Four Dimensional Superconformal Index and AdS_5 Black Hole Entropy

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Video-Poster Session

Prelude

In this video-poster we will first discuss how to compute the Super-Conformal Index (SCI) $\mathcal{I}$ of a class of $d = 4$, $\mathcal{N} = 1$ holographic quiver theories at large N using a Bethe Ansätz (BA) approach

Secondly, we will extract predictions for the entropy of still unknown AdS_5 Black Holes (BH) and compare them with a near-horizon Supergravity computation

It is based on the following joint work

[1] F. Benini, E. Colombo, S. Soltani, A. Zaffaroni and Z. Zhang, "Superconformal indices at large N and the entropy of $\mathrm{AdS}_5 \times \mathrm{SE}_5$ black holes", arXiv:2005.12308 [hep-th]

Black Holes: a Quantum Gravity Laboratory

Quantum Gravity (QG) is one of the biggest challenges we have nowadays in theoretical physics

Weakness of gravity makes experimental QG tests very difficult $\Longrightarrow$ The search for a QG theory is often guided by theoretical constraints and self-consistency

A QG theory has to reproduce the Bekenstein-Hawking BH entropy from a microscopical viewpoint

$$S_{BH} = \frac{A_{BH}}{4G_N} = \log N_{\text{micro}}$$

Our aim is to understand how the AdS/CFT correspondence (non-perturbative definition of QG on AdS space) accounts for the BH microstates

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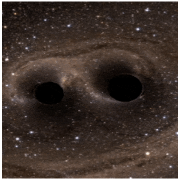

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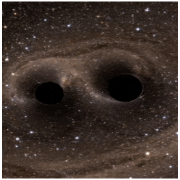

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Holographic Principle

Gravity on asymptotically AdS_5 space $\Longleftrightarrow$ CFT on the boundary ∂AdS_5

BH Entropy, $S_{BH} = \frac{A_{BH}}{4G_N}$ $\Longleftrightarrow$ Legendre transform of $\log \mathcal{I}$

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Why Superconformal Indices?

SUSY indices play an important role in studying non-perturbative aspects of QFTs. They are generalizations of the standard Witten index $\mathrm{Tr}_{\mathcal{H}}(-1)^F$

Many interesting features, among which

- they count with a sign ground states of SUSY theories $\Longrightarrow$ very robust
- they can be computed exactly via localization techniques

Superconformal Index in $d = 4$

$$\mathcal{I} = \mathrm{Tr}\left[(-1)^F e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}} \prod_{i=1}^{G_F} v_i^{Q_i}\right]$$

Counts with a sign 1/16-BPS states on $S^1 \times S^3$ of a superconformal theory

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AdS/CFT and Black Hole Entropy

AdS_5 BPS BHs preserve same number of supercharges [Gutowski et al. (2004)]

- Natural question: can one microscopically explain BHs macroscopic entropy using SCI via the AdS/CFT correspondence?

Non-trivial question! BH entropy counts BPS states without signs, can be captured only if boson-fermion cancellations are obstructed

Decisive idea from AdS_4 [Benini et al. (2016)] $\Longrightarrow$ complex chemical potentials. Later clarified in AdS_5 context by SUGRA analysis [Cabo-Bizet et al. (2018)]. This allowed people to compute AdS_5 BHs entropy via SCI in various limits

- Cardy-like limit (small chemical potentials) + Saddle point method [Choi et al. (2018), Kim et al., Cabo-Bizet et al. (2019)]
- Bethe Ansätz + Large N, $J_1 = J_2$ [Benini and Milan (2018)]

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Our Results using the Bethe Ansätz Approach

- Evaluation of SCI at large N for generic angular momenta
- Extension from $\mathcal{N} = 4$ SYM to a broad class of $\mathcal{N} = 1$ holographic quivers, including toric ones
- Non trivial checks via SUGRA computations

Superconformal Index Definition

$$\mathcal{I} = \mathrm{Tr}\left[(-1)^F e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}} p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}} \prod_{a=1}^{G_F} v_a^{Q_a}\right]$$

$$(p, q, v_a) = \exp\left[2\pi i\left(\tau, \sigma, \Delta_a - r_a\frac{\tau+\sigma}{2}\right)\right]$$

- Only 1/16-BPS ground states of H on $S^1 \times S^3$ contribute to $\mathcal{I}$
- $\mathcal{I}$ does not depend on β
- Mild boson-fermion cancellations $\implies$ micro-canonical BH entropy is naturally captured by Legendre transform of $\log\mathcal{I}$: $\mathcal{I}$-extremization
  - Why these cancellations do not take place was proven in the AdS$_4$ case in [Benini et al. (2016)]

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Bethe Ansätz Approach

SCI has an exact integral representation using localization. Via residue theorem it becomes a sum over Bethe vacua [Benini et al. (2018)]

Bethe Ansätz Formulation

$$\mathcal{I} = \kappa \sum_{\widehat{u}_i \in \mathfrak{M}_{\text{BAEs}}} \sum_{\{m_i\}=1}^{ab} \mathcal{Z}(\widehat{u}_i - m_i\omega; \Delta, \tau = a\omega, \sigma = b\omega)\, H^{-1}(\widehat{u}_i; \Delta, \omega)$$

$$\mathfrak{M}_{\text{BAEs}} = \left\{\widehat{u} \in \mathrm{T}^2_\omega \mid Q(\widehat{u}; \Delta, \omega) = 1\right\}$$

We will focus on contribution from "basic solution" [Hong et al. (2018)] for a specific m_i

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$\mathcal{N} = 4$ Super-Yang-Mills: Results

SCI of SU(N) $\mathcal{N} = 4$ SYM

$$\log \mathcal{I} \simeq -i\pi N^2 \frac{\Delta_1 \Delta_2 \Delta_3}{\tau \, \sigma}$$

$$\Delta_1 + \Delta_2 + \Delta_3 - \tau - \sigma = -1$$

- The (constrained) Legendre transform of $\log \mathcal{I}$ is

$$\mathcal{S}(Q_I, J_1, J_2) = 2\pi \sqrt{\sum_{I<J} Q_I Q_J - \frac{N^2}{2}(J_1 + J_2)} = \frac{A}{4G_N} \equiv S_{BH}(Q_I, J_1, J_2)$$

  that is exactly the Bekenstein-Hawking entropy of the BHs!

- The constraint precisely matches the BPS condition of Euclidean BH solutions [Cabo-Bizet et al. (2018)]

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Holographic Quivers: Results

The result is more complicated than $\mathcal{N} = 4$ SYM, but it simplifies in particular domains of chemical potentials

SCI of SU(N) Holographic Quivers

$$\log \mathcal{I} \simeq -\frac{4\pi i}{27} \frac{(\tau+\sigma-1)^3}{\tau\sigma} a(\widehat{\Delta})$$

$$a(\widehat{\Delta}) = \frac{9}{32} \operatorname{Tr} R(\widehat{\Delta})^3 \qquad \widehat{\Delta} = \frac{\Delta}{\tau+\sigma-1}$$

Notice: $\mathcal{I}$-extremization $\sim$ a-maximization [Intriligator and Wecht (2003)]!

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SCI of SU(N) Toric Quivers

$$\log \mathcal{I} \simeq -\pi i N^2 \sum_{a,b,c=1}^{D} \frac{C_{abc}}{6} \frac{\Delta_a \Delta_b \Delta_c}{\tau\sigma}$$

$$a(\widehat{\Delta}) = \frac{9}{32}\,\mathrm{Tr} R^3(\widehat{\Delta}) = \frac{9N^2}{64} C_{abc} \widehat{\Delta}_a \widehat{\Delta}_b \widehat{\Delta}_c$$

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Notice: C_{abc} are 't Hooft anomaly coefficients!

A Supergravity Check

Toric quivers are dual to gravity on $\mathrm{AdS}_5 \times \mathrm{SE}_5$ for which BH solutions are not known yet: how do we compare our QFT predictions? Near-horizon analysis

Let's take the conifold dual (i.e. gravity on $\mathrm{AdS}_5 \times \mathrm{T}^{1,1}$) and focus on consistent truncation on AdS_5 called "second model" in [Cassani et al. (2011)]

Following [Hosseini at al. (2017)] we will search for BHs with $J_1 = J_2$ and horizon with $\mathrm{AdS}_2 \times \mathrm{S}^3$ topology

Reducing now the theory down to 4d along Hopf fiber we can try and solve BPS equations near the horizon for static BHs (since $J_1 = J_2$)

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Near-horizon 4d BPS equations

- fix hypers scalars
- fix massive vectors scalars
- extremization principle for massless vectors scalars: attractor mechanism
  - function to be extremized is the horizon area, i.e. BH entropy!

Using AdS/CFT dictionary to match charges on the two sides, the extremization in gravity and CFT exactly match: attractor mechanism $= \mathcal{I}$-extremization

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