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NLL small-x resummation for Higgs induced DIS

Video contribution for
"Cortona young 2021"

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Work in progress with Simone Marzani and Giovanni Ridolfi (INFN Genova) and
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Motivation

Perturbative QCD

$$O = \sum_n \alpha_s^n c_n$$

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Small enough

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Small enough

Not divergent

How does c_n behave?

$$c_n \supset \ln^k \left(\frac{1-x}{x} \right) \qquad k = 0, \ldots, n$$

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Threshold limit

$x \to 1$

"Large-x" limit

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$$c_n \supset \ln^k\left(\frac{1-x}{x}\right) \qquad k = 0, \ldots, n$$

Threshold limit

$x \to 1$

"Large-x" limit

High energy limit

$x \to 0$

"Small-x" limit

Problem

$$O^{(n)} \sim \alpha_s^n \left[\ln^n(x) + \ln^{n-1}(x) + \ldots\right]$$

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$$\begin{array}{cc} \downarrow & \downarrow \\ \text{LL} & \text{NLL} \end{array}$$

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$\ln^n(x)$ → LL

$\ln^{n-1}(x)$ → NLL

→ Spoil the convergence of the perturbative series

↓

Theoretical prediction are no longer reliable

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$\ln^n(x)$ ↓ LL

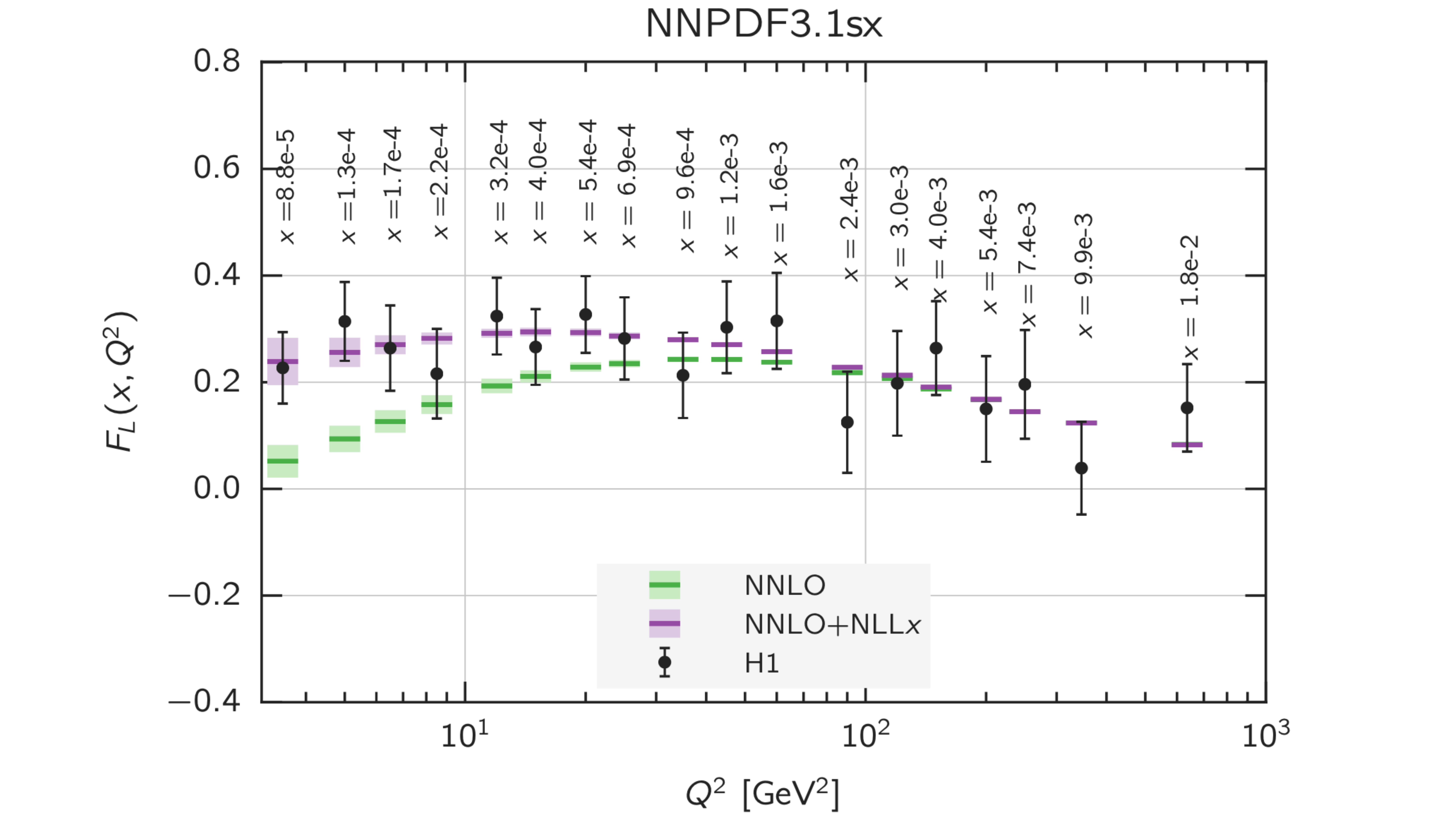
$\ln^{n-1}(x)$ ↓ NLL

Solution?

Find out the all-order structure of these logarithmic contributions by writing them as a series and summing it

$\boxed{\text{Resummation}}$

How much small-x resummation improves theoretical predictions?



Ball, R. D., Bertone, V., Bonvini, M., Marzani, S., Rojo, J., & Rottoli, L. (2018). Parton distributions with small-x resummation: evidence for BFKL dynamics in HERA data. The European Physical Journal C, 78(4), 1-52.

Small-x resummation:
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- Resummation ⟷ Factorization

We need some factorisation properties

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- Resummation $\longleftrightarrow$ Factorization

We need some factorisation properties

- Mellin Transform

$$g(N,Q^2) = \int_0^1 dx\, x^N g\left(x,Q^2\right) \qquad \ln^k(x) \to \frac{1}{N^{k+1}}$$

Collinear factorization theorem

$$\sigma(N, Q^2) = \sum_{i=q,g} C_i\left(N, \alpha_s(Q^2)\right) f_i\left(N, Q^2\right)$$

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Coefficient function

Parton distribution function (PDF)

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Our goal: resum NLL logarithms in the coefficient function

High energy factorization theorem

$$\sigma(N, Q^2) = \int_0^\infty dk_\perp^2 \, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right) \, \mathscr{F}_g\left(N, k_\perp^2\right)$$

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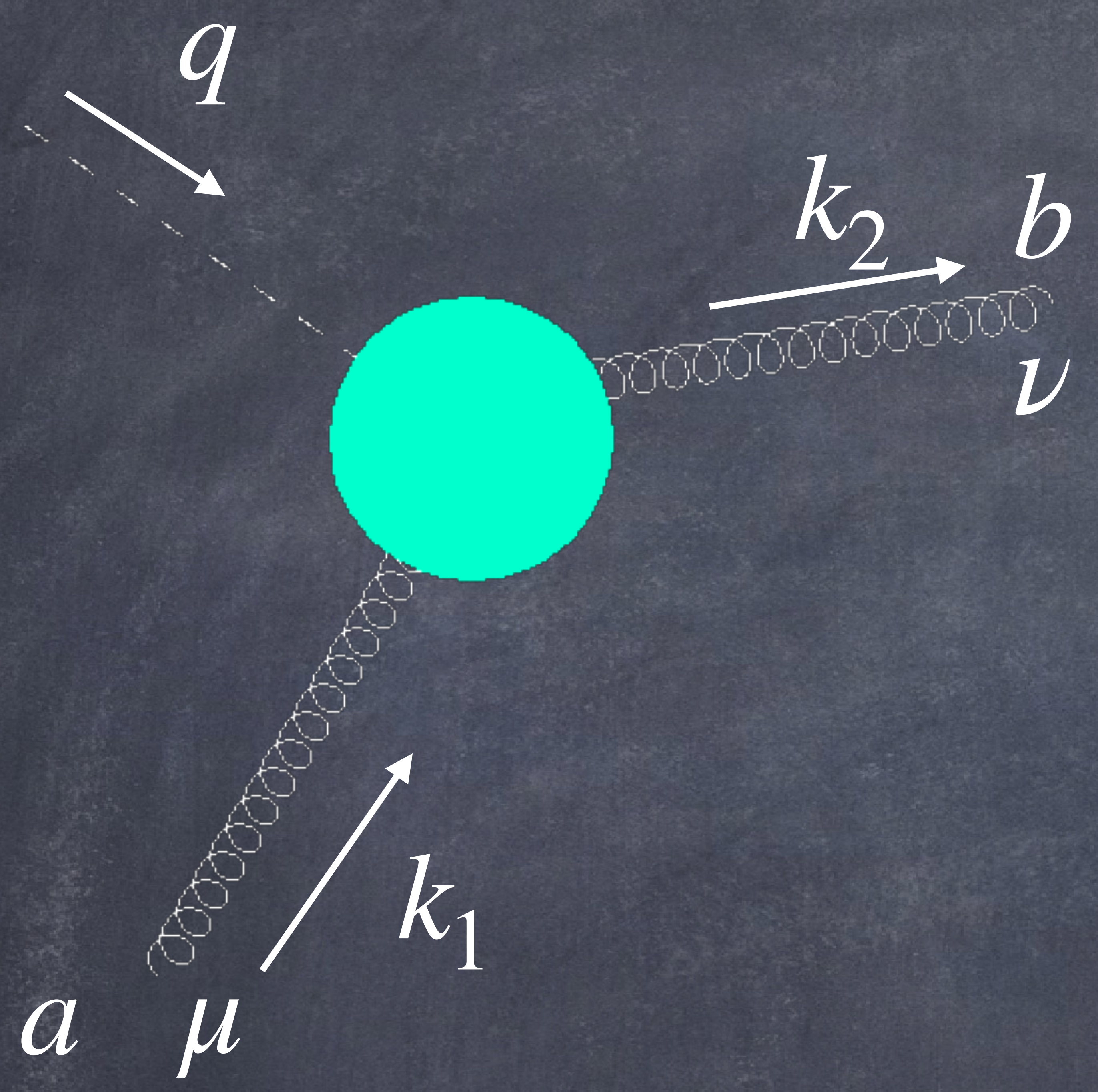
Bonvini M., Marzani S., and Peraro T. (2016). Small-x resummation from HELL. The European Physical Journal C, 76(11), 1-28.

Higgs induced DIS: What we want to compute?

Higgs DIS



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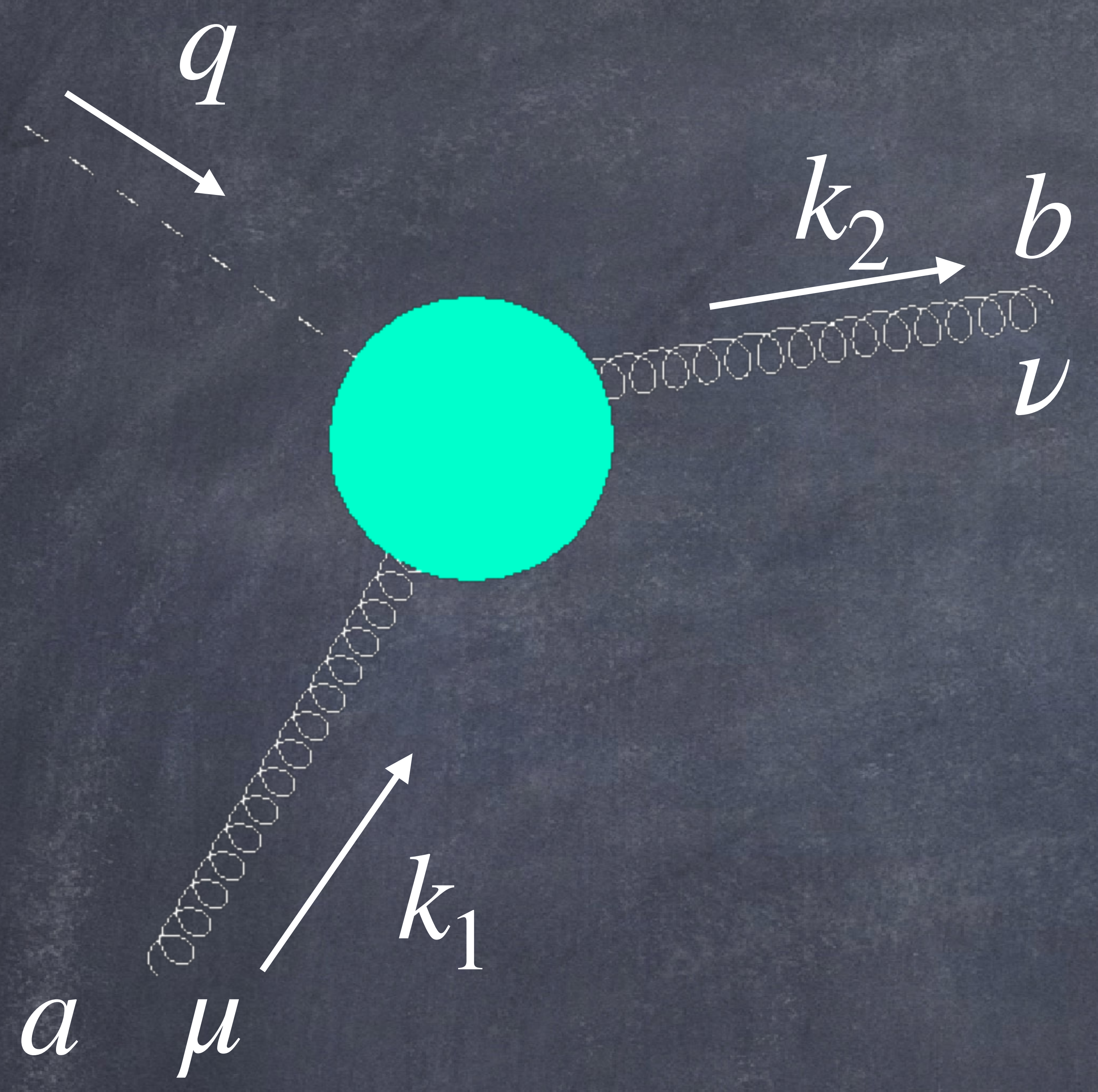

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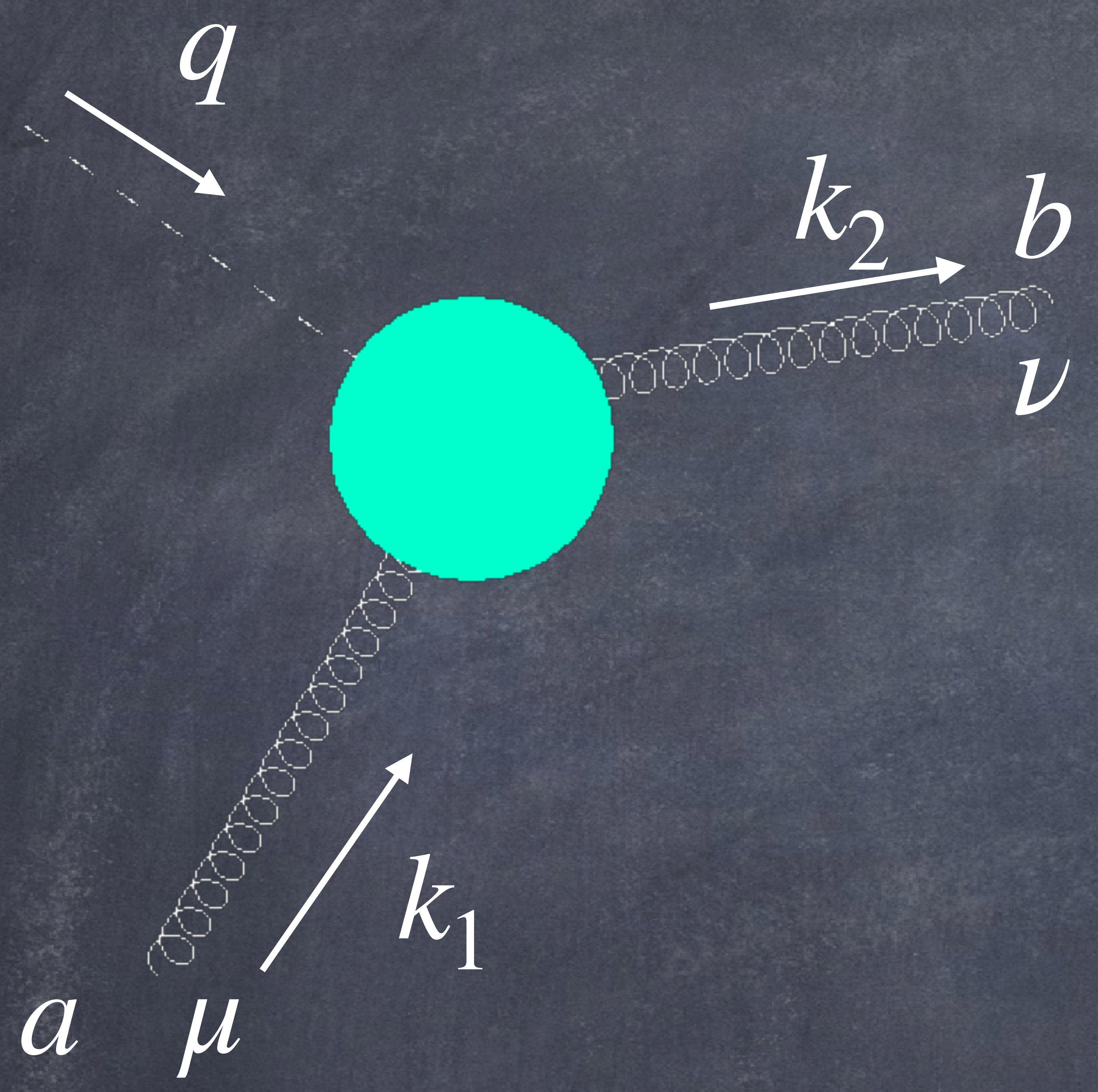
Higgs DIS



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- Higgs gluon effective vertex:

$$M^{\mu\nu} = i\, c\, \delta_a^b \left[k_2^\mu k_1^\nu - g^{\mu\nu} k_1 \cdot k_2 \right]$$

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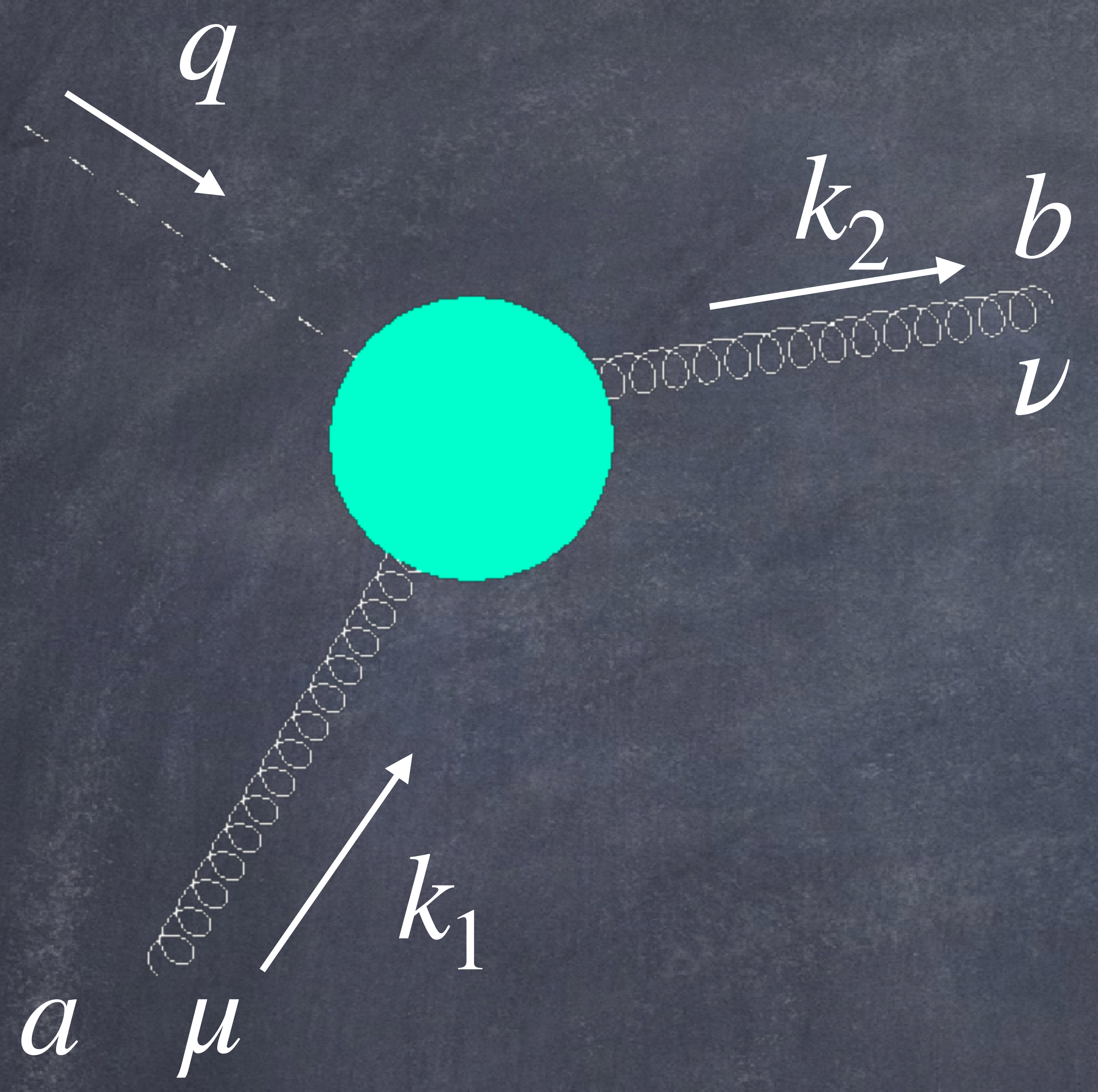

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$$k_1^2 = -\vec{k}_1^{\,2}$$

Higgs DIS

We want to resum NLL terms in the coefficient function

$$C_g(N,\alpha_s) = \int_0^\infty dk_\perp^2\, \mathscr{C}\left(N,k_\perp^2,Q^2,\alpha_s\right)\, \mathscr{U}\left(N,k_\perp^2,Q^2\right)$$

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We have to compute the one-loop off-shell coefficient function

Axial gauge

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- Growing number of terms due to gauge choice

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- Final stages of the calculation

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Thank you!