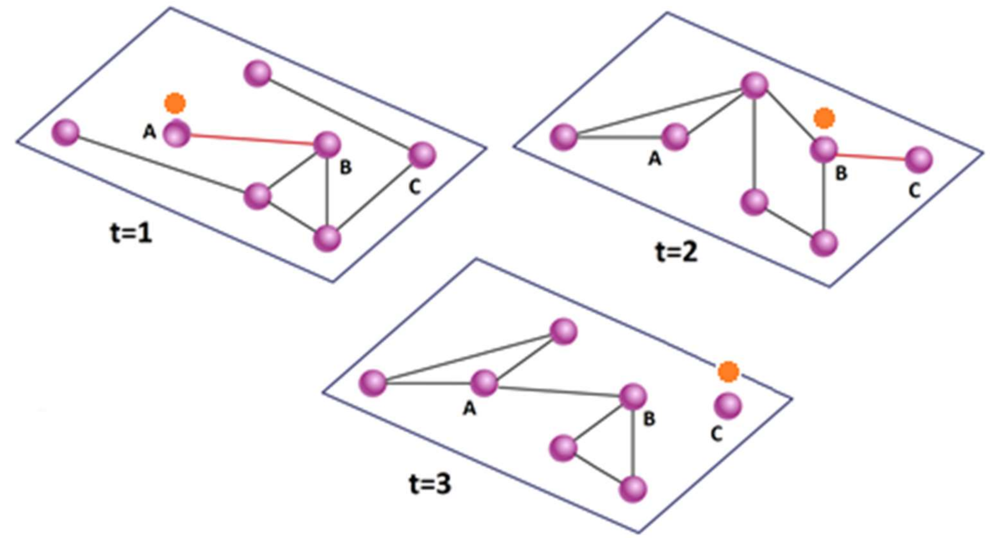
Temporal Networks



Perra et al., Sci. Rep. (2012)

Network dynamics (τ_N) ⟶ **Dynamical processes (τ_D)**



Static snapshots

$$\mathcal{T} = \{\mathcal{G}_t\}$$

Integrated Network

$$\mathcal{G}_T = \cup_{t=0}^{T} \mathcal{G}_t$$

Three cases:

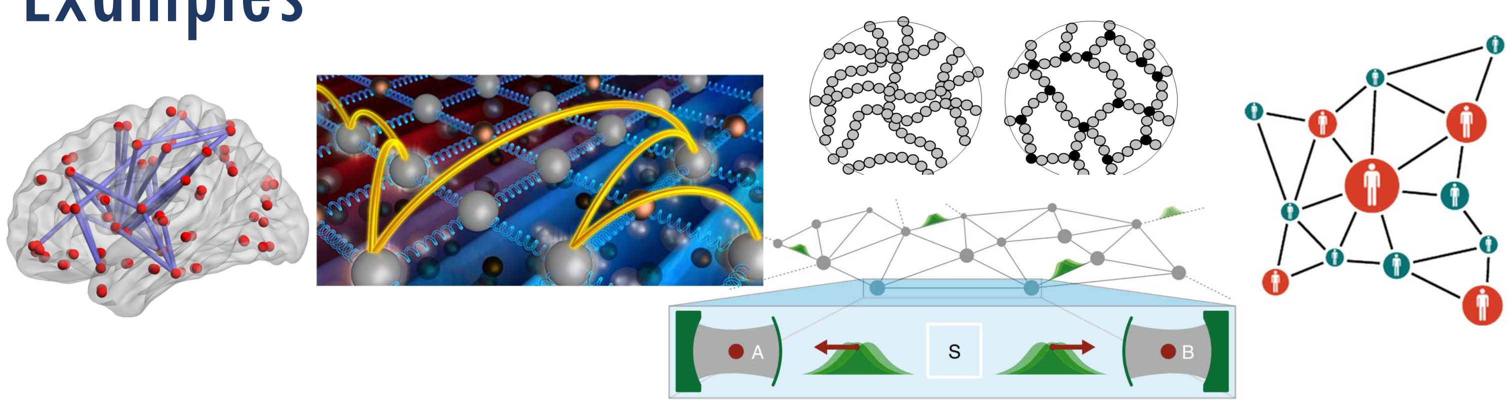
- $\tau_D \ll \tau_N$
- $\tau_D \gg \tau_N$

Static Network

- $\tau_D \sim \tau_N$

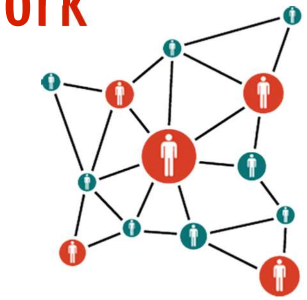
Temporal Network

Examples



Phase transition in epidemic model

Epidemics ⟶ Stochastic processes coupled to network evolution of social interactions



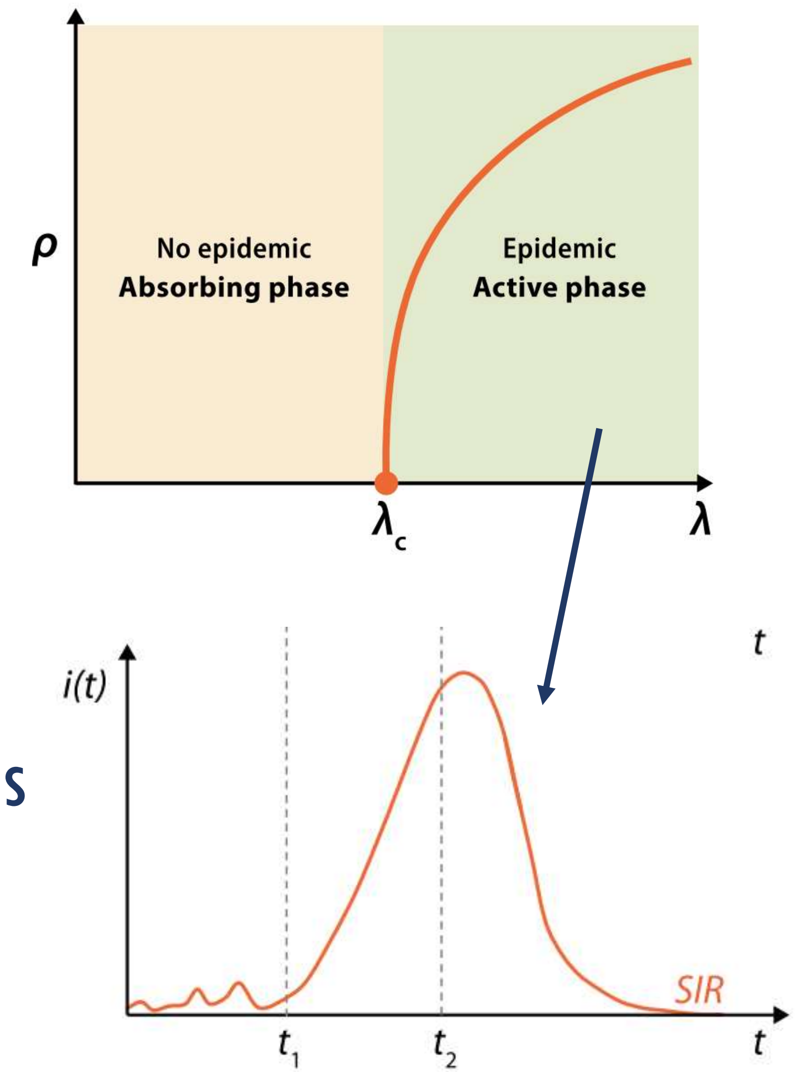
SIR = Susceptible – Infected – Recovered

Dynamical phase transition ⟶ Non-equilibrium system with two phases for epidemic evolution

⟶ $r = \lambda/\mu$ Control parameter

r_C Epidemic threshold ⟶ $r < r_C$ Absorbing phase

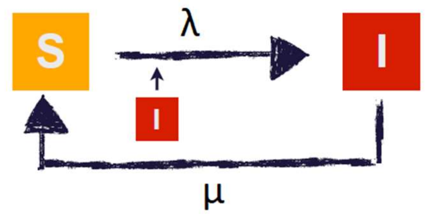
$r > r_C$ Active phase



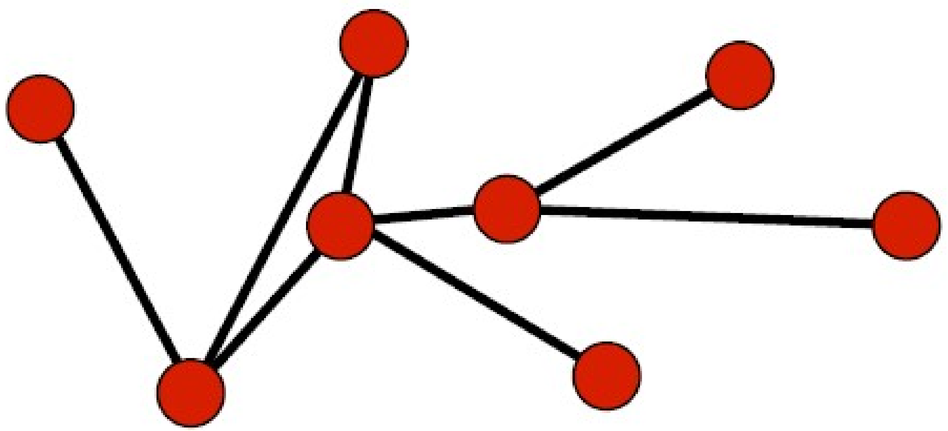
Pastor-Satorras R. et al., Rev. Mod. Phys. (2015)

Phase transition on networks

SIS model



Static networks



$$\lambda_c^{\mathrm{IBMF}} = \frac{1}{\Lambda_1} \qquad \lambda_c^{\mathrm{DBMF,unc}} = \frac{\langle k \rangle}{\langle k^2 \rangle}$$

Max eigenvalue of adjacency matrix

Degree distribution

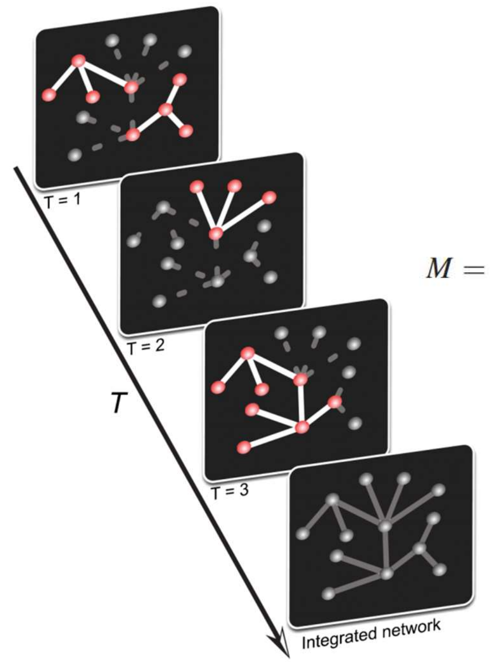
Topology affects epidemic spreading and threshold

Temporal networks

Adjacency matrix evolves in time

$$\mathbf{M}_{ij}^{tt'} = \delta^{t,t'+1}[(1-\mu)\delta_{ij} + \lambda A_{ij}^{(t)}].$$

$$M = \begin{pmatrix} 0 & 1-\mu+\lambda A^{(1)} & 0 & \cdots & 0 \\ 0 & 0 & 1-\mu+\lambda A^{(2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-\mu+\lambda A^{(T-1)} \\ 1-\mu+\lambda A^{(T)} & 0 & 0 & \cdots & 0 \end{pmatrix}.$$



Perra et al., Sci. Rep. (2012)

$$\boxed{\rho(M^{\dagger}) = 1}$$

Infection propagator approach

Valdano E. et al., PRX (2015)

Network evolution and structure affect epidemic spreading and threshold

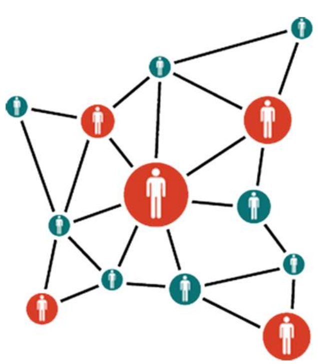
Adaptive temporal networks

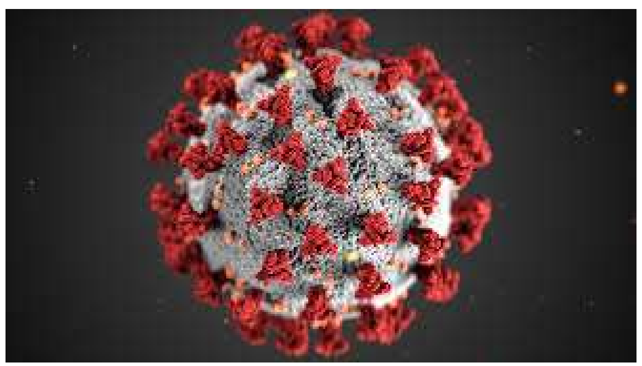
Network dynamics affects dynamical processes

↑ Coupled dynamics ↓

Dynamical processes can affect network evolution

Response to epidemics and control measures change the network





THE WALL STREET JOURNAL.

WORLD | EUROPE

As Virus Spreads, Italy Locks Down Country

CDC, Alissa Eckert, Dan Higgins

Symptoms of the disease

Risk perception and awareness

Containment and control measures

↓

Adaptive behaviors

Impact on **network dynamics**

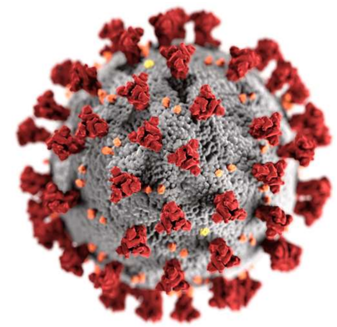
↓

Adaptive Temporal Networks

COVID-19 Pandemic

→ **Lockdowns and restrictions to mobility**

Stop network evolution and link formation



CDC, Alissa Eckert, Dan Higgins

Contact tracing

→ **Contact Tracing dynamics**

Break chains of infection without stopping network evolution → **Manual and Digital CT**

Contacts exploration over interaction dynamics

↓

Isolation of infected contacts affects epidemic spreading →

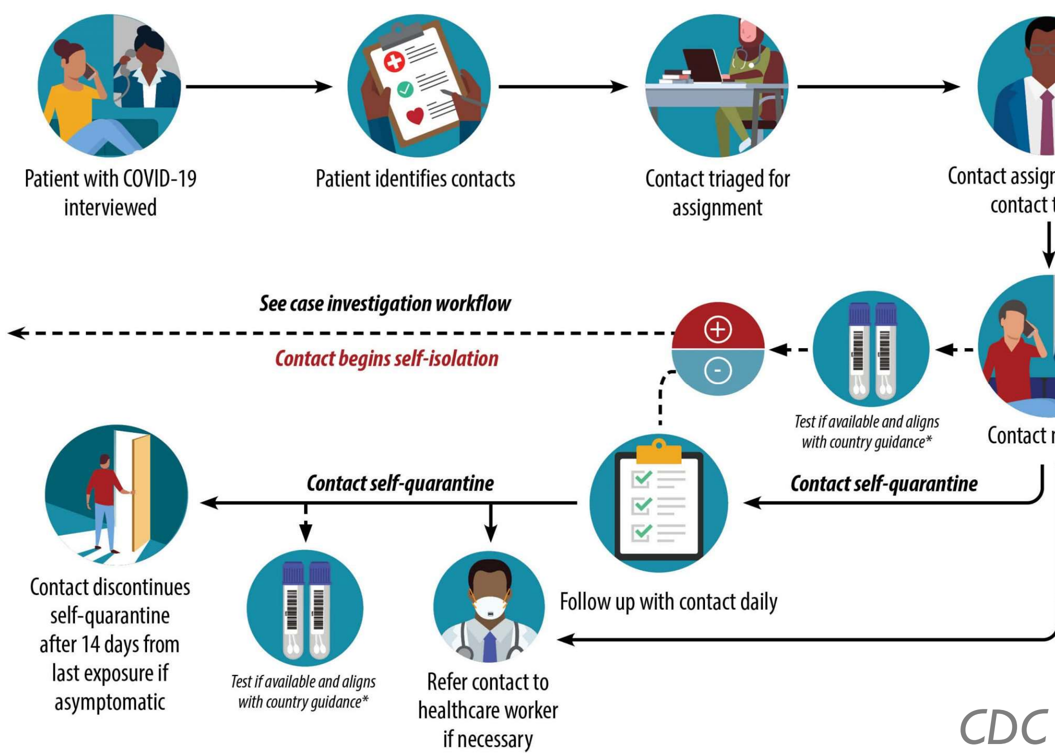
Network Evolution
Epidemic spreading
Contact tracing process

Coupled dynamics

Effective control measures increase the threshold

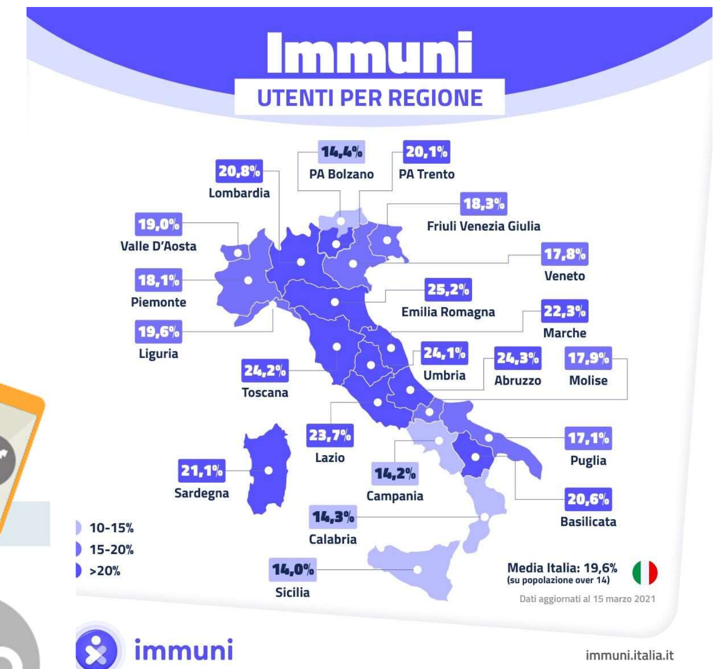
Contact tracing protocols

Manual contact tracing



- Delays in tracing and isolation
- Individual memory ε

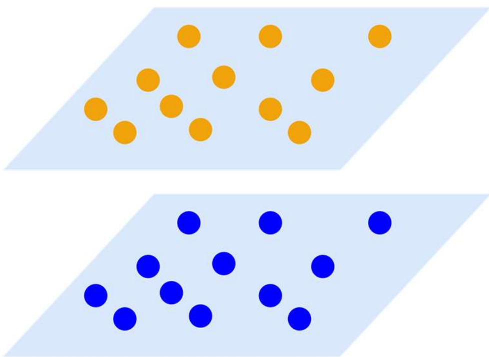
Digital contact tracing



- Small delays
- Adoption level f

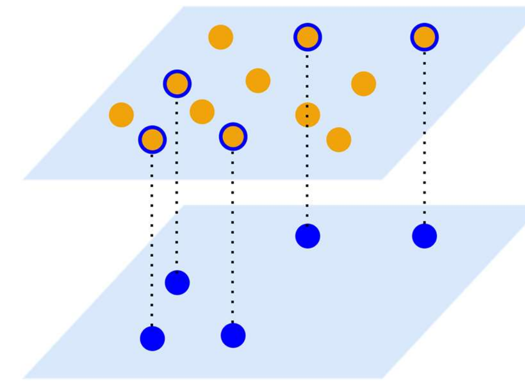
Intrinsic difference in contacts sampling

Effects of stochasticity



Intrinsic difference in contacts exploration and sampling

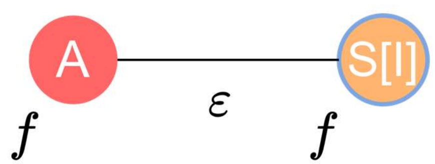
$$\boxed{\varepsilon = f^2}$$



Manual CT = Stochastic sampling
All links are traceable
Annealed

Digital CT = Localized sampling
Prearranged traceable links
Quenched

Two nodes interaction

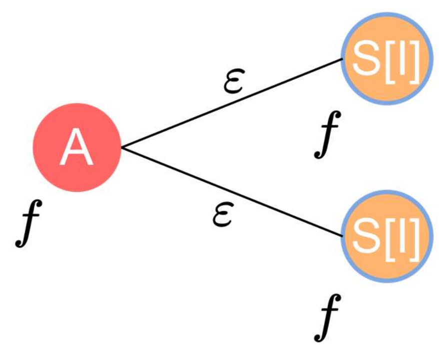


Manual CT: ε

App-based CT: f^2

→ Same probability to trace the link

Three nodes interaction

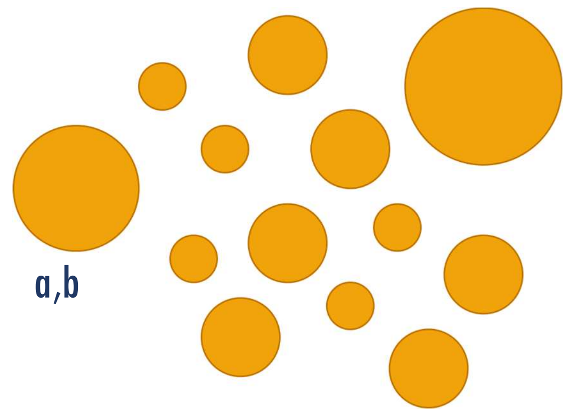


Manual CT: $1-(1-\varepsilon)^2 = 2f^2 - f^4$

App-based CT: $f(1-(1-f)^2) = 2f^2 - f^3$

Always higher probability for manual CT

Activity-Driven Network

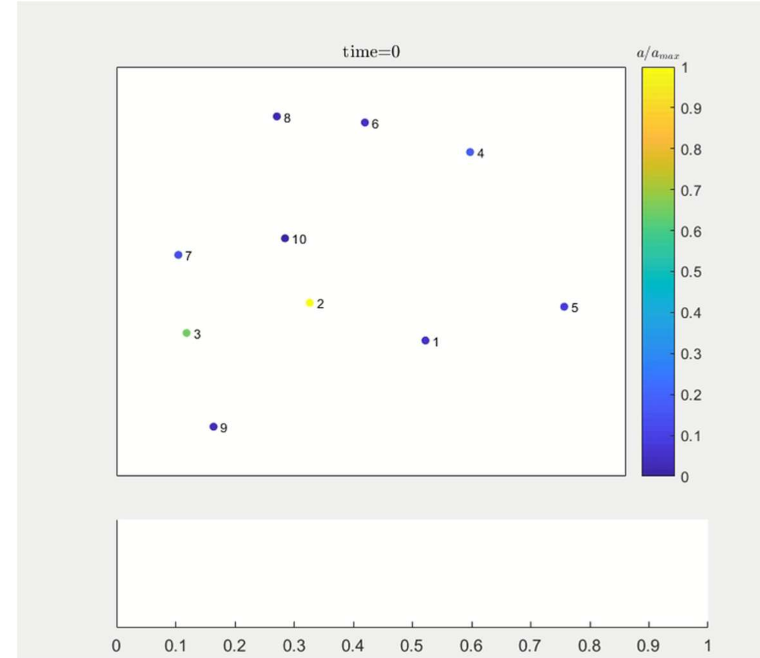


Activity-Driven with attractiveness

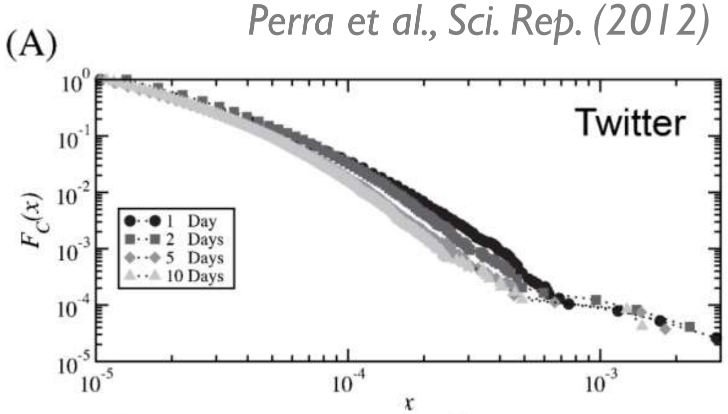
a - activity of node → Poissonian dynamics

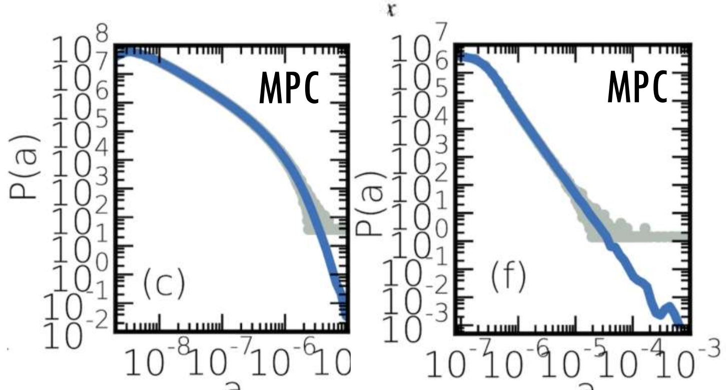
b - attractiveness of a node → Probability to contact a node ∝ b

$\rho(a,b)$ joint distribution



Perra et al., Sci. Rep. (2012)





Karsai et al., Sci. Rep. (2014)

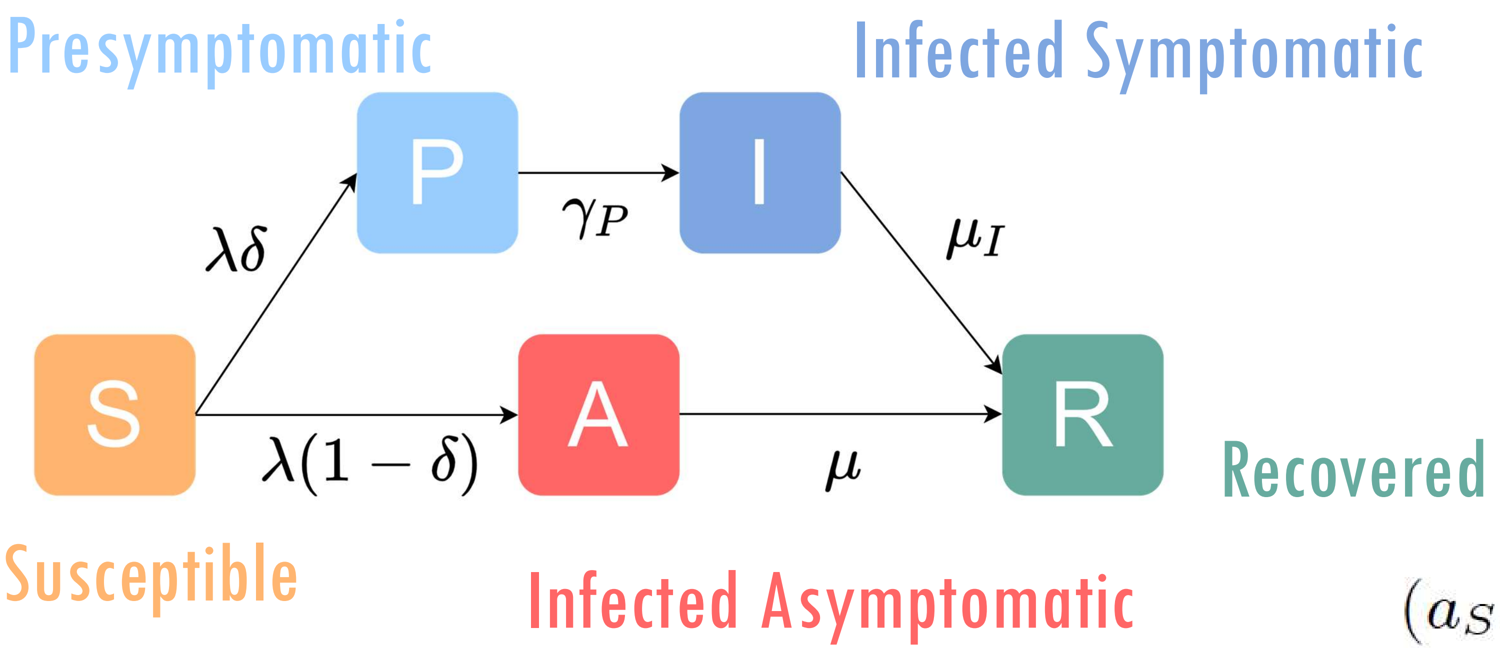
"Heavy tails", **heterogeneities,** activity and attractiveness

correlations → $\rho(a,b) \sim a^{-(\nu+1)}\delta(b-ca)$ $\quad \nu \in [0.5, 1.5]$

→ Hubs and **superspreaders**

Adaptive behavior with a change in activity and attractiveness

Epidemic model for COVID-19



Covid-19 parameters

World Health Organization

$$\tau = 1/\mu = 14\,days$$

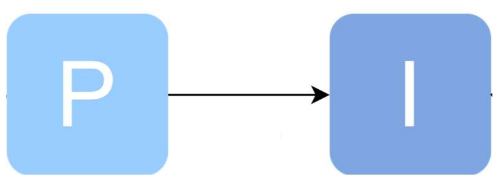
$$(1-\delta) = 43\%$$

$$\tau_P = 1/\gamma_P = 1.5\,days$$

$$\mu_I = \gamma_P\mu/(\gamma_P - \mu)$$

$$(a_S, b_S) = (a_P, b_P) = (a_A, b_A) = (a_R, b_R)$$

↓ Asymptomatic/Presymptomatic transmissions



→ Immediately isolated

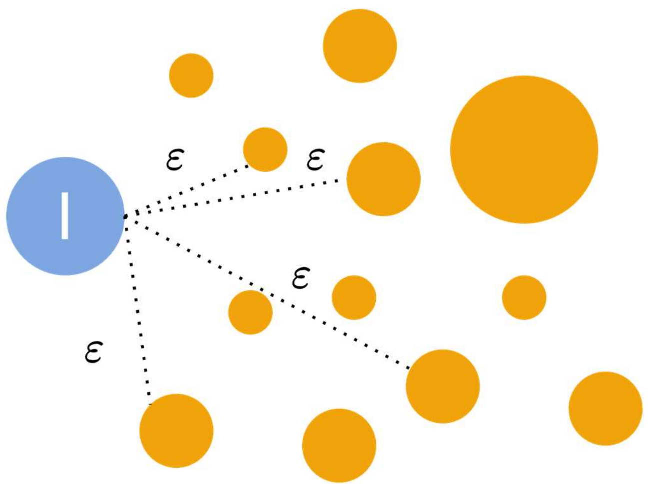
$$(a_I, b_I) = (0, 0)$$

→ Index case

Contact tracing

Isolation of asymptomatic contacts

Contact Tracing (CT)



Manual CT

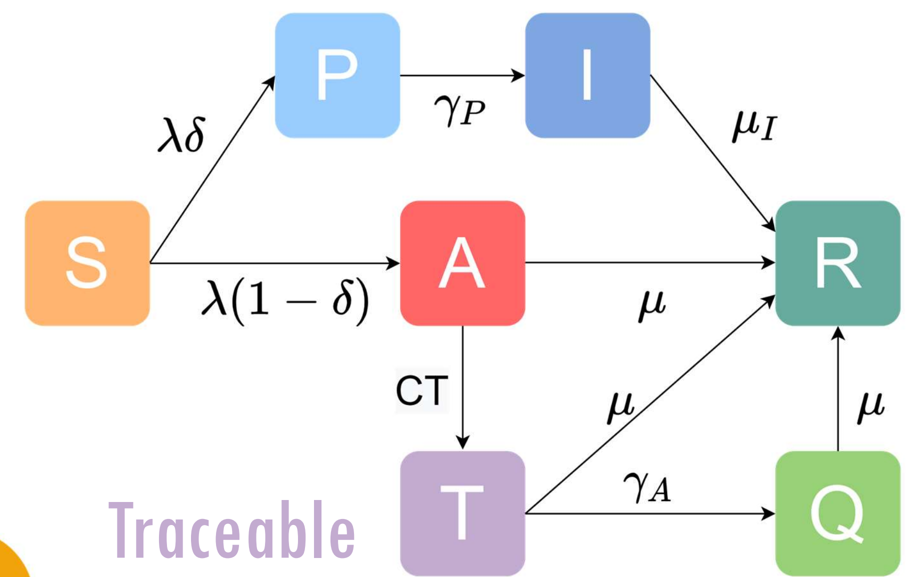
All contacts traced with prob. $\varepsilon(a_S)$

$\tau_C \longrightarrow$ Delay in tracing and isolation

$$\boxed{(a_T, b_T) = (a_S, b_S)}$$

$$\boxed{(a_Q, b_Q) = (0, 0)}$$

$$\gamma_A = 1/(\tau_C + \tau_P)$$



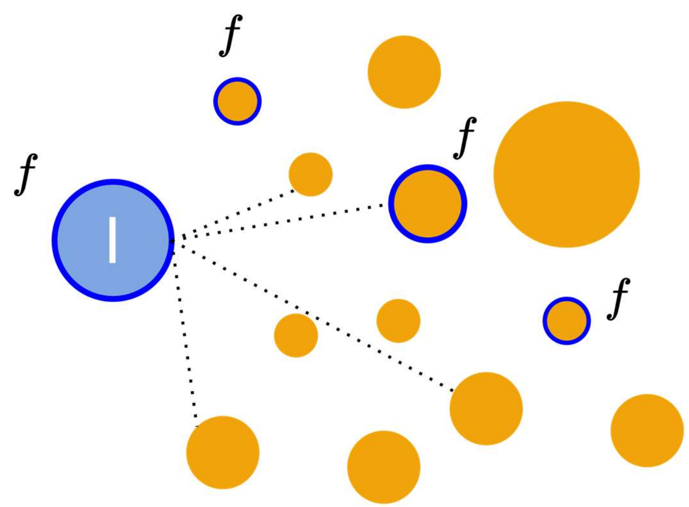
Traceable Asymptomatic

Isolated Asymptomatic

Digital CT

$f(a_S)$ prob. to download the app

$\tau_C = 0 \longrightarrow$ Instantaneous tracing



Mean-field equations – Manual CT

Activity-attractiveness based mean-field approach → Network evolution, Epidemic spreading, CT dynamics

$$\begin{aligned}\partial_t P_{a_S,b_S}(t) = {} & -\gamma_P P_{a_S,b_S}(t) + \lambda a_S(1 - I_{a_S,b_S}(t) - P_{a_S,b_S}(t))\frac{\delta\overline{b_S P}(t) + (1-\delta)[\overline{b_S T}(t) + \overline{b_S A}(t)]}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)} \\ & + \lambda b_S(1 - I_{a_S,b_S}(t) - P_{a_S,b_S}(t))\frac{\delta\overline{a_S P}(t) + (1-\delta)[\overline{a_S T}(t) + \overline{a_S A}(t)]}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)}\end{aligned}$$

$$\partial_t I_{a_S,b_S}(t) = -\mu_I I_{a_S,b_S}(t) + \gamma_P P_{a_S,b_S}(t) \qquad \partial_t Q_{a_S,b_S}(t) = -\mu Q_{a_S,b_S}(t) + \gamma_A T_{a_S,b_S}(t)$$

$$\begin{aligned}\partial_t A_{a_S,b_S}(t) = {} & -\mu A_{a_S,b_S}(t) + \lambda a_S(1 - A_{a_S,b_S}(t) - T_{a_S,b_S}(t) - Q_{a_S,b_S}(t))\frac{\delta[\overline{b_S P}(t) - \overline{\varepsilon b_S P}(t)] + (1-\delta)[\overline{b_S T}(t) + \overline{b_S A}(t)]}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)} \\ & + \lambda b_S(1 - A_{a_S,b_S}(t) - T_{a_S,b_S}(t) - Q_{a_S,b_S}(t))\frac{\delta[\overline{a_S P}(t) - \overline{\varepsilon a_S P}(t)] + (1-\delta)[\overline{a_S T}(t) + \overline{a_S A}(t)]}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)} \\ & - \lambda a_S \delta A_{a_S,b_S}(t)\frac{\overline{\varepsilon b_S} - \overline{\varepsilon b_S I}(t) - \overline{\varepsilon b_S P}(t)}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)} - \lambda b_S \delta A_{a_S,b_S}(t)\frac{\overline{\varepsilon a_S} - \overline{\varepsilon a_S I}(t) - \overline{\varepsilon a_S P}(t)}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)}\end{aligned}$$

CT terms

$$\begin{aligned}\partial_t T_{a_S,b_S}(t) = {} & -(\mu + \gamma_A)T_{a_S,b_S}(t) + \lambda b_S(1 - A_{a_S,b_S}(t) - T_{a_S,b_S}(t) - Q_{a_S,b_S}(t))\frac{\delta\overline{\varepsilon a_S P}(t)}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)} \\ & + \lambda a_S(1 - A_{a_S,b_S}(t) - T_{a_S,b_S}(t) - Q_{a_S,b_S}(t))\frac{\delta\overline{\varepsilon b_S P}(t)}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)} \\ & + \lambda b_S \delta A_{a_S,b_S}(t)\frac{\overline{\varepsilon a_S} - \overline{\varepsilon a_S I}(t) - \overline{\varepsilon a_S P}(t)}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)} \\ & + \lambda a_S \delta A_{a_S,b_S}(t)\frac{\overline{\varepsilon b_S} - \overline{\varepsilon b_S I}(t) - \overline{\varepsilon b_S P}(t)}{\overline{b_S} - (1-\delta)\overline{b_S Q}(t) - \delta\overline{b_S I}(t)}\end{aligned}$$

Mean-field equations – Manual CT

$$\rho(a_S, b_S) = \rho_S(a_S)\delta(b_S - a_S)$$

Realistic activity/attractiveness

positive correlation

$$\partial_t \overline{I}(t) = -\mu_I \overline{I}(t) + \gamma_P \overline{P}(t)$$

$$\partial_t \overline{P}(t) = -\gamma_P \overline{P}(t) + 2\lambda[\delta \overline{a_S P}(t) + (1-\delta)(\overline{a_S T}(t) + \overline{a_S A}(t))]$$

$$\partial_t \overline{Q}(t) = -\mu \overline{Q}(t) + \gamma_A \overline{T}(t)$$

$$\partial_t \overline{T}(t) = -(\mu + \gamma_A)\overline{T}(t) + 2\lambda\delta \overline{\varepsilon a_S P}(t) + 2\lambda\delta \overline{a_S A}(t) \frac{\overline{\varepsilon a_S}}{\overline{a_S}}$$

$$\partial_t \overline{A}(t) = -\mu \overline{A}(t) + 2\lambda[\delta(\overline{a_S P}(t) - \overline{\varepsilon a_S P}(t)) + (1-\delta)(\overline{a_S T}(t) + \overline{a_S A}(t))] - 2\lambda\delta \overline{a_S A}(t) \frac{\overline{\varepsilon a_S}}{\overline{a_S}}$$

$$\partial_t \overline{a_S P}(t) = -\gamma_P \overline{a_S P}(t) + 2\lambda \frac{\overline{a_S^2}}{\overline{a_S}}[\delta \overline{a_S P}(t) + (1-\delta)(\overline{a_S T}(t) + \overline{a_S A}(t))]$$

$$\partial_t \overline{\varepsilon a_S P}(t) = -\gamma_P \overline{\varepsilon a_S P}(t) + 2\lambda \frac{\overline{\varepsilon a_S^2}}{\overline{a_S}}[\delta \overline{a_S P}(t) + (1-\delta)(\overline{a_S T}(t) + \overline{a_S A}(t))]$$

$$\partial_t \overline{a_S T}(t) = -(\mu + \gamma_A)\overline{a_S T}(t) + 2\lambda\delta \frac{\overline{a_S^2}}{\overline{a_S}} \overline{\varepsilon a_S P}(t) + 2\lambda\delta \overline{a_S^2 A}(t) \frac{\overline{\varepsilon a_S}}{\overline{a_S}}$$

$$\partial_t \overline{a_S A}(t) = -\mu \overline{a_S A}(t) + 2\lambda \frac{\overline{a_S^2}}{\overline{a_S}}[\delta(\overline{a_S P}(t) - \overline{\varepsilon a_S P}(t)) + (1-\delta)(\overline{a_S T}(t) + \overline{a_S A}(t))] - 2\lambda\delta \overline{a_S^2 A}(t) \frac{\overline{\varepsilon a_S}}{\overline{a_S}}$$

→ Set of linearized equations around the absorbing state

→ Linear stability analysis of absorbing state

→ Analogous for Digital CT with additional compartments

$\boxed{\text{Epidemic threshold}}$

Epidemic thresholds

Manual CT

$$8r^3\delta^2(1-\delta)\frac{\overline{\varepsilon a_S^2}\,\overline{\varepsilon a_S}}{\overline{a_S}}\,\overline{\frac{a_S^3}{1+2r\delta a_S\frac{\overline{\varepsilon a_S}}{\overline{a_S}}}}\,\frac{\gamma_A}{\mu} - 4r^2\delta(1-\delta)\left[\overline{\varepsilon a_S^2}\,\overline{a_S^2} + \frac{\gamma_P}{\mu}\overline{\varepsilon a_S}\,\overline{\frac{a_S^3}{1+2r\delta a_S\frac{\overline{\varepsilon a_S}}{\overline{a_S}}}}\right]\frac{\gamma_A}{\mu}$$
$$+ 2r\overline{a_S^2}\,\overline{a_S}\left(\frac{\gamma_A}{\mu}+1\right)\left(\delta+\frac{\gamma_P}{\mu}(1-\delta)\right) - \overline{a_S}^2\frac{\gamma_P}{\mu}\left(\frac{\gamma_A}{\mu}+1\right) = 0$$

Digital CT

$$8r^3\delta^2(1-\delta)\frac{\overline{f a_S^2}\,\overline{f a_S}}{\overline{a_S}}\,\overline{\frac{f a_S^3}{1+2r\delta a_S\frac{\overline{f a_S}}{\overline{a_S}}}}\,\frac{\gamma_P}{\mu} - 4r^2\delta(1-\delta)\left[\overline{f a_S^2}^2 + \frac{\gamma_P}{\mu}\overline{f a_S}\,\overline{\frac{f a_S^3}{1+2r\delta a_S\frac{\overline{f a_S}}{\overline{a_S}}}}\right]\frac{\gamma_P}{\mu}$$
$$+ 2r\overline{a_S^2}\,\overline{a_S}\left(\frac{\gamma_P}{\mu}+1\right)\left(\delta+\frac{\gamma_P}{\mu}(1-\delta)\right) - \overline{a_S}^2\frac{\gamma_P}{\mu}\left(\frac{\gamma_P}{\mu}+1\right) = 0$$

Exact analytical closed relations $\longrightarrow$ Hold for arbitrary ρ_S, $\varepsilon(a_S)$, $f(a_S)$

$\longrightarrow$ Solvable numerically $\longrightarrow$ Epidemic threshold

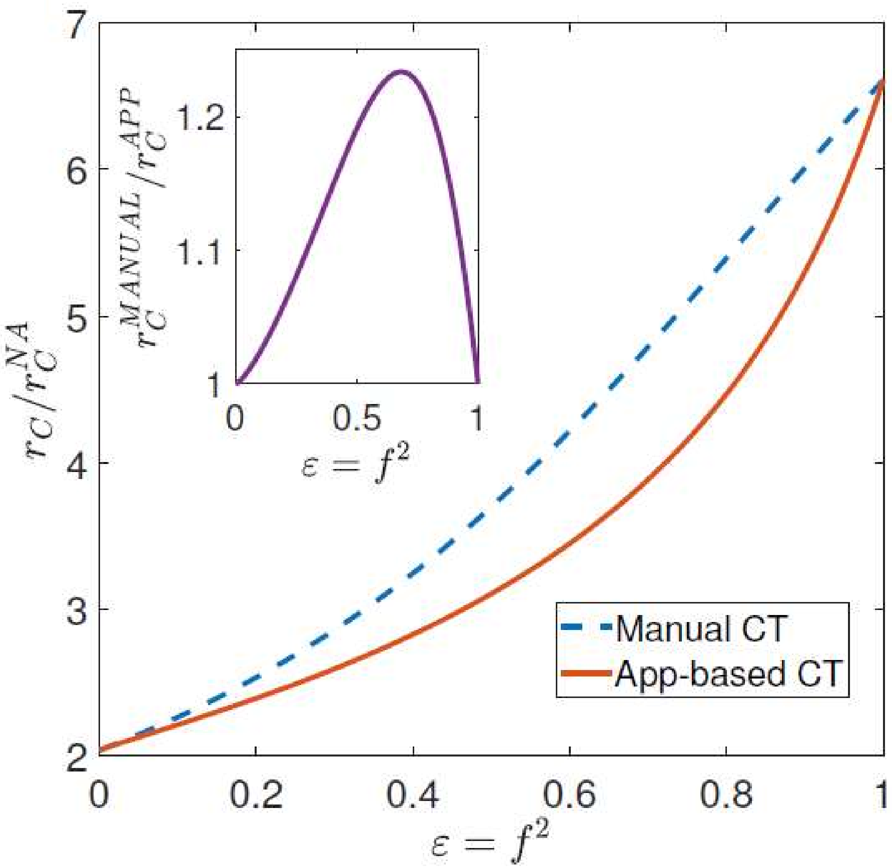
Effects of stochasticity

$$\rho(a_S, b_S) = \delta(a_S - a)\delta(b_S - b)$$

Homogeneous population

$$r_C^{APP} = r_C^{NA} \frac{2\frac{\gamma_P}{\mu}}{\delta + (1-\delta-f\delta)\frac{\gamma_P}{\mu} + \sqrt{(\delta + (1-\delta-f\delta)\frac{\gamma_P}{\mu})^2 + 4\delta f \frac{\gamma_P}{\mu}(\delta + \frac{\gamma_P}{\mu}(1-f)(1-\delta))}}$$

$$r_C^{MANUAL} = r_C^{NA} \frac{2\frac{\gamma_P}{\mu}}{\delta + (1-\delta-\varepsilon\delta)\frac{\gamma_P}{\mu} + \sqrt{(\delta + (1-\delta-\varepsilon\delta)\frac{\gamma_P}{\mu})^2 + 4\delta^2\varepsilon\frac{\gamma_P}{\mu}}}$$



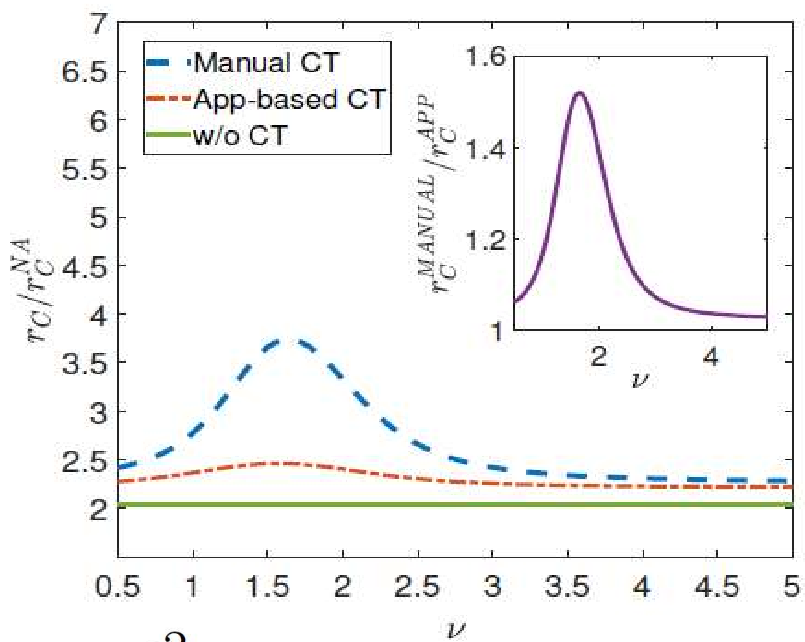
Increase in the epidemic threshold ⟶ CT effectiveness

$\varepsilon = f^2$ ⟶ Same probability to trace a contact

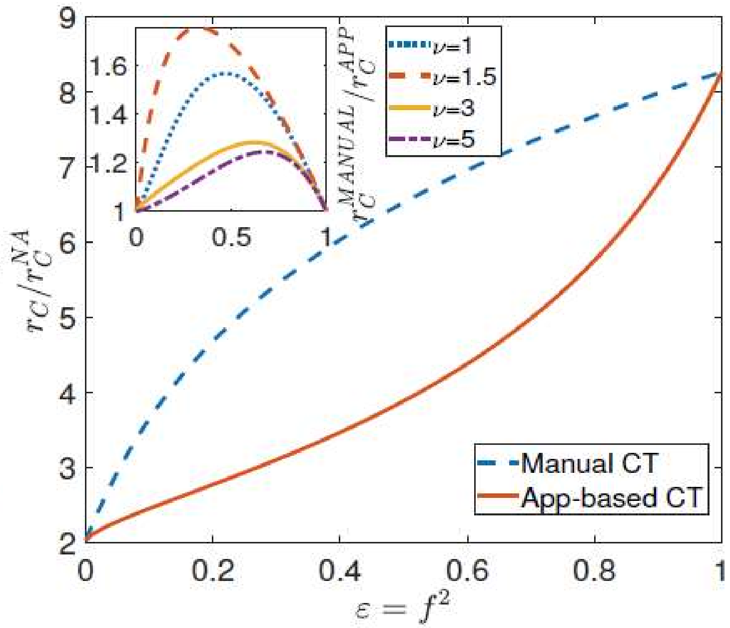
Manual CT is more effective in increasing the epidemic threshold ⟶ Intrinsic difference in contact exploration

Effects of heterogeneity

$$\rho_S(a_S, b_S) \sim a_S^{-(\nu+1)}\delta(b_S - a_S)$$



$\varepsilon = f^2 = 0.1$



$\nu = 1.5$

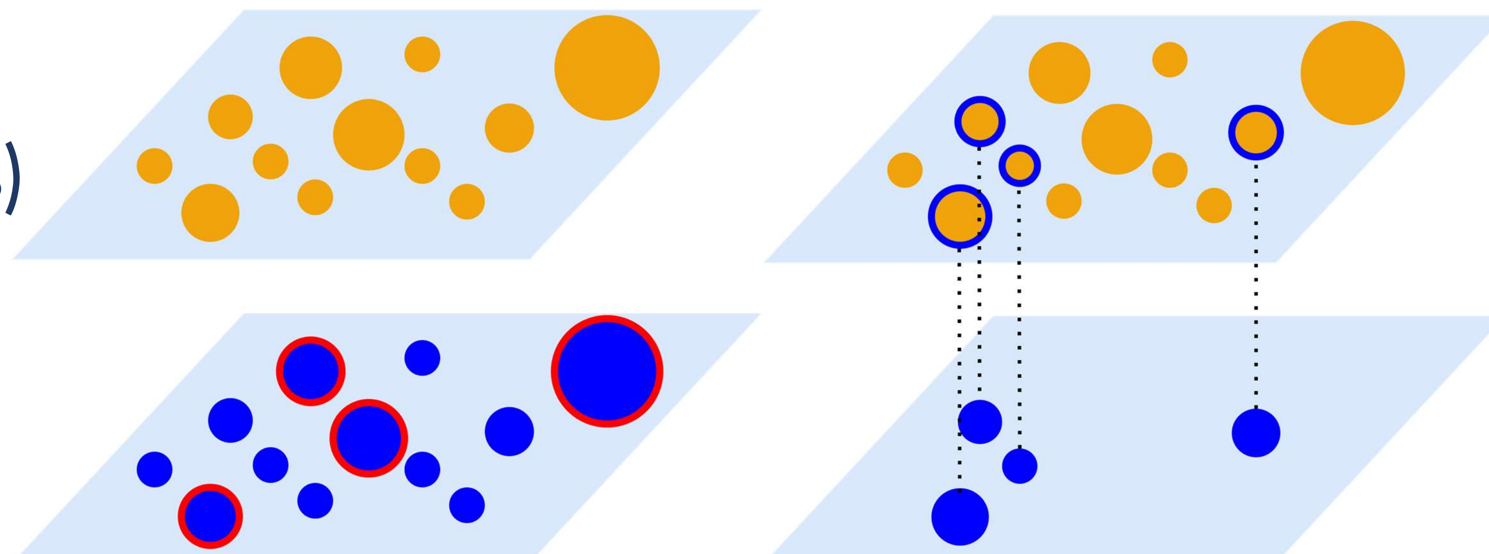
→ Manual CT much more effective in heterogeneous population

Heterogeneities amplify the intrinsic difference

→ Tracing of super-spreaders (hubs)

→ Hubs without app are not traced in digital CT

→ Hubs easily traced in manual CT



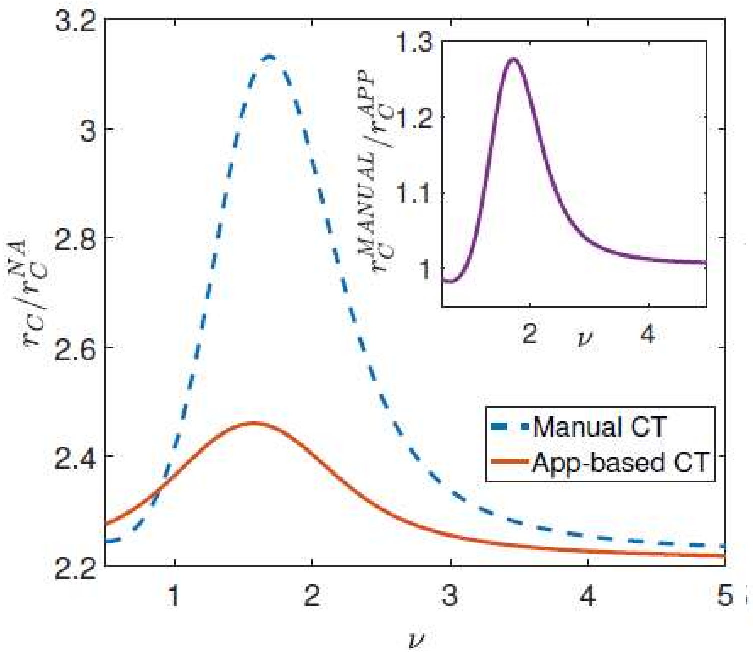
Manual CT **Digital CT**

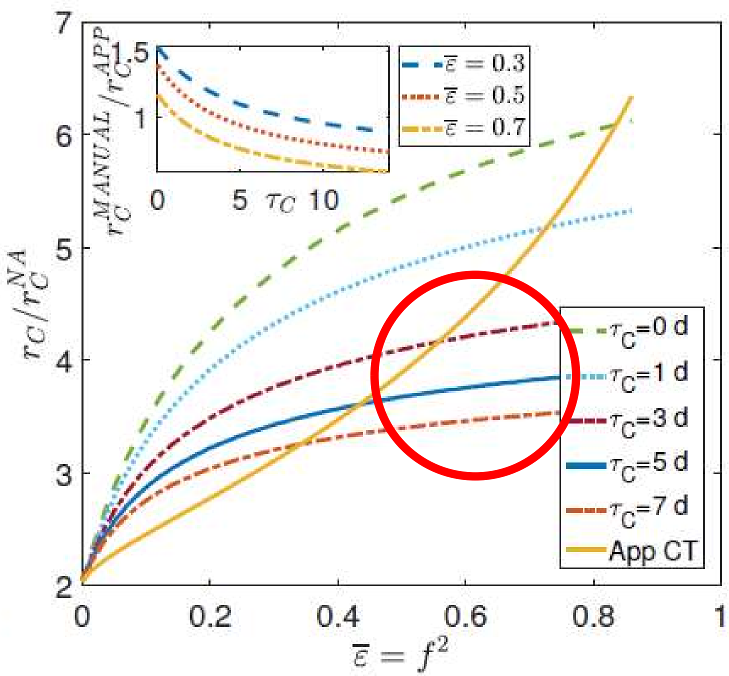
Effects of delays and limited scalability

$$\varepsilon(a_S) = \begin{cases} \varepsilon^*, & \text{if}\, a_S \leq a^* \\ \varepsilon^* \frac{a^*}{a_S} = \frac{k_c}{2T_{CT} a_S}, & \text{if}\, a_S > a^* \end{cases}$$

→ Limited scalability

τ_C → Delay in manual CT





$\tau_C = 3\, days \quad \overline{\varepsilon} = f^2 = 0.1 \qquad \nu = 1.5$

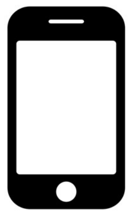
→ Differences reduced by delays and limited scalability

→ Strong delays and high app adoption rate to obtain digital CT more effective than manual CT

The intrinsic advantage of manual CT, amplified by heterogeneities, holds also in the presence of limited scalability and delays

Hybrid CT

Hybrid CT protocols → Nodes traced both digitally or manually

 & 

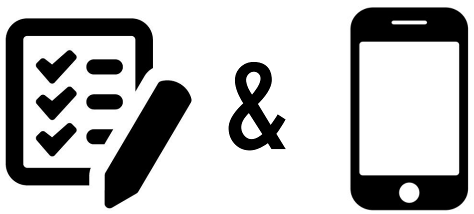
→ Role of Manual and Digital CT in realistic hybrid protocols

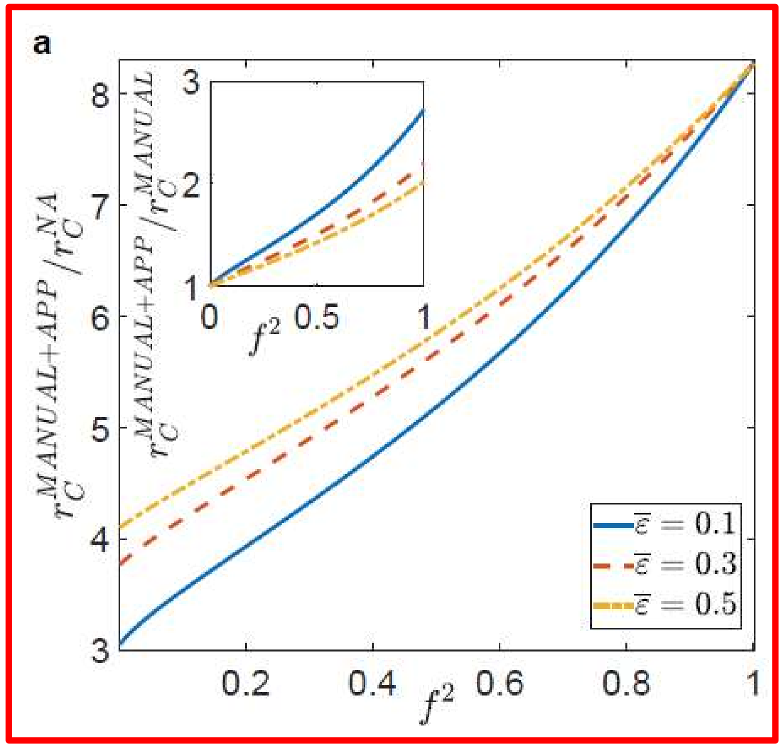
→ Additional compartments and analogous (but more complex) mean-field equations

Hybrid CT

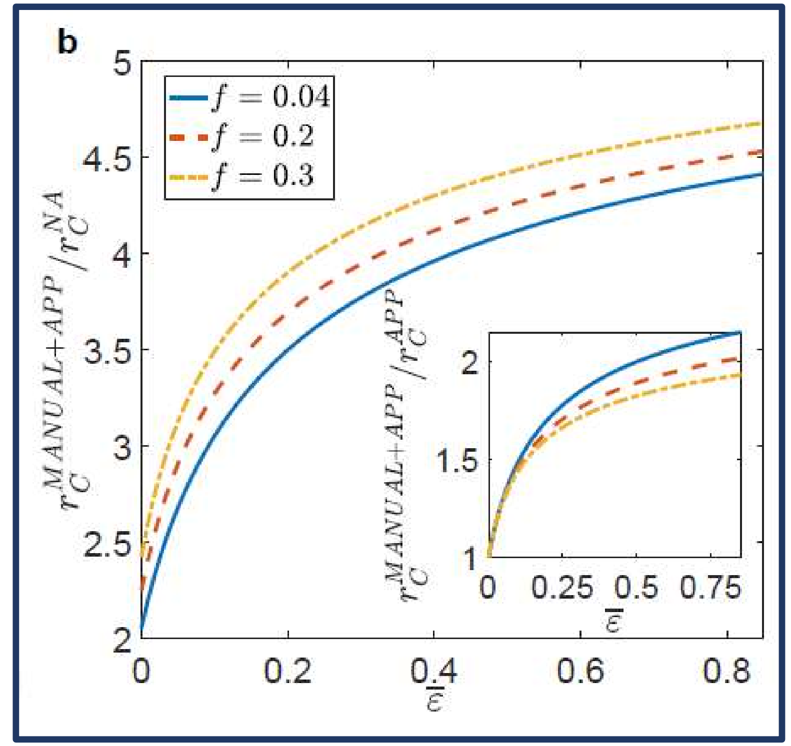
$$\rho_S(a_S, b_S) \sim a_S^{-(\nu+1)} \delta(b_S - a_S)$$

Hybrid CT protocols





Effects of Digital CT



Effects of Manual CT

→ Manual CT strongly increases the threshold for realistic app adoption levels

→ Digital CT increases more slowly with adoption rate for realistic Manual CT implementation

→ **Manual CT must play a prominent role in any CT strategy**

Conclusions

- We model epidemic spreading on adaptive temporal networks with contact tracing (CT)
  - ⟶ Coupled dynamics
  - ⟶ Dynamical phase transition on temporal networks
- Manual CT is more effective than digital CT in increasing the epidemic threshold
  - ⟶ Intrinsic difference due to contacts sampling (annealed vs. quenched)
  - ⟶ Heterogeneities amplify differences
- Manual CT must play a prominent role in any hybrid CT strategy to mitigate epidemic spreading
- **Future perspective:** optimize integration of protocols (policy for resources allocation), multiple steps CT

M. Mancastroppa, C. Castellano, A. Vezzani, R. Burioni, ***"Stochastic sampling effects favor manual over digital contact tracing"***, *Nature Communications* **12**, 1919 (2021)

M. Mancastroppa, R. Burioni, V. Colizza, A. Vezzani, ***"Active and inactive quarantine in epidemic spreading on adaptive activity-driven networks"***, *Physical Review E* **102**, 020301(R) (2020)