Phases of five-dimensional supersymmetric gauge theories

Based on [arXiv:2103.14049]

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Video-poster

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Introduction

Today's focus: 5d $\mathcal{N} = 1$ gauge theories.

They admit UV completion in a superconformal field theory (SCFT) *[Seiberg, Intriligator-Morrison-Seiberg, many others]*, which is

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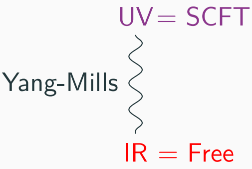
The gauge theories are realised via mass deformation of these SCFTs.



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Compute supersymmetry-protected quantities, e.g. sphere partition function $\mathcal{Z}_{\mathbb{S}^5}$ or Wilson loop vev $\langle \mathcal{W} \rangle$, in *weakly coupled* gauge theory, extract info about *strongly coupled* UV SCFT.

Matrix model viewpoint

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Matrix model $\leftrightarrow$ Statistical ensemble

Starting from localization on $\mathbb{S}^5$, we ask new questions about the theory (or old questions from new angles), such as phase diagram and universality.

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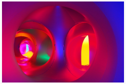
- We present a systematic study for theories with simple gauge group.
- A $U(N)$ motivation: $\mathcal{N} = 1$ theories with unitary gauge group *[Collinucci-Valandro]*. As a preliminary step to understand their UV behaviour, we compare their phase diagram with the one of theories with known UV SCFT.

5d theories from CY threefolds

Geometric construction via M-theory compactifications



$$\begin{aligned} \text{SCFT} &\longleftrightarrow \text{singular CY threefold } X \\ \text{gauge theory} &\longleftrightarrow \text{resolution } \widetilde{X} \to X \text{ of CY threefold} \\ \text{Extended CB moduli} &\longleftrightarrow \text{curves in } \widetilde{X} \\ \text{operations on gauge theory} &\longleftrightarrow \text{operations on geometry } \widetilde{X} \end{aligned}$$

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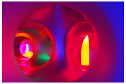

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- SCFT limit: volume of all curves $\to 0$.
- 't Hooft limit: # of *compact* divisors $\to \infty$.
- Veneziano limit: # of *non-compact* divisors $\to \infty$ linearly with # of *compact* divisors.

Phases of $U(N)$ and $SU(N)$ theories: Setup

We consider a 5d theory with gauge group $U(N)$ or $SU(N)$ and hypers in the fundamental rep. We assign masses m_α to n_α of them.
The $\mathbb{S}^5$ partition function is [Källén-Qiu-Zabzine]

$$\mathcal{Z}_{\mathbb{S}^5} = \frac{1}{N!} \int_{\mathbb{R}^N} \mathrm{d}\phi \, e^{-\sum_{a=1}^{N} V(\phi_a)} Z_{\text{1-loop}}^{\text{vec}}(\phi) \prod_\alpha \left[Z_{\text{1-loop}}^{\text{hyp}}(\phi, m_\alpha) \right]^{n_\alpha} (1 + \text{non-pert.})$$

with classical piece from BPS configuration

$$\frac{1}{\pi r^3} V(\phi) = \underbrace{h\phi^2}_{\text{YM}} + \underbrace{\frac{k}{3}\phi^3}_{\text{CS}} + \underbrace{\xi\phi}_{\text{FI}} .$$

Wick-rotated FI term serves as a Lagrange multiplier: set to 0 for $U(N)$ or use to enforce traceless condition on $SU(N)$.

Phases of $U(N)$ and $SU(N)$ theories: 't Hooft limit

We take the large N 't Hooft and Veneziano limit, $N \to \infty$ with

$$\frac{N}{h} = \lambda \text{ fixed}, \qquad \frac{N}{k} = t \text{ fixed}, \qquad \frac{n_\alpha}{N} = \zeta_\alpha \text{ fixed}.$$

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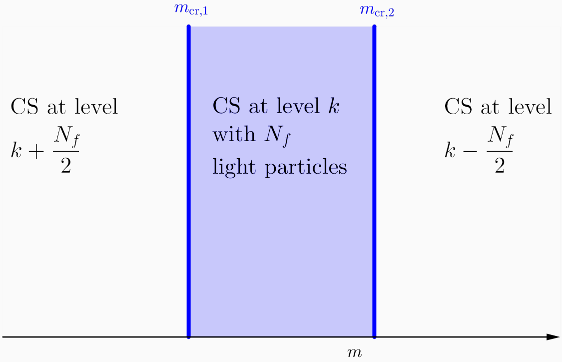
Instead of facing directly the SPE, we take a further simplification: small curvature limit $\frac{1}{r} \to 0$ *[Russo-Zarembo]*.

Example 1: One mass scale

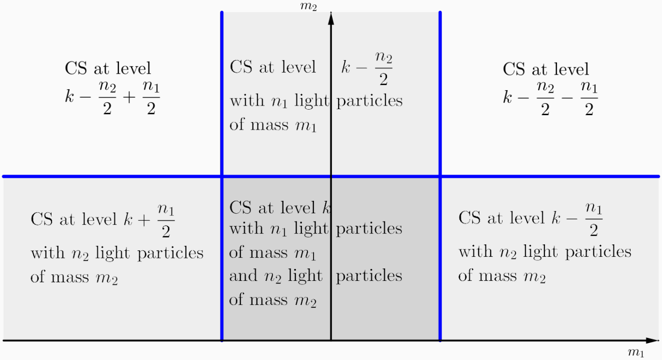
N_f hypers, all of equal mass m.



Phase transitions are second order if $U(N)$, third order if $SU(N)$.

Example 2: Two mass scales

n_1 hypers of mass m_1 and n_2 hypers of mass m_2.



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Fundamental Wilson loops

Wilson loops are order operators in Yang–Mills theories $\Rightarrow$ their vev $\langle \mathcal{W} \rangle$ are order parameters of the theories $\Rightarrow$ $\langle \mathcal{W} \rangle$ control the phase diagram.

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Evaluate Wilson loops in the fundamental rep. $\Rightarrow$ prove that $\langle \mathcal{W}_{\mathsf{F}} \rangle$ is always continuous $\Rightarrow$ transitions are always at least second order.
Studying $\partial_{m_\alpha} \langle \mathcal{W}_{\mathsf{F}} \rangle$, the traceless condition is used to guarantee that the derivative is continuous $\Rightarrow$ transitions are always third order for $SU(N)$.

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Apply same strategy to other theories: different matter content and/or gauge group.

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  Study Wilson loops $\rightsquigarrow$ confirm spontaneous one-form symmetry breaking.
- $SO(2N+1), SO(2N), USp(2N)$: analogous to $SU(N)_{k=0}$.

Summary: Classical groups

For classical gauge group we obtained:

- For theories with fundamental hypers, there are third order phase transitions each time a mass parameters crosses a critical hypersurface (function of 't Hooft parameters).
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- For theories with fundamental hypers, there are third order phase transitions each time a mass parameters crosses a critical hypersurface (function of 't Hooft parameters).
- The F-theorem is always satisfied.
- For theories with adjoint or anti-symmetric hypers the phase transitions are second order.

Summary: Unitary group

For unitary gauge group we found that

- For theories with fundamental hypers there are phase transitions that can be both second or third order, depending on the assignment of masses.
- For balanced theories our method does not distinguish between $U(N)$ or $SU(N)$.

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For both $U(N)$ and $SU(N)$ and $\lambda = \infty$, the transitions are third order at zero mass, matching the flop transition in the CY.

5d SCFT and large N: A visual summary