A falling magnetic monopole as a holographic local quench

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Based on a work in progress in collaboration with
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Cortona Young
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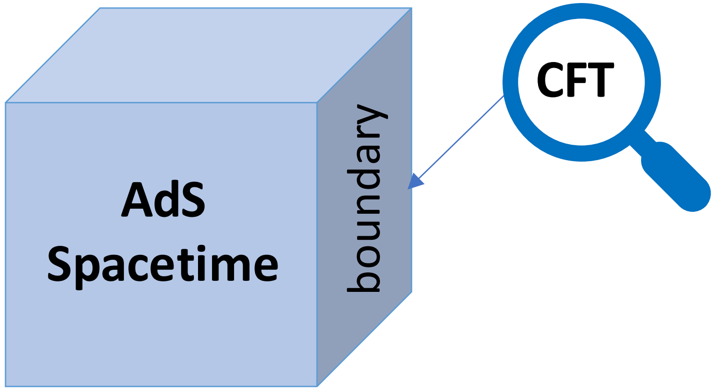
AdS/CFT

Weakly-coupled gravity
in asimptotically
Anti-de Sitter spacetime

[J. M. Maldacena, 1997]

Strongly-coupled
Conformal Field Theory
on AdS boundary



Opportunity to explore quantum systems
at strong coupling far from equilibrium

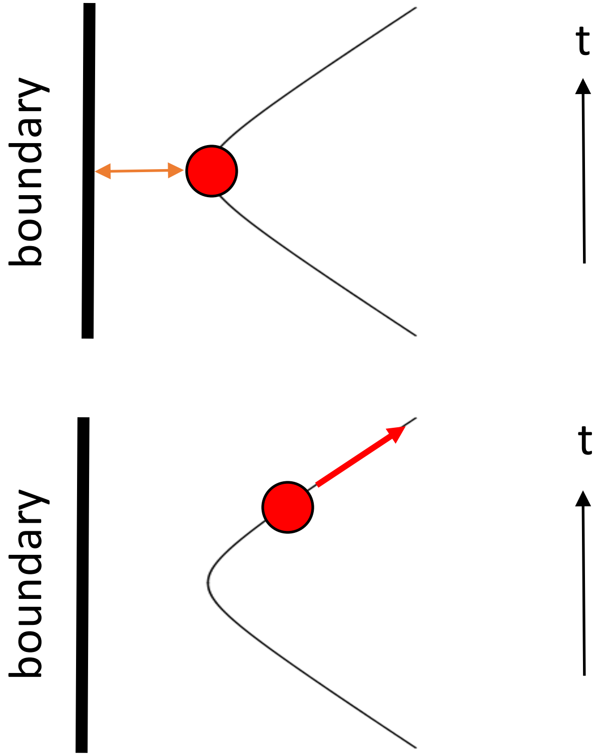
Local quench: Evolution of a system triggered by a localized injection of energy

Holographic local quench

[M. Nozaki, T. Numasawa and T. Takayanagi, 2013]

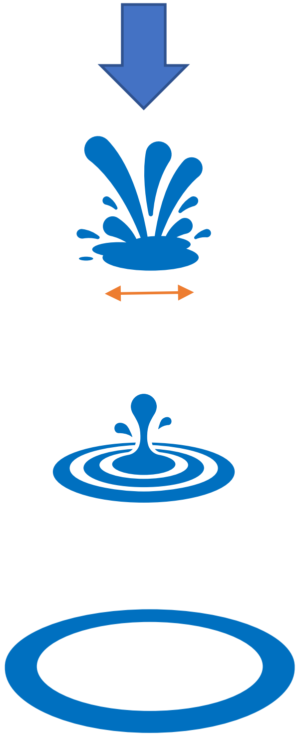
Falling particle-like **object**
in Poincaré AdS

Local quench
in boundary CFT



t = 0

t > 0



Monopole in global AdS

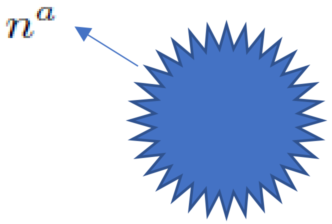
Bulk action: $$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R+\frac{6}{L^2}\right)+\int d^4x\sqrt{-g}\left[-\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu}-\frac{1}{2}D_\mu\phi^a D^\mu\phi^a+\frac{1}{L^2}\phi^a\phi^a\right]$$

matter action for massive scalar and SU(2) gauge field

No backreaction:

$$\frac{G}{L^2}\to 0 \qquad ds^2 = L^2\left(-(1+r^2)d\tau^2+\frac{dr^2}{1+r^2}+r^2(d\theta^2+\sin^2\theta d\varphi^2)\right)$$ global AdS_4

We consider a generalization of **t'Hooft-Polyakov ansatz** in spherical coordinates:



$$\phi^a = \frac{1}{L}H(r)n^a\,, \qquad A^a_l = F(r)r\,\epsilon^{aik}n^k\partial_l(n^i)$$

$$r\to 0 \quad : \qquad H(r)\propto r\,, \qquad F(r)\propto r$$ Smooth solution

$$r\to\infty : \quad H(r)\approx\frac{\alpha_H}{r}+\frac{\beta_H}{r^2}+\ldots\,, \qquad F(r)\approx\cancel{\frac{\alpha_F}{r}}+\frac{\beta_F}{r^2}+\ldots$$

No explicit breaking of SU(2) symmetry in the boundary CFT

Backreaction on the metric

Ansatz for the **metric with backreaction** [A. Esposito, S. Garcia-Saenz, A. Nicolis and R. Penco, 2017]:

$$ds^2 = L^2\left(-(1+r^2)h(r)g(r)dt^2 + \frac{h(r)}{g(r)}\frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right)$$

$$r \to \infty \; : \quad h(r) \approx 1 + \frac{h_2}{r^2} + \frac{h_3}{r^3} + \ldots\,, \qquad g(r) \approx 1 + \frac{g_2}{r^2} + \frac{g_3}{r^3} + \ldots$$

Schwarzschild black hole in global AdS$_4$

$$h(r) = 1\,, \qquad g(r) = \frac{1 + r^2 - \frac{M}{r}}{1 + r^2}$$

→ $g_2 = h_2 = h_3 = 0\,, \qquad g_3 = -M$

→ Vanishing matter fields

Magnetic monopole in global AdS$_4$

First order perturbative solution:

$$\varepsilon = \frac{G}{L^2}$$

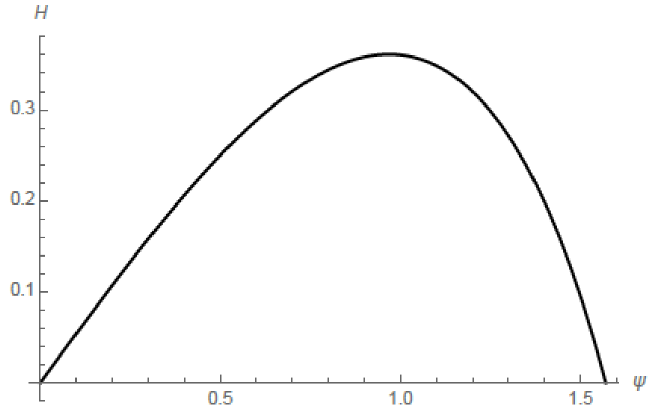
$$h(r) = 1 + \varepsilon\, h_\varepsilon(r) + \mathcal{O}(\varepsilon^2)$$

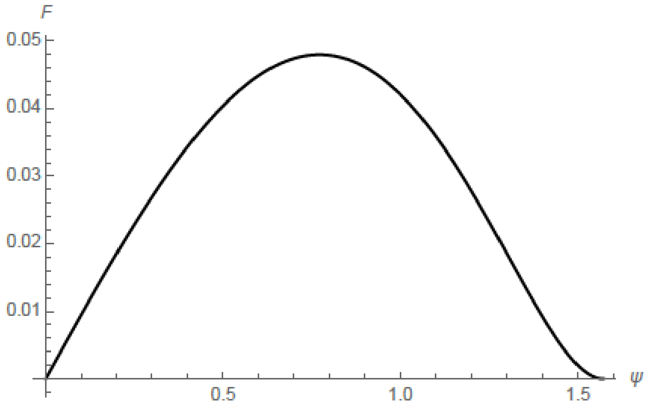
$$g(r) = 1 + \varepsilon\, g_\varepsilon(r) + \mathcal{O}(\varepsilon^2)$$

Monopole numerical solutions

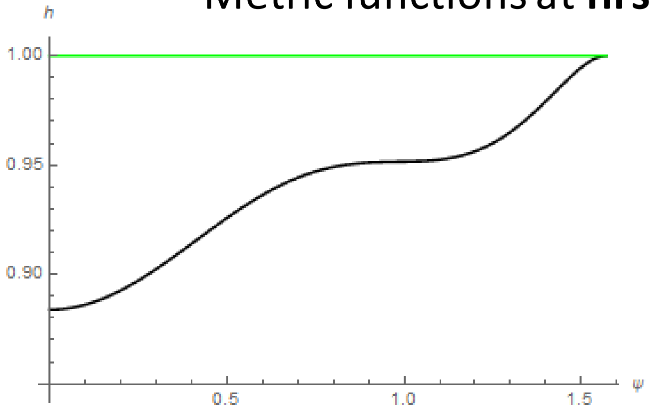
Unperturbed profile functions with $\alpha_H = 1.5\,, \quad \beta_H = -2.26\,, \quad \beta_F = 0.47$

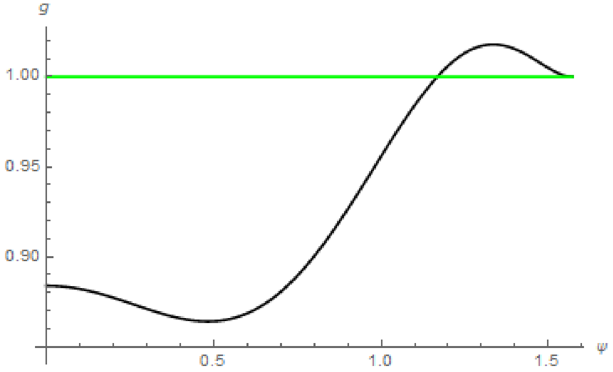
$r = \tan\psi$





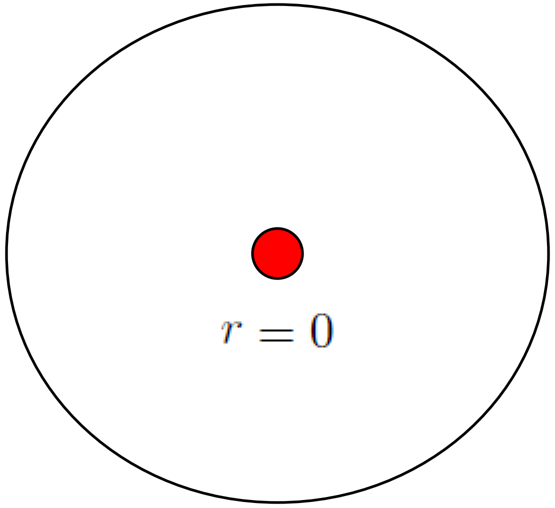
Metric functions at **first order** with $\varepsilon = 0.1$





Free falling particle in AdS

Particle-like object
in **global AdS**



Particle position

$$r = 0$$

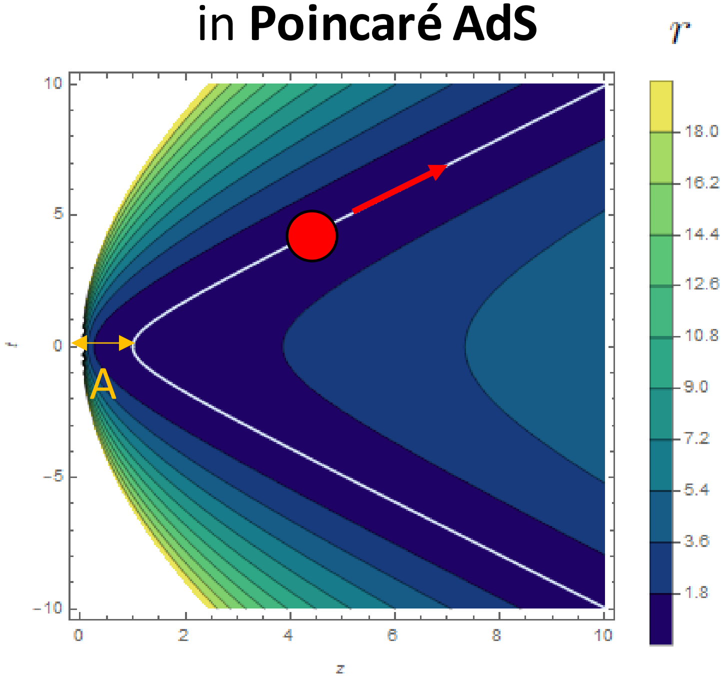
[M. Nozaki, T. Numasawa and T. Takayanagi, 2013]

$$\tau = \arctan \frac{2A\,t}{A^2 + z^2 + x^2 - t^2}$$

$$\theta = \arctan \frac{2A\,x}{-A^2 + z^2 + x^2 - t^2}$$

$$r = \frac{\sqrt{A^4 + (z^2 + x^2 - t^2)^2 + 2A^2(t^2 + x^2 - z^2)}}{2Az}$$

Particle-like object
in **Poincaré AdS**



Particle position

$$x = 0\,, \qquad t = \pm\sqrt{z^2 - A^2}$$

Boundary conditions of fields

Near the AdS boundary:

$$r = \frac{a(t,x)}{z} + b(t,x)\, z + \mathcal{O}(z^3)$$

The **boundary expansions** of the profile functions become:

$$H(t,z,x) \approx \frac{\alpha_H}{a} z + \frac{\beta_H}{a^2} z^2 + \dots\,, \qquad F(t,z,x) \approx \cancel{\frac{\alpha_F}{a}} z + \frac{\beta_F}{a^2} z^2 + \dots$$

$\frac{\alpha_H}{a} \to \tilde{\alpha}_H$ $\qquad$ $\frac{\beta_H}{a^2} \to \tilde{\beta}_H$

$\frac{\alpha_F}{a}$ → Still no explicit symmetry breaking

$\frac{\beta_F}{a^2}$ → Expectation value of currents

$$\tilde{\beta}_H = \kappa(\tilde{\alpha}_H)^2\,, \qquad \kappa = \frac{\beta_H}{\alpha_H^2}$$

Poincaré AdS

boundary

$$\Delta S_{CFT} = \frac{\kappa}{3}\int d^3x\, \mathcal{O}_1^3$$

$$\langle \mathcal{O}_1 \rangle = \tilde{\alpha}_H$$

Triple-trace deformation

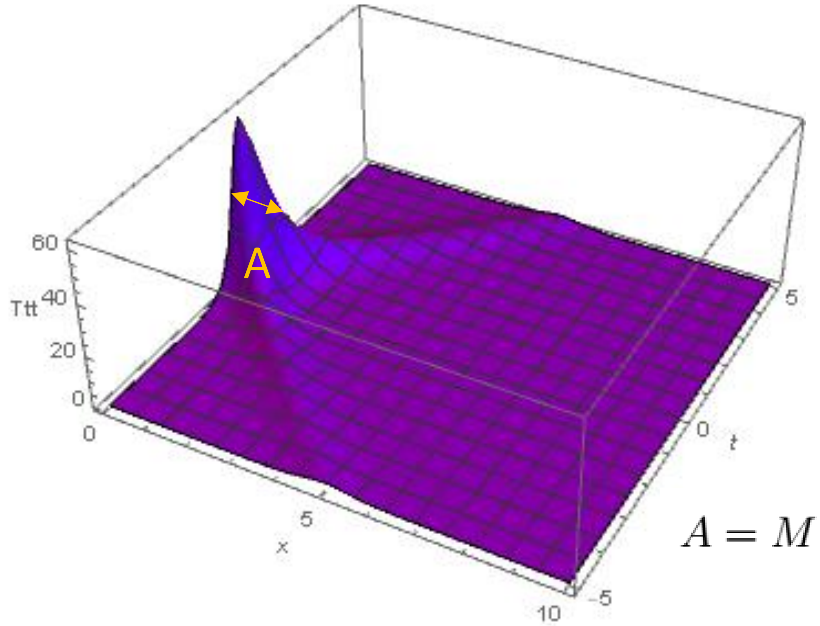
[E. Witten, 2002]

Holographic energy-momentum tensor

- ✓ **Fefferman-Graham** gauge: $$ds^2 = L^2\left(\frac{d\hat{z}^2}{\hat{z}^2} + \frac{1}{\hat{z}^2} g_{ab}(\hat{z},\hat{x}^a)d\hat{x}^a d\hat{x}^b\right)$$
- ✓ **Holographic energy-stress tensor** for $J_D = \tilde{\alpha}_H\,, \qquad \langle \mathcal{O}_2 \rangle = \tilde{\beta}_H$: $$T^{(D)}_{ij} = \frac{L}{8\pi G}\lim_{\hat{z}\to 0}\frac{1}{\hat{z}}\left(K_{ij} - \gamma_{ij}K - \frac{2}{L}\gamma_{ij} - \frac{4\pi G}{L}\gamma_{ij}\,\phi^a\phi^a\right)$$
- ✓ Change of boundary conditions
- ✓ Triple-trace: $T_{ij} = T^{(D)}_{ij} - \eta_{ij}\,\frac{\kappa}{3}\tilde{\alpha}^3_H$

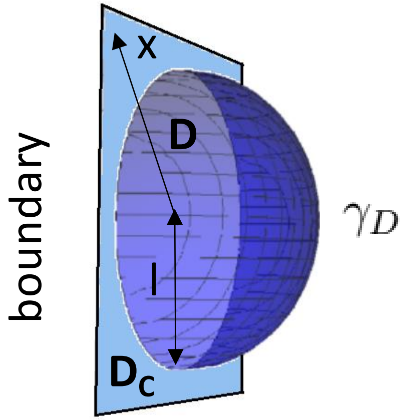
$$T_{ij} = T^{(BH)}_{ij}\left(\frac{16G\pi\alpha_H\beta_H - 3L^2 g_3}{3L^2 M}\right)$$

$$\partial^i T_{ij} = 0\,, \qquad T^i_i = 0$$



$A = M = L = 1$

Holographic entanglement entropy

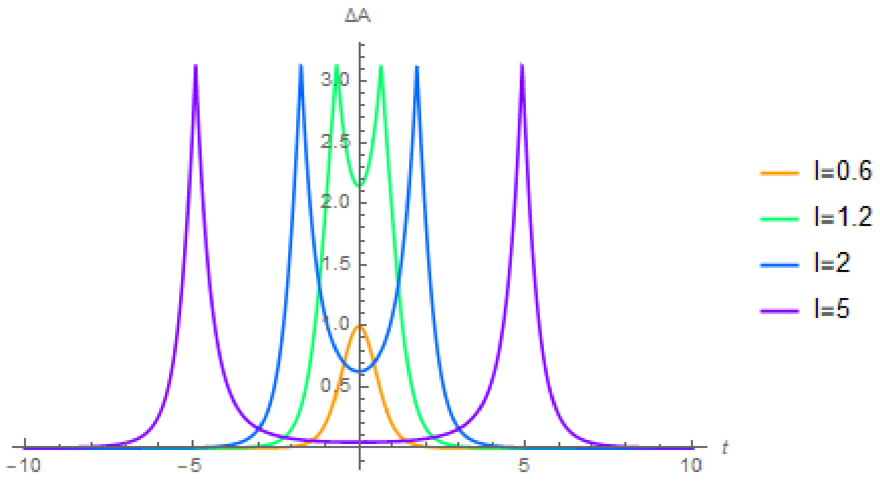


$$\Delta S_D = \frac{\Delta \mathcal{A}(\gamma_D)}{4G}$$

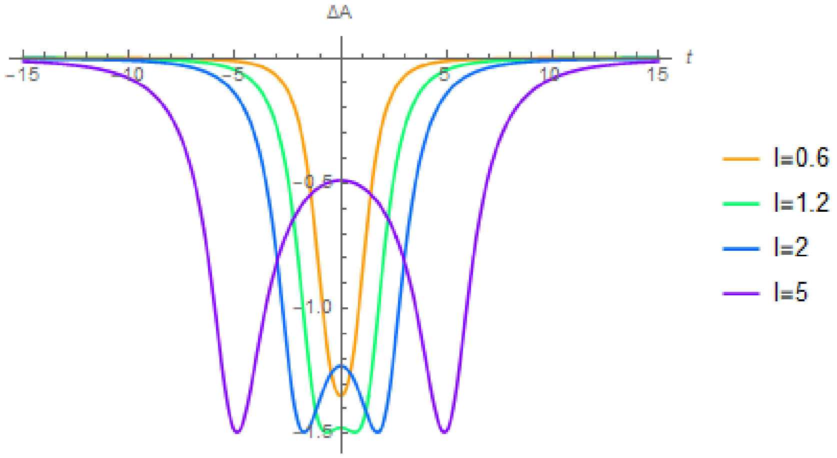
Perturbed holographic entanglement entropy

At first order in the backreaction parameter:

- Unperturbed Ryu-Takayanagi surface $\gamma_D : z = \sqrt{l^2 - x^2}$
- Metric with backreaction at first order



Black hole: $A = M = L = 1$



Magnetic monopole: $A = L = 1\,, \quad \varepsilon = 0.1$

First law of entanglement entropy

Consider two nearby states, one of which is the vacuum: $\sigma = \rho_0\,, \qquad \rho = \rho_0 + \epsilon\rho_1 + \mathcal{O}(\epsilon^2)$

For a spherical entangling region in a CFT: $\mathcal{K}_\sigma = 2\pi \int_{|\vec{x}|<l} d^2x \, \frac{l^2 - x^2}{2l} T_{tt}(\vec{x})$ **Modular Hamiltonian**

At first order in the perturbation parameter: $\boxed{\Delta\langle\mathcal{K}_\sigma\rangle = \Delta S}$ **First law of entanglement entropy**

Black hole

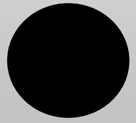
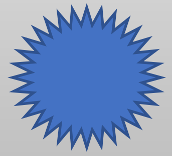
For small size l
[M. Nozaki, T. Numasawa and T. Takayanagi, 2013]

Magnetic monopole

$$\Delta\langle\mathcal{K}_\sigma\rangle > 0\,, \qquad \Delta S < 0$$

FLEE is **violated** in the presence of scalar operators acquiring an expectation value
[D. D. Blanco, H. Casini, L. Y. Hung and R. C. Myers, 2013]

Conclusions and open questions

		
T_{ij}	Same functional form	
ΔS	Positive	Negative

? Holographic entanglement entropy for **different subregions**

? Interpretation of the results from the **CFT point of view**

Thank you
for your
attention!