

# The partonic structure of protons and nuclei: from current facilities to the EIC

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“Frontiers in Nuclear and Hadronic Physics”

Galileo Galilei Institute, Florence, Italy

20-24 February 2017



# Plan of the lectures

## □ PART 1: QCD factorization and global PDF fitting

- Lecture 1 – Hadrons, partons and Deep Inelastic Scattering
- Lecture 2 – Parton model
- Lecture 3 – The QCD factorization theorem
- Lecture 4 – Global PDF fits

## □ PART 2: Parton distributions from nucleons to nuclei

- Lecture 5 / 6

## □ PART 3: The next QCD frontier – The Electron-Ion collider

- Lectures 7 / 8

# Plan of Part 1

## □ Lecture 1 – Motivation

- Quarks, gluons, hadrons
- Deep Inelastic Scattering (DIS)

## □ Lecture 2 – Parton model

- DIS revisited
- Collinear factorization and Parton Distribution Functions (PDFs)
- Limitations

## □ Lecture 3 – The QCD factorization theorem

- QCD factorization, universality of PDFs
- DIS, Drell-Yan (DY) lepton pairs,  $W$  and  $Z$  production, hadronic jets

## □ Lecture 4 – Global PDF fits

- How to make a fit, and use its results
- Fits as community service (*e.g.*, measure PDFs, apply to LHC)
- Fits as a tool to study hadron and nuclear structure

# Resources

## □ Textbooks

- Povh et al., “Particles and Nuclei,” Springer, 1999
- Halzen, Martin, “Quarks and leptons,” John Wiley and sons, 1984
- Lenz et al. (Eds.), “Lectures on QCD. Applications,” Springer, 1997
  - esp. lectures by Levy, Rith, Jaffe
- Devenish, Cooper-Sarkar, “Deep Inelastic Scattering,” Oxford U.P., 2004
- Feynman, “Photon-hadron interactions,” Addison Wesley, 1972
- Collins, “Foundations of perturbative QCD”, Oxford U.P, 2011

## □ PDFs and Global QCD fitting

- Jimenez-Delgado, Melnitchouk, Owens,  
“Momentum and helicity distributions in the nucleon”, arXiv:1306.6515
- Forte, Watt, “Progress in partonic structure of proton”, arXiv:1301.6754
- J.Owens, “PDF and global fitting”, 2007 / 2013 CTEQ summer school

## □ Lectures (from the [CTEQ pedagogical page](#))

- W.K. Tung, “pQCD and parton structure of the nucleon”
- B. Poetter, “Calculational Techniques in pQCD: The Drell-Yan Process”

# Lecture 1 - Motivation

## □ An illustrated introduction

- Hadrons are made of quarks and gluons
- How to probe the partonic structure of hadrons

## □ Deep Inelastic Scattering (DIS)

- A bit (!) of kinematics
- Cross section
- Structure function

## □ A taste of the parton model

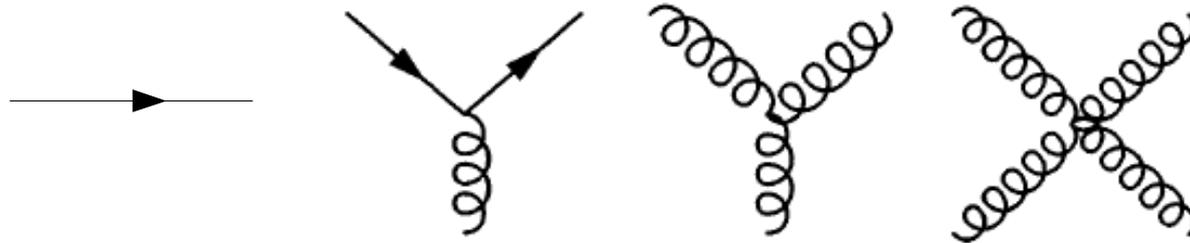
# An illustrated introduction

# Motivation: quarks, gluon, hadrons...

- The strong force is described in terms of colored quarks and gluons:

$$L_{QCD} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - g (\bar{\psi} \gamma^\mu T_a \psi) A_\mu^a - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

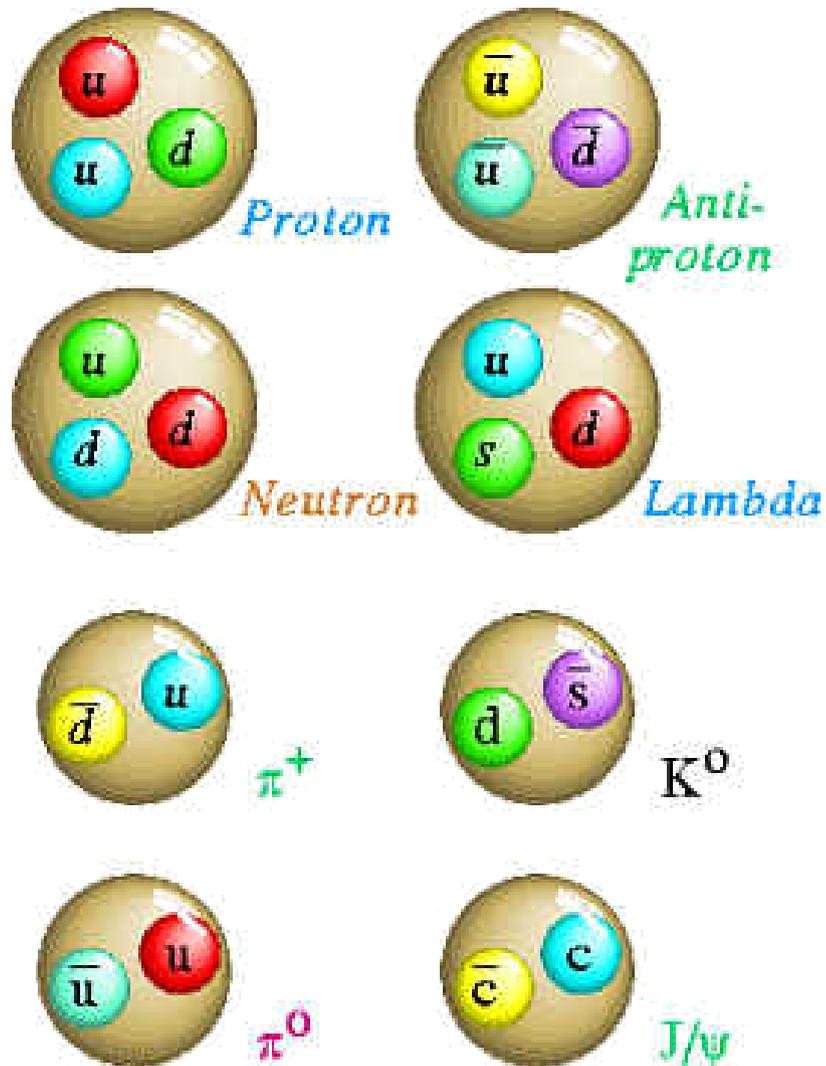
a quark                      "color"                      a gluon



- But only color neutral hadrons can be detected – **color confinement**

- How can one understand, say, proton and neutrons in terms of quark and gluons?
- And, for that matter, what's the evidence for quarks and gluons?

# Hadrons are made of quarks



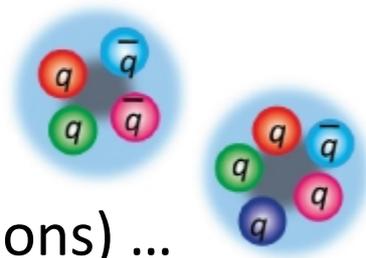
## 6 flavors (and 3 colors):

up, down, strange – light  
charm, bottom, top – heavy

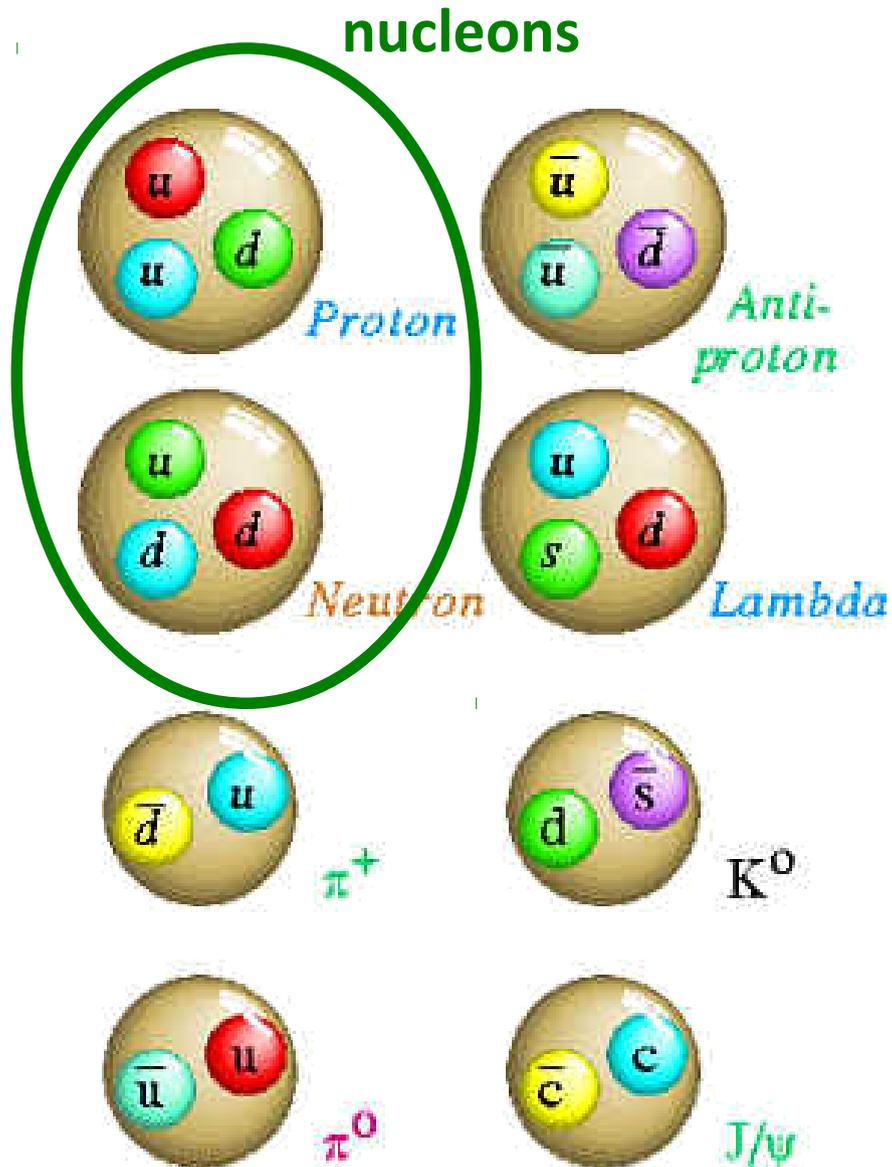
spin  $\frac{1}{2}$ ; charge (elm, weak)  
isospin ( $u = \frac{1}{2}$ ,  $d = -\frac{1}{2}$ )  
strangeness ( $s = 1$ )

## Confined in “colorless” hadrons

- mesons – 2 quarks
- baryons – 3 quarks
- Tetraquarks (?)
- Pentaquarks (???)
- Hybrids (quarks+gluons) ...
- Glueballs ...



# Hadrons are made of quarks



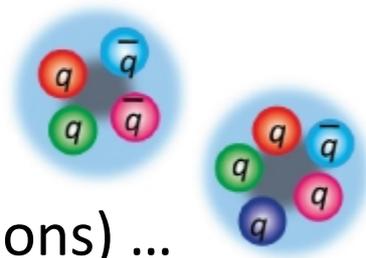
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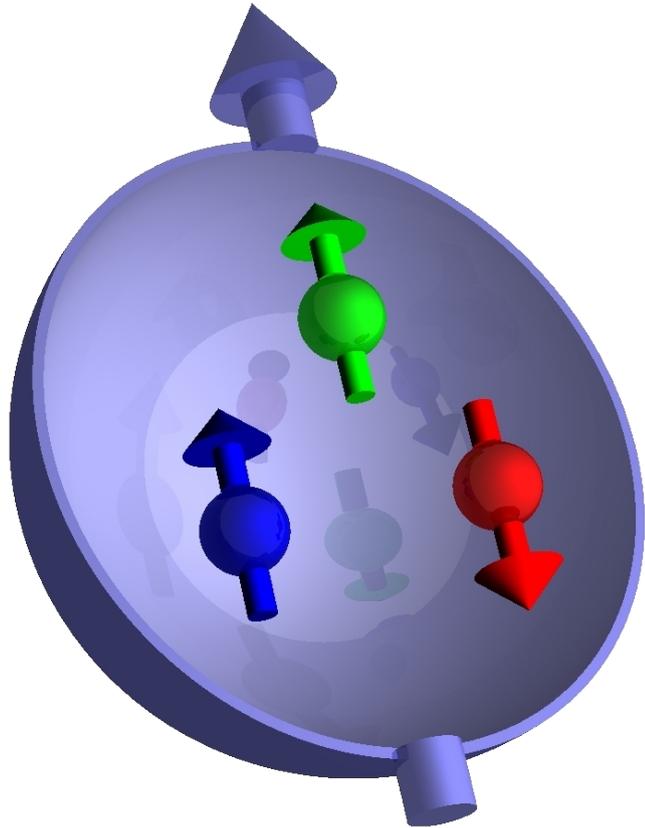
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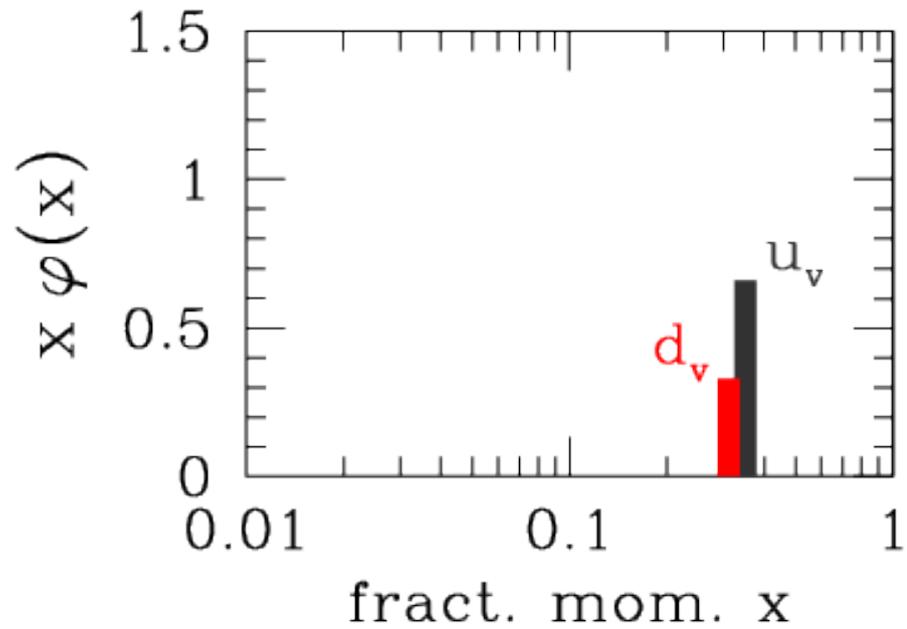
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# Nucleons are made of 3 quarks...



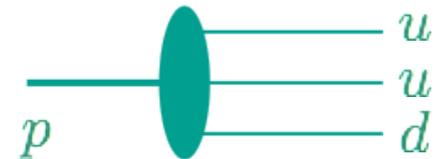
## Parton Distribution Functions (PDF)



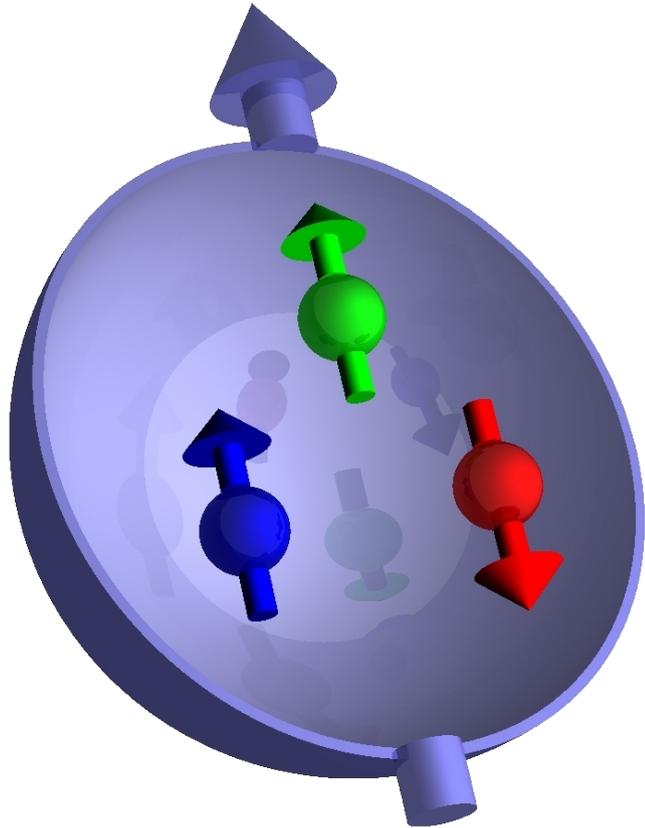
Fractional momentum:

$$x = \frac{p_{\text{parton}}^+}{p_{\text{nucleon}}^+}$$

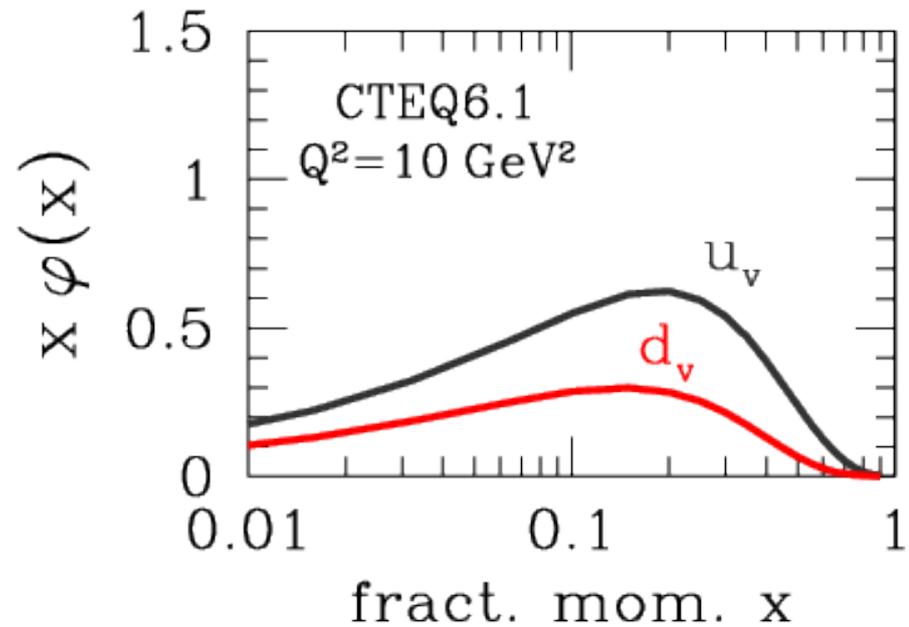
$$p^\pm = \frac{1}{\sqrt{2}} (E \pm p_z)$$



# Nucleons are made of 3 quarks...



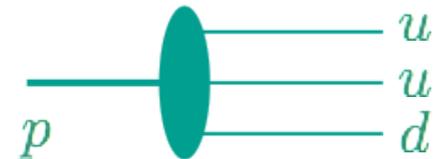
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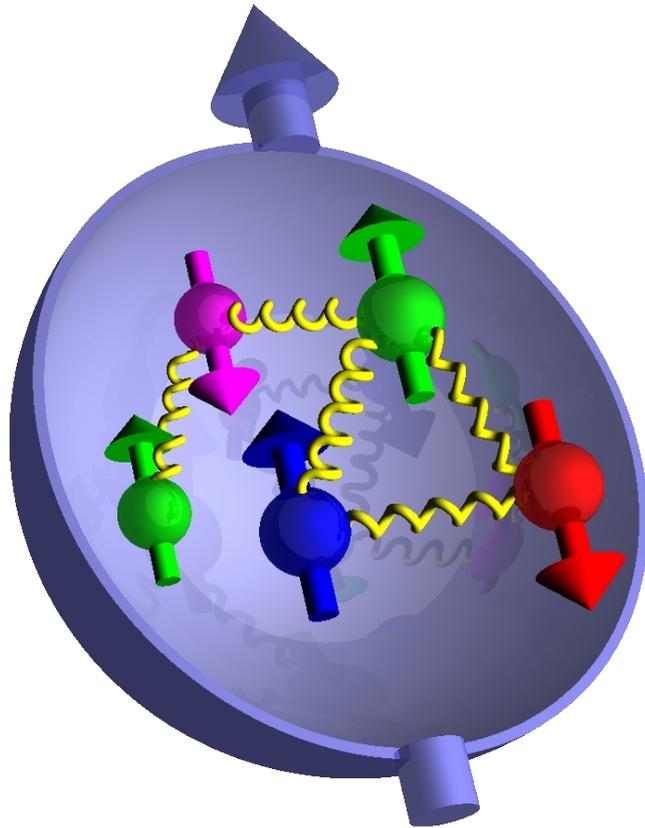
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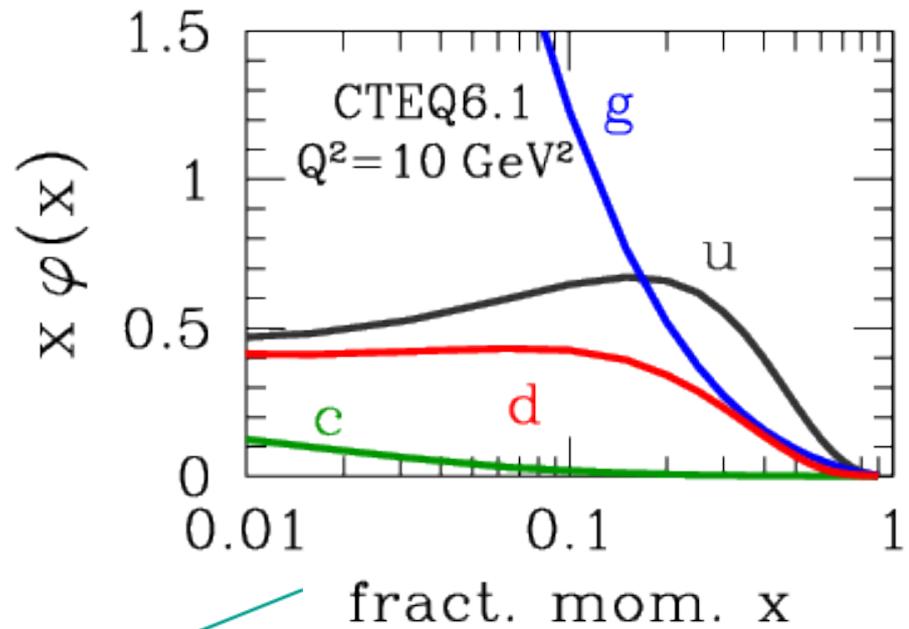
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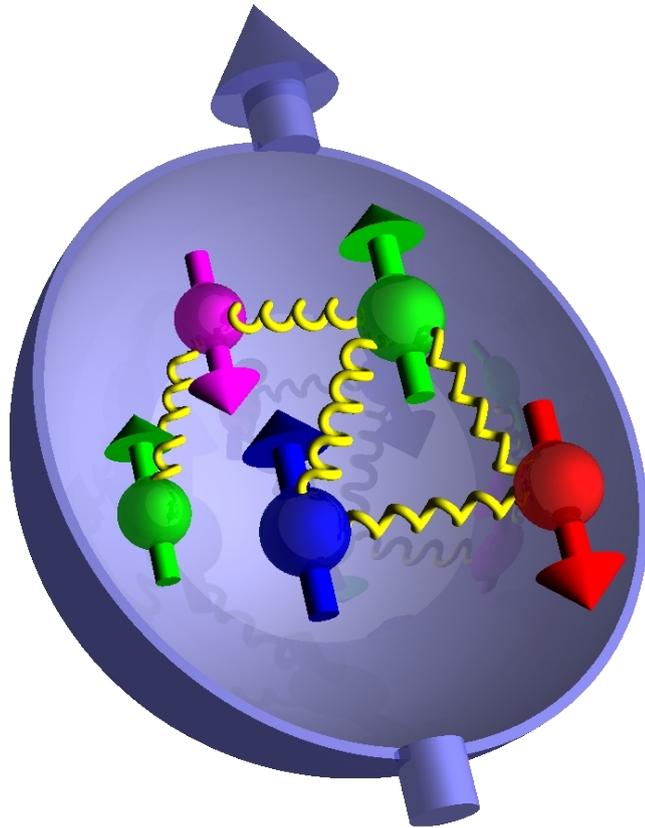
# ... and gluons, and sea quarks ...



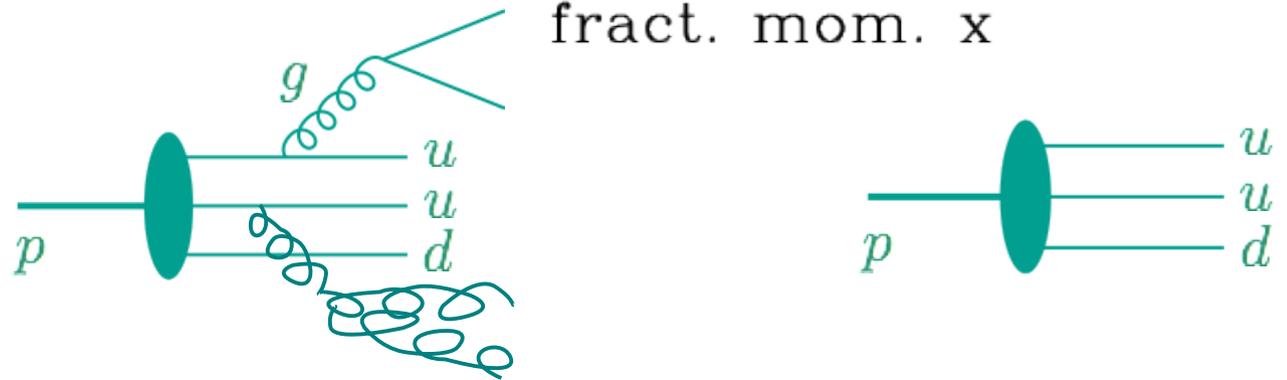
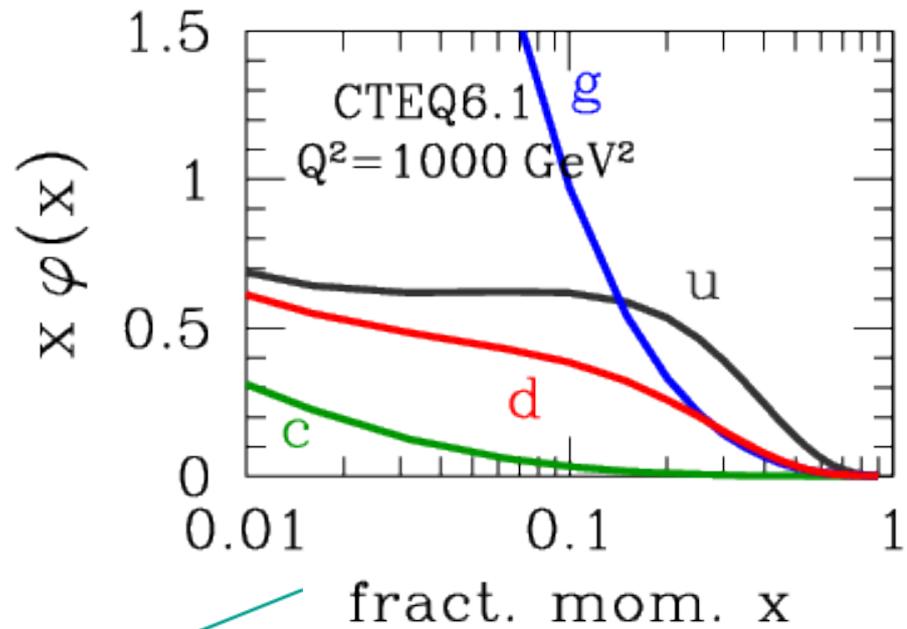
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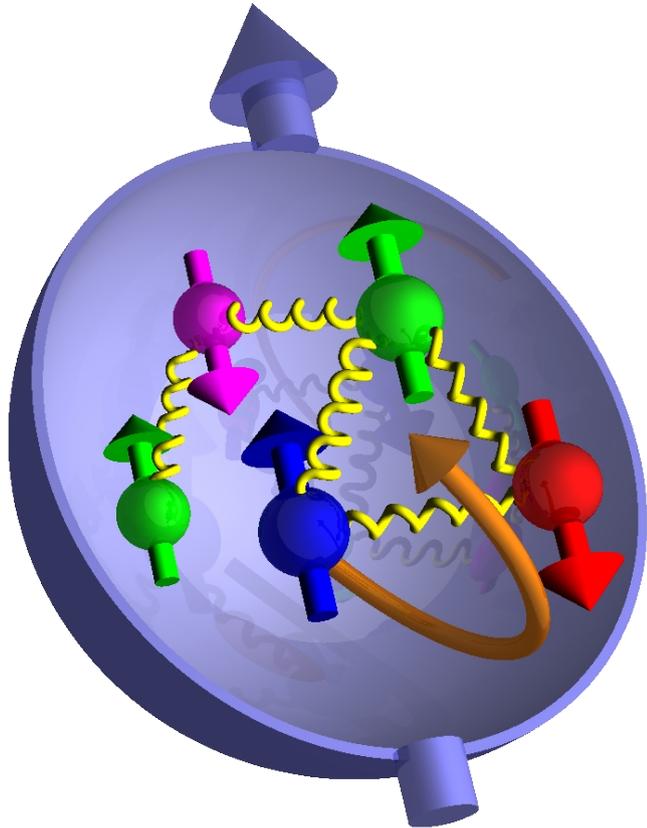
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## Parton Distribution Functions (PDF)



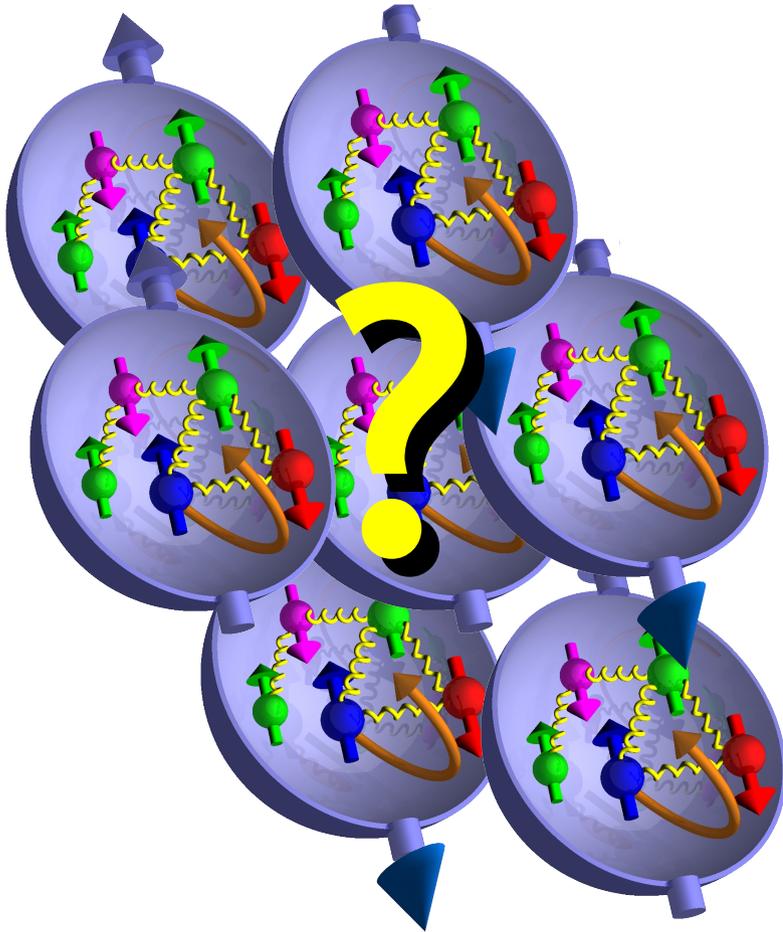
... spinning and orbiting around...



Much studied at:  
JLab, HERMES, COMPASS, RHIC

Fundamental topic at:  
JLab 6, Electron-Ion-Collider (EIC)

## ...and interacting inside nuclei



### □ EMC effect

discovered more than 30 years ago:

- $A \neq \sum p, n$
- quarks / hadrons modified inside a nucleus
- still a theoretical mystery

# Evidence for quarks and gluons

# Evidence for quarks and gluons - a whirlwind tour

## □ Baryon spectroscopy – light sector ( $u, d, s$ ), ground state

- $J=3/2^+$ :  $|q_1^\uparrow, q_2^\uparrow, q_3^\uparrow\rangle$  totally symmetric w.fn.
- $J=1/2^+$ :  $|q_1^\uparrow, q_2^\uparrow, q_3^\downarrow\rangle$

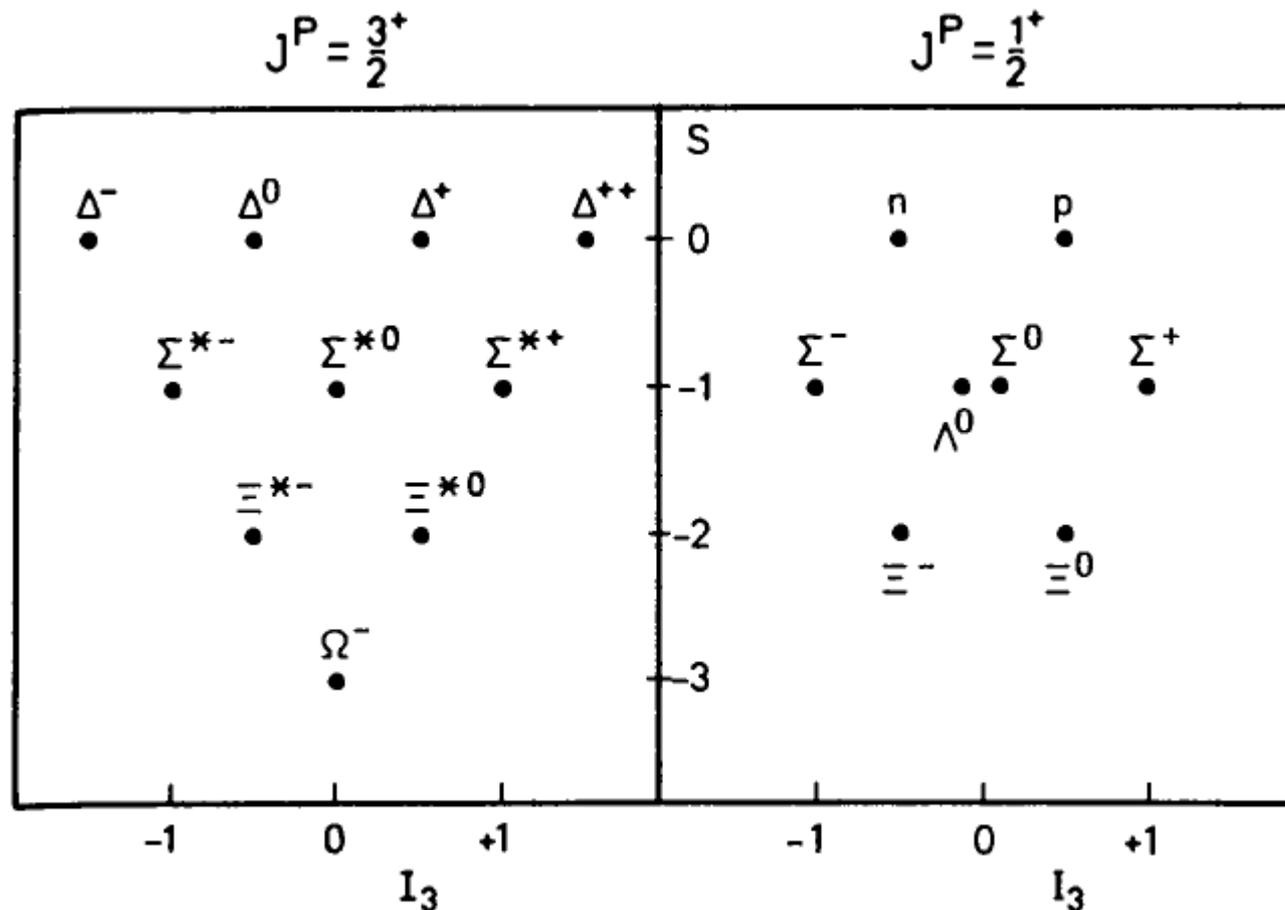


Fig:  
[from Povh et al.]

# Evidence for quarks and gluons - a whirlwind tour

## □ Baryon spectroscopy – light sector ( $u, d, s$ ), ground state

- $J=3/2^+$ :  $|q_1^\uparrow, q_2^\uparrow, q_3^\uparrow\rangle$  spin symmetric, color antisymmetric
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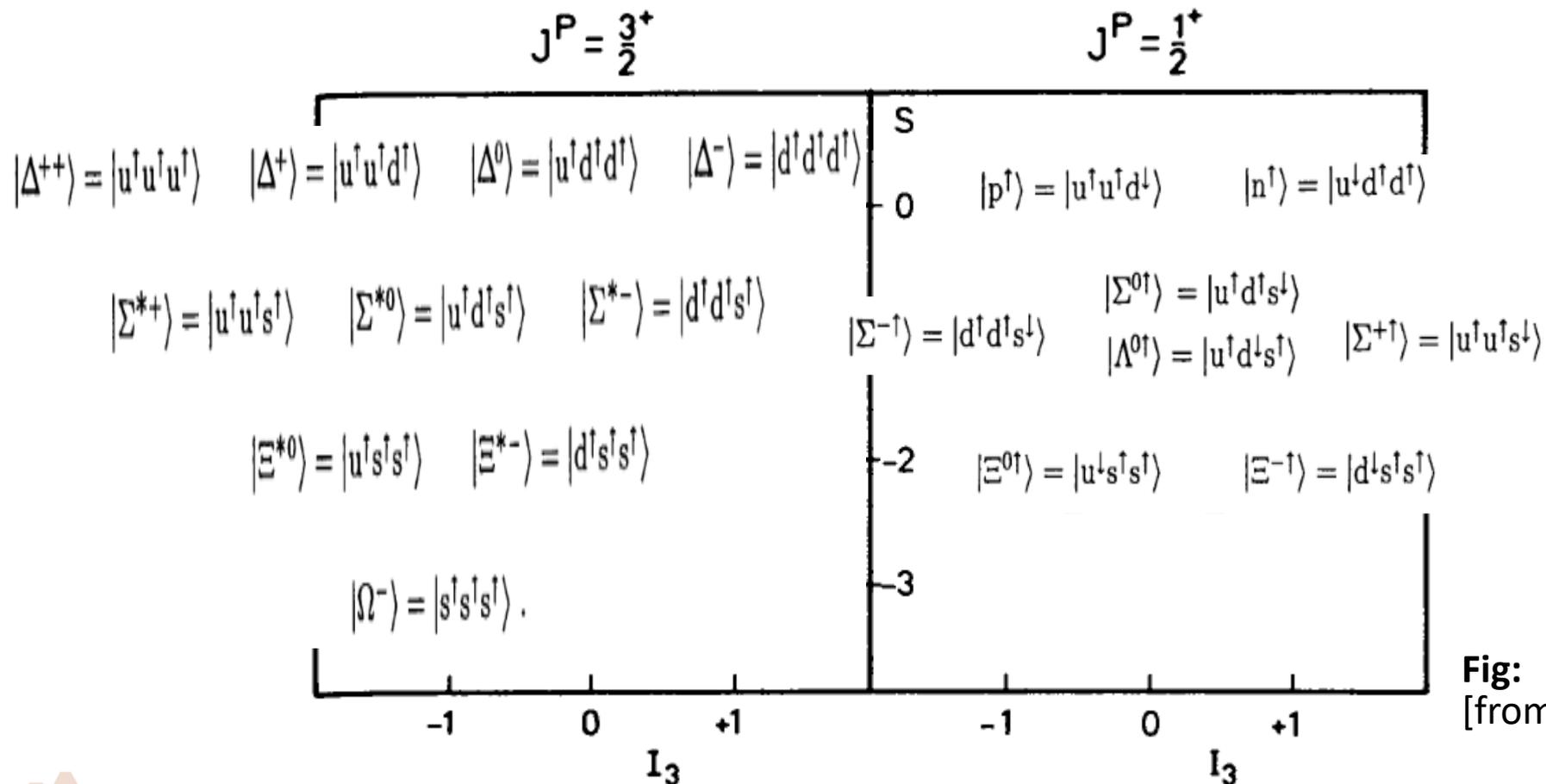


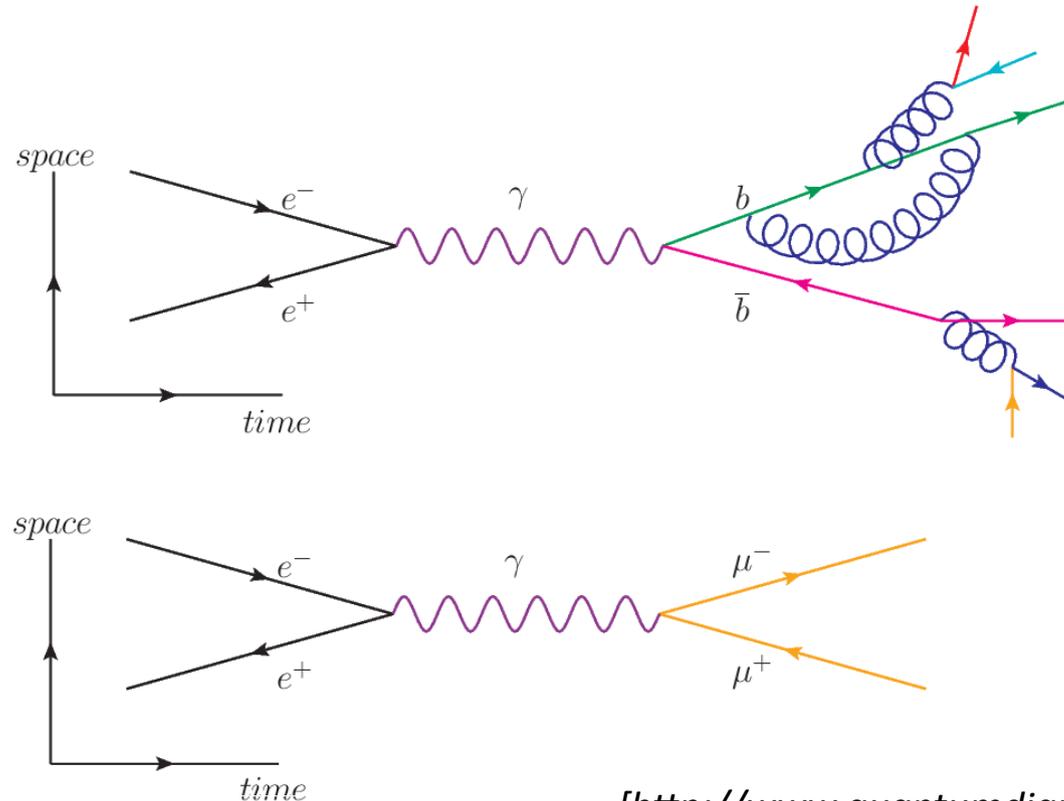
Fig:  
[from Povh et al.]

# Evidence for quarks and gluons - a whirlwind tour

## □ $e^+ + e^-$ annihilation into hadrons

– quark-mediated process  $e^+ + e^- \rightarrow q + \bar{q} \rightarrow hadrons$

$$R = \frac{\sigma(e^+e^- hadrons)}{\sigma(e^+e^- \mu^+\mu^-)} = N_{colors} \sum_q e_q^2$$



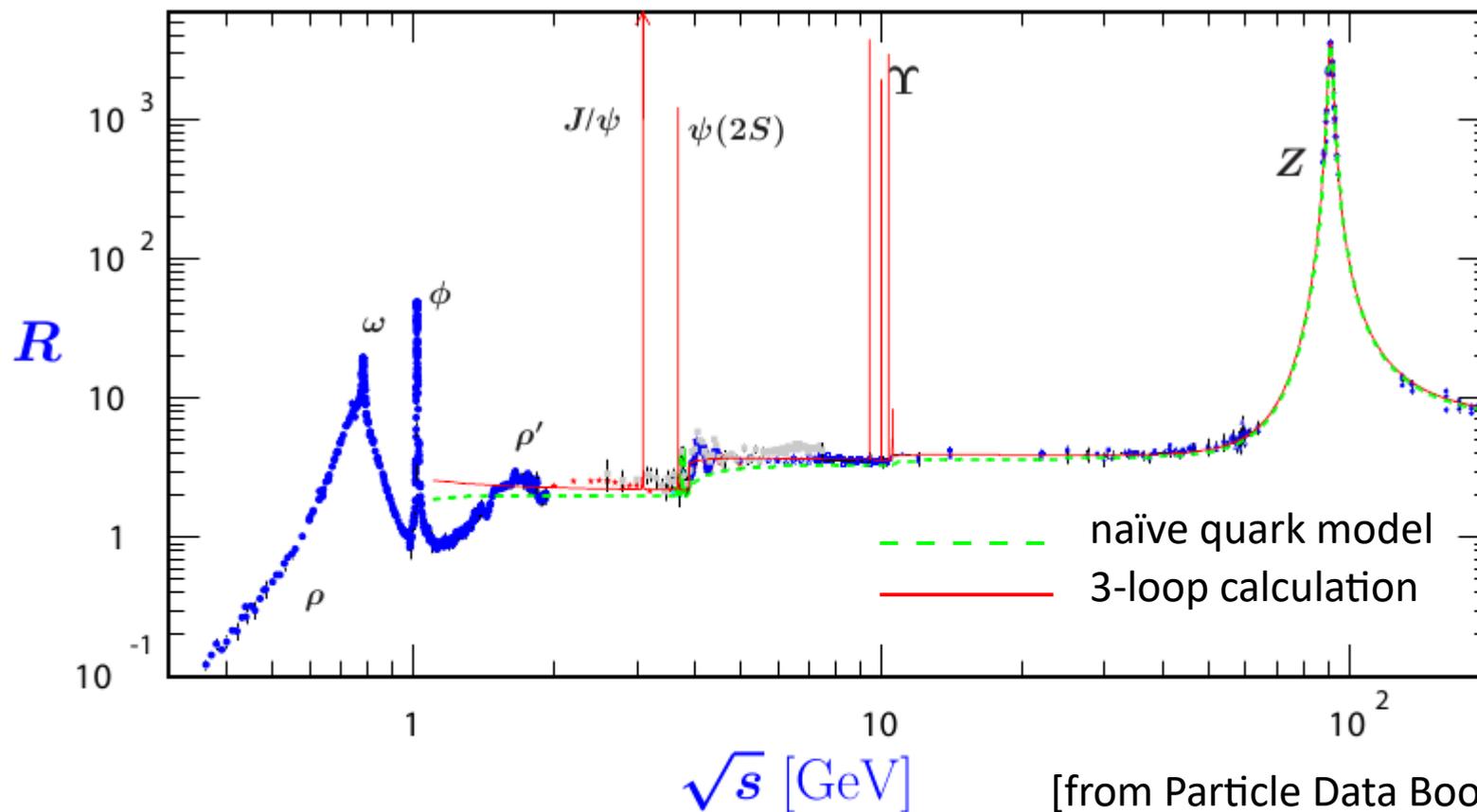
[<http://www.quantumdiaries.org/author/richard-ruiz/>]

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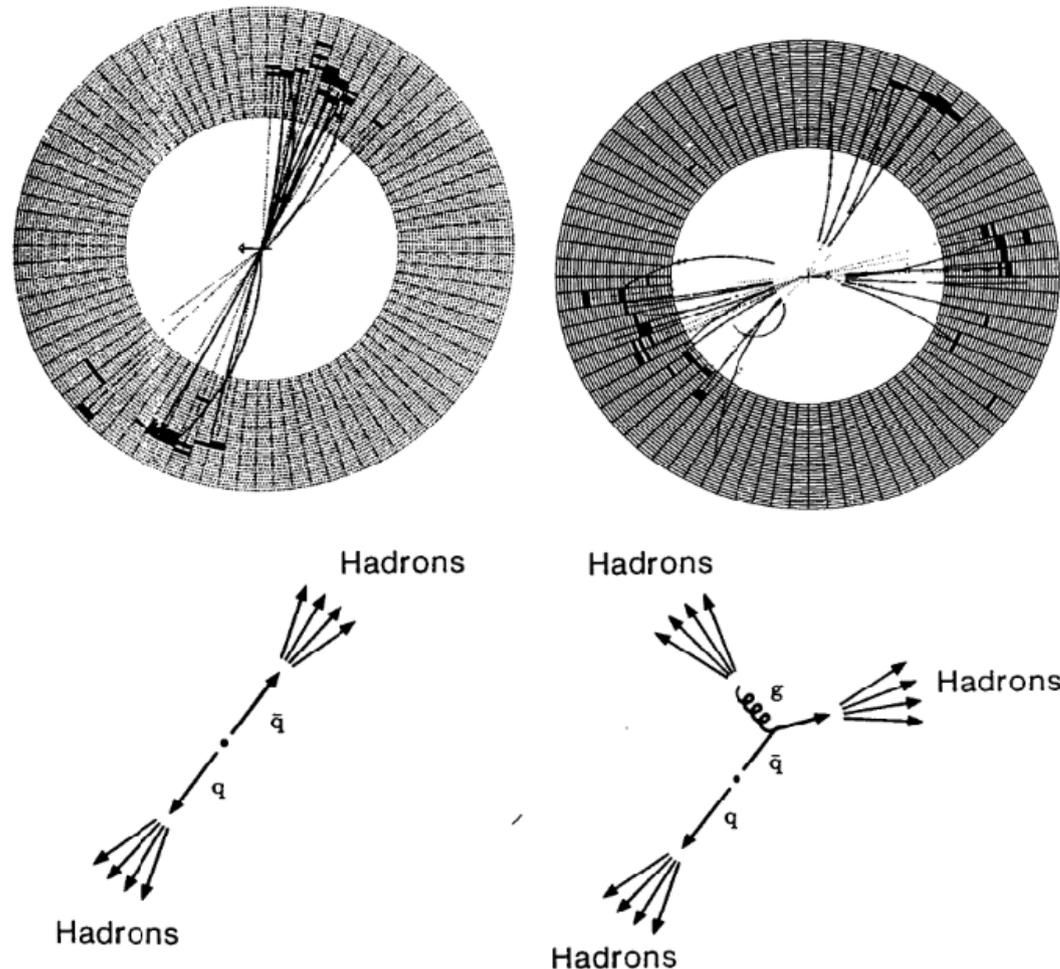


[from Particle Data Book, [pdg.lbl.gov](http://pdg.lbl.gov)]

# Evidence for quarks and gluons - a whirlwind tour

## □ Jets in high-energy $e^+ + e^-$ collisions

- Hadron produced in 2, 3, ... N, high-energy collimated “jets”
- Evidence of common origin from a parton



**Fig.:** 2- and 3-jet events observed by the JADE detector at PETRA [from Povh et al.]

# Probing the quark and gluon structure of a hadron

# Probing the nucleon parton structure

□ Need large momentum transfer  $Q^2 = q_\mu q^\mu$  to “resolve” partons

□ Example 1: **Deep Inelastic Scattering (DIS)**  $e^\pm + p \rightarrow e^\pm + X$

– Photon wave-length in rest frame, neglect proton mass  $M/Q \ll 1$ :

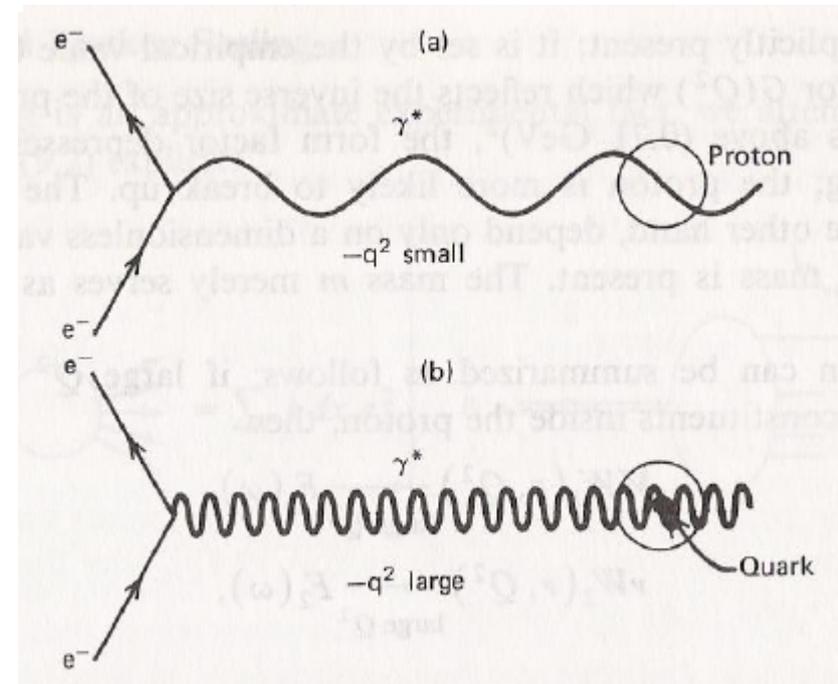
$$\lambda = \frac{1}{|\vec{q}|} = \frac{1}{\sqrt{\nu^2 + Q^2}} \approx \frac{1}{\nu} = \frac{2Mx}{Q^2}$$

– *E.g.*, for  $x=0.1$ ,  $Q^2=4 \text{ GeV}^2$   
(and putting back  $c$  and  $\hbar$ ),

$$\lambda = 10^{-17} \text{ m} = 10^{-2} \text{ fm}$$

to be compared with

$$R_p \approx 1 \text{ fm}$$



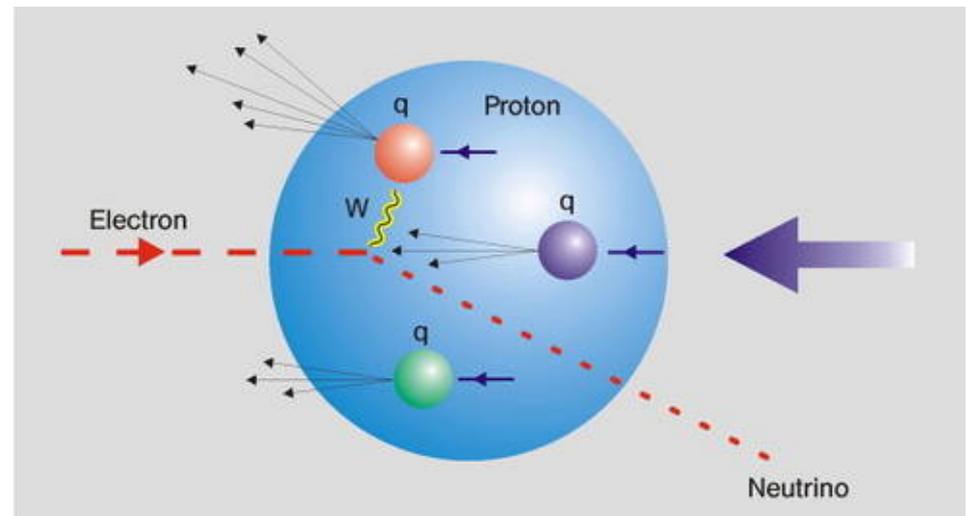
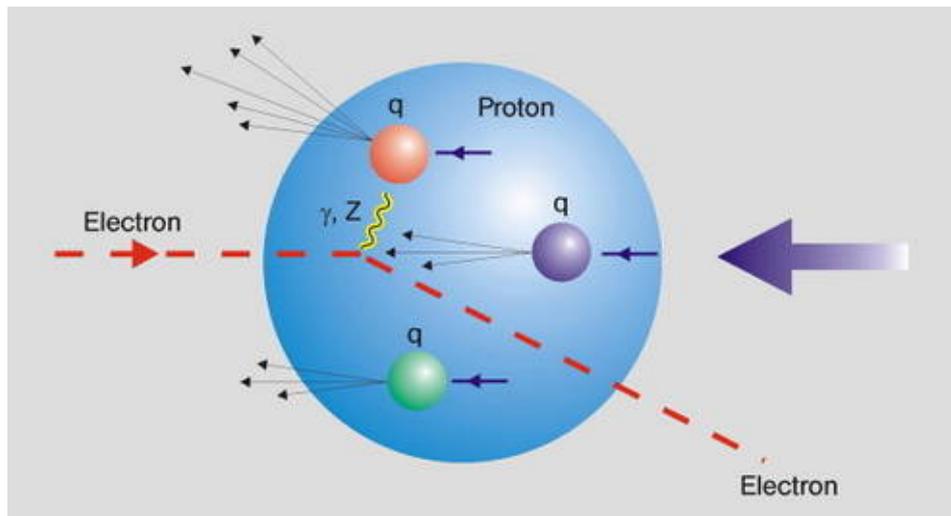
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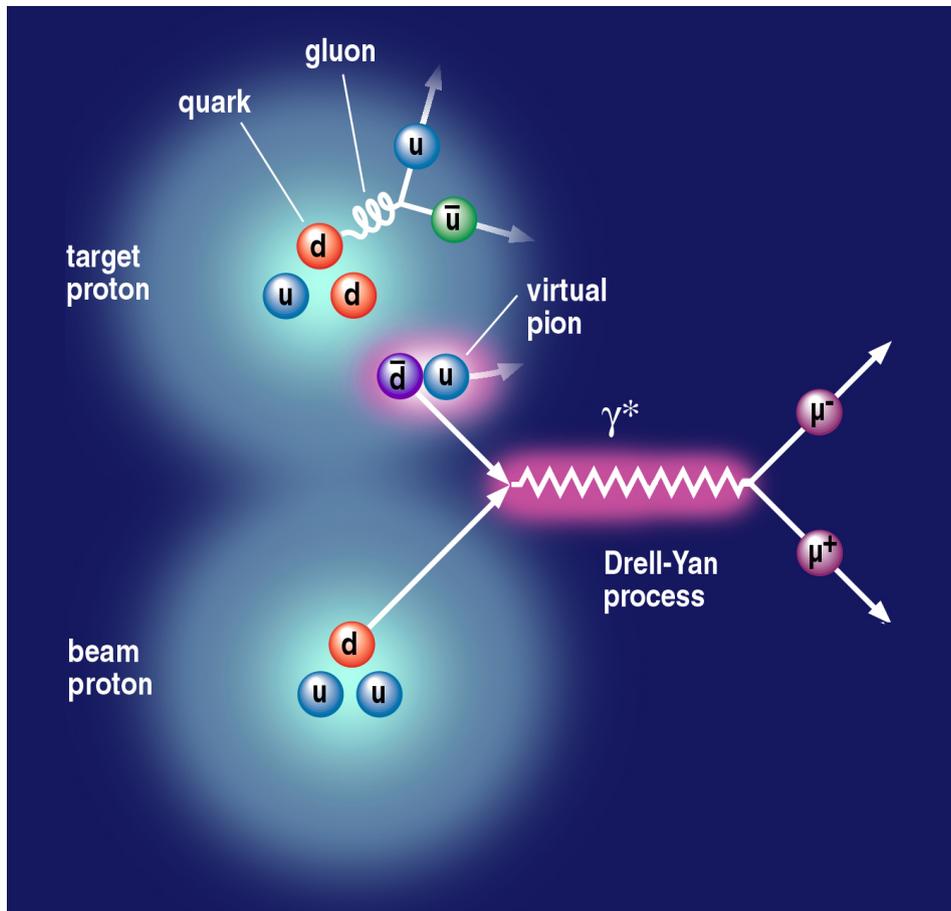
$$e^\pm + p \rightarrow e^\pm + X$$
$$e^+(e^-) + p \rightarrow \bar{\nu}(\nu) + X$$

$$Q^2 = -p_{\gamma,Z}^2$$



# Probing the nucleon parton structure

- Need large momentum transfer  $Q^2 = q_\mu q^\mu$  to “resolve” partons
- Example 2: **Lepton-pair production (“Drell-Yan” process)**

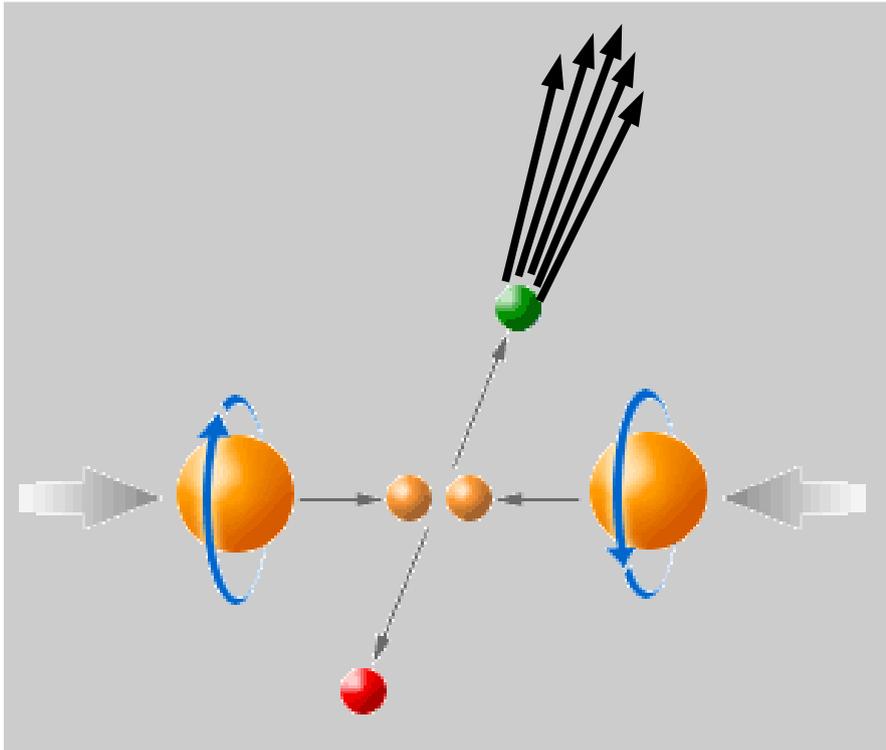


$$p + p (\bar{p}) \rightarrow \ell + \bar{\ell} + X$$

$$Q^2 = (p_\ell + p_{\bar{\ell}})^2$$

# Probing the nucleon parton structure

- Need large momentum transfer  $Q^2 = q_\mu q^\mu$  to “resolve” partons
- Example 3: **jet production in p+p collisions**



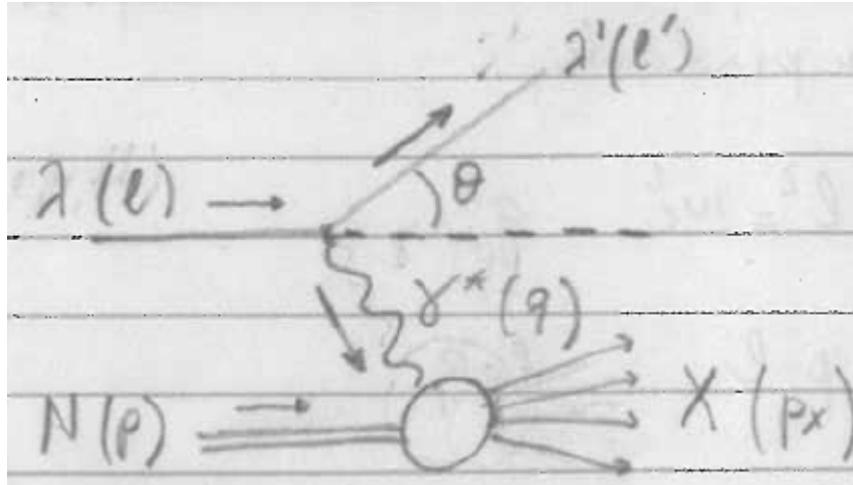
$$p + p (\bar{p}) \rightarrow jet + X$$

$$Q^2 = E_{jet}^2$$

# Deep Inelastic Scattering

# Kinematics

## □ Inclusive lepton-hadron scattering:



$$\lambda = e, \mu, \nu, \dots$$

$$N = p, n, \dots$$

$$q^\mu = \ell^\mu - \ell'^\mu$$

Virtual photon  
momentum

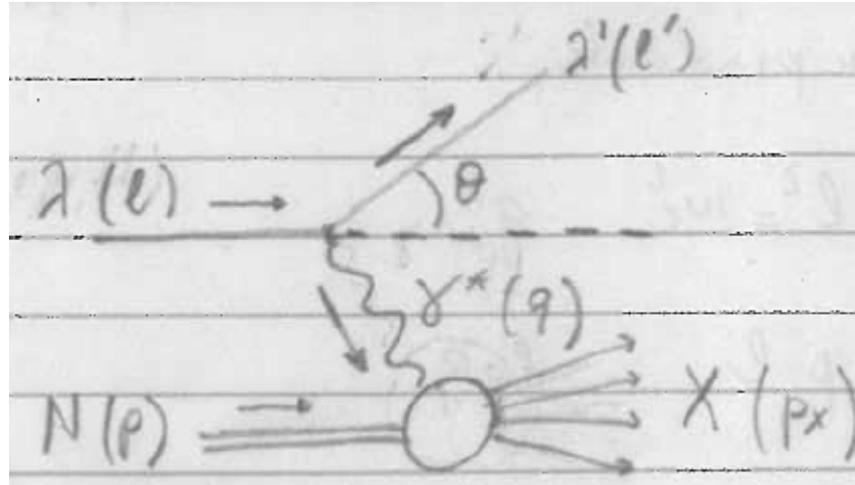
$$p_X^\mu = \sum_{i \in X} p_i^\mu$$

Hadronic final state  
momentum

- Notation:  $p^2 = p_\mu p^\mu$      $p \cdot q = p_\mu q^\mu$
- Masses:  $p^2 = M^2$      $\ell^2 = m_\lambda^2$

# Kinematics

## □ Inclusive lepton-hadron scattering:



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## □ The photon is virtual: $q^2 < 0$

Neglect compared to  $q^2$  (MeV vs. GeV)

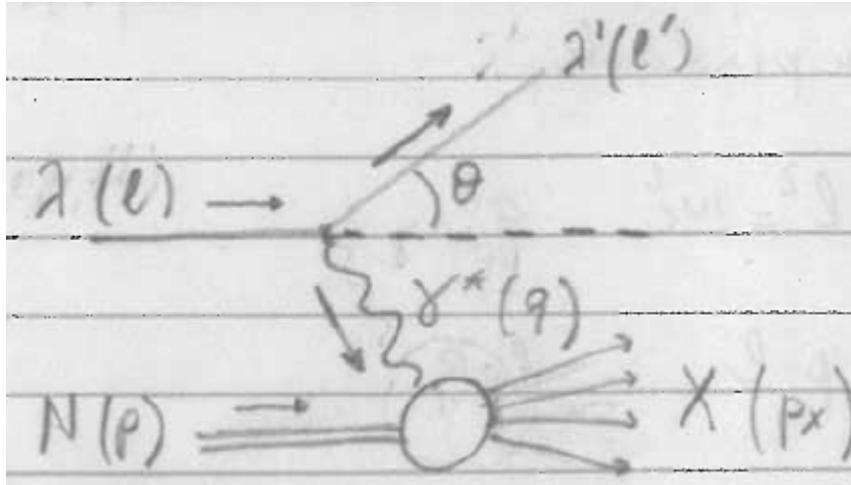
$$q^2 = (\ell - \ell')^2 = m_\lambda^2 + m_\lambda^2 - 2\ell \cdot \ell'$$

$$= -2EE' + |\vec{\ell}| |\vec{\ell}'| \cos \theta = -2EE'(1 - \cos \theta) \leq 0$$

$$m_\lambda^2 = 0 \Rightarrow |\vec{\ell}| = E$$

# Kinematics

## □ Inclusive lepton-hadron scattering:



$$\lambda = e, \mu, \nu, \dots$$

$$N = p, n, \dots$$

$$q^\mu = \ell^\mu - \ell'^\mu$$

Virtual photon  
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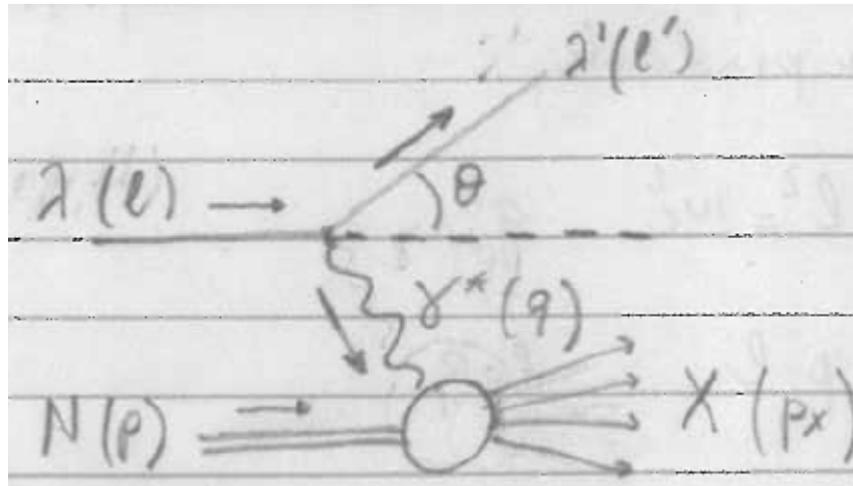
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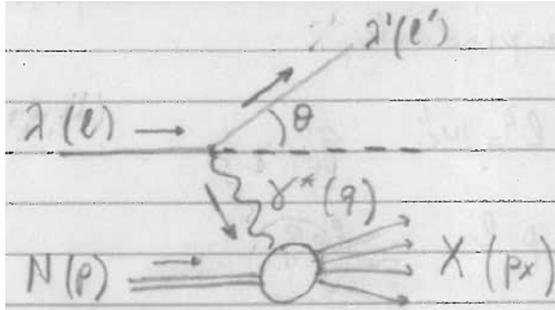
## □ Measuring $q^2$ (see [Levy]):

- Scattered lepton
- Hadronic final state
- Mixed methods

$$q = \ell - \ell' = p_X - p$$

# Kinematics

## □ Lorentz invariants



$$Q^2 = -q^2$$

virtuality

$$x_B = \frac{-q^2}{2p \cdot q}$$

Bjorken x

$$\nu = \frac{p \cdot q}{\sqrt{p^2}}$$

beam energy loss \*

$$y = \frac{p \cdot q}{p \cdot \ell}$$

inelasticity \*

$$W^2 = (p + q)^2$$

(final state)  
invariant mass

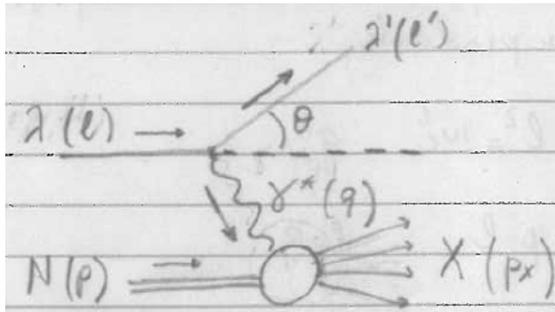
$$s = (p + \ell)^2$$

center-of-mass  
energy

\* interpretation valid in hadron rest frame, see later

# Kinematics

## □ Lorentz invariants



$$Q^2 = -q^2$$

virtuality

$$x_B = \frac{-q^2}{2p \cdot q}$$

Bjorken x

$$\nu = \frac{p \cdot q}{\sqrt{p^2}}$$

beam energy loss \*

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inelasticity \*

$$W^2 = (p + q)^2$$

(final state)  
invariant mass

$$s = (p + \ell)^2$$

center-of-mass  
energy

- **Ex.1** (easy) – Do these 6 invariants exhaust all possibilities?
- **Ex.2** (easy) – Are all 6 independent of each other? Prove that:

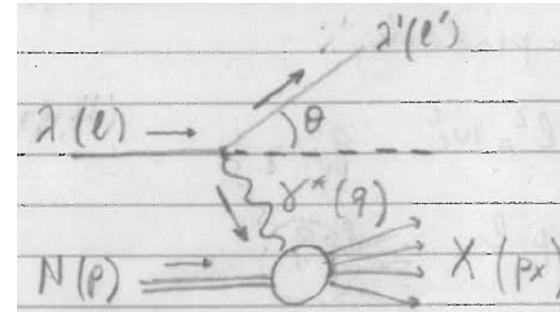
$$\nu = \frac{Q^2}{2Mx_B} \quad s - M^2 = \frac{Q^2}{2x_B y} \quad W^2 = M^2 + Q^2 \left( \frac{1}{x_B} - 1 \right)$$

\* interpretation valid in hadron rest frame, see later

# Kinematics

## □ Kinematic limits on invariants

- $Q^2 > 0$
- $0 < x_B < 1$



- By baryon number conservation the final state must contain at least 1 proton, if the target is a proton:  $p_X = p_{proton} + \sum_i p_i$

$$\text{Then, } p_X^2 = M^2 + \sum_i p_{proton} \cdot p_i + \sum_{i,j} p_i \cdot p_j$$

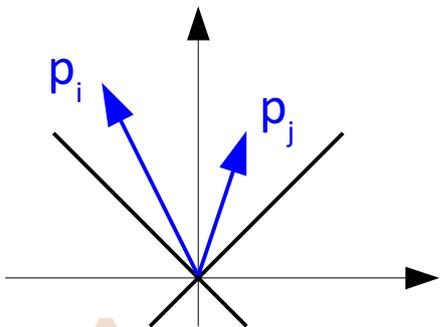
- All final state hadrons are on-shell:  $p_i^2 = M_i^2 \Rightarrow E_i \geq |\vec{p}_i|$

Then,

$$p_j \cdot p_i = E_i E_j - |\vec{p}_i| |\vec{p}_j| \cos \theta_{ij} \geq E_i E_j - |\vec{p}_i| |\vec{p}_j| \geq 0$$

so that

$$W^2 = p_X^2 \geq M^2 \Rightarrow Q^2 \frac{1 - x_B}{x_B} \geq 0 \Rightarrow 0 \leq x_B \leq 1$$



# Kinematics

## □ Kinematic limits on invariants, cont'd

–  $v \geq 0$

- **Ex.3** (easy) Hint: use definition and  $W^2 - M^2 \geq 0$

–  $0 \leq y \leq 1$

Consider the “target rest frame” such that  $\vec{p} = 0$

$$p = (M, \vec{0}_T, 0) \quad \ell = (E_\ell, \vec{0}_T, \ell_z) \quad \ell' = (E'_\ell, \vec{\ell}'_T, \ell'_z)$$

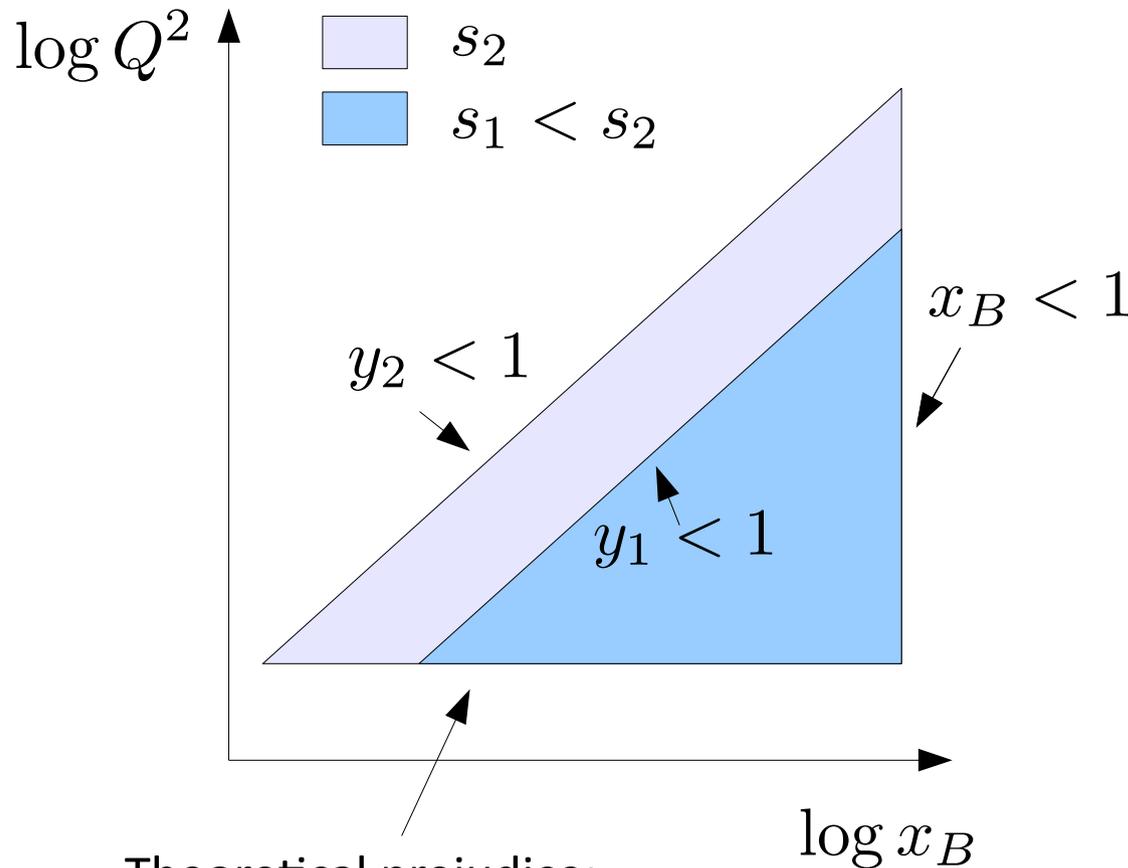
$$\text{Then, } y = \frac{M(E_\ell - E'_\ell)}{ME_\ell} = 1 - \frac{E'_\ell}{E_\ell}$$

- **Ex.4** (hard): find a Lorentz invariant proof

# Kinematics

## □ Kinematic plane – theoretical

- Pick  $x_B$ ,  $Q^2$ ,  $y$  as independent variables



$$y_i \approx \frac{Q^2}{x s_i}$$

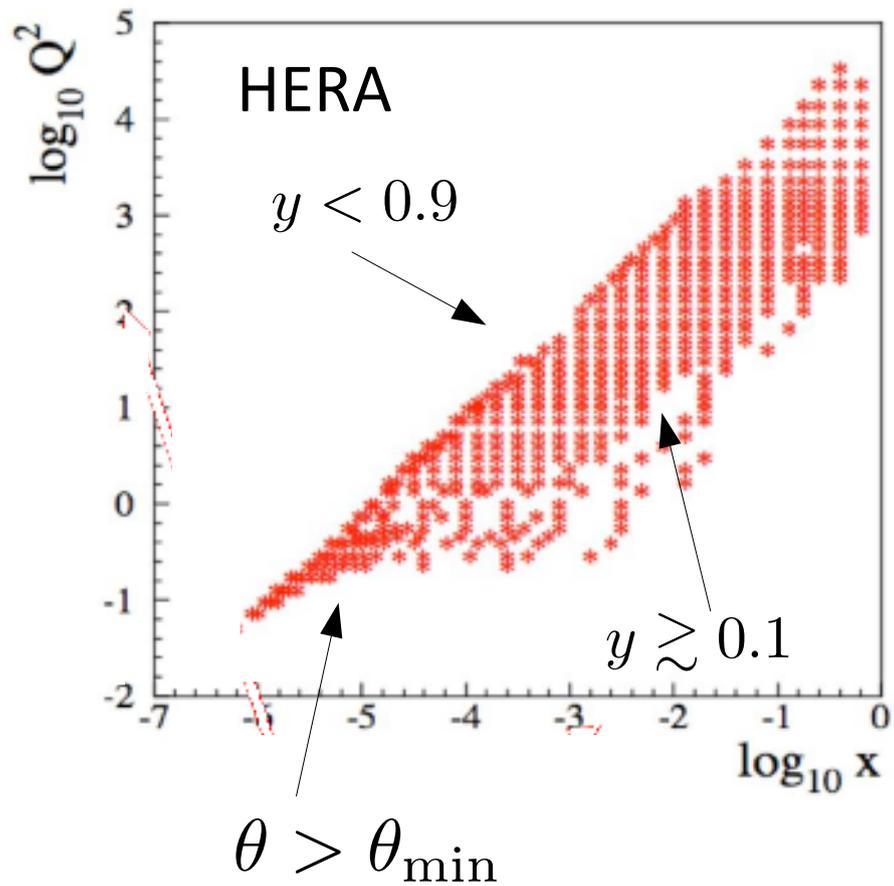
$$s_2 > s_1$$

Theoretical prejudice:  
can use pQCD only if  $Q^2$  large “enough”

# Kinematics

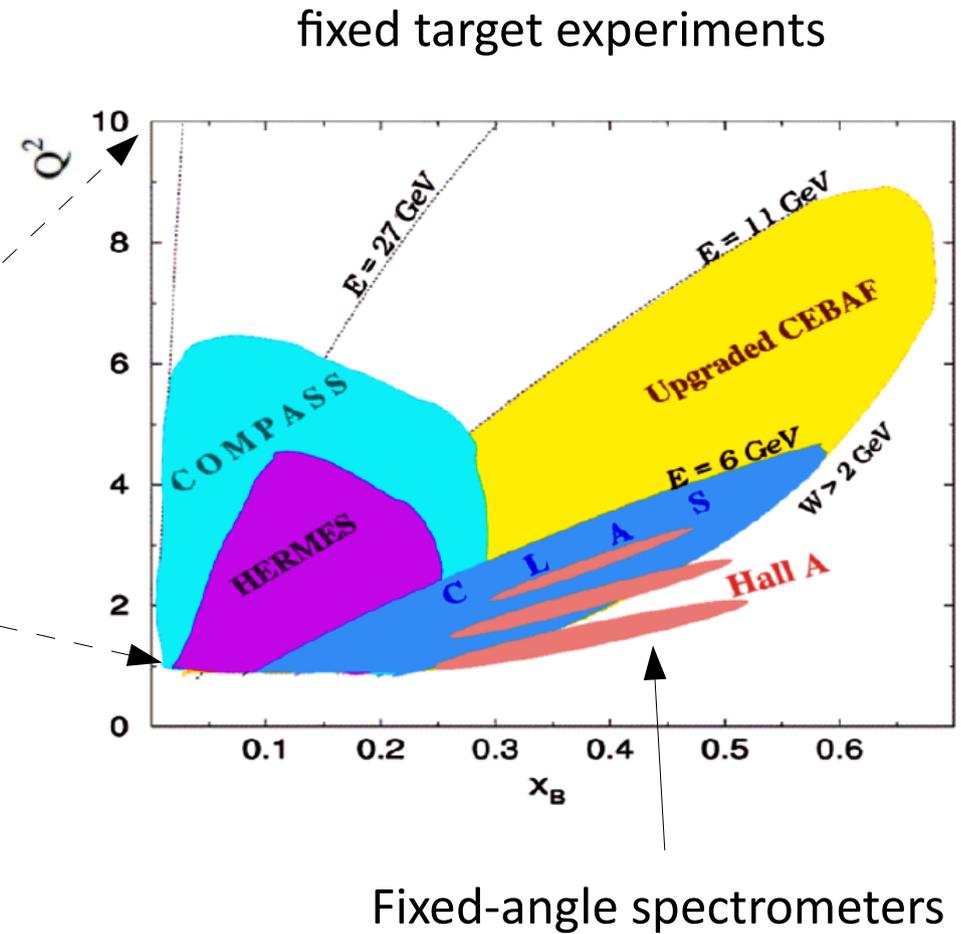
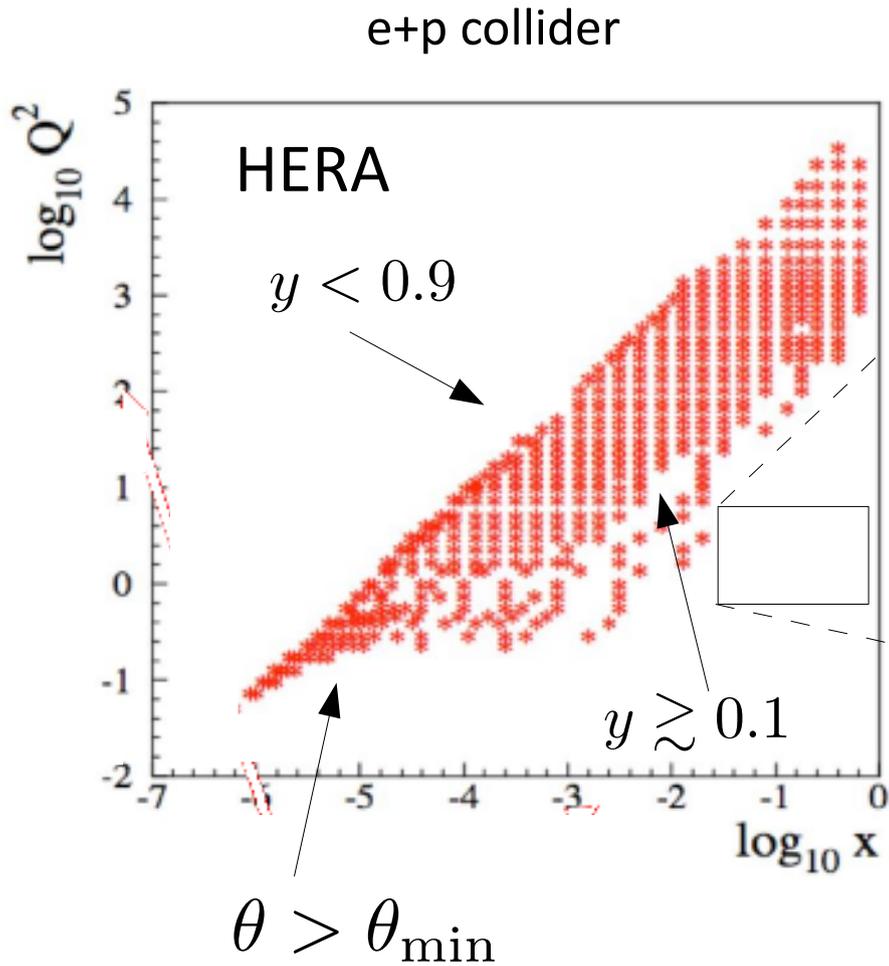
## □ Kinematic plane – in practice

e+p collider



# Kinematics

## □ Kinematic plane – in practice



# Cross section

## DIS in the target rest frame

$$p = (M, \vec{0}_T, 0) \quad \ell = (E_\ell, \vec{0}_T, \ell_z)$$

$$q = (q_0, \vec{q}_T, q_z) \quad \ell' = (E'_\ell, \vec{\ell}'_T, \ell'_z)$$

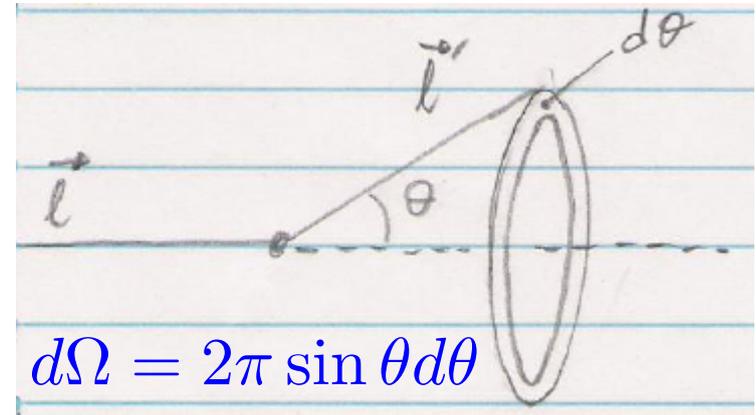
– invariants

$$\nu = p \cdot q / M = q_0 = E_\ell - E'_\ell$$

$$y = p \cdot q / p \cdot \ell = \nu / E_\ell$$

$$Q^2 = 2E_\ell E'_\ell (1 - \cos \theta) = 4E_\ell E'_\ell \sin^2(\theta/2)$$

(for collider, Breit frames see [Levy])



## Cross section

$$\frac{d\sigma}{dx_B dQ^2} = \frac{y}{x_B} \frac{\pi}{E'_\ell} \frac{d\sigma}{dE'_\ell d\Omega}$$

– **Ex.5** (med) : prove this (hint: work out  $dQ^2/d\theta$ )

– **Ex.6** (hard) : directly show that r.h.s. is Lorentz invariant

# Cross section

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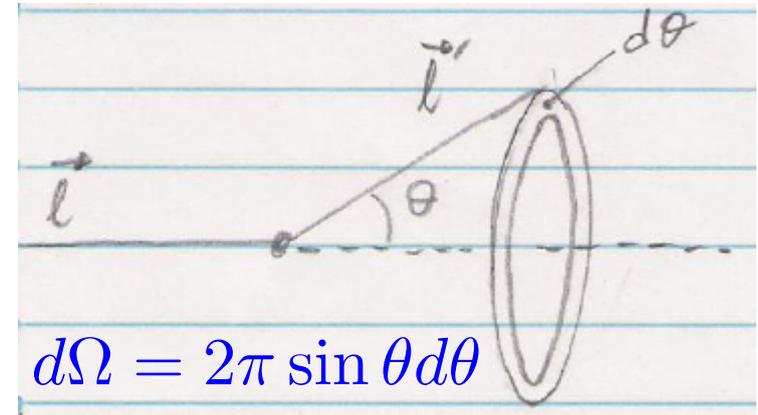
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## Cross section

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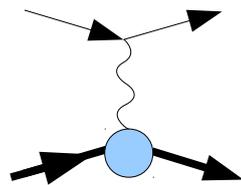
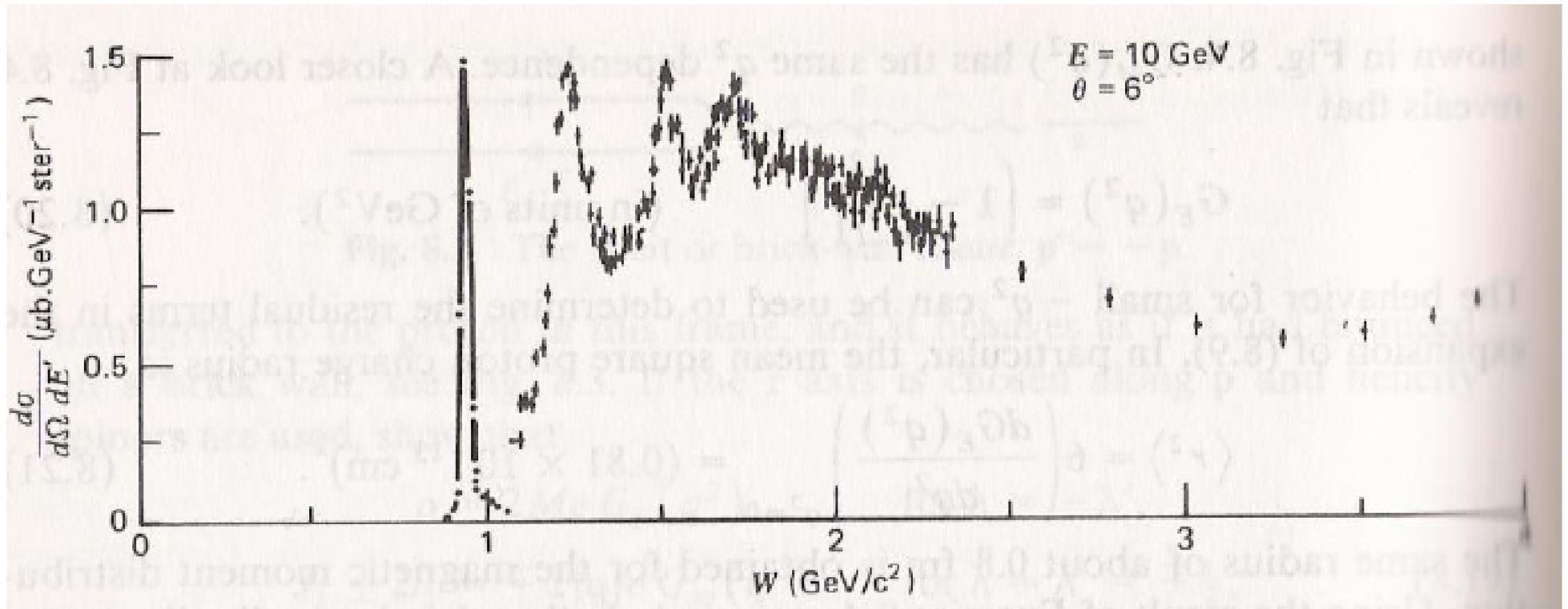
$$\begin{aligned} \frac{d\sigma}{dx_B dQ^2} &= \frac{\nu}{x_B} \frac{d\sigma}{d\nu dQ^2} \\ &= \frac{y}{Q^2} \frac{d\sigma}{dx_B dy} = \frac{y}{x_B} \frac{d\sigma}{dy dQ^2} \end{aligned}$$

– and also

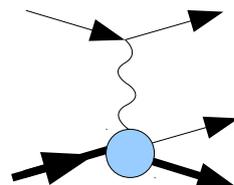
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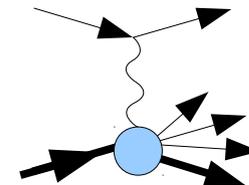
# Cross section



**Elastic:**  $W=M$   
 (no energy to produce  
 any other particle)



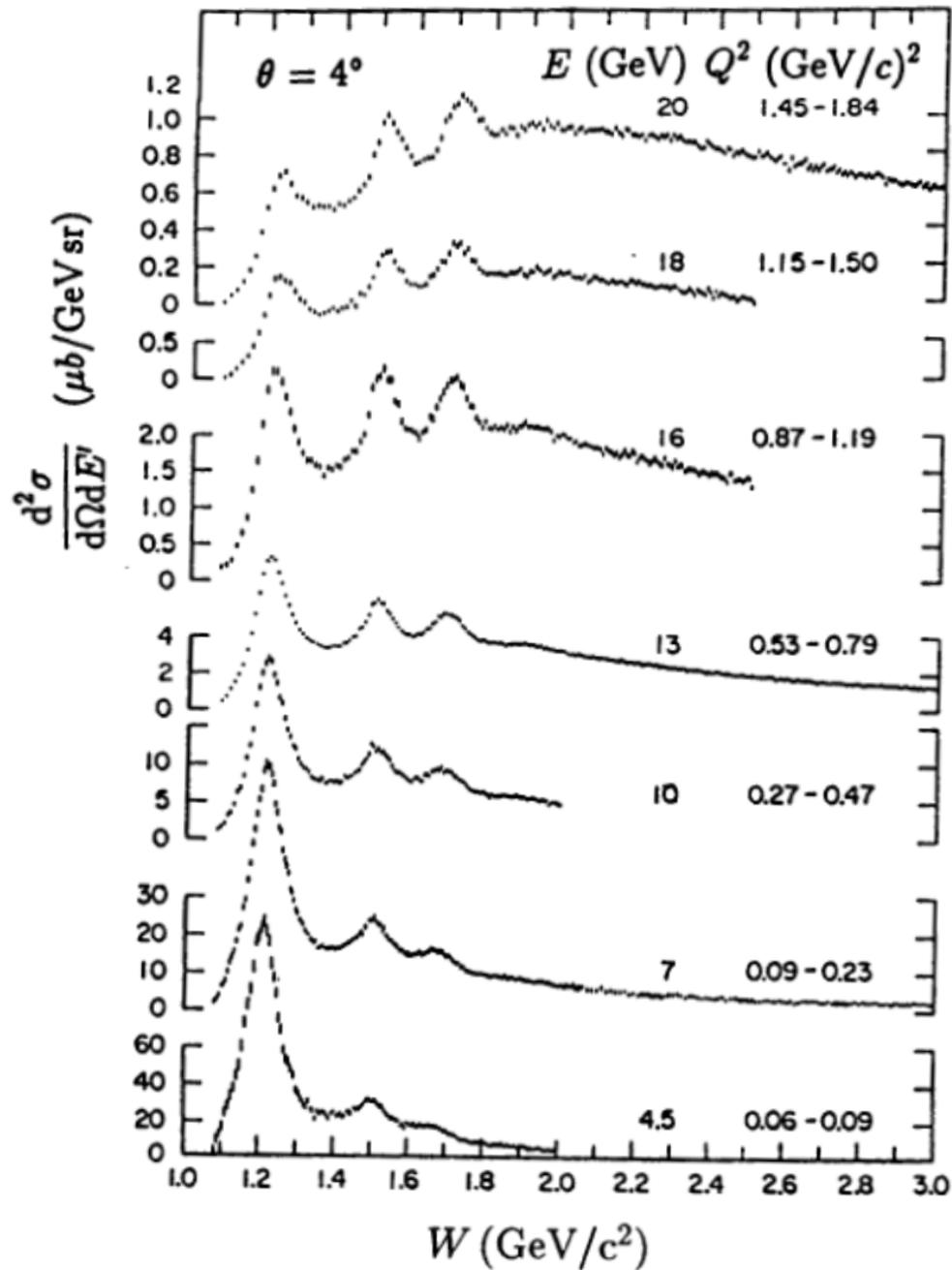
**Inelastic  
 (resonance region)**  
 ( $W$  for only a few particles)



**Deep inelastic**  
 (large  $W$ , more particles)

$$Q^2, W^2 \rightarrow \infty \quad x_B \text{ fixed}$$

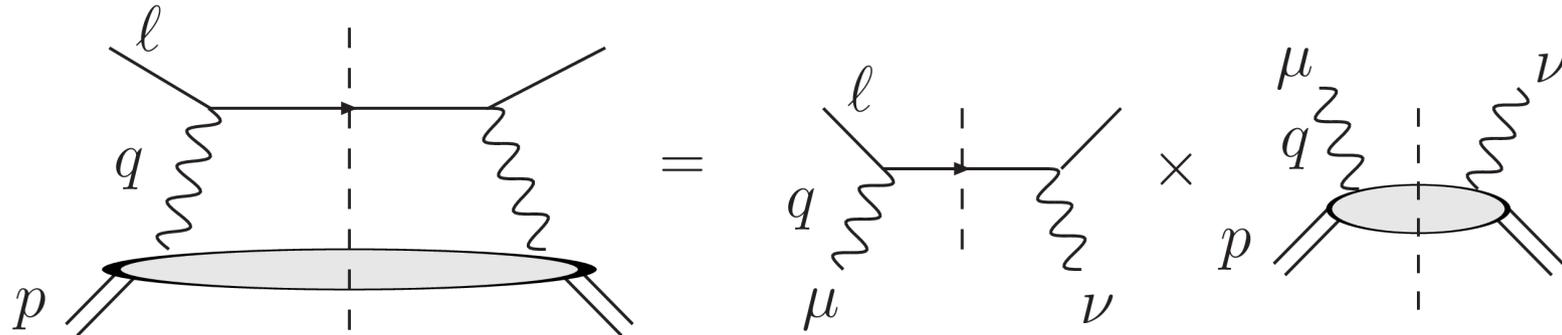
# Cross section



# Structure functions

- Leptonic and hadronic tensors – 1 photon exchange

$$d\sigma \propto L_{\mu\nu}(\ell, q) W^{\mu\nu}(p, q)$$



- Electron is elementary:  $L_{\mu\nu}$  can be calculated perturbatively

- Lorentz decomposition + gauge invariance = structure functions

$$W^{\mu\nu}(p, q) = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x_B, Q^2) \\ + \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) F_2(x_B, Q^2) \\ + i\varepsilon_{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{2p \cdot q} F_3(x_B, Q^2)$$

←  $\neq 0$  only for  $W^\pm, Z^0$  boson exchanges

# Structure functions

- Lorentz decomposition + gauge invariance = structure functions

$$\begin{aligned} W^{\mu\nu}(p, q) = & \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x_B, Q^2) \\ & + \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) F_2(x_B, Q^2) \\ & + i\varepsilon_{\mu\nu\alpha\beta} \frac{p^\alpha q^\beta}{2p \cdot q} F_3(x_B, Q^2) \end{aligned}$$

- Note:

- $F_1, F_2, F_3$  are Lorentz invariants  
(the tensor structure is explicitly factorized out)
- Most general structure compatible with Lorentz and gauge invariance, no missing functions
- **Ex.7** (med) : convince yourself of these 2 statements  
(gauge invariance implies  $q_\mu W^{\mu\nu} = 0$ )

# Structure functions and cross section

□ The cross section (for a  $\gamma$  exchange) reads

$$\frac{d\sigma}{dx dQ^2} = \frac{2\pi\alpha^2 y^2}{Q^4} L_{\mu\nu}(\ell, q) W^{\mu\nu}(p, q)$$

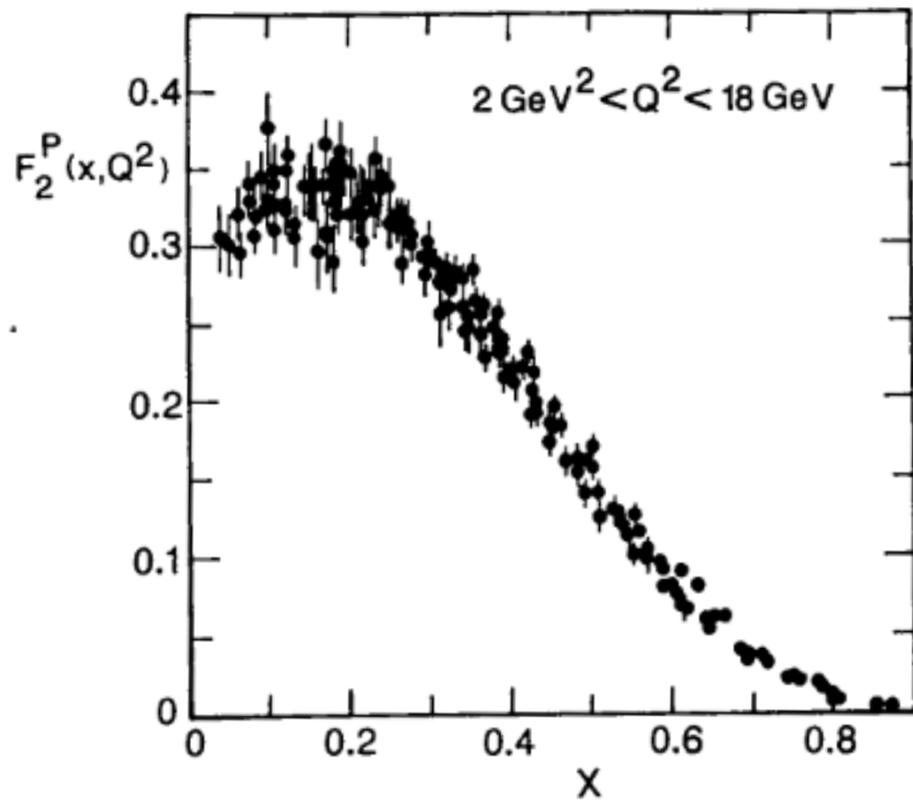
□ Using

$$L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - \ell \cdot \ell' g_{\mu\nu})$$

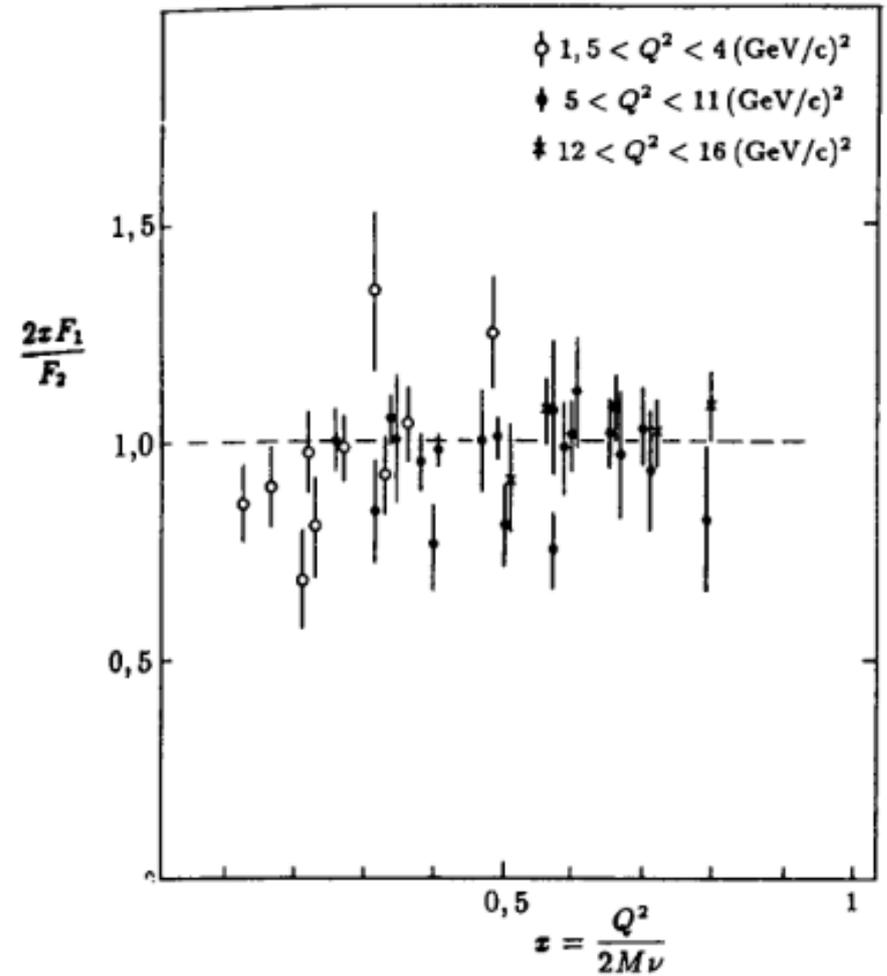
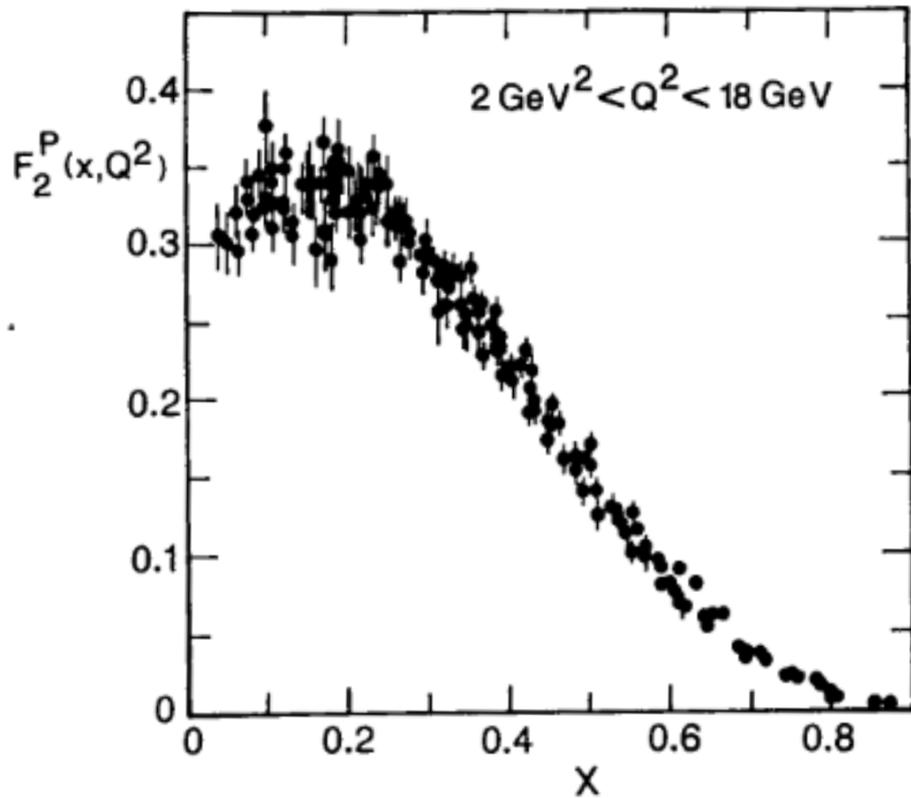
one obtains

$$\begin{aligned} \frac{d\sigma}{dx_B dQ^2} = & \frac{4\pi\alpha^2}{x_B Q^4} \left\{ \left( 1 - y - x_B^2 y^2 \frac{M^2}{Q^2} \right) F_2(x_B, Q^2) \right. \\ & \left. + y^2 x_B F_1(x_B, Q^2) \mp \left( y - \frac{y^2}{2} \right) x_B F_3(x_B, Q^2) \right\} \end{aligned}$$

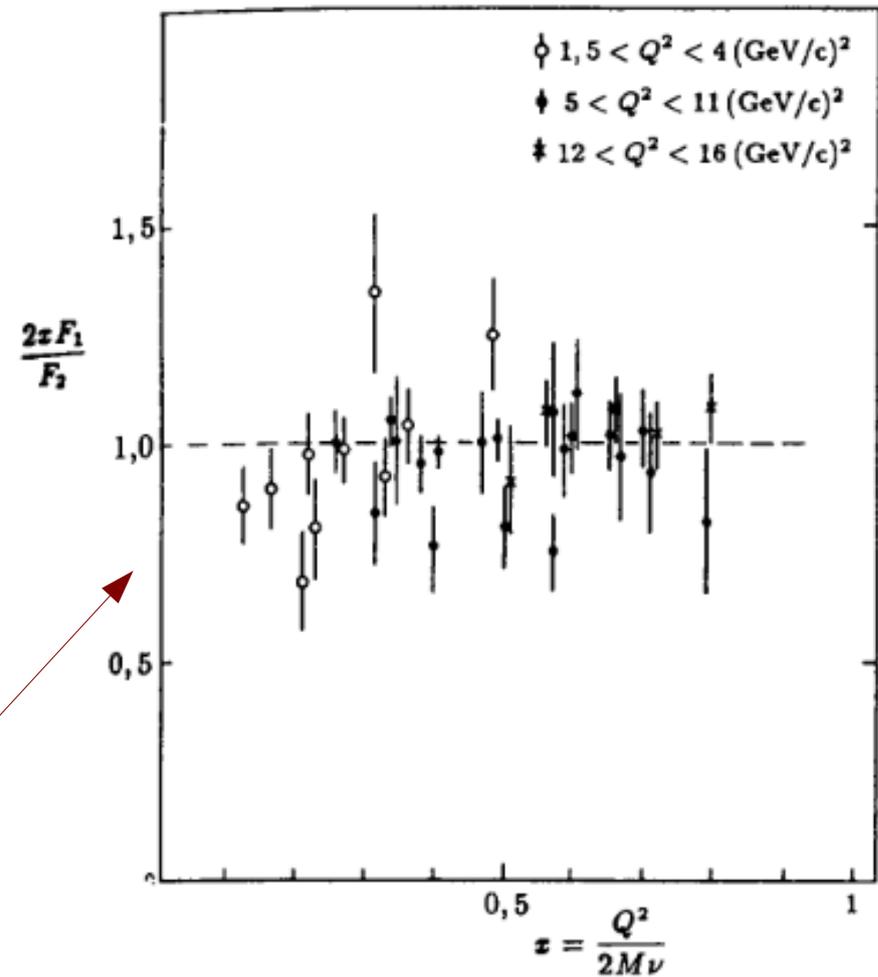
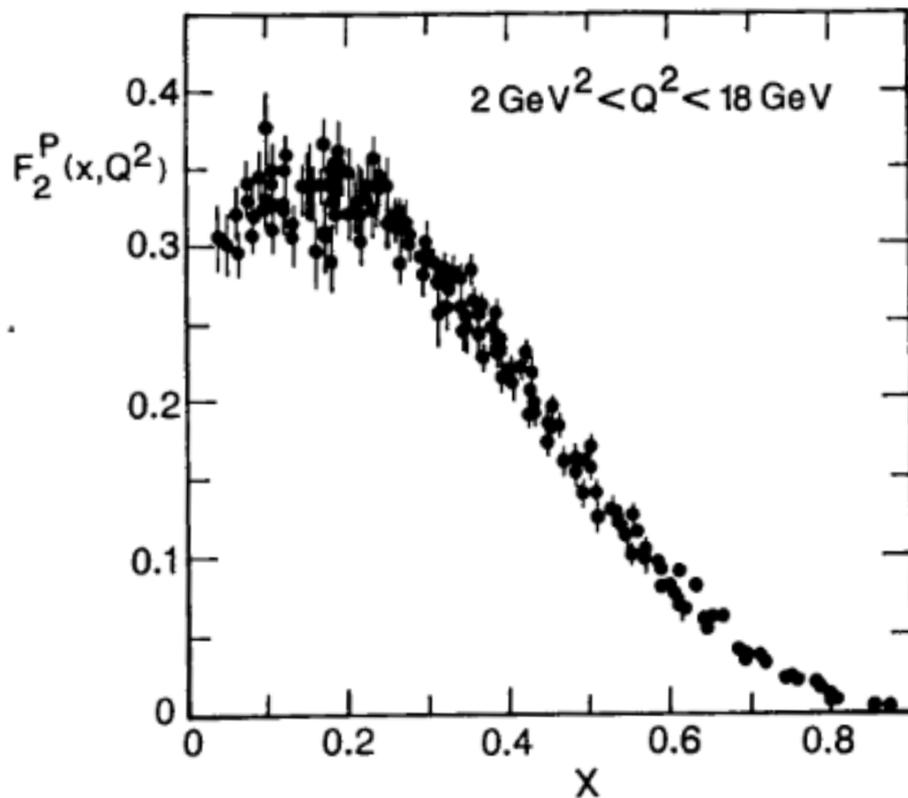
# Structure functions



# Structure functions

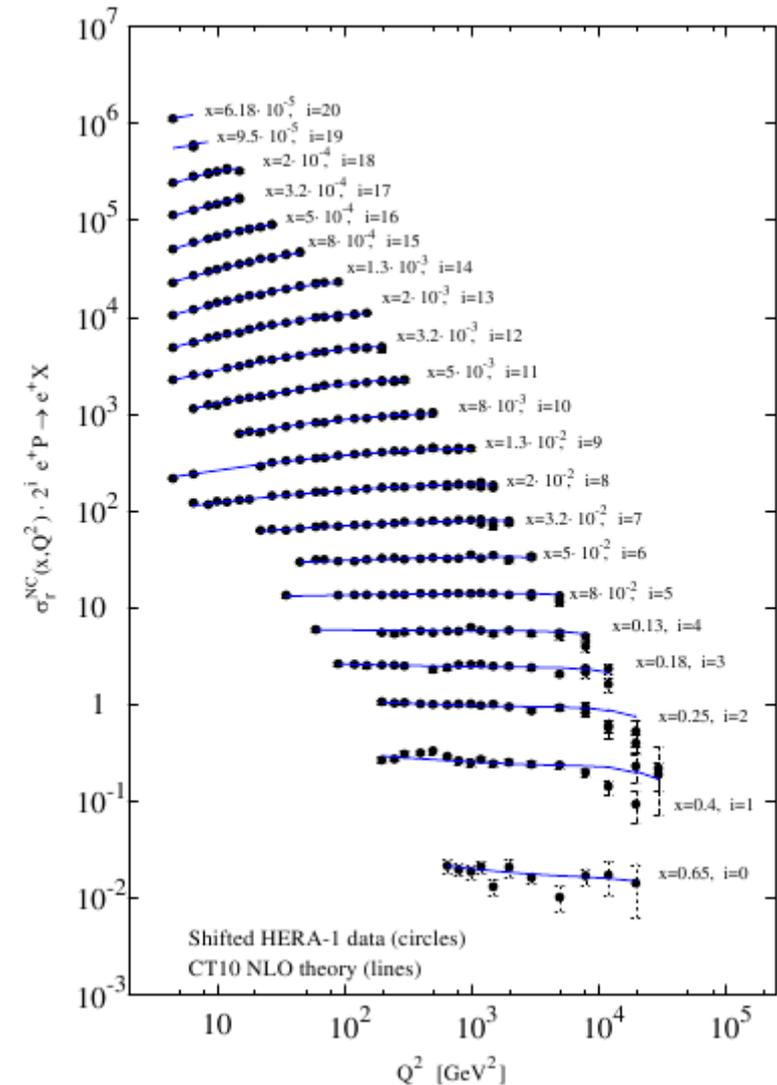
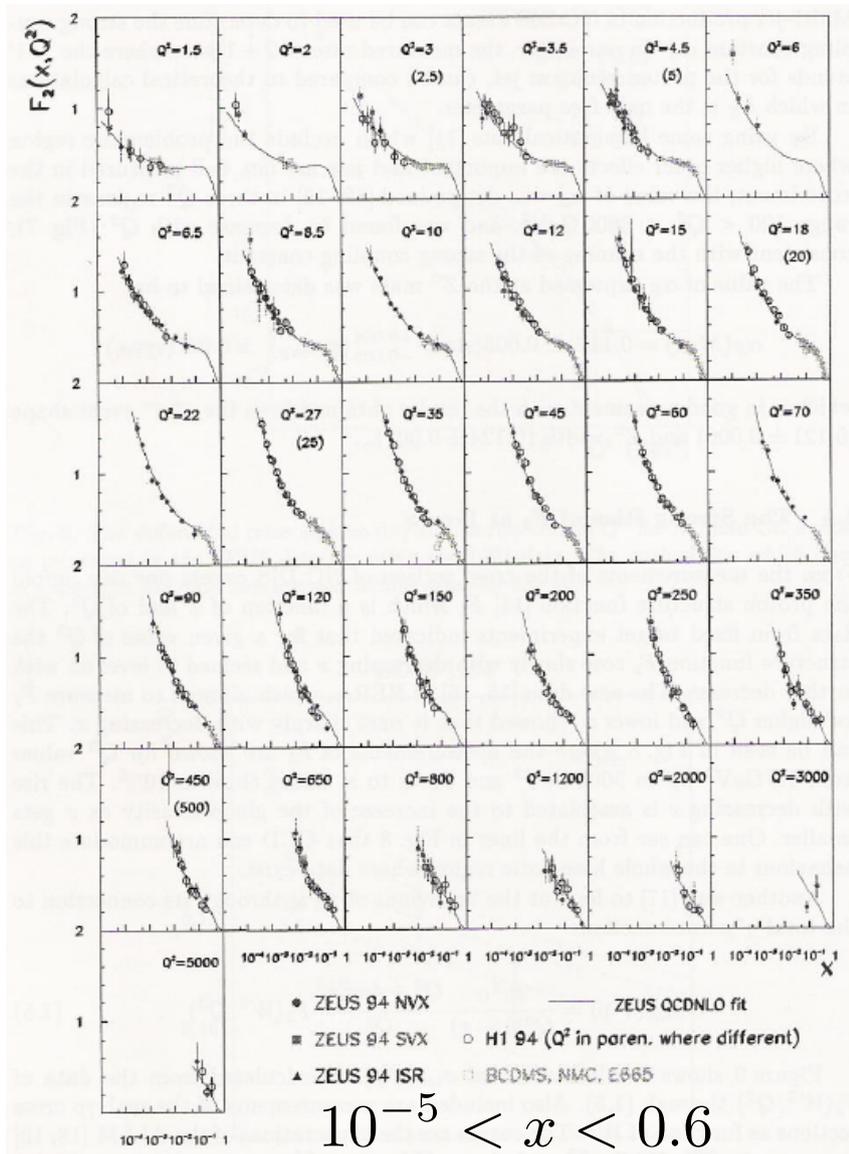


# Structure functions



Same as if target was a free spin  $\frac{1}{2}$  particle:  
**The photon is scattering on quasi-free quarks**

# Structure functions



□ Note: evolution with  $Q^2$  and rise at low- $x$  (here come the gluons!)

# A taste of the parton model

# A taste of the parton model

## □ The photon scatters on quasi-free quarks

– Empirical evidence:  $F_2 = 2x_B F_1$

– Photon wave-length in rest frame, neglect proton mass  $M/Q \ll 1$ :

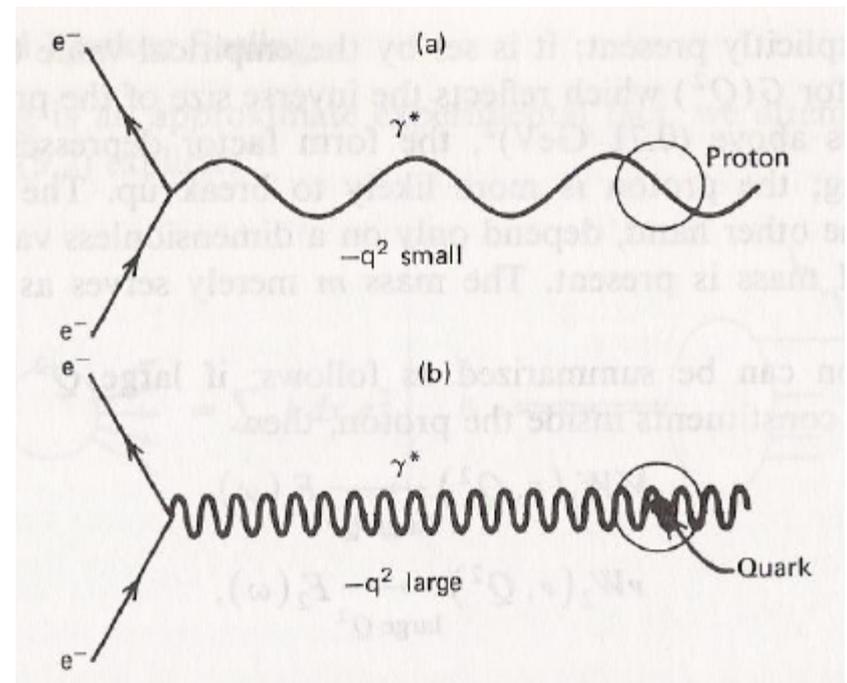
$$\lambda = \frac{1}{|\vec{q}|} = \frac{1}{\sqrt{\nu^2 + Q^2}} \approx \frac{1}{\nu} = \frac{2Mx}{Q^2}$$

– E.g., for  $x=0.1$ ,  $Q^2=4 \text{ GeV}^2$   
(and putting back  $c$  and  $\hbar$ ),

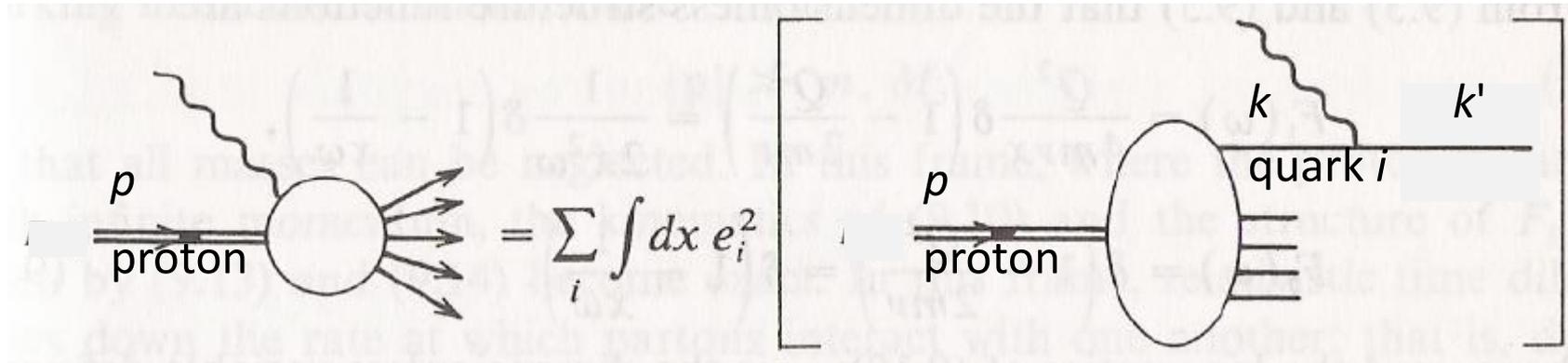
$$\lambda = 10^{-17} \text{ m} = 10^{-2} \text{ fm}$$

to be compared with

$$R_p \approx 1 \text{ fm}$$



# A taste of the parton model



□ DIS  $\approx$  photon-quark *elastic* scattering

□ Interpretation of  $x_B$

- Parton carries fraction  $x$  of proton's momentum:  $k^\mu = x p^\mu$
- 4-momentum conservation:  $k' = k + q$
- Partons have zero mass:  $k^2 = k'^2 = 0$

$$x = \frac{Q^2}{2p \cdot q} = x_B$$

□ **The virtual photon probes quarks with  $x = x_B$**

## A taste of the parton model

□ **Beware:** There is an inconsistency in the derivation:

- From  $k = xp$  follows that quarks are massive,  $M_q = xM$  !!

□ A heuristic way out is to work in the “infinite momentum frame”, where  $|\vec{p}| \rightarrow \infty$  so that one can neglect the proton's mass:

$$E_p = \sqrt{\vec{p}^2 + M^2} \rightarrow |\vec{p}|$$

- This frame is also important to better justify the parton model
- But quarks should be massless in any frame
  - The problem lies in the definition of  $x$
  - We'll see a better solution in tomorrow's lecture
- Similarly, the vector 3-momentum is not a Lorentz-invariant scale
  - In fact,  $M$  can be neglected compared to  $Q$ , not  $|\vec{p}|$ , as we shall see

# Lecture 1 - recap

- **Hadrons are made of quarks and gluons**
  - Partonic structure probed in DIS, DY, jets, .... (we'll see more)
- **Deep Inelastic Scattering (DIS)**
  - The master method to measure quarks and gluons
  - Invariant kinematics, target rest frame
  - Cross section, parametrized in terms of structure functions
- **A taste of the parton model**
  - Phenomenological evidence for quasi-free quarks, gluons
  - Some trouble in heuristic, textbook arguments
- **Next lecture: Parton model, parton distributions (PDF)**
  - (QCD improved) parton model
  - More kinematics
  - Factorization, universality: Drell-Yan, W and Z, jets