Thermoelectric Conductivities in Charge Density Waves States from Hydrodynamics

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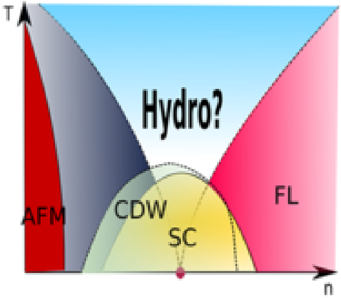
Motivation: cuprates

- Cuprates are a class of $2D$ high-T_c superconductors (CuO_2 layers)
- Strange metal phase is *strongly coupled*
- Near optimal doping $\Longrightarrow$ CDW order [Peng, 2018] $\Longrightarrow$ broken translation invariance



Why Hydrodynamics?

Weakly coupled metal phases described by Fermi liquid theory $\Longleftrightarrow$ Wiedemann–Franz law

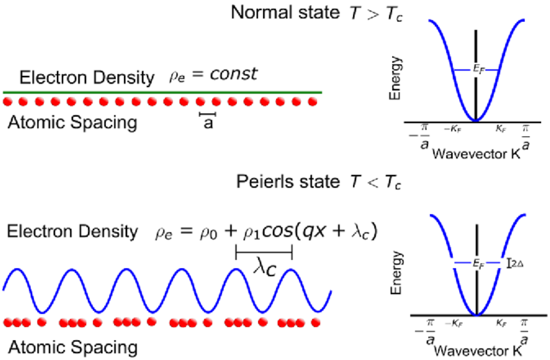
$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2}{3}$$

Wiedemann–Franz law violation $\Longrightarrow$ non-metallic strongly correlated system $\Longrightarrow$ no well defined quasi-particles (non-FL) $\Longrightarrow$ *Hydrodynamics*

Systems with non-FL phases

Graphene [Crossno, 2016], Weyl semimetals [Gooth, 2017], High-T_c superconductors [Grissonnanche, 2015], ...

Charge Density Waves



To describe cuprate phases, in summary:

- Strongly coupled $\Longrightarrow$ Hydrodynamics
- Charge Density Waves $\Longrightarrow$ Broken translation symmetry

Symmetry breaking

System with scalar operators O^I with non-zero v.e.v. $\langle O^I \rangle = x^I$ [Delacrétaz, 2015]. This breaks Lorentz boost (K_i), rotations (J_i) and translations (P_i).

To recover isotropy and homogeneity in the EFT: internal translational (Q_i) and rotational ($\widetilde{Q}_i$) symmetries on the fields must be broken

$$O^I \longrightarrow O^I + a^I \qquad O^I \longrightarrow SO(2) \cdot O^I$$

Ground state breaks 5 generators (2 K_i, 2 Q_i and 1 $\widetilde{Q}_i$)

$$\bar{P}_0 = P_0, \qquad \bar{P}_i = P_i + Q_i, \qquad \bar{J}_i = J_i + \widetilde{Q}_i$$

inverse-Higgs constraints $\Longrightarrow$ only 2 Goldstone modes are independent

Hydrodynamics

Hydrodynamics

A many-body EFT at non-zero temperature in the long wavelength/time scale regime that describes the dynamics of conserved charges close to thermal equilibrium

Two main ingredients:

- Equations of hydrodynamics

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \nabla_\mu J^\mu = 0$$

- Constitutive relations, i.e. expressions for $T^{\mu\nu}$ and J^μ w.r.t. hydrodynamic fields (μ, T and u^μ) and sources ($g_{\mu\nu}, F_{\mu\nu}$) + constraints (symmetries and 2nd law of thermodynamics) + gradient expansion

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \mathcal{O}(\partial) \qquad J^\mu = q u^\mu + \mathcal{O}(\partial)$$

Viscoelastic hydrodynamics - spontaneous case (SC)

Define $e^I_\mu = \partial_\mu O^I$, then the most general constitutive relations allowed by symmetries [Armas, 2020]

$$J^\mu = qu^\mu - P^{I\mu}\sigma^q_{IJ}P^{J\nu}\left(T\partial_\nu\frac{\mu}{T} - E_\nu\right) - P^{I\mu}\gamma_{IJ}u^\nu e^J_\nu$$

$$T^{\mu\nu} = (\epsilon + P)\,u^\mu u^\nu + Pg^{\mu\nu} - r_{IJ}e^{I\mu}e^{J\nu} - P^{I(\mu}P^{J\nu)}\eta_{IJKL}P^{K(\rho}P^{L\sigma)}\nabla_\rho u_\sigma$$

The EoM for the fields are

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda}J_\lambda - K^{\text{ext}}_I e^{I\nu} \qquad \nabla_\mu J^\mu = 0$$

and also the configuration equation, i.e. EoM for O^I

$$\sigma^\phi_{IJ}u^\mu e^I_\mu - \gamma_{JK}P^{K\mu}\left(T\partial_\mu\frac{\mu}{T} - E_\mu\right) + \nabla_\mu\left(r_{JK}e^{K\mu}\right) = K^{\text{ext}}_J$$

Viscoelastic hydrodynamics - explicit case (EC)

In the pseudo-spontaneous case the Goldstone bosons acquire a small mass $\Longrightarrow$ momentum is not conserved

$$\nabla_\mu T^{\mu i} = F^{i\lambda} J_\lambda - K_I^{\text{ext}} e^{Ii} + \omega_0^2 \chi_{\pi\pi} O^I \delta^{Ii}$$

We can also add a relaxation term in the configuration equation $\sim \Omega_{IJ} O^I$

Modulo these corrections, the EoM and the constitutive relations are the same as in the spontaneous case

Green functions computation

Decompose the transport coefficients w.r.t. the external B field

$$(\gamma, \sigma^q, \sigma^\phi)_{IJ} = (\gamma, \sigma^q, \sigma^\phi)_L \delta_{IJ} + (\gamma, \sigma^q, \sigma^\phi)_H F_{IJ}$$

Linearize around equilibrium (vanishing sources)

$$T \to T + \delta T \qquad \mu \to \mu + \delta\mu \qquad u^\mu \to (1,0) + \delta u^\mu$$
$$O^I \to O^I - \delta O^I \qquad F^{0i} \to \delta E^i \qquad g_{\mu\nu} \to \eta_{\mu\nu} + \delta h_{\mu\nu}$$
$$K_I^{\text{ext}} \to \delta K_I^{\text{ext}}$$

Linear response theory (Martin-Kadanoff) $\Longrightarrow$ Green functions

$$\langle J^i J^j \rangle \qquad \langle J^i Q^j \rangle \qquad \langle Q^i Q^j \rangle$$
$$\langle O^i O^j \rangle \qquad \langle O^i J^j \rangle \qquad \langle O^i Q^j \rangle$$

Results

Expressions for the correlators are very ugly! Nonetheless

- Green functions $\Longrightarrow$ AC conductivities both in SC and EC
- Perfect match with Ward Identities
- Transport coefficients in terms of DC conductivities
- EC, from Onsager relations new constraint

$$\Omega_{IJ} = \omega_0^2 \chi_{\pi\pi} \delta_{IJ}$$

- Good match with holographic computation
- At $B = 0$ a new gapped pole $\omega \sim -i/P_I$ currently under study

Conclusions

We computed the AC conductivities for a system with broken translations in $d = 2 + 1$ dimensions and in the presence of a constant background magnetic field B.

- It is possible to check the theoretical predictions with experimental tests
- Generalize the result to further symmetry breaking patterns
- Work in $d = 3 + 1$ in order to consider chiral anomaly (dissipation terms could help avoid known runaway behaviour)

Thanks for the attention!