Defining the Mellin transform in 1d CFT

based on arXiv 2105.xxxx with L. Bianchi, G. Bliard and V. Forini

"Using the Mellin transform in 1d CFT"

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RTG 2575: Rethinking Quantum Field Theory

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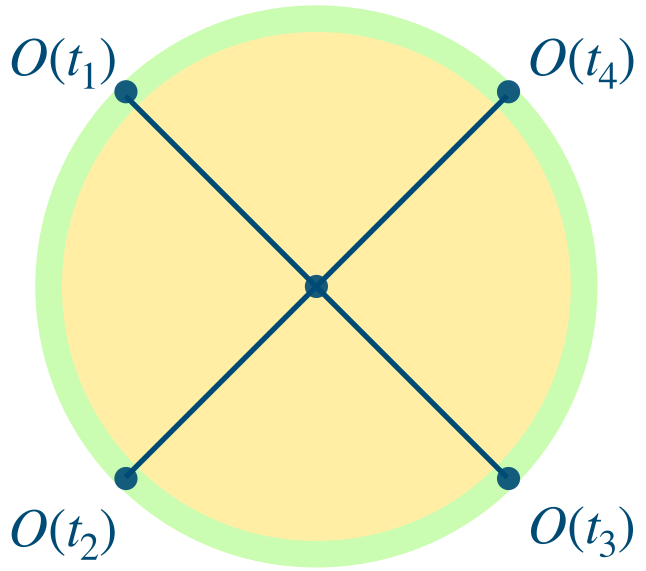
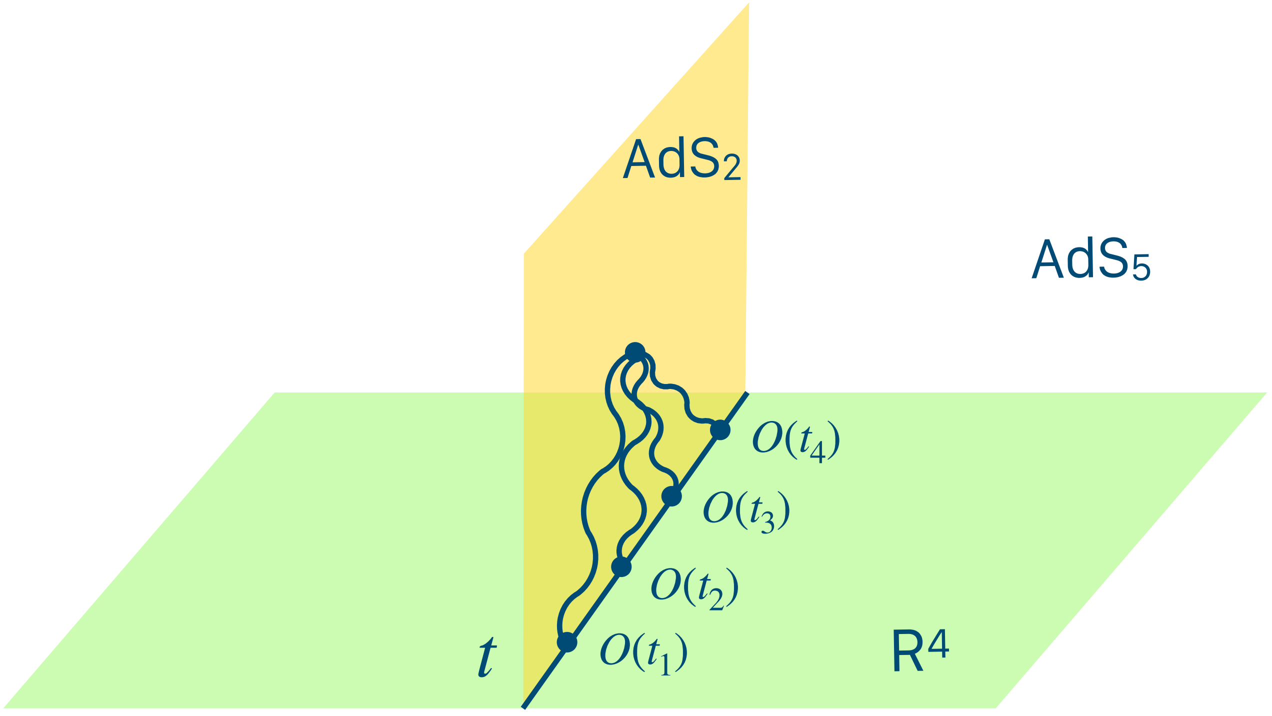
Motivation



1d

- AdS/CFT correspondence
- Defect CFT
- Every higher-d CFT is a 1d CFT
- Simpler but still constraining setting for testing ideas about higher-d CFT



Mellin

- Effective description of correlation functions in higher dimensional CFT
- Similarity with scattering in flat space

[Mack; 2009]

[Penedones, Silva, Zhiboedov; 2019]

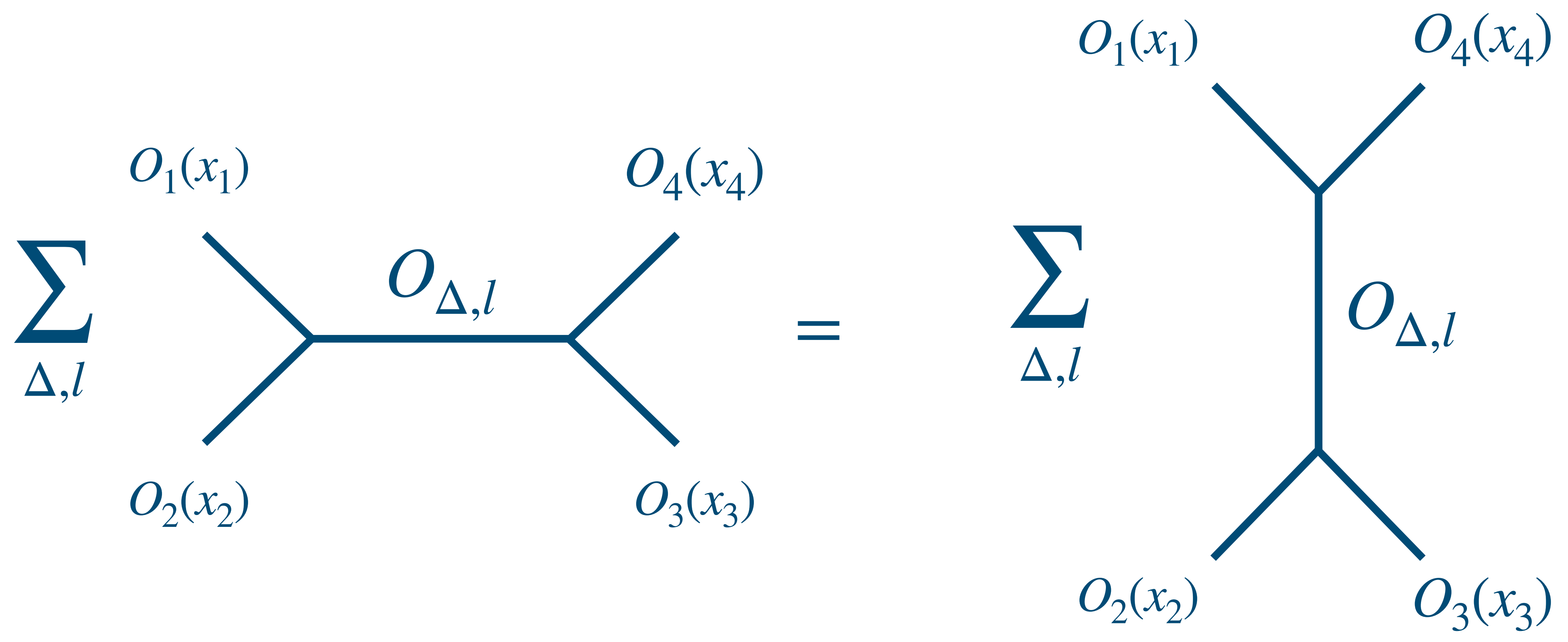
OPE: $\phi_{\Delta_\phi}(x_i)\phi_{\Delta_\phi}(x_j) \overset{x_i \to x_j}{=} \sum_{\Delta} c^2_{\Delta_\phi \Delta_\phi \Delta} (x_{ij}^2)^{\frac{\Delta - 2\Delta_\phi}{2}} [\phi_\Delta(x_j) + \partial \ldots]$ $\left\{\Delta_i, c_{\Delta_\phi \Delta_\phi \Delta}\right\}$

OPE consequences —Mellin transform→ simple analytical properties

- correspondence between physical exchanged operators and poles of the Mellin
- $\Delta = 2\Delta_\phi + 2n + g\,\gamma_n^{(1)} + \ldots$ ⟶ double poles

  $c_{\Delta_\phi \Delta_\phi \Delta} = c_n^{(0)}(\Delta_\phi) + g\, c_n^{(1)}(\Delta_\phi) + \ldots$ ⟶ simple poles

Crossing-symmetry:



Mellin Definition

$$\langle \phi_{\Delta_\phi}(x_1)\ldots\phi_{\Delta_\phi}(x_4)\rangle = \frac{1}{(x_{12}x_{34})^{2\Delta_\phi}} f(t)$$

$$f(t) = \sum_{\Delta} c^2_{\Delta_\phi\Delta_\phi\Delta}\, t^{\Delta}\,{}_2F_1(\Delta,\Delta;2\Delta;-t)$$

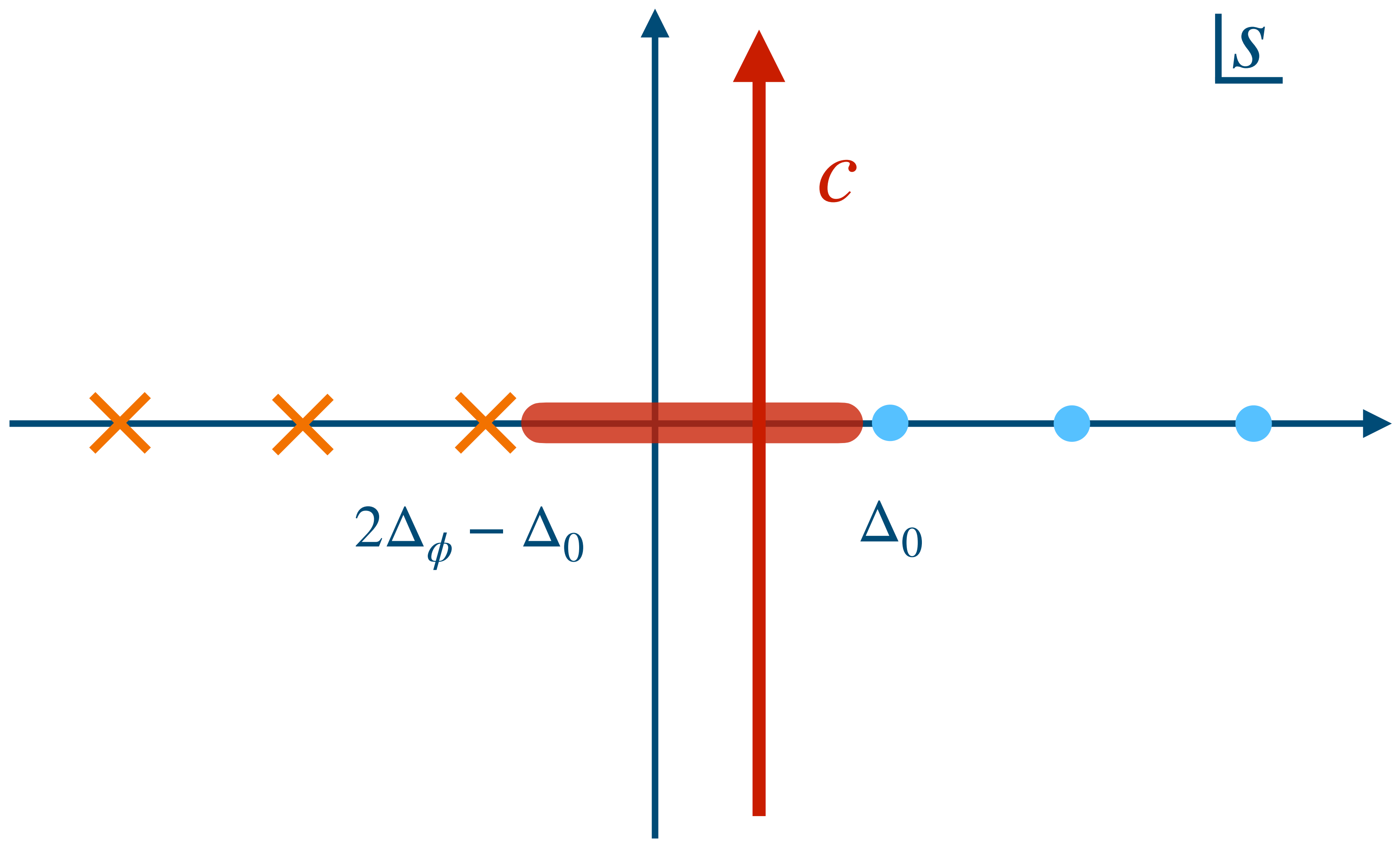
$$z = \frac{x_{12}x_{34}}{x_{14}x_{24}}\,, \qquad 0 < z < 1$$

$$t = \frac{z}{1-z}\,, \qquad 0 < t < \infty$$

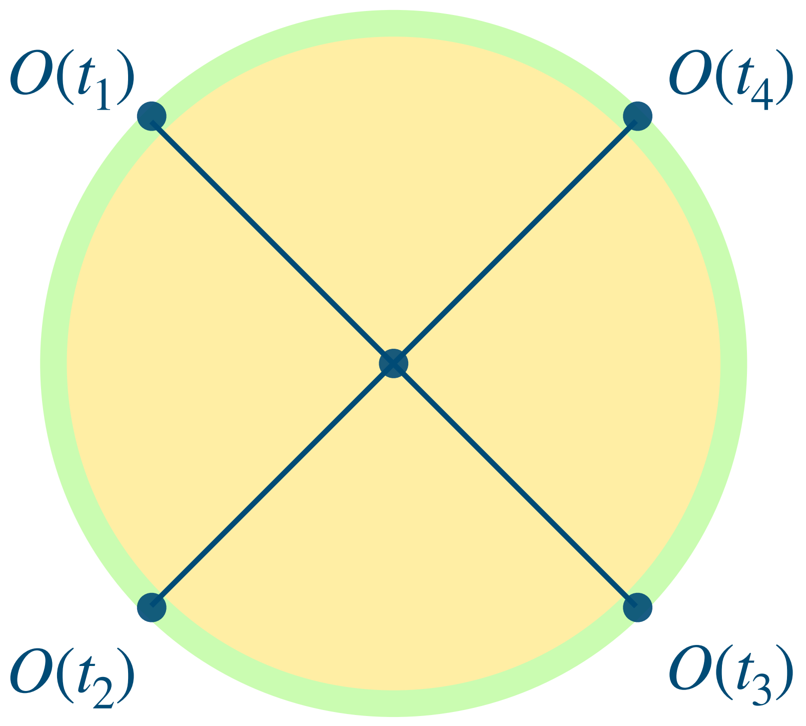
$$\hat{M}_a(s) = \int_0^\infty dt\, f(t)\left(\frac{t}{1+t}\right)^a t^{-1-s} \quad \longleftrightarrow \quad f(t) = \int_{c-i\infty}^{c+i\infty}\frac{ds}{2\pi i}\hat{M}_a(s)\left(\frac{t}{1+t}\right)^{-a} t^s$$

$$\hat{M}(s) = M(s)\,\Gamma(s)\,\Gamma(2\Delta_\phi + a - s)$$

- $2\Delta_\phi - \Delta_0 < Re(s) < \Delta_0\,, \qquad \Delta_0 > \Delta_\phi$
- $a = 0 \qquad (a = -2\Delta_\phi + 1)$
- $f(t) = t^{2\Delta_\phi} f\left(\frac{1}{t}\right) \quad \longleftrightarrow \quad \hat{M}(s) = \hat{M}(2\Delta_\phi - s)$



D-functions



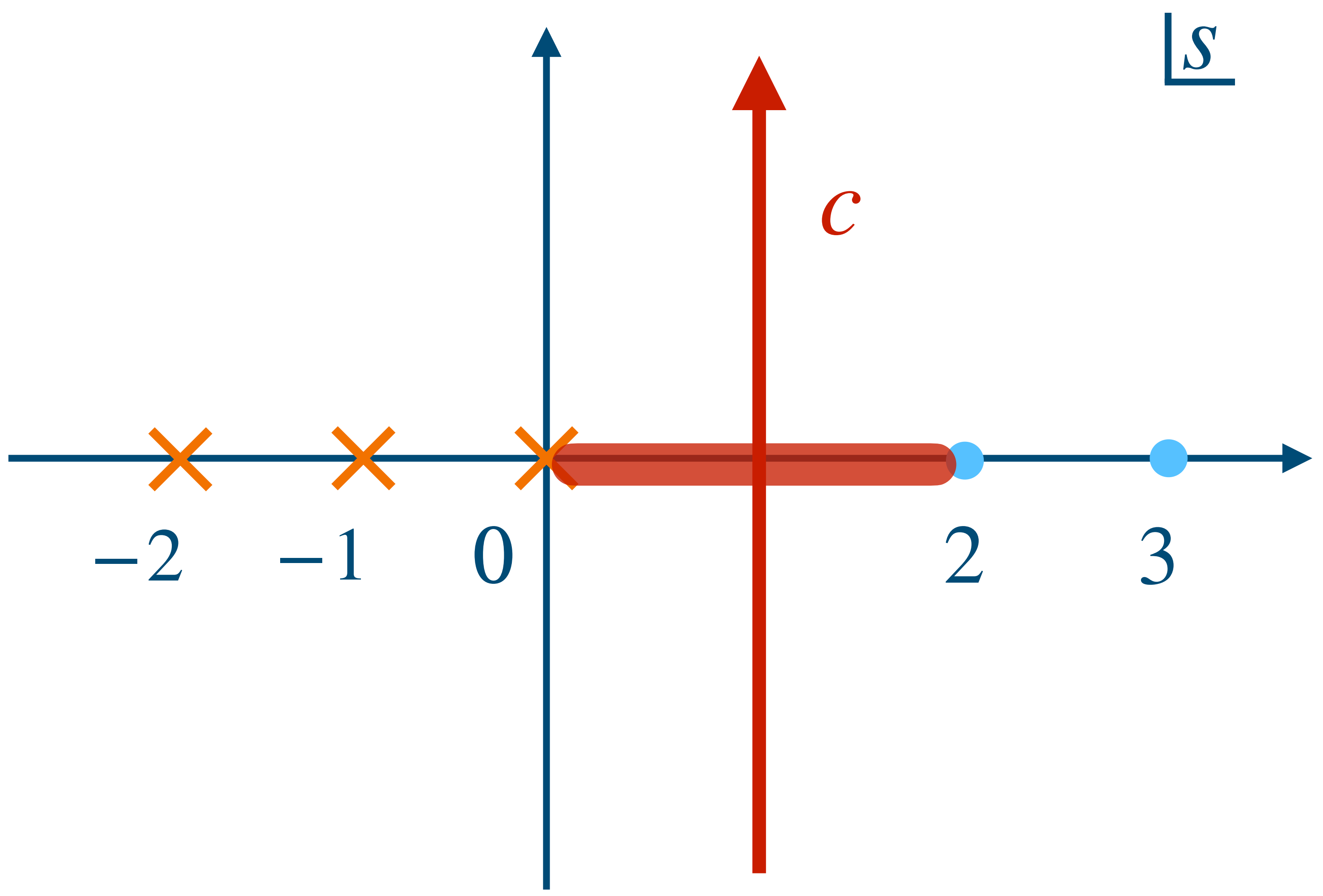
$$D_{\Delta_\phi\Delta_\phi\Delta_\phi\Delta_\phi} = \int \frac{dzdx}{z^2} \tilde{K}_{\Delta_\phi}(z,x;x_1)\tilde{K}_{\Delta_\phi}(z,x;x_2)\tilde{K}_{\Delta_\phi}(z,x;x_3)\tilde{K}_{\Delta_\phi}(z,x;x_4)$$

at perturbative level : $\Delta_0 = 2\Delta_\phi$ $\qquad 0 < Re(s) < 2\Delta_\phi$

$$\bar{D}_{1111}(z) = -\frac{2\,log(1-z)}{z} - \frac{2\,log(z)}{1-z} \quad \longleftrightarrow \quad \hat{M}_{1111}(s) = 2\pi^2\,\mathrm{Cot}(\pi\,s)\mathrm{Csc}(\pi\,s) - \frac{2\,\pi\,\mathrm{Csc}(\pi\,s)}{s-1}$$

$$\hat{M}_{\Delta_\phi}(s) = \pi\,\mathrm{Csc}(\pi s)\left(\pi\,\mathrm{Cot}(\pi s)P_{\Delta_\phi}(s) - \sum_{s_i=1}^{2\Delta_\phi-1}\frac{P_{\Delta_\phi}(s_i)}{s-s_i}\right)$$

$$P_{\Delta_\phi}(s) = 2\frac{\Gamma(\Delta_\phi)^4}{\Gamma(2\Delta_\phi)}\,{}_4F_3\left(s, 2\Delta_\phi - s, 1-\Delta_\phi;\,1\,,1\,,\Delta_\phi+\frac{1}{2};1\right)$$

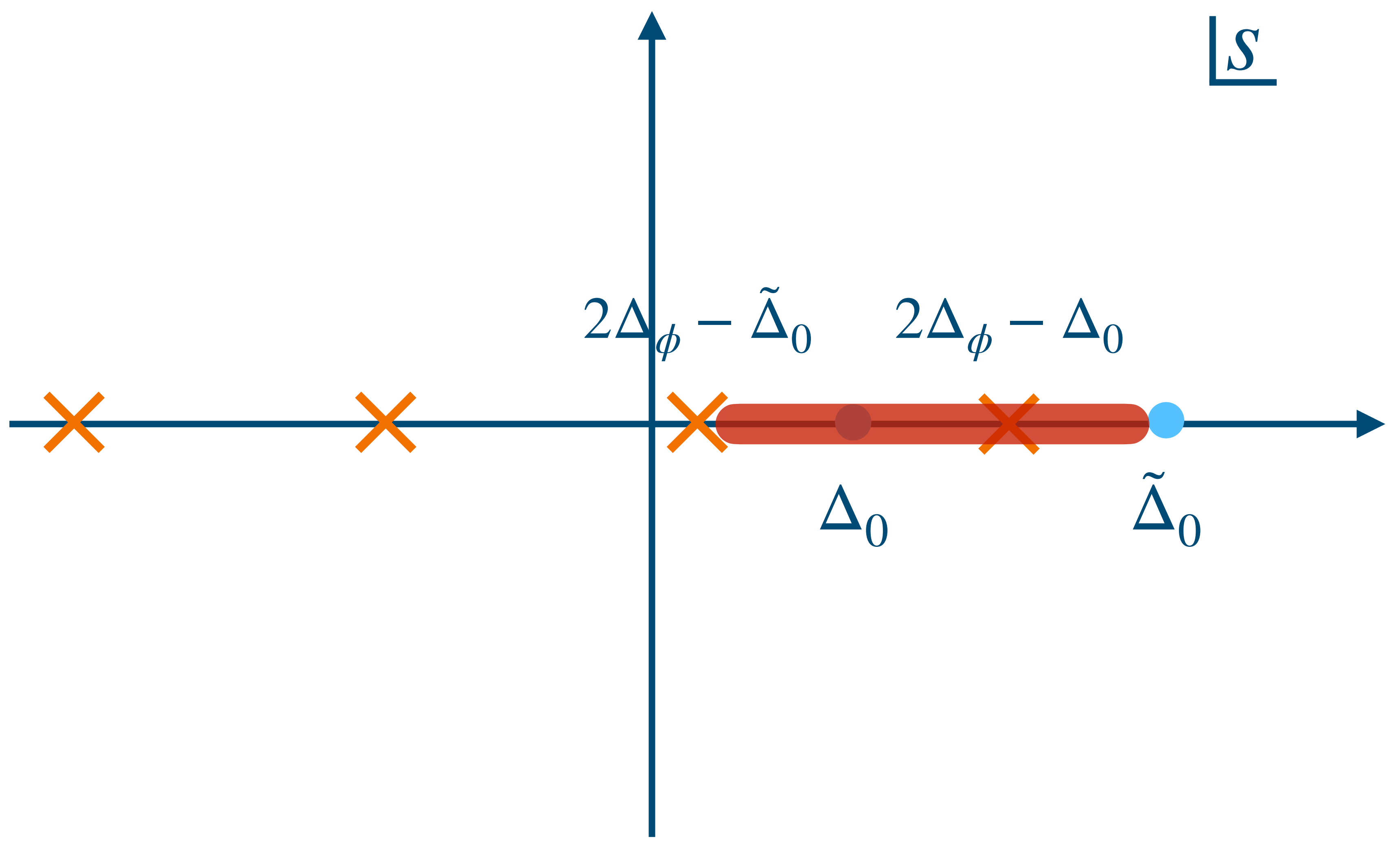


Subtractions

[Penedones, Silva, Zhiboedov; 2019]

At non-perturbative level : operators lighter that $2\Delta_\phi$ can be exchanged

$$f(t) \underset{t\to 0}{\sim} t^{\Delta_0} \qquad f(t) \underset{t\to\infty}{\sim} t^{2\Delta_\phi-\Delta_0}$$



$$f_0(t) = f_{conn}(t) - \sum_{\Delta=\Delta_0}^{\Delta_\phi} \sum_{k=0}^{[\Delta_\phi-\Delta]} c^2_{\Delta_\phi\Delta_\phi\Delta} \frac{\Gamma(\Delta+k)^2\Gamma(2\Delta)}{\Gamma(\Delta)^2\Gamma(2\Delta+k)}\, t^{\Delta+k}$$

$$f_\infty(t) = f_{conn}(t) - \sum_{\Delta=\Delta_0}^{\Delta_\phi} \sum_{k=0}^{[\Delta_\phi-\Delta]} c^2_{\Delta_\phi\Delta_\phi\Delta} \frac{\Gamma(\Delta+k)^2\Gamma(2\Delta)}{\Gamma(\Delta)^2\Gamma(2\Delta+k)}\, t^{2\Delta_\phi-\Delta-k}$$

$$\psi_0(s) = \int_0^1 dz\, f_0(t) \left(\frac{t}{1+t}\right)^{-2\Delta_\phi} t^{-1-s} + \sum_{\Delta=\Delta_0}^{\Delta_\phi} \sum_{k=0}^{[\Delta_\phi-\Delta]} c^2_{\Delta_\phi\Delta_\phi\Delta} \frac{\Gamma(\Delta+k)^2\Gamma(2\Delta)}{\Gamma(\Delta)^2\Gamma(2\Delta+k)} \frac{(-1)^k}{k!} \frac{1}{s-\Delta-k} \qquad Re(s) < \tilde{\Delta}_0$$

$$\psi_\infty(s) = \int_1^\infty dz\, f_\infty(t) \left(\frac{t}{1+t}\right)^{-2\Delta_\phi} t^{-1-s} + \sum_{\Delta=\Delta_0}^{\Delta_\phi} \sum_{k=0}^{[\Delta_\phi-\Delta]} c^2_{\Delta_\phi\Delta_\phi\Delta} \frac{\Gamma(\Delta+k)^2\Gamma(2\Delta)}{\Gamma(\Delta)^2\Gamma(2\Delta+k)} \frac{(-1)^k}{k!} \frac{1}{s-2\Delta_\phi+\Delta+k} \qquad Re(s) > 2\Delta_\phi - \tilde{\Delta}_0$$

Subtractions

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At non-perturbative level : operators lighter that $2\Delta_\phi$ can be exchanged

$$f(t) \underset{t\to 0}{\sim} t^{\Delta_0} \qquad f(t) \underset{t\to\infty}{\sim} t^{2\Delta_\phi - \Delta_0}$$

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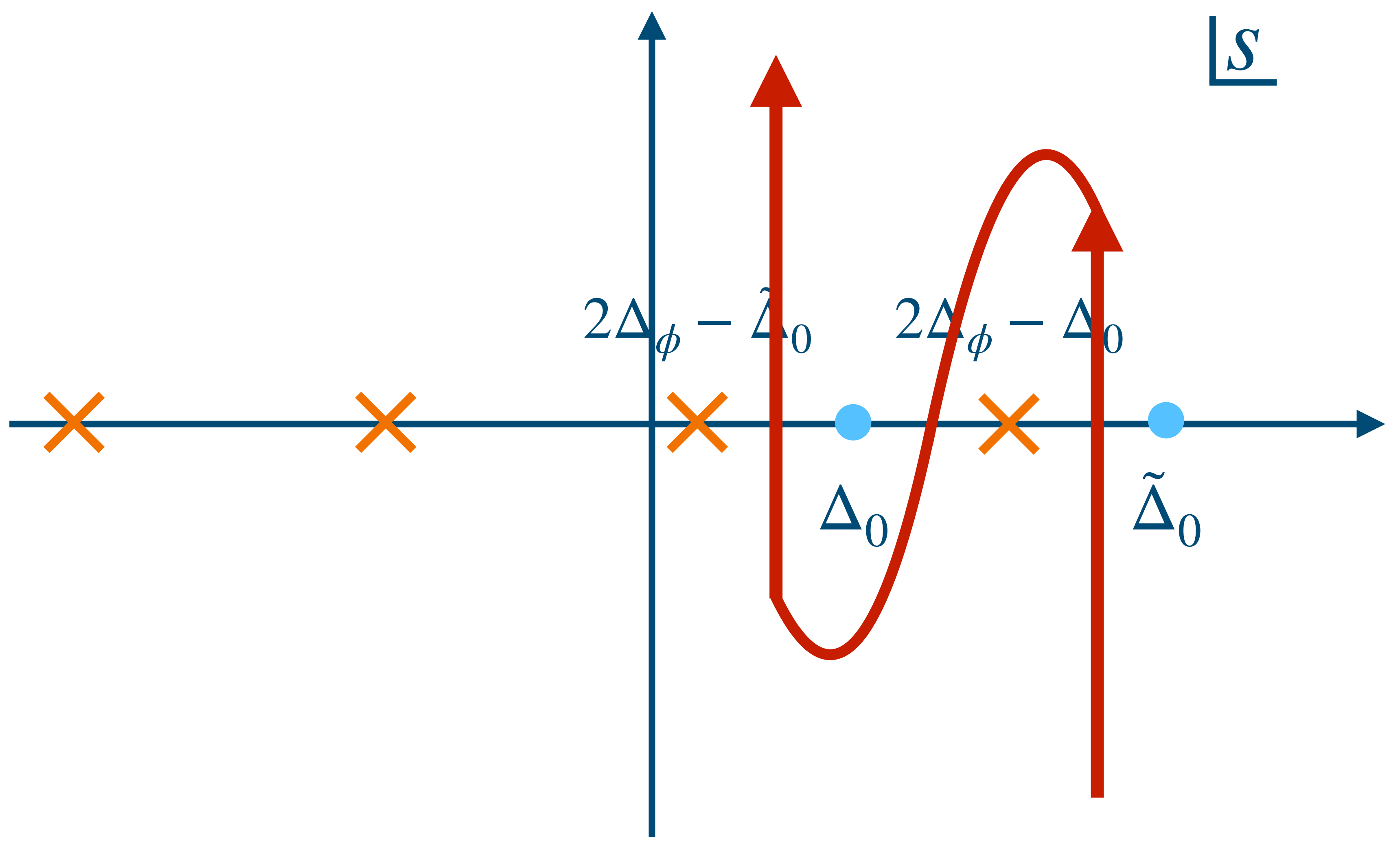

$$\hat{M}(s) = \psi_0(s) + \psi_\infty(s)$$

$\tilde\Delta_0 \to \infty$: $\hat{M}(s)$ on $\mathbb{C}$

$$2\Delta_\phi - \tilde\Delta_0 < Re(s) < \tilde\Delta_0$$

$$\hat{M}(s) = \sum_{\Delta} c^2_{\Delta_\phi\Delta_\phi\Delta} \Big(F_\Delta(s) + F_\Delta(2\Delta_\phi - s) \Big)$$

$$F_\Delta = \frac{{}_3F_2(\Delta, \Delta, \Delta - s; 2\Delta, 1 + \Delta - s, -1)}{\Delta - s}$$

⌘ poles at the physical exchanged operators

$$s = \Delta + k\,, \quad k \in \mathbb{Z}$$

+ crossing sym.

- Gabriel's Talk : "Using the Mellin Transform in 1d CFT" → Sum rules
  → Extraction of CFT data for derivative interactions

→ Generalise to n-point correlation functions and to non-identical scalars

→ Flat-space limit, scattering amplitudes link

→ Higher loop computations

THANK YOU!