SCHRÖDINGER CONNECTION WITH SELFDUAL NON-METRICITY VECTOR IN 2+1 DIMENSIONS

Based on: S. Klemm, L. Ravera, 2008.12740

1918: Weyl attempts to unify EM with gravity geometrically → Connection with non-metricity ⇝ Weyl vector ⟷ gauge pot.

- Physical inconsistencies ("2nd clock")

1940s: Schrödinger → Symmetric connection that involves non-metricity but preserves the length of vectors under parallel transport! → No "2nd clock"

Schrödinger connection

$$\hat{\Gamma}^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} + g^{\lambda\rho} Z_{\rho\mu\nu}$$

$\hookrightarrow$ LC

$$Z_{\lambda\mu\nu} = Z_{\lambda\nu\mu}\,, \quad Z_{(\lambda\mu\nu)} = 0$$

Now, consider a generic conn. $\Gamma^{\lambda}{}_{\mu\nu} = \tilde{\Gamma}^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}$

$N^{\lambda}{}_{\mu\nu}$: deflection $(Q_{\lambda\mu\nu})$ – contorsion $(T_{\lambda\mu}{}^{\nu})$

$\rightsquigarrow$ If $T^\lambda{}_{\mu\nu} = 0$ and $N_{(\lambda\mu\nu)} = 0$

$$\Rightarrow \Gamma^\lambda{}_{\mu\nu} = \tilde{\Gamma}^\lambda{}_{\mu\nu} - g^{\lambda\rho} Q_{\rho\mu\nu}, \quad Q_{(\lambda\mu\nu)} = 0$$

Taking $Z_{\lambda\mu\nu} = -Q_{\lambda\mu\nu} \Rightarrow \Gamma = \hat{\Gamma}$, Schr.

$$\Rightarrow \hat{\Gamma}^\lambda{}_{\mu\nu} = \tilde{\Gamma}^\lambda{}_{\mu\nu} - g^{\lambda\rho} Q_{\rho\mu\nu}$$

More physical than Weyl's connection!

Non-metricity decomposition in 3-dim.

$$Q_{\lambda\mu\nu} = \frac{2}{5} Q_\lambda g_{\mu\nu} - \frac{1}{5} \tilde{Q}_\lambda g_{\mu\nu} + \frac{3}{5} g_{\lambda(\nu} \tilde{Q}_{\mu)}$$

$$- \frac{1}{5} g_{\lambda(\nu} Q_{\mu)} + \Omega_{\lambda\mu\nu}$$

$\hookrightarrow$ traceless

$$Q_\lambda = Q_{\lambda\mu}{}^{\mu} \qquad \tilde{Q}_\lambda = Q^{\mu}{}_{\mu\lambda}$$

Our model (3 dim., 2+1)

$$S = \frac{1}{2\kappa^2}\int d^3x \left(\sqrt{-g}\, f(R) + \frac{1}{2\mu}\, \epsilon^{\mu\nu\rho} Q_\rho \hat{R}_{\nu\mu}\right)$$

↳ CS term

$$+ \int d^3x\, \epsilon^{\mu\nu\rho}\, \zeta_{\nu\sigma}\, T_{\rho\mu}{}^{\sigma}$$

↳ LM

$$R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$$

$$\hat{R}_{\mu\nu} = R^{\lambda}{}_{\lambda\mu\nu} = \partial_{[\mu} Q_{\nu]}$$

$$\epsilon^{\mu\nu\rho} = \sqrt{-g}\, \varepsilon^{\mu\nu\rho}$$

E.O.M.:

$\delta \Gamma_{\mu\nu}$: $\quad T_{\rho\sigma}{}^{\lambda} = 0$

$\delta g^{\mu\nu}$ $\quad f'(R)\, R_{(\mu\nu)} - \frac{1}{2} f(R)\, g_{\mu\nu} = 0$

Trace: $\quad \frac{f}{2f'} = \frac{R}{3}$

$\to \quad f(R) = C R^{3/2}$

inv. $\quad g_{\mu\nu} \to e^{2\Omega} g_{\mu\nu}$

$\to \quad R = c_k$, const.

(would be EW if Weyl non-metricity)

back $\downarrow$

$$R_{(\mu\nu)} - \frac{R}{3} g_{\mu\nu} = 0$$

$$\delta\Gamma^{\lambda}{}_{\mu\nu} : \quad P_{\lambda}{}^{\mu\nu} + \delta_{\lambda}{}^{\nu} \frac{\partial^{\mu} f'}{f'} - g^{\mu\nu} \frac{\partial_{\lambda} f'}{f'}$$

$$+ \frac{2}{\mu f'} \varepsilon^{\nu\rho\sigma} \delta_{\lambda}{}^{\mu} \hat{R}_{\rho\sigma} + \frac{2\kappa^2}{f'} \varepsilon^{\mu\nu\rho} \zeta_{\rho\lambda} = 0$$

$$\rightsquigarrow \quad \Omega_{\lambda\mu\nu} = 0 \,, \quad \zeta_{\rho\sigma} = \frac{3}{\kappa^2 \mu} \hat{R}_{\rho\sigma}$$

$$\hat{R}_{\rho\sigma} = \frac{\mu f'}{20} \varepsilon^{\mu\nu\lambda} g_{\rho\mu} g_{\sigma\nu} \left(Q_{\lambda} - 3\tilde{Q}_{\lambda} \right)$$

And $\frac{\partial_\lambda f'}{f'} = \frac{1}{10}\left(Q_\lambda + 2\tilde{Q}_\lambda\right)$

$$\Rightarrow \Gamma^\lambda{}_{\mu\nu} = \tilde{\Gamma}^\lambda{}_{\mu\nu} - \frac{3}{10} g_{\mu\nu} Q^\lambda + \frac{2}{5}\delta^\lambda_{(\mu} Q_{\nu)} + \frac{2}{5} g_{\mu\nu}\tilde{Q}^\lambda - \frac{1}{5}\delta^\lambda_{(\mu}\tilde{Q}_{\nu)}$$

- For $C=1$: Generalized monopole eq.

$$\star\left(d\Sigma + h\Sigma\right) = dh$$

$\Sigma = 3\mu\sqrt{R}/8$ ↵ ↳ $h_\lambda = -Q_\lambda/6$

And $\partial_\mu \ln R = \frac{1}{5}\left(Q_\mu + 2\tilde{Q}_\mu\right)$

- If Σ const. (achieved by $\Sigma \to e^{-\Omega}\Sigma$)

$\Rightarrow$ $dh = * h\Sigma$

Self - duality in 3 dimensions!

In this case $\tilde{Q}_\mu = -\frac{1}{2} Q_\mu$

- Setting Σ const $=$ Sol. $R = c_R \Rightarrow f(R)$ const

and $f'(R) = C_0$

- Consider $f'(R) = C_0$

$\partial_\lambda f' = 0$

$$R_{(\mu\nu)} = \frac{C_\kappa}{3} g_{\mu\nu}$$

and $\zeta_{\rho\sigma} = \dfrac{3C_0}{8\kappa^2} \varepsilon_{\rho\sigma\tau} Q^\tau$

$$\hat{R}_{\rho\sigma} = \frac{C_0\mu}{8} \varepsilon_{\rho\sigma\tau} Q^\tau$$

$\downarrow$ can be dualized as

$$-\frac{\omega^\mu}{4} Q_\alpha = \varepsilon_{\alpha\rho\sigma}\,\partial^\rho Q^\sigma$$

which implies $\tilde{\nabla}^\mu Q_\mu = 0$

In particular we have $Q_{(\lambda\mu\nu)} = 0$

and $Q_{\lambda\mu\nu} = \frac{1}{2} Q_\lambda g_{\mu\nu} - \frac{1}{2} Q_\mu g_{\lambda\nu} - \frac{1}{4} Q_\nu g_{\lambda\mu}$

$$\Rightarrow \Gamma^\lambda{}_{\mu\nu} = \tilde{\Gamma}^\lambda{}_{\mu\nu} - g^{\lambda\rho} Q_{\rho\mu\nu}$$

Schrödinger with selfdual Q_λ

Self-duality in odd dim. and inh. Maxwell eqs.

Townsend - Pilch - van Nieuwenhuizen :

dim. $n = 4k - 1$, $k = 1, 2, 3, \ldots$

"Square root" of Proca eq. for a massive AS tensor field → Self dual field

- Our model : $k = 1$

- We have:

$$Q_\mu = \frac{1}{2M} \varepsilon_{\mu\nu\rho} F^{\nu\rho} \quad , \quad F_{\mu\nu} = 2\partial_{[\mu} Q_{\nu]} = 2\hat{R}_{\mu\nu}$$

$$M = -C_0\mu/4$$

Dual form: $F_{\alpha\beta} = M\, \varepsilon_{\mu\beta\alpha} Q^\mu$

$\Rightarrow \tilde{\nabla}^\mu Q_\mu = 0$ and covariant Proca eq.

$$\tilde{\nabla}^\mu F_{\mu\nu} - M^2 Q_\nu = 0$$

$$\rightarrow \partial^\mu F_{\mu\nu} - \frac{1}{2} g^{\alpha\beta} \partial^\mu g_{\alpha\beta} F_{\mu\nu} - M^2 Q_\nu = 0$$

$\Rightarrow Q_\mu$: massive photon gauge field

- Furthermore, cov. Proca = inhom. EM wave eq. following from inhom. Maxwell eqs.

$\hookrightarrow Q_\mu$: massless and "source of itself"

See this explicitly:

- Wave eq. in our model $(\Box - M^2) Q^\mu = 0$

$$\Box \equiv \tilde{\nabla}_\alpha \tilde{\nabla}^\alpha$$

- Inhom. EM wave eq for Q^μ in the gauge

$\tilde{\nabla}^\mu Q_\mu = 0$: $\Box Q^\mu = \mu_0 J^\mu \Leftrightarrow \Box Q^\mu = \tilde{\nabla}_\rho F^{\rho\mu}$

μ_0: vacuum permeability

J^μ: current

- Using S-d in our model & $(\Box - M^2) Q^\mu = 0$:

$$\Box Q^\mu = \frac{M}{2} \varepsilon^{\mu\rho\sigma} F_{\rho\sigma} = \tilde{\nabla}_\rho (M \varepsilon^{\mu\rho\sigma} Q_\sigma) = \tilde{\nabla}_\rho F^{\rho\mu}$$

$\Rightarrow \Box Q^\mu = \tilde{\nabla}_\rho F^{\rho\mu}$ in our model

$\rightarrow$ We can write $\Box Q^\mu = \mu_0 J^\mu$,

$$\underline{J^\mu} = \frac{1}{\mu_0} \tilde{\nabla}_\rho F^{\rho\mu} = \frac{M}{2\mu_0} \varepsilon^{\mu\rho\sigma} F_{\rho\sigma} = \frac{M^2}{\mu_0} Q^\mu$$

- The source current is covariantly cons. : $\tilde{\nabla}_\mu J^\mu = 0$

↳ Cannot def globally cons. charge, since we need a local cons. law for this

⇝ Take Proca : $\partial^\mu F_{\mu\nu} - \frac{1}{2} g^{\alpha\beta} \partial^\mu g_{\alpha\beta} F_{\mu\nu} - M^2 Q_\nu = 0$

→ Total current (source + self) $j^\mu = J^\mu + \frac{1}{2\mu_0} g^{\alpha\beta} \partial_\rho g_{\alpha\beta} F^{\mu\rho}$

locally conserved : $\partial_\mu j^\mu = 0$

Conclusions

- Our Proca, massive photon $\rightsquigarrow$ interpreted as Inh. Maxwell eqs., with corresponding wave eq.
  - ↳ Photon "source of itself" due to self-duality
- Gauge inv. of connection and inh. Maxwell eqs follows from self-duality

$\Rightarrow$ Q_μ "photon"

- In our framework: Inh. Maxwell eqs. from a purely gravitational setup!

- CS and Lagr. mult. break proj. inv. of S

$\Rightarrow$ Consistent matter couplings allowed

- Dynamical d.o.f.: 1 for Q_μ ($\partial^\mu Q_\mu = 0$ + S-d.)

⇝ Extension to higher dimensions ?

THANK

you !