Linearized Geodesic Light-Cone Gauge

Based on JCAP02(2021)014

In collaboration with G. Fanizza, G. Marozzi, G. Schiaffino

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Outline

- Introduction
- GLC Perturbations
- GLC Gauge
- Observables
- Final remarks

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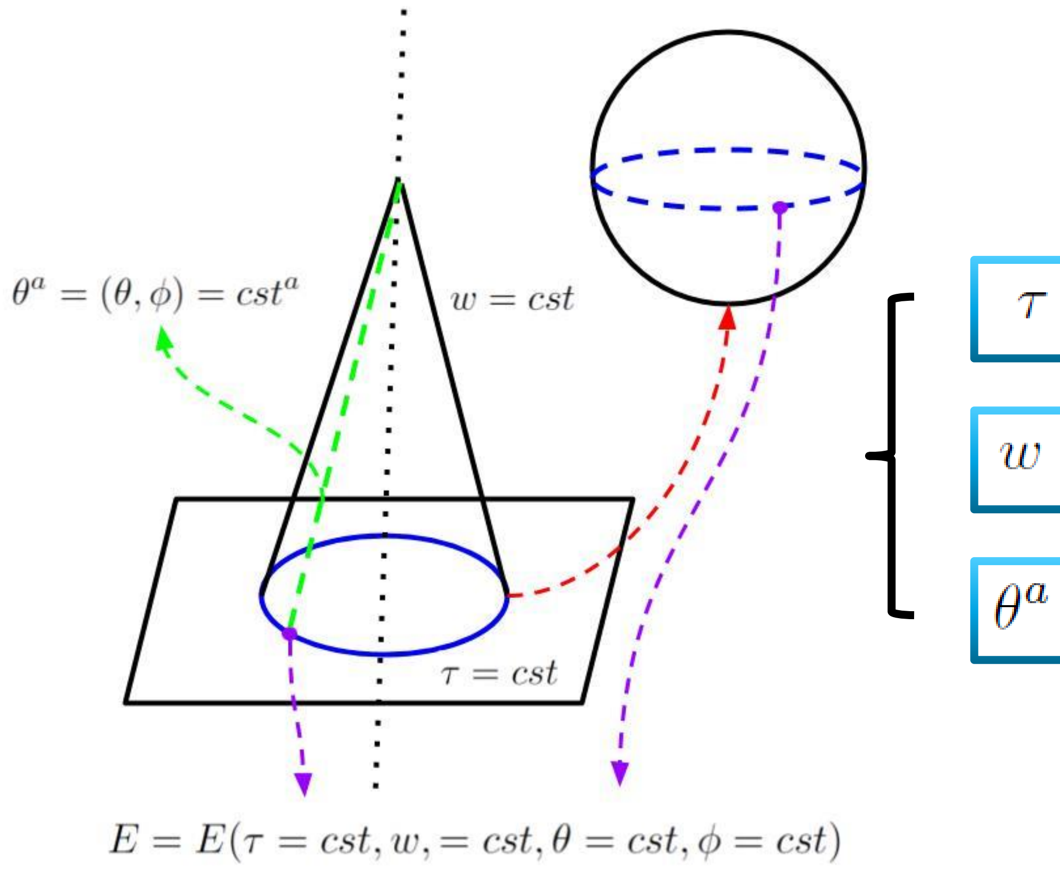
Introduction

- Non-linear GLC gauge



$\tau = cst$ → Comoving Observer Sinchronous time

$w = cst$ → Past Light Cone

$\theta^a = cst$ → Light Geodesics

Introduction

- Non-linear GLC gauge

$$\boxed{ds^2 = -2\Upsilon dw d\tau + \Upsilon^2 dw^2 + \gamma_{ab}\left(d\tilde{\theta}^a - \tilde{U}^a dw\right)\left(d\tilde{\theta}^b - \tilde{U}^b dw\right)}$$

- Simplify the light-like geodesics

$$\boxed{u_\mu = -\partial_\mu \tau}$$

$$\boxed{(1+z) = \frac{\Upsilon_o}{\Upsilon_s}}$$

$$\boxed{k^\mu = \omega \delta^\mu_\tau}$$

$$\boxed{d_A^2 = \frac{\sqrt{\gamma}}{\left(\frac{det\dot{\gamma}_{ab}}{4\sqrt{\gamma}}\right)_o}}$$

Exactly solves the Jacobi Map

→ M. Gasperini, G. Marozzi, F. Nugier, G. Veneziano, JCAP07(2011)008

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Geometrical Construction → P. Fleury, F. Nugier, G. Fanizza, JCAP06(2016)008

Introduction

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Exactly solves the Jacobi Map

Jacobi Map → G. Fanizza, M. Gasperini, G. Marozzi, G. Veneziano, JCAP11(2013)019

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Introduction

- General procedure so far

Non-linear GLC Gauge → Cosmological Observables → Coordinate Transformation GLC → FLRW

- Our objective

Non-linear GLC Gauge → Cosmological Observables → GLC Linearized

In order to achieve this objective, we will build general perturbations in an isotropic/homogeneous GLC background, then we fix the GLC Gauge and present an equation for the gauge invariant angular distance-redshift relation.

→ G. Fanizza, G. Marozzi, MM, G. Schiaffino, JCAP02(2021)014

GLC Perturbations

$$\bar{f}_{\mu\nu} + \delta f_{\mu\nu} = a^2(\tau)\left\{\begin{pmatrix} 0 & -a^{-1} & \vec{0} \\ -a^{-1} & 1 & \vec{0} \\ \vec{0}^{\mathbf{T}} & \vec{0}^{\mathbf{T}} & \bar{\gamma}_{ab} \end{pmatrix} + \begin{pmatrix} L & M & V_b \\ M & N & U_b \\ V_a^{\mathbf{T}} & U_a^{\mathbf{T}} & \delta\gamma_{ab} \end{pmatrix}\right\}$$

- Scalar/Pseudo-Scalar Decomposition

$$V_a = r^2\left(D_a v + \tilde{D}_a \hat{v}\right)$$

$$U_a = r^2\left(D_a u + \tilde{D}_a \hat{u}\right)$$

$$\delta\gamma_{ab} = 2r^2\left(q_{ab}\nu + D_{ab}\mu + \tilde{D}_{ab}\hat{\mu}\right)$$

- **Gauge Freedom**

$$\tilde{x}^\mu = x^\mu + \xi^\mu$$

$$\delta \tilde{f}_{\mu\nu} = \delta f_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$$

$$\tilde{L} = L + \frac{2}{a}\partial_\tau \xi^w$$

$$\tilde{M} = M + \frac{1}{a} H \xi^\tau + \frac{1}{a}\partial_\tau \xi^\tau - \partial_\tau \xi^w + \frac{1}{a}\partial_w \xi^w$$

$$\tilde{N} = N - 2H\xi^\tau + 2\partial_w \left(\frac{\xi^\tau}{a} - \xi^w \right)$$

$$\tilde{V}_a = V_a + \frac{1}{a}\partial_a \xi^w - \bar{\gamma}_{ab}\partial_\tau \hat{\xi}^b$$

$$\tilde{U}_a = U_a + \partial_a \left(\frac{\xi^\tau}{a} - \xi^w \right) - \bar{\gamma}_{ab}\partial_w \hat{\xi}^b$$

$$\delta\tilde{\gamma}_{ab} = \delta\gamma_{ab} - \frac{1}{a^2}\xi^\tau \partial_\tau \left(a^2 \bar{\gamma}_{ab} \right) - \xi^w \partial_w \bar{\gamma}_{ab} - \left(\bar{\gamma}_{ac} D_b + \bar{\gamma}_{bc} D_a \right) \hat{\xi}^c$$

GLC Gauge Fixing

$$\begin{pmatrix} 0 & -\Upsilon & \vec{0} \\ -\Upsilon & \Upsilon^2 + U^2 & -U^a \\ \vec{0}^{\mathrm{T}} & -\left(U^a\right)^{\mathrm{T}} & \bar{\gamma}_{ab} \end{pmatrix} = a^2(\tau)\left\{\begin{pmatrix} 0 & -a^{-1} & \vec{0} \\ -a^{-1} & 1 & \vec{0} \\ \vec{0}^{\mathrm{T}} & \vec{0}^{\mathrm{T}} & \bar{\gamma}_{ab} \end{pmatrix} + \begin{pmatrix} L & M & V_b \\ M & N & U_b \\ V_a^{\mathrm{T}} & U_a^{\mathrm{T}} & \delta\gamma_{ab} \end{pmatrix}\right\}$$

$$\boxed{\tilde{L} = 0}$$

$$\boxed{\tilde{N} + 2a\tilde{M} = 0}$$

$$\boxed{\tilde{V}_a = 0}$$

$$\boxed{\Upsilon = a\left(1 + \frac{1}{2}N\right)}$$

$$\boxed{\tilde{U}_a = -a^2 U_a}$$

GLC Gauge Fixing

$$ds^2 = a^2(\eta)\left[-(1+2\phi)\,d\eta^2 - 2\mathcal{B}_r dr d\eta - 2\mathcal{B}_a d\theta^a d\eta + (1+\mathcal{C}_{rr})\,dr^2 + 2\mathcal{C}_{ra} dr d\theta^a + (\bar{\gamma}_{ab} + \mathcal{C}_{ab})\,d\theta^a d\theta^b\right]$$

$\tilde{L} = 0$

$\tilde{N} + 2a\tilde{M} = 0$

$\tilde{V}_a = 0$

GLC

$\dashrightarrow$

$\phi = 0$

$\mathcal{B}_r = -\frac{1}{2}\mathcal{C}_{rr}$

$\mathcal{B}_a = -\mathcal{C}_{ra}$

OSG

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Angular distance-redshift relation

- Redshift

$$1+z=\frac{a_o}{a_s}\left(1+\frac{1}{2}N|_s^o\right)$$

- General Prescription for Gauge Invariant GLC Observables

I. Linearize the observable
II. Write the observable in terms of the redshift

$$\delta\tau_z=\frac{N|_s^o}{2H_z}$$

$$\delta\tau_z\equiv\tau-\tau_z$$

$$1+z\equiv\frac{a_o}{a(\tau_z)}$$

$$\mathcal{O}=\bar{\mathcal{O}}(\tau)+\delta\mathcal{O}(\tau,w,\theta^a)=\bar{\mathcal{O}}(\tau_z)+\delta\mathcal{O}(\tau_z,w,\theta^a)+\delta\tau_z\partial_\tau\bar{\mathcal{O}}$$

- **Angular distance-redshift relation**

$$\boxed{d_A^2 = \frac{\sqrt{\gamma}}{\left(\frac{det\dot{\gamma}_{ab}}{4\sqrt{\gamma}}\right)_o}}$$

Linearize

$$\boxed{d_A = ar\frac{(1+\nu)}{lim_{\tau\to\tau_o}\left[(arH-1)\left(1+\nu+\frac{ar}{arH-1}\partial_\tau\nu\right)\right]}}$$

Angular distance-redshift relation

$$ar = a_z r_z \left[1 + \frac{1}{2}\left(1 - \frac{1}{a_z r_z H_z}\right) N\Big|_z^o\right]$$

Write the observable in terms of the redshift

$$d_A = ar \frac{\left[1 + \tilde{\nu} - \frac{1}{2}\left(1 - \frac{1}{a_z r_z H_z}\right)\tilde{N}\Big|_o^z\right]}{\left(1 + \tilde{\nu} - ar\partial_\tau \tilde{\nu}\right)_o}$$

Angular distance-redshift relation

$$
\begin{aligned}
d_A(z) = a_z r_z \Bigg[& 1 + \nu_z + \frac{1}{2}\int_{\tau_z}^{\tau_o} d\tau' \left(D^2 v + \frac{1}{2ar^2}\int_{\tau'}^{\tau_o} d\tau'' a D^2 L \right) - \frac{1}{2r_z}\int_{\tau_z}^{\tau_o} d\tau' a L \\
& + \frac{1}{2}\left(1 - \frac{1}{a_z r_z H_z}\right)\frac{1}{a_z}\int_{\tau_{in}}^{\tau_z} d\tau' \partial_w \left(N + 2aM + a^2 L\right) - \frac{1}{2}\left(1 - \frac{1}{a_z r_z H_z}\right) N_o^z \\
& + \frac{1}{2}\left(1 - \frac{1}{a_z r_z H_z}\right)\int_{\tau_z}^{\tau_o} d\tau' a \partial_w L + \frac{1}{2 a_z r_z H_z}\frac{1}{a_o}\int_{\tau_{in}}^{\tau_o} d\tau' \partial_w \left(N + 2aM + a^2 L\right) \\
& - \nu_o - \frac{1}{2} N_o - a_o M_o + (a r \dot{\nu})_o + \frac{1}{2}\left(H_o - \frac{H_o}{a_z r_z H_z} + \frac{1}{a_o r_z}\right)\int_{\tau_{in}}^{\tau_o} d\tau' \left(N + 2aM + a^2 L\right) \Bigg]
\end{aligned}
$$

Without fix GLC Gauge

- **Angular distance-redshift relation**

$$\int_{\tau_z}^{\tau_o} d\tau' a\partial_w L = -2\int_{\eta}^{\eta_o} d\eta' \partial_{\eta'}\left(\phi - \frac{1}{2}C_{rr} - \mathcal{B}_r\right) + 2\left(\phi - \frac{1}{2}C_{rr} - \mathcal{B}_r\right)$$

Integrated and Local Sachs-Wolfe effect

$$\int_{\tau}^{\tau_o} d\tau' aL = -2\int_{\eta}^{\eta_o} d\eta' \left(\phi - \frac{1}{2}C_{rr} - \mathcal{B}_r\right)$$

Time delay

- **Angular distance-redshift relation**

$$\int_{\tau_{in}}^{\tau_z} d\tau' \partial_w \left(N + 2aM + a^2 L\right) = -\frac{1}{a_z}\int_{\eta_{in}}^{\eta_z} d\eta' a \partial_r \phi$$

Peculiar velocity

$$\int_{\tau_z}^{\tau_o} d\tau' \left(D^2 v + \frac{1}{2ar^2}\int_{\tau'}^{\tau_o} d\tau'' a D^2 L\right) = -\int_{\eta_z}^{\eta_o} \frac{d\eta'}{r^2}\left[D^a\left(\mathcal{B}_a + \mathcal{C}_{ar}\right)\right.$$
$$\left.\int_{\eta'}^{\eta_o} d\eta'' D^2 \left(\phi - \frac{1}{2}C_{rr} - \mathcal{B}_r\right)\right]$$

Lensing

Final remarks

- We developed a Cosmological Perturbation Theory directly in a homogeneous and isotropic GLC background.
- We showed how to fix the GLC Gauge as well as the GLC gauge conditions on the standard perturbation theory.
- We presented a general procedure for gauge invariant observables in GLC.
- We applied this general procedure to the angular distance-redshift relation.
- The linearized angular distance-redshift relation preserves the simplicity of the full non-linear GLC Gauge. Where all the integrated terms vanish for the GLC or OSG gauge.

Thank you!!!

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