Istituto Nazionale di Fisica Nucleare

SAPIENZA UNIVERSITÀ DI ROMA

Small-x resummation at the LHC:

multi-differential cross-sections for heavy quark production.

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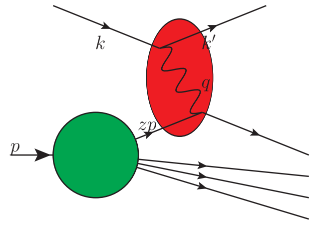
A video poster for Cortona Young 2021 virtual conference
Based on: *"Differential heavy-quark production at small-x"*,
F.Silvetti and M. Bonvini, in preparation

Theoretical description of proton scattering

Example: Deep Inelastic Scattering

$$\sigma\left(x, Q^2\right) = \sum_{i \in \{q, \bar{q}, g\}} \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(\frac{x}{z}, \alpha_s, Q^2\right) f_i\left(z, Q^2\right)$$

$$x = \frac{Q^2}{2(q \cdot p)}, \quad Q^2 = -q^2$$



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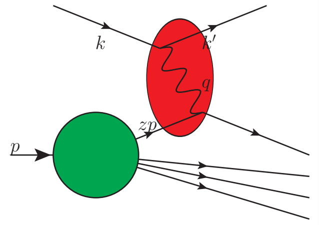

Coefficient function C:

- Computed from QCD in perturbation theory

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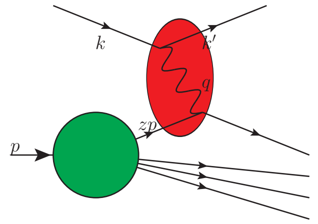

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- Q^2 scale dependence is governed by DGLAP equations

$$Q^2 \frac{\mathrm{d}}{\mathrm{d}Q^2} f_i\left(z, Q^2\right) = \int_z^1 \frac{\mathrm{d}w}{w} P_{ij}(w, \alpha_s) f_j\left(\frac{z}{w}, Q^2\right) = P_{ij}(\alpha_s) \otimes f_j\left(Q^2\right),$$

with the splitting functions P being perturbatively determined themselves

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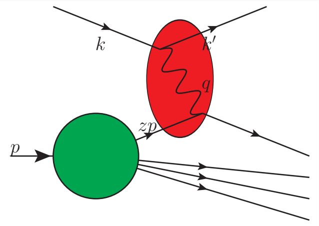

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hadronic-cross section σ

- can be predicted given the two above
- must be provided as experimental input in order to determine PDFs
- PDFs are a **large source of theoretical uncertainty** $\rightarrow$ refine theory and include more data to improve results

Collider physics at small-x

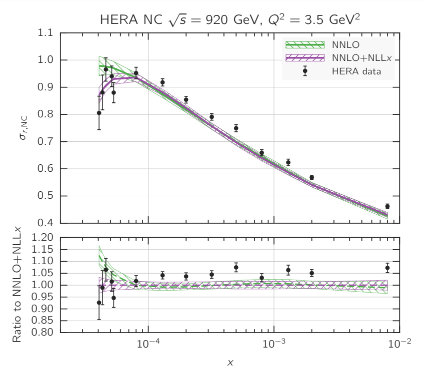

Figure: from Eur.Phys.J.C 78 (2018) 4, 321

- High-energy precision physics from LHC $\leftrightarrow$ constrain PDFs to lower regions of x
- Proper description of **small-x** requires accounting for a special class of logarithms

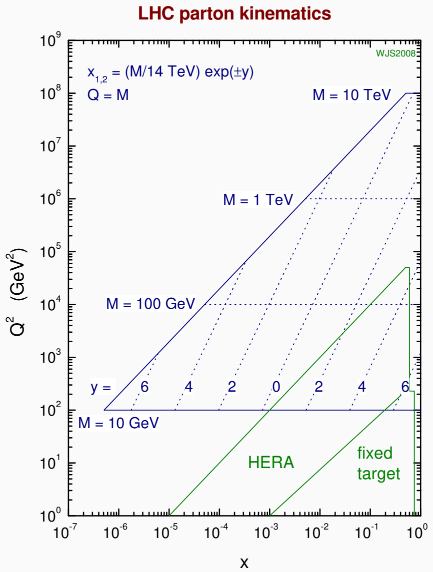

Figure: "W.J. Stirling, private communication"

Features of small-x logarithms

Single logarithm enhancement: one extra power of the logarithm at each successive order of perturbation theory.

- When $\alpha_s \ln(x) = \mathcal{O}(1)$ $\rightarrow$ breakdown of fixed order pert. theory $\rightarrow$ resum to all orders in α_s
- In DGLAP splitting functions
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  - Known up to NLL

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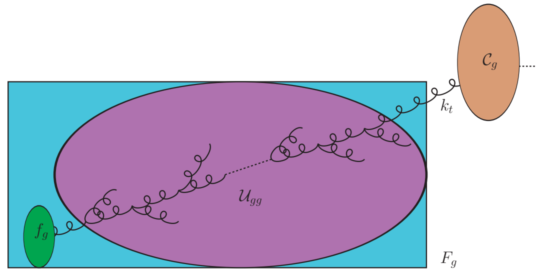
- In coefficient functions
  - The resummation algorithm is based on the k_t-factorization, an **alternative factorization scheme**
  - This operation can be carried out exclusively to LL accuracy

$$\begin{aligned} C = {} & \alpha_s \ln(x) b_{11} + \alpha_s b_{10} \\ & + \alpha_s^2 \ln^2(x) b_{22} + \alpha_s^2 \ln(x) b_{21} + \alpha_s^2 b_{20} \\ & + \alpha_s^3 \ln^3(x) b_{33} + \alpha_s^3 \ln^2(x) b_{32} + \alpha_s^3 \ln(x) b_{31} + \alpha_s^3 b_{30} \\ & \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \ddots \\ & \text{(LL)} \qquad \text{(NLL)} \qquad \text{(NNLL)} \qquad \cdots \end{aligned}$$

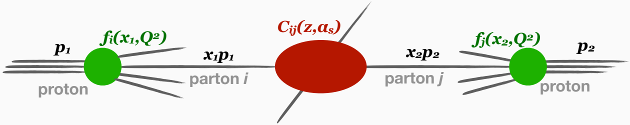


- k_t-factorisation[1]: $\sigma\left(x, Q^2\right) = \int \mathrm{d}k_t^2\, \mathfrak{C}_g\left(\alpha_s, Q^2, k_t^2\right) \otimes \mathfrak{F}_g\left(Q^2, k_t^2\right)$,
- Definition of PDF-evolutor[2]: $\mathfrak{F}_g\left(z, Q^2, k_t^2\right) = \mathcal{U}_{gg}\left(k_t^2, Q^2\right) \otimes f_g\left(Q^2\right)$
- By comparison: $C_g\left(z, Q^2, \alpha_s\right) = \int \mathrm{d}k_t^2\, \mathfrak{C}_g\left(Q^2, k_t^2, \alpha_s\right) \otimes \mathcal{U}_{gg}\left(k_t^2, Q^2\right)$

[1] Catani and Hautmann: hep-ph/9405388
[2] Bonvini, Marzani and Peraro: hep-ph/1607.02153

Differential cross sections in proton-proton collisions

At hadron colliders, two PDFs and suitable variables must be introduced



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}Y\,\mathrm{d}q_t^2}(x) = \sum_{i,j\in\{q,\bar{q},g\}} \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}y\, \frac{\mathrm{d}C_{ij}}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_t^2}\left(\frac{x}{z}, y\right) L_{ij}(z, Y-y)$$

$$= \sum_{i,j\in\{q,\bar{q},g\}} \frac{\mathrm{d}C_{ij}}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_t^2} \otimes L_{ij}$$

$$L_{ij}(z,\hat{y}) = f_i\left(\sqrt{z}\mathrm{e}^{-\hat{y}}, Q^2\right) f_j\left(\sqrt{z}\mathrm{e}^{+\hat{y}}, Q^2\right) \vartheta(\mathrm{e}^{-2|\hat{y}|} - x)$$

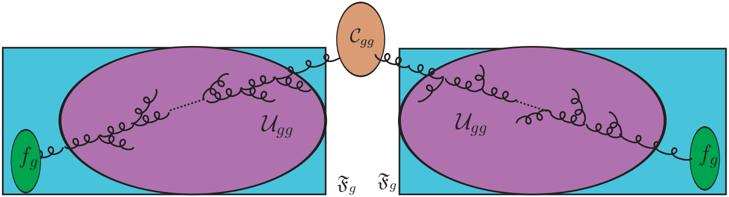
$x = \dfrac{Q^2}{S}$ $\quad$ $Q^2 =$ final state invariant mass $\quad$ $\sqrt{S} =$ collider energy

$$x_{1,2} = \sqrt{z}\mathrm{e}^{\mp\hat{y}} \to z = x_1 x_2\,, \quad \hat{y} = \frac{1}{2}\ln\left(\frac{x_2}{x_1}\right)$$

Resummation for triple differential cross sections [3]

$$\frac{d\sigma}{dQ^2\,dY\,dq_t^2}(x,Y) = \int dk_{1t}^2 \int dk_{2t}^2\, \frac{d\mathfrak{C}_{gg}}{dQ^2\,dy\,dq_t^2}\left(k_{1t}^2, k_{2t}^2\right) \otimes \mathfrak{L}_{gg}\left(k_{1t}^2, k_{2t}^2\right)$$

$$\mathfrak{L}_{gg}\left(z,\hat{y},k_{1t}^2,k_{2t}^2\right) = \mathfrak{F}_g\left(\sqrt{z}e^{-\hat{y}}, k_{1t}^2\right)\mathfrak{F}_g\left(\sqrt{z}e^{+\hat{y}}, k_{2t}^2\right)\vartheta(e^{-2|\hat{y}|} - x)$$

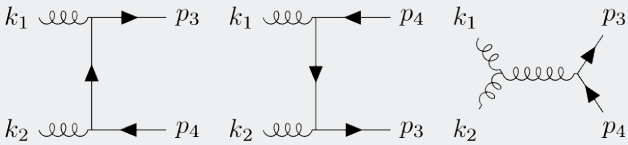


$$\mathfrak{F}_g\left(\sqrt{z}e^{\mp\hat{y}}, Q^2, k_t^2\right) = \mathcal{U}_{gg}\left(k_{1,2t}^2, Q^2\right) \otimes f_g\left(Q^2\right)$$

$$\frac{dC_{gg}}{dQ^2\,dy\,dq_t^2}(z,y) = \int dk_{1t}^2 \int dk_{2t}^2\, \frac{d\mathfrak{C}_{gg}}{dQ^2\,dy\,dq_t^2}\left(k_{1t}^2, k_{2t}^2\right) \otimes \mathcal{U}_{gg}\left(k_{1t}^2, Q^2\right) \otimes \mathcal{U}_{gg}\left(k_{2t}^2, Q^2\right)$$

[3] A similar result, achieved with an equivalent approach is reported in hep-ph/1010.2743 and hep-ph/1710.0937

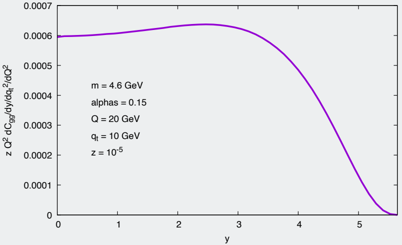
Application to $Q\overline{Q}$ production



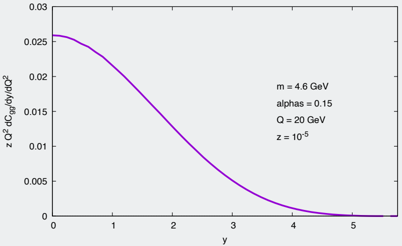
- Why? Recent measurements from LHCb for charmed mesons down to $x \sim 10^{-6}$ + resummed prediction $\rightarrow$ accurate small-x PDF fit
- Final state has been studied as both
  - quark-antiquark pair $\rightarrow$ simplified kinematics
  - single quark $\rightarrow$ useful for phenomenology

Application to $Q\overline{Q}$ production

New result: resummed triple differential coefficient function



(a) parton-level triple distribution for fixed Q^2 and transverse momentum



(b) parton-level double distribution for fixed Q^2

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Conclusions and outlook

New results

- Expressions for the resummed differential coefficient function in any combination of invariant mass, rapidity and transverse momentum was devised
- A direct application of the previous result to heavy-flavor pair production for both single quark and pair final state kinematics

[4] see hep-ex/1302.2864

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Outlook

- Small-x determination of PDFs including LHCb B/D mesons data [4]
- Extension of resummation strategy to NLL
  - currently underway in collaboration with Anna Rinaudo, Simone Marzani and Giovanni Ridolfi (University of Genova) as well as Marco Bonvini (INFN Roma 1)

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Thank you for your attention!

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