Exact mean values at equilibrium with rotation and acceleration by analytic distillation

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Global equilibrium in special relativity

The most general operator of global equilibrium in special relativity is:

$$\widehat{\rho} = \frac{1}{Z} e^{-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu}}$$

The **generators of the Poincaré** group appear.

ϖ is the thermal vorticity, a constant antisymmetric tensor. It describes rotation and acceleration.

$$\varpi^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{\omega_\rho}{T}u_\sigma + \left(\frac{A^\mu}{T}u^\nu - \frac{A^\nu}{T}u^\mu\right)$$

Existing approaches to compute mean values: perturbation theory, curvilinear coordinates.

To compute mean values we analytically continue $\widehat{\rho}$ to imaginary vorticity $\varpi \mapsto -i\phi$: it becomes a Poincaré transformation!

$$\widehat{\rho} = \frac{1}{Z} e^{-b_\mu \widehat{P}^\mu - \frac{i}{2}\phi_{\mu\nu}\widehat{J}^{\mu\nu}} = \frac{1}{Z} e^{-\widetilde{b}_\mu(\phi)\widehat{P}^\mu} \underbrace{e^{-\frac{i}{2}\phi_{\mu\nu}\widehat{J}^{\mu\nu}}}_{\equiv \widehat{\Lambda}}$$

With (Baker-Campbell-Haussdorff):

$$\widetilde{b}^\mu(\phi) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!} \underbrace{\left(\phi^{\mu\nu_1}\phi_{\nu_1\nu_2}\dots\phi^{\nu_{k-1}\nu_k}\right)}_{k-\mathrm{times}} b_{\nu_k}$$

We can use group theory to compute thermal expectation values.

$$\mathrm{Tr}\left(\widehat{\rho}\,\widehat{a}_s^\dagger(p)\widehat{a}_r(p')\right) \equiv \langle \widehat{a}_s^\dagger(p)\widehat{a}_r(p')\rangle$$

Thermal expectation values

Using the density operator as a Poincaré transformation:

$$\langle \widehat{a}_s^\dagger(p)\widehat{a}_r(p')\rangle = (-1)^{2S}\sum_t W^{(S)}(\Lambda,p)_{ts} e^{-\widetilde{b}\cdot\Lambda p}\langle \widehat{a}_t^\dagger(\Lambda p) a_r(p')\rangle +$$

$$+ 2\varepsilon e^{-\widetilde{b}\cdot\Lambda p} W^{(S)}(\Lambda,p)_{rs}\delta^3(\Lambda p - p')$$

$W(\Lambda, p)$ is the "Wigner rotation". A particular solution is found by iteration:

$$\langle \widehat{a}_s^\dagger(p)\widehat{a}_r(p')\rangle = 2\epsilon' \sum_{n=1}^{\infty}(-1)^{2S(n+1)}\delta^3(\Lambda^n p - p') W_{rs}^{(S)}(\Lambda^n, p) e^{-\widetilde{b}\cdot\sum_{k=1}^n \Lambda^k p}$$

$$\langle \widehat{a}^\dagger(p)_s \widehat{a}_r(p')\rangle|_{\phi=0} = \delta_{rs}\frac{2\varepsilon\delta^3(p-p')}{e^{b\cdot p} + (-1)^{2S+1}}$$

Thermal expectation values are expressed as series.
E.g. energy density massless scalar field with acceleration.

$$\rho_{\text{scalar}} = \frac{3T_0^4}{16\pi^2}\sum_{n=1}^{\infty}\frac{\phi^4}{\sinh^4\left(\frac{n\phi}{2}\right)}$$

The series diverges when $\phi \mapsto i\varpi$, because it contains undesired non-analytic terms. We remove them by analytic distillation.

After the distillation we find agreement with the previous literature, (perturbative and curvilinear approaches).

$$\rho_{\text{scalar}} = T^4 \left(\frac{\pi^2}{30} - \frac{1}{12T^2} A^\mu A_\mu - \frac{11}{480\pi^2 T^4} (A^\mu A_\mu)^2 \right)$$

The method is applicable also when other techniques fail, such as the case of equilibrium with both acceleration and rotation. E.g. massless Dirac field:

$$\alpha^\mu = \frac{A^\mu}{T} \qquad w^\mu = \frac{\omega^\mu}{T}$$

$$\rho_D = T^4 \left(\frac{7\pi^2}{60} - \frac{\alpha^2}{24} - \frac{w^2}{8} - \frac{17\alpha^4}{960\pi^2} + \frac{w^4}{64\pi^2} + \frac{23\alpha^2 w^2}{1440\pi^2} + \frac{11(\alpha \cdot w)^2}{720\pi^2} \right)$$

The technique can be used also to study polarization induced by thermal vorticity in relativistic fluids.

Exact mean spin vector for free massive Dirac fermions:

$$S^\mu(p) = \frac{1}{2m} \frac{\sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\widetilde{b}(-n\varpi)\cdot p} \mathrm{tr}\left(\gamma^\mu \gamma_5 e^{n\frac{\varpi}{2}:\Sigma} \not{p}\right)}{\sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\widetilde{b}(-n\varpi)\cdot p} \mathrm{tr}\left(e^{n\frac{\varpi}{2}:\Sigma}\right)}$$

The first order in vorticity agrees with the literature. The result can be used to study the Λ polarization induced by vorticity in the quark gluon plasma.

THANK YOU FOR THE ATTENTION