Can topology regularise $\mathcal{N}=2$ AdS$_4$ pure Supergravity?

Riccardo Matrecano

Politecnico di Torino, INFN

Cortona Young 2021

9th-11th June 2021

based on "$\mathcal{N}=2$ AdS$_4$ Supergravity, Holography and Ward Identities"

[L.Andrianopoli, B.L.Cerchiai, R.M., O.Miskovic, R.Noris, R.Olea, L.Ravera, M.Trigiante]

2010.02119, JHEP 02 (2021)

Starting point, Objectives & Motivation

Starting point:

- [Andrianopoli and D'Auria '14]: Restore SUSY in $\mathcal{N} = 2$ AdS_4 pure Supergravity theory with boundary by adding topological terms.

Starting point, Objectives & Motivation

Starting point:

- [Andrianopoli and D'Auria '14]: Restore SUSY in $\mathcal{N} = 2$ AdS_4 pure Supergravity theory with boundary by adding topological terms.

Objectives:

- Consistent boundary theory in the framework of AdS/CFT correspondence

Starting point, Objectives & Motivation

Starting point:

- [Andrianopoli and D'Auria '14]: Restore SUSY in $\mathcal{N} = 2$ AdS$_4$ pure Supergravity theory with boundary by adding topological terms.

Objectives:

- Consistent boundary theory in the framework of AdS/CFT correspondence
  - Perform a holographic analysis of the bulk theory;
  - Check Ward identities.

Starting point, Objectives & Motivation

Starting point:

- ▷ [Andrianopoli and D'Auria '14]: Restore SUSY in $\mathcal{N} = 2$ AdS$_4$ pure Supergravity theory with boundary by adding topological terms.

Objectives:

- ▷ Consistent boundary theory in the framework of AdS/CFT correspondence
  - Perform a holographic analysis of the bulk theory;
  - Check Ward identities.

Motivation:

- ▷ The added topological terms are the supersymmetric generalisation of Gauss–Bonnet term and the obtained action can be rewritten in a MacDowell-Mansouri form [MacDowell and Mansouri '77];

Starting point, Objectives & Motivation

Starting point:

- [Andrianopoli and D'Auria '14]: Restore SUSY in $\mathcal{N} = 2$ AdS_4 pure Supergravity theory with boundary by adding topological terms.

Objectives:

- Consistent boundary theory in the framework of AdS/CFT correspondence
  - Perform a holographic analysis of the bulk theory;
  - Check Ward identities.

Motivation:

- The added topological terms are the supersymmetric generalisation of Gauss–Bonnet term and the obtained action can be rewritten in a MacDowell-Mansouri form [MacDowell and Mansouri '77];
- Future purpose: Obtain a holographic model for 2D materials, like graphene (Unconventional Supersymmetry).

Background: AdS/CFT Correspondence

- The relation between the gravitational theory and the CFT

$$Z_{\text{gravity}}[\Phi_{i(0)}] = Z_{\text{CFT}}[\mathcal{J} = \Phi_{i(0)}]$$

where $\Phi_{i(0)}$ are the boundary values of the gravity fields Φ_i.

Background: AdS/CFT Correspondence

- The relation between the gravitational theory and the CFT

$$Z_{\mathrm{gravity}}[\Phi_{i(0)}] = Z_{\mathrm{CFT}}[\mathcal{J} = \Phi_{i(0)}]$$

  where $\Phi_{i(0)}$ are the boundary values of the gravity fields Φ_i.

- Low-energy approximation:

$$W[\mathcal{J} \equiv \Phi_{i(0)}] \simeq I_{\mathrm{on-shell}}[\Phi_{i(0)}]$$

Background: The 4D Lagrangian

We use the following supercurvatures in $\mathcal{N}=2$, D=4 superspace

$$
\begin{aligned}
\hat{\mathcal{R}}^{ab} &= \mathrm{d}\hat{\omega}^{ab} + \hat{\omega}^{ac} \wedge \hat{\omega}_c{}^b \\
\hat{\rho}_A &= \hat{\mathcal{D}}\Psi_A - \frac{1}{2\ell}\hat{A}\epsilon_{AB} \wedge \Psi^B = \mathrm{d}\Psi_A + \frac{1}{4}\Gamma_{ab}\hat{\omega}^{ab} \wedge \Psi_A - \frac{1}{2\ell}\hat{A}\epsilon_{AB} \wedge \Psi^B \\
F &= \mathrm{d}\hat{A} - \overline{\Psi}^A \wedge \Psi^B \epsilon_{AB}
\end{aligned}
$$

Background: The 4D Lagrangian

We use the following supercurvatures in $\mathcal{N}=2$, D=4 superspace

$$\begin{aligned}
\hat{\mathcal{R}}^{ab} &= \mathrm{d}\hat{\omega}^{ab} + \hat{\omega}^{ac} \wedge \hat{\omega}_c{}^{b} \\
\hat{\rho}_A &= \hat{\mathcal{D}}\Psi_A - \frac{1}{2\ell}\hat{A}\epsilon_{AB} \wedge \Psi^B = \mathrm{d}\Psi_A + \frac{1}{4}\Gamma_{ab}\hat{\omega}^{ab} \wedge \Psi_A - \frac{1}{2\ell}\hat{A}\epsilon_{AB} \wedge \Psi^B \\
F &= \mathrm{d}\hat{A} - \overline{\Psi}^A \wedge \Psi^B \epsilon_{AB}
\end{aligned}$$

to write the bulk Lagrangian

$$\begin{aligned}
\mathcal{L}^{\text{bulk}} =& \frac{1}{4}\hat{\mathcal{R}}^{ab} V^c V^d \epsilon_{abcd} + \overline{\Psi}^A \Gamma_a \Gamma_5 \hat{\rho}_A V^a \\
&+ \frac{\mathrm{i}}{2}\left(\hat{F} + \frac{1}{2}\overline{\Psi}^A \Psi^B \epsilon_{AB}\right) \overline{\Psi}^C \Gamma_5 \Psi^D \epsilon_{CD} \\
&- \frac{\mathrm{i}}{2\ell}\overline{\Psi}^A \Gamma_{ab}\Gamma_5 \Psi_A V^a V^b - \frac{1}{8\ell^2} V^a V^b V^c V^d \epsilon_{abcd} \\
&+ \frac{1}{4}\left(\tilde{F}^{cd} V^a V^b \hat{F} - \frac{1}{12}\tilde{F}_{lm}\tilde{F}^{lm} V^a V^b V^c V^d\right)\epsilon_{abcd}
\end{aligned}$$

A consistent way to deal with a boundary is to add the following terms to the bulk Lagrangian

$$\mathcal{L}^{\text{boundary}} = -\frac{\ell^2}{8}\left(\hat{\mathcal{R}}^{ab}\hat{\mathcal{R}}^{cd}\epsilon_{abcd} + \frac{8\mathrm{i}}{\ell}\,\hat{\overline{\rho}}^A\Gamma_5\hat{\rho}_A - \frac{2\mathrm{i}}{\ell}\,\hat{\mathcal{R}}^{ab}\overline{\Psi}^A\Gamma_{ab}\Gamma_5\Psi_A + \frac{4\mathrm{i}}{\ell^2}\,\mathrm{d}\hat{A}\,\overline{\Psi}^A\Gamma_5\Psi^B\epsilon_{AB}\right)$$

A consistent way to deal with a boundary is to add the following terms to the bulk Lagrangian

$$\mathcal{L}^{\text{boundary}} = -\frac{\ell^2}{8}\left(\hat{\mathcal{R}}^{ab}\hat{\mathcal{R}}^{cd}\epsilon_{abcd} + \frac{8\mathrm{i}}{\ell}\,\bar{\hat{\rho}}^A\Gamma_5\hat{\rho}_A - \frac{2\mathrm{i}}{\ell}\,\hat{\mathcal{R}}^{ab}\overline{\Psi}^A\Gamma_{ab}\Gamma_5\Psi_A + \frac{4\mathrm{i}}{\ell^2}\,\mathrm{d}\hat{A}\,\overline{\Psi}^A\Gamma_5\Psi^B\epsilon_{AB}\right)$$

and require the following constraints

$$\hat{\boldsymbol{R}}^{ab}\Big|_{\partial\mathcal{M}} = 0, \quad \hat{\boldsymbol{\rho}}_A\Big|_{\partial\mathcal{M}} = 0, \quad \hat{\boldsymbol{F}}\Big|_{\partial\mathcal{M}} = 0\,,$$

where the field strengths above are the OSp(2|4) curvatures defined as

$$\begin{aligned}
\widehat{\mathbf{R}}^{ab} &= \hat{\mathcal{R}}^{ab} - \frac{1}{\ell^2}\,V^aV^b - \frac{1}{2\ell}\,\delta^{AB}\overline{\Psi}_A\Gamma^{ab}\Psi_B \\
\widehat{\mathbf{R}}^{a} &= \hat{\mathcal{D}}V^a - \frac{\mathrm{i}}{2}\,\overline{\Psi}^A\Gamma^a\Psi_A \\
\hat{\boldsymbol{\rho}}_A &= \hat{\rho}_A - \frac{\mathrm{i}}{2\ell}\,\delta_{AB}\Gamma_a\Psi^BV^a \\
\widehat{\mathbf{F}} &= F
\end{aligned}$$

A consistent way to deal with a boundary is to add the following terms to the bulk Lagrangian

$$\mathcal{L}^{\text{boundary}} = -\frac{\ell^2}{8}\left(\hat{\mathcal{R}}^{ab}\hat{\mathcal{R}}^{cd}\epsilon_{abcd} + \frac{8\mathrm{i}}{\ell}\,\hat{\bar{\rho}}^A\Gamma_5\hat{\rho}_A - \frac{2\mathrm{i}}{\ell}\,\hat{\mathcal{R}}^{ab}\overline{\Psi}^A\Gamma_{ab}\Gamma_5\Psi_A + \frac{4\mathrm{i}}{\ell^2}\,\mathrm{d}\hat{A}\,\overline{\Psi}^A\Gamma_5\Psi^B\epsilon_{AB}\right)$$

and require the following constraints

$$\hat{\boldsymbol{R}}^{ab}\Big|_{\partial\mathcal{M}} = 0, \quad \hat{\boldsymbol{\rho}}_A\Big|_{\partial\mathcal{M}} = 0, \quad \hat{\boldsymbol{F}}\Big|_{\partial\mathcal{M}} = 0\,,$$

where the field strengths above are the OSp(2|4) curvatures defined as

$$\begin{aligned}
\widehat{\mathbf{R}}^{ab} &= \hat{\mathcal{R}}^{ab} - \frac{1}{\ell^2}\,V^aV^b - \frac{1}{2\ell}\,\delta^{AB}\overline{\Psi}_A\Gamma^{ab}\Psi_B \\
\widehat{\mathbf{R}}^{a} &= \hat{\mathcal{D}}V^a - \frac{\mathrm{i}}{2}\,\overline{\Psi}^A\Gamma^a\Psi_A \\
\hat{\boldsymbol{\rho}}_A &= \hat{\rho}_A - \frac{\mathrm{i}}{2\ell}\,\delta_{AB}\Gamma_a\Psi^BV^a \\
\widehat{\mathbf{F}} &= F
\end{aligned}$$

The full Lagrangian acquires the following form à la MacDowell-Mansouri

$$\mathcal{L} = -\frac{\ell^2}{8}\hat{\boldsymbol{R}}^{ab}\wedge\hat{\boldsymbol{R}}^{cd}\epsilon_{abcd} - \mathrm{i}\ell\hat{\bar{\boldsymbol{\rho}}}^A\Gamma_5\wedge\hat{\boldsymbol{\rho}}_A + \frac{1}{4}\hat{\boldsymbol{F}}\wedge{}^*\hat{\boldsymbol{F}}$$

Holographic description: Gauge-fixing Conditions near the Boundary

The transformation laws of the 4D fields depend on the local parameters

$$\begin{aligned} p^a &\to \text{diffeomorphisms} \\ j^{ab} &\to \text{Lorentz} \\ \hat{\lambda} &\to \text{R}-\text{symmetry} \\ \epsilon_A &\to \text{SUSY} \end{aligned}$$

and we use this freedom to fix the radial components of the fields themselves near the boundary (at $z = 0$)

$$\begin{aligned} V^3{}_z &= \frac{\ell}{z}\,, \qquad & \hat{\omega}_z^{i3} &= w^i(x,z)\,, \qquad & \Psi_{\pm Az} &= \left(\frac{z}{\ell}\right)^{\pm\frac{1}{2}} \varphi_{\pm Az}(x,z)\,, \\ V^i{}_z &= 0\,, & \hat{\omega}_z^{ij} &= \frac{z}{\ell}\, w^{ij}(x,z)\,, & \hat{A}_z &= \frac{\ell}{z} A_{(-1)z}(x) + \frac{z}{\ell} A_{(1)z}(x) + \mathcal{O}(z^3) \end{aligned}$$

Holographic description: Boundary Fields Expansion

The boundary fields are then expanded in the following way:

$$\begin{aligned}
V^i{}_\mu &= \frac{\ell}{z} E^i{}_\mu + \frac{z}{\ell} S^i{}_\mu + \frac{z^2}{\ell^2} \tau^i{}_\mu + \mathcal{O}(z^3) \\
\hat{\omega}^{i3}_\mu &= \frac{1}{z} E^i{}_\mu - \frac{z}{\ell^2} \tilde{S}^i{}_\mu - \frac{2z^2}{\ell^3} \tilde{\tau}^i{}_\mu + \mathcal{O}(z^3) \\
\hat{\omega}^{ij}_\mu &= \omega^{ij}_\mu(x,z) = \omega^{ij}_\mu + \frac{z}{\ell} \omega^{ij}_{(1)\mu} + \frac{z^2}{\ell^2} \omega^{ij}_{(2)\mu} + \mathcal{O}(z^3) \\
\hat{A}_\mu &= A_\mu(x,z) = A_\mu + \frac{z}{\ell} A_{(1)\mu} + \frac{z^2}{\ell^2} A_{(2)\mu} + \mathcal{O}(z^3) \\
\Psi^A_{\mu+} &= \sqrt{\frac{\ell}{z}}\, \varphi^A_{\mu+}(x,z) = \sqrt{\frac{\ell}{z}} \left[\begin{pmatrix} \psi^A_{\mu+} \\ 0 \end{pmatrix} + \frac{z}{\ell} \begin{pmatrix} \zeta^A_{\mu+} \\ 0 \end{pmatrix} + \frac{z^2}{\ell^2} \begin{pmatrix} \Pi^A_{\mu+} \\ 0 \end{pmatrix} + \mathcal{O}(z^3) \right] \\
\Psi^A_{\mu-} &= \sqrt{\frac{z}{\ell}}\, \varphi^A_{\mu-}(x,z) = \sqrt{\frac{z}{\ell}} \left[\begin{pmatrix} 0 \\ \psi^A_{\mu-} \end{pmatrix} + \frac{z}{\ell} \begin{pmatrix} 0 \\ \zeta^A_{\mu-} \end{pmatrix} + \mathcal{O}(z^2) \right]
\end{aligned}$$

with the super-Schouten expressed as

$$\mathcal{S}^i{}_\mu = \frac{1}{\ell^2} \left(S^i{}_\mu + \tilde{S}^i{}_\mu \right)$$

Holographic description: Residual Symmetries

We can now look for the residual symmetries that leave the gauge fixing unaltered:

$$\delta V^{a}_{\ z} = 0\,, \quad \delta\hat{\omega}^{ij}_{z} = \mathcal{O}(z)\,, \quad \delta\hat{\omega}^{i3}_{z} = \mathcal{O}(z^2)\,, \quad \delta\hat{A}_z = 0\,, \quad \delta\Psi_{\pm Az} = 0\,,$$

which imply

$$\begin{aligned}
p^3 &= -\ell\sigma + \mathcal{O}(z^2) \\
p^i &= \frac{\ell}{z}\,\xi^i + \frac{z}{\ell}\,b^i + \frac{z^2}{\ell^2}\,p^i_{(2)} + \mathcal{O}(z^3) \\
j^{i3} &= \frac{1}{z}\,\xi^i - \frac{z}{\ell^2}\,\tilde{b}^i + \frac{z^2}{\ell^2}\,j^{i3}_{(2)} + \mathcal{O}(z^3) \\
j^{ij} &= \theta^{ij} + \mathcal{O}(z^2) \\
\hat{\lambda} &= \lambda + \frac{z}{\ell}\,A_{(1)\mu}\xi^\mu + \mathcal{O}(z^2) \\
\epsilon^A_+ &= \sqrt{\frac{\ell}{z}}\begin{pmatrix}\eta^A_+ \\ 0\end{pmatrix} + \mathcal{O}(z^{1/2}) \\
\epsilon^A_- &= \sqrt{\frac{z}{\ell}}\begin{pmatrix}0 \\ \eta^A_-\end{pmatrix} + \mathcal{O}(z^{3/2})
\end{aligned}$$

OSp(2|4) Transformation Laws of Holographic Fields

From the bulk transformation law and by writing the result in terms of boundary 1-forms, we find $(K^i \equiv (b^i + \tilde{b}^i)/\ell^2)$

$$
\begin{aligned}
\delta E^i &= \mathcal{D}\xi^i + \sigma E^i - \theta^{ij} E_j + \mathrm{i}\,\overline{\eta}_+^A \gamma^i \psi_{+A} \\
\delta \omega^{ij} &= \mathcal{D}\theta^{ij} + 2\xi^{[i}\mathcal{S}^{j]} + 2\,K^{[i}E^{j]} + \frac{1}{\ell}\,\overline{\eta}_+^A \gamma^{ij}\psi_{-A} + \frac{1}{\ell}\,\overline{\eta}_-^A \gamma^{ij}\psi_{+A} \\
\delta A &= \mathrm{d}\lambda + 2\epsilon_{AB}\,\overline{\eta}_+^A \psi_-^B + 2\epsilon_{AB}\,\overline{\eta}_-^A \psi_+^B \\
\delta \psi_{+A} &= \mathcal{D}\eta_{A+} + \frac{\mathrm{i}}{\ell}\,E^i \gamma_i \eta_{A-} - \frac{\mathrm{i}}{\ell}\,\xi^i \gamma_i \psi_{-A} + \frac{1}{2}\,\sigma\psi_{+A} \\
&\quad - \frac{1}{4}\,\theta^{ij}\gamma_{ij}\varphi_{A+} + \frac{1}{2\ell}\,\lambda\epsilon_{AB}\,\psi_+^B - \frac{1}{2\ell}\,A\,\epsilon_{AB}\eta_+^B\,.
\end{aligned}
$$

OSp(2|4) Transformation Laws of Holographic Fields

From the bulk transformation law and by writing the result in terms of boundary 1-forms, we find ($K^i \equiv (b^i + \tilde{b}^i)/\ell^2$)

$$\begin{aligned}
\delta E^i &= \mathcal{D}\xi^i + \sigma E^i - \theta^{ij} E_j + \mathrm{i}\,\overline{\eta}^A_+ \gamma^i \psi_{+A} \\
\delta\omega^{ij} &= \mathcal{D}\theta^{ij} + 2\xi^{[i}\mathcal{S}^{j]} + 2\,K^{[i}E^{j]} + \frac{1}{\ell}\,\overline{\eta}^A_+ \gamma^{ij}\psi_{-A} + \frac{1}{\ell}\,\overline{\eta}^A_- \gamma^{ij}\psi_{+A} \\
\delta A &= \mathrm{d}\lambda + 2\epsilon_{AB}\,\overline{\eta}^A_+ \psi^B_- + 2\epsilon_{AB}\,\overline{\eta}^A_- \psi^B_+ \\
\delta\psi_{+A} &= \mathcal{D}\eta_{A+} + \frac{\mathrm{i}}{\ell}\,E^i\gamma_i\eta_{A-} - \frac{\mathrm{i}}{\ell}\,\xi^i\gamma_i\psi_{-A} + \frac{1}{2}\,\sigma\psi_{+A} \\
&\quad -\frac{1}{4}\,\theta^{ij}\gamma_{ij}\varphi_{A+} + \frac{1}{2\ell}\,\lambda\epsilon_{AB}\,\psi^B_+ - \frac{1}{2\ell}\,A\,\epsilon_{AB}\eta^B_+ \,.
\end{aligned}$$

Furthermore, since the OSp(2|4) supercurvatures vanish at the boundary, we can compute

$$\begin{aligned}
\delta\mathcal{S}^i &= \mathcal{D}K^i - \sigma\mathcal{S}^i - \theta^{ij}\mathcal{S}_j + \frac{2\mathrm{i}}{\ell^2}\,\overline{\eta}^A_- \Gamma^i\psi_{-A} + \mathcal{E}^i \\
\delta\psi_{-A} &= \mathcal{D}\eta_{A-} + \frac{\mathrm{i}\ell}{2}\,\mathcal{S}^i\gamma_i\eta_{A+} - \frac{\mathrm{i}\ell}{2}\,K^i\gamma_i\psi_{+A} - \frac{1}{2}\,\sigma\psi_{-A} \\
&\quad -\frac{1}{4}\,\theta^{ij}\gamma_{ij}\varphi_{-A} + \frac{1}{2\ell}\,\lambda\epsilon_{AB}\psi^B_- - \frac{1}{2\ell}\,A\epsilon_{AB}\eta^B_- + \Sigma_A \,,
\end{aligned}$$

where $\mathcal{E}^i$ and Σ_A are related to the contraction of the super-Cotton and the Cottino tensors.

The $\mathcal{N} = 2$ supersymmetric extension of the Cotton tensor (${\mathcal{C}^i}_{\mu\nu}$) and its superpartner, the Cottino ($\Omega^A_{\mu\nu}$), are

$$\begin{aligned} \widehat{\mathbf{R}}^{i3}_{\mu\nu} &= -z\,{\mathcal{C}^i}_{\mu\nu} + \mathcal{O}(z^2)\,, \\ \hat{\boldsymbol{\rho}}^A{}_{-\mu\nu} &= \sqrt{\frac{z}{\ell}}\begin{pmatrix} 0 \\ \Omega^A_{\mu\nu} \end{pmatrix} + \mathcal{O}(z^{3/2})\,. \end{aligned}$$

They represent the field strengths of $\mathcal{S}^i$ and ψ_{A-}, respectively.

The $\mathcal{N}=2$ supersymmetric extension of the Cotton tensor $({\mathcal{C}^i}_{\mu\nu})$ and its superpartner, the Cottino $(\Omega^A_{\mu\nu})$, are

$$\begin{aligned} \widehat{\mathbf{R}}^{i3}_{\mu\nu} &= -z\,{\mathcal{C}^i}_{\mu\nu} + \mathcal{O}(z^2)\,, \\ {\hat{\boldsymbol{\rho}}^A}_{-\mu\nu} &= \sqrt{\frac{z}{\ell}}\begin{pmatrix} 0 \\ \Omega^A_{\mu\nu} \end{pmatrix} + \mathcal{O}(z^{3/2})\,. \end{aligned}$$

They represent the field strengths of $\mathcal{S}^i$ and ψ_{A-}, respectively.
Together with the transformation law of $B \equiv {V^3}_\mu \mathrm{d}x^\mu$

$$\delta B_\mu \equiv \delta {V^3}_\mu = -\ell\partial_\mu\sigma - \ell {E^i}_\mu K_i + \ell\xi_i {\mathcal{S}^i}_\mu + \overline{\eta}_{A+}\psi_{-A\mu} - \overline{\eta}_{A-}\psi_{+A\mu} + \mathcal{O}(z)\,,$$

the previous relations define the full set of $\mathcal{N}=2$ superconformal transformations on the boundary 1-forms E^i, B, $\mathcal{S}$, ω^{ij}, A, $\psi_{\pm A}$.

Asymptotic Symmetries

A summary of local parameter, source and current associated to each symmetry transformation:

Transformation	Local parameter	Source	Current
Lorentz	θ^{ij}	ω^{ij}_{μ}	$J^{\mu}{}_{ij} = 0$
Translation	ξ^{i}	$E^{i}{}_{\mu}$	$J^{\mu}{}_{i}$
Dilatation	σ	$B_{\mu} = 0$	$J^{\mu}{}_{(D)} = 0$
Special conformal	K^{i}	$\mathcal{S}^{i}{}_{\mu}$	$J^{\mu}_{(K)i} = 0$
Abelian R-symmetry	λ	A_{μ}	J^{μ}
Supersymmetry	η_{A+}	$\psi_{A+\mu}$	$J^{\mu}{}_{A+}$
Superconformal	η_{A-}	$\psi_{A-\mu}$	$J^{\mu}{}_{A-} = 0$

Asymptotic Symmetries

A summary of local parameter, source and current associated to each symmetry transformation:

Transformation	**Local parameter**	**Source**	**Current**
Lorentz	θ^{ij}	ω_μ^{ij}	$J^\mu{}_{ij} = 0$
Translation	ξ^i	$E^i{}_\mu$	$J^\mu{}_i$
Dilatation	σ	$B_\mu = 0$	$J^\mu{}_{(D)} = 0$
Special conformal	K^i	$\mathcal{S}^i{}_\mu$	$J^\mu_{(K)i} = 0$
Abelian R-symmetry	λ	A_μ	J^μ
Supersymmetry	η_{A+}	$\psi_{A+\mu}$	$J^\mu{}_{A+}$
Superconformal	η_{A-}	$\psi_{A-\mu}$	$J^\mu{}_{A-} = 0$

▷ When all boundary fields are treated as independent superconformal fields, then the quantum operators are all non vanishing;

Asymptotic Symmetries

A summary of local parameter, source and current associated to each symmetry transformation:

Transformation	Local parameter	Source	Current
Lorentz	θ^{ij}	ω^{ij}_{μ}	$J^{\mu}{}_{ij} = 0$
Translation	ξ^{i}	$E^{i}{}_{\mu}$	$J^{\mu}{}_{i}$
Dilatation	σ	$B_{\mu} = 0$	$J^{\mu}{}_{(D)} = 0$
Special conformal	K^{i}	$\mathcal{S}^{i}{}_{\mu}$	$J^{\mu}_{(K)i} = 0$
Abelian R-symmetry	λ	A_{μ}	J^{μ}
Supersymmetry	η_{A+}	$\psi_{A+\mu}$	$J^{\mu}{}_{A+}$
Superconformal	η_{A-}	$\psi_{A-\mu}$	$J^{\mu}{}_{A-} = 0$

- When all boundary fields are treated as independent superconformal fields, then the quantum operators are all non vanishing;
- When $\mathcal{S}^{i}$, ψ_{-A} are expressed non-linearly as composite fields in terms of other boundary fields on the 3D manifold (i.e. E^{i}, ψ_{+A}, A, ω^{ij}), the associated quantum operator vanish.

Conserved Currents of the SCFT

According to the AdS/CFT conjecture

$$\mathcal{J}^{\Lambda}(x) = \{E^{i}{}_{\mu}(x),\, \omega^{ij}_{\mu}(x),\, \psi_{+A\,\mu}(x),\, A_{\mu}(x)\} \iff J^{\mu}_{\Lambda} = \{J^{\mu}_{i},\, J^{\mu}_{ij},\, J^{\mu}_{+A},\, J^{\mu}\} \ .$$

Conserved Currents of the SCFT

According to the AdS/CFT conjecture

$$\mathcal{J}^\Lambda(x) = \{E^i{}_\mu(x),\, \omega^{ij}_\mu(x),\, \psi_{+A\,\mu}(x),\, A_\mu(x)\} \iff J^\mu_\Lambda = \{J^\mu_i,\, J^\mu_{ij},\, J^\mu_{+A},\, J^\mu\} \ .$$

This identification allows to obtain the explicit expression of the currents

$$\delta W = \delta I_{\text{on}-\text{shell}} = \int_{\partial\mathcal{M}} \left(-\frac{\ell^2}{4}\, \delta\hat{\omega}^{ab}\hat{\boldsymbol{R}}^{cd}\epsilon_{abcd} - 2\mathrm{i}\ell\delta\overline{\Psi}^A\Gamma_5\hat{\boldsymbol{\rho}}_A + \frac{1}{2}\,\delta\hat{A}\ {}^*\hat{\boldsymbol{F}} \right) \Bigg|^{\text{on-shell}}_{z=\mathrm{d}z=0} =$$

$$= \int_{\partial\mathcal{M}} \delta\mathcal{J}^\Lambda \wedge J_\Lambda = \int_{\partial\mathcal{M}} \left(\delta E^i \wedge J_i + \frac{1}{2}\,\delta\omega^{ij} \wedge J_{ij} + \overline{J}^A_+ \wedge \delta\psi_{A+} + J \wedge \delta A \right)$$

$$J_i = \frac{1}{2}\,\epsilon_{ijk}\left[\frac{2}{\ell}\, E^j\wedge(\tau^k + 2\tilde{\tau}^k) + \overline{\psi}^A_+\wedge\gamma^{jk}\zeta_{A-}\right]$$

$$J_{ij} = 0$$

$$J = \frac{1}{2}\,\epsilon_{ijk}\,\tilde{F}^{i3}\, V^j\wedge V^k\Big|_{z=0}$$

$$J^A_+ = -2\,\mathrm{i}\, E^i\wedge\gamma_i\zeta^A_- + A_{(1)}\wedge\epsilon_{AB}\,\psi^B_+$$

The Hodge dual of the obtained expressions yields the Noether currents

$$J^\mu_i = -\frac{1}{\ell}\left[(\tau^\mu_i + 2\tilde{\tau}^\mu_i) - E^\mu_i(\tau^k_k + 2\tilde{\tau}^k_k)\right] + \frac{\mathrm{i}}{e_3}\,\epsilon^{\mu\nu\rho}\,\bar{\psi}^A_{+\nu}\gamma_i\zeta_{A-\rho}$$

$$J^\mu_{A+} = -\frac{2\mathrm{i}}{e_3}\,\epsilon^{\mu\nu\rho}\gamma_\nu\zeta_{A-\rho} + \frac{1}{e_3}\,\epsilon^{\mu\nu\rho}\,A_{(1)\nu}\epsilon_{AB}\,\psi^B_{+\rho}$$

$$J^\mu = -g^{\mu\nu}_{(0)}\,\tilde{F}_{\nu z} = \frac{1}{2\ell}\,g^{\mu\nu}_{(0)}\,A_{(1)\nu}$$

Ward Identities of the SCFT

The conservation laws of the operators read

$$
\begin{aligned}
\mathcal{D}J_i &= \mathcal{S}^j\, J_{ij} - \frac{\mathrm{i}}{\ell}\bar{J}^A_{+}\gamma_i\psi_{A-} + \mathcal{S}^k{}_i\, J_{kj}\, E^j - \frac{\mathrm{i}\ell}{2}\,\mathcal{S}^j{}_i\,\bar{J}^A_{-}\gamma_j\psi_{A+}\\
\mathcal{D}J_{ij} &= 2\,E_{[i}\,J_{j]} - \frac{\mathrm{i}}{2}\,\bar{J}^A_{+}\gamma_{ij}\psi_{A+} - \frac{\mathrm{i}}{2}\,\bar{J}^A_{-}\gamma_{ij}\psi_{A-}\\
0 &= \partial_\mu\left[E^{\mu i}\left(J_{ij}\,E^j - \frac{\mathrm{i}\ell}{2}\,\bar{J}^A_{-}\gamma_i\psi_{A+}\right)\right] + E^i\,J_i + \frac{1}{2}\,\bar{J}^A_{+}\,\psi_{A+} - \frac{1}{2}\,\bar{J}^A_{-}\,\psi_{A-}\\
\mathrm{d}J &= \frac{1}{2\ell}\,\epsilon_{AB}\left(\bar{J}^A_{+}\,\psi_{B+} + \bar{J}^A_{-}\,\psi_{B-}\right)\\
\nabla J_{A+} &= \frac{1}{2\ell}\,\gamma^{ij}\psi_{A-}\,J_{ij} + \mathrm{i}\,\gamma^i\,\psi_{A+}\,J_i - \frac{\mathrm{i}\ell}{2}\,\mathcal{S}^i\,\gamma_i\,J_{A-} + 2\,\epsilon_{AB}\,\psi_{B-}\,J + \frac{1}{\ell}\,\psi^i_{A-}\,J_{ij}\,E^j\\
&\quad - \frac{\mathrm{i}}{2}\,\psi^i_{A-}\,\bar{J}_{B-}\gamma_i\psi_{B+}\\
\nabla J_{A-} &= \frac{1}{2\ell}\,\gamma^{ij}\psi_{A+}\,J_{ij} + 2\,\epsilon_{AB}\,\psi_{B+}\,J - \frac{\mathrm{i}}{\ell}\,E^i\,\gamma_i\,J_{A+} - \frac{1}{\ell}\,\psi^i_{A+}\,J_{ij}\,E^j + \frac{\mathrm{i}}{2}\,\psi^i_{A+}\,\bar{J}_{B-}\gamma_i\psi_{B+}
\end{aligned}
$$

Let's prove that the Ward identities are satisfied:

- ▷ We integrate by parts δW

$$\delta W = \int_{\partial\mathcal{M}} \Bigg[\frac{\ell^2}{4} j^{ab} \mathcal{D} \hat{\boldsymbol{R}}^{cd} \epsilon_{abcd} - \frac{\ell^2}{4} \left(\frac{2}{\ell^2} p^a V^b + \frac{1}{\ell} \overline{\epsilon}^A \Gamma^{ab} \Psi_A \right) \hat{\boldsymbol{R}}^{cd} \epsilon_{abcd} + 2\mathrm{i}\ell \overline{\epsilon}^A \Gamma_5 \mathcal{D} \hat{\boldsymbol{\rho}}_A$$
$$- 2\mathrm{i}\ell \left(\frac{1}{4} j^{ab} \overline{\Psi}^A \Gamma_{ab} + \frac{\mathrm{i}}{2\ell} p^a \overline{\Psi}^A \Gamma_a + \frac{1}{2\ell} \lambda \epsilon^{AB} \overline{\Psi}_B - \frac{1}{2\ell} \hat{A} \epsilon^{AB} \overline{\epsilon}_B - \frac{\mathrm{i}}{2\ell} \overline{\epsilon}^A \Gamma_a V^a \right) \Gamma_5 \hat{\boldsymbol{\rho}}_A$$
$$- \frac{1}{2} \lambda \, \mathrm{d} \, {}^*\hat{\boldsymbol{F}} + \overline{\epsilon}^A \Psi^B \epsilon_{AB} \, {}^*\hat{\boldsymbol{F}} \Bigg] \Bigg|_{z=\mathrm{d}z=0}^{\text{on-shell}}$$

▷ We make use of the Bianchi identities to manipulate the previous equation

$$\delta W = \int_{\partial\mathcal{M}} \Bigg[\frac{\ell}{4} j^{ab} \overline{\Psi}^A \Gamma^{cd} \hat{\boldsymbol{\rho}}_A \epsilon_{abcd} - \frac{\ell^2}{4} \left(\frac{2}{\ell^2} p^a V^b + \frac{1}{\ell} \overline{\epsilon}^A \Gamma^{ab} \Psi_A \right) \hat{\boldsymbol{R}}^{cd} \epsilon_{abcd}$$

$$+ 2\mathrm{i}\ell \left(\frac{1}{2\ell} \hat{A} \epsilon^{AB} \overline{\epsilon}_A \Gamma_5 \hat{\boldsymbol{\rho}}_B - \frac{\mathrm{i}}{2\ell} \overline{\epsilon}^A \Gamma_5 \Gamma_a \hat{\boldsymbol{\rho}}_A V^a + \frac{1}{4} \hat{\boldsymbol{R}}^{ab} \overline{\epsilon}^A \Gamma_5 \Gamma_{ab} \Psi_A - \frac{1}{2\ell} \epsilon^{AB} \hat{\boldsymbol{F}} \overline{\epsilon}_A \Gamma_5 \Psi_B \right)$$

$$- 2\mathrm{i}\ell \left(\frac{1}{4} j^{ab} \overline{\Psi}^A \Gamma_{ab} + \frac{\mathrm{i}}{2\ell} p^a \overline{\Psi}^A \Gamma_a + \frac{1}{2\ell} \lambda \epsilon^{AB} \overline{\Psi}_B - \frac{1}{2\ell} \hat{A} \epsilon^{AB} \overline{\epsilon}_B - \frac{\mathrm{i}}{2\ell} \overline{\epsilon}^A \Gamma_a V^a \right) \Gamma_5 \hat{\boldsymbol{\rho}}_A$$

$$- \frac{1}{2} \lambda \, \mathrm{d}^* \hat{\boldsymbol{F}} + \overline{\epsilon}^A \Psi^B \epsilon_{AB} \, {}^* \hat{\boldsymbol{F}} \Bigg] \Bigg|_{z=\mathrm{d}z=0}^{\text{on-shell}} = 0$$

▷ We make use of the Bianchi identities to manipulate the previous equation

$$\delta W = \int_{\partial\mathcal{M}} \Bigg[\frac{\ell}{4} j^{ab} \overline{\Psi}^A \Gamma^{cd} \hat{\boldsymbol{\rho}}_A \epsilon_{abcd} - \frac{\ell^2}{4} \left(\frac{2}{\ell^2} p^a V^b + \frac{1}{\ell} \overline{\epsilon}^A \Gamma^{ab} \Psi_A \right) \hat{\boldsymbol{R}}^{cd} \epsilon_{abcd}$$

$$+ 2\mathrm{i}\ell \left(\frac{1}{2\ell} \hat{A} \epsilon^{AB} \overline{\epsilon}_A \Gamma_5 \hat{\boldsymbol{\rho}}_B - \frac{\mathrm{i}}{2\ell} \overline{\epsilon}^A \Gamma_5 \Gamma_a \hat{\boldsymbol{\rho}}_A V^a + \frac{1}{4} \hat{\boldsymbol{R}}^{ab} \overline{\epsilon}^A \Gamma_5 \Gamma_{ab} \Psi_A - \frac{1}{2\ell} \epsilon^{AB} \hat{\boldsymbol{F}} \overline{\epsilon}_A \Gamma_5 \Psi_B \right)$$

$$- 2\mathrm{i}\ell \left(\frac{1}{4} j^{ab} \overline{\Psi}^A \Gamma_{ab} + \frac{\mathrm{i}}{2\ell} p^a \overline{\Psi}^A \Gamma_a + \frac{1}{2\ell} \lambda \epsilon^{AB} \overline{\Psi}_B - \frac{1}{2\ell} \hat{A} \epsilon^{AB} \overline{\epsilon}_B - \frac{\mathrm{i}}{2\ell} \overline{\epsilon}^A \Gamma_a V^a \right) \Gamma_5 \hat{\boldsymbol{\rho}}_A$$

$$- \frac{1}{2} \lambda \, \mathrm{d}^* \hat{\boldsymbol{F}} + \overline{\epsilon}^A \Psi^B \epsilon_{AB} \, {}^* \hat{\boldsymbol{F}} \Bigg] \Bigg|_{z=\mathrm{d}z=0}^{\text{on-shell}} = 0$$

▷ All these expressions vanish as a consequence of the bulk equations of motion.

Summary and Future Development

Summary:

- We developed in detail the holographic framework for an $\mathcal{N} = 2$ pure AdS supergravity model in four dimensions, including all the contributions from the fermionic fields;

Summary and Future Development

Summary:

- We developed in detail the holographic framework for an $\mathcal{N} = 2$ pure AdS supergravity model in four dimensions, including all the contributions from the fermionic fields;
- We kept manifest the full superconformal structure, even if only a part of it is realized as a symmetry on the boundary;

Summary and Future Development

Summary:

- We developed in detail the holographic framework for an $\mathcal{N} = 2$ pure AdS supergravity model in four dimensions, including all the contributions from the fermionic fields;
- We kept manifest the full superconformal structure, even if only a part of it is realized as a symmetry on the boundary;
- We constructed the corresponding superconformal currents and show that they satisfy the related Ward identities when the bulk equations of motion are imposed.

Summary and Future Development

Summary:

- We developed in detail the holographic framework for an $\mathcal{N} = 2$ pure AdS supergravity model in four dimensions, including all the contributions from the fermionic fields;
- We kept manifest the full superconformal structure, even if only a part of it is realized as a symmetry on the boundary;
- We constructed the corresponding superconformal currents and show that they satisfy the related Ward identities when the bulk equations of motion are imposed.

Future Development:

- Implement the Unconventional Supersymmetry and apply the holographic analysis to the AVZ model as boundary field theory. [Álvarez, Valenzuela and Zanelli '12]

Thank you for your attention!