Disconnected gauge theories

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Based on 1804.01108 with A.Bourget and A.Pini

- Gauge theories lie at the core of Theoretical Physics… and as such a hughe effort has been/is dedicated to their study

- It is probably fair to say that most studies are for connected gauge groups (at least comparatively).

- However interesting things may be hidding in the wild forest of disconnected gauge groups

  - More generic N=2 theories? Perhaps with exotic properties?

  - N=3 SUSY theories?

  - …

- One natural context where they appear is when gauging discrete global symmetries (such as e.g. charge conjugation)

- This is subtle…

See *e.g.* Argyres & Martone

- A natural alternative approach is to consider a gauge group which, *ab initio* includes the gauging of charge conjugation

- While the existence of this is not obvious *a priori*, it is clear that, if exists, the standard technology can be directly imported

- Today we will argue for the existence of such gauge groups: in the math literature they are called Principal Extensions

- They naturally implement a version of charge conjugation

- They lead to very surprising consequences

    - Non-freely generated Coulomb branches (contrary to standard lore, first example of such thing!!!)

    - An “exotic” pattern of global symmetries

- Note that, starting with these “new” gauge groups we may consider gauge theories in arbitrary dimensions…

- Today we will concentrate on the 4d N=2 case for definitness…

- …but a whole new world to explore!

Contents

- Introduction
- A primer in Principal Extensions (including an integration formula)
- 4d N=2 theories based on Principal Extensions
- Coulomb branches
- Higgs branches
- ~~Conclusions~~ Open questions

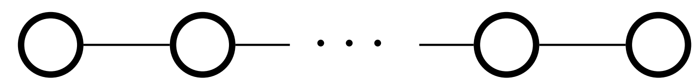
A primer in Principal Extensions

- Charge conjugation is essentially complex conjugation. It mixes nontrivially with gauge transformations

$$(G_2 \circ C \circ G_1)\psi = G_2 C(e^{i\alpha_1}\psi) = G_2(e^{-i\alpha_1}\psi^\star) = e^{i(-\alpha_1+\alpha_2)}\psi^\star$$
$$(G_1 \circ C \circ G_2)\psi = G_1 C(e^{i\alpha_2}\psi) = G_1(e^{-i\alpha_2}\psi^\star) = e^{i(\alpha_1-\alpha_2)}\psi^\star$$

- So the combination of G and C cannot simply be the direct product G x C

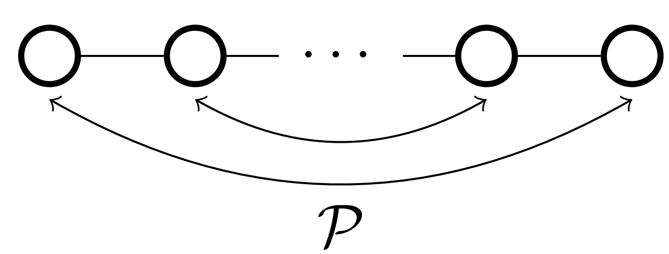
- Let us take instead a fresh start…Let's consider the group SU(N). Its Dynkin diagram is

A_{N-1} 

- As a graph, it has an automorphism group Γ of order 2

$$\Gamma = \{1,\, \mathcal{P}\} \sim \mathbb{Z}_2$$

- In the graph

A_{N-1} 

- One may imagine representing Γ as an automorphism of SU(N)

$$\varphi : \Gamma \to \mathrm{Aut}(SU(N))$$

- It turns out that one can construct a Lie group (the Principal Extension) as

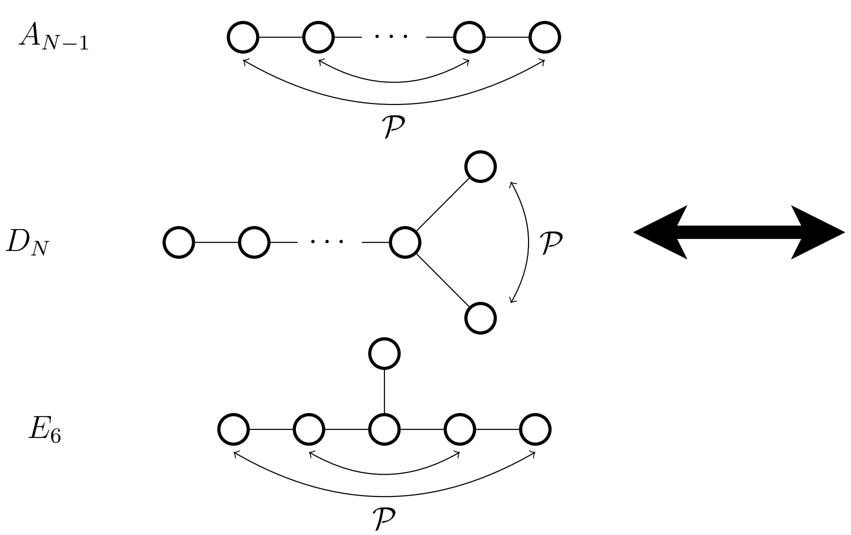
$$\widetilde{SU}(N) = SU(N) \rtimes_{\varphi} \Gamma$$

E.g. Wendt'01

- Crucially, the Principal Extension is a semidirect product

$$(g_1,\, h_1) \circ (g_2,\, h_2) = (g_1 \cdot \varphi_{h_1}(g_2),\, h_1\, h_2)$$

- Of course, one may imagine doing the same thing starting with any other Lie group whose Dynkin diagram has a symmetry



In this case the Principal Extension is well-known!
This is just the corresponding O (vs. SO) group!

- In fact, the $\widetilde{SU}(N)$ have also appeared in the past: branes on group manifolds

Bachas, Douglas & Schweigert'00
Maldacena, Moore & Seiberg'01
Stanciu'01

- Using hermitean generators, the Lie bracket is i[,]. Consistency demands complex conjugation to be defined as

$$\mathcal{C}(T_a) = \alpha\, T_a^\star \rightsquigarrow \mathcal{C}\Big(i[T_a, T_b]\Big) = -i\alpha[T_a, T_b]^\star = i[T_a, T_b]^\star \Rightarrow \mathcal{C}(T_a) = -\,T_a^\star$$

- It turns out that one can represent $\mathcal{P}$ as

$$\mathcal{P}(\mathfrak{M}) = A\,\mathcal{C}(\mathfrak{M})\,A^{-1} \qquad A = \begin{pmatrix} & & & 1 \\ & & (-1) & \\ & \dots & & \\ (-1)^{N-1} & & & \end{pmatrix}$$

- This indeed satisfies

$$\mathcal{P}(H_i) = H_{N-i} \longleftrightarrow$$ Exchange of Cartans *i.e.* flipping of the Dynin diagram

- Let us briefly discuss some relevant representations
  - **Adjoint**: this is just the action of the group on its algebra, and so it is equal to the adjoint of SU(N).

$$\dim(\mathrm{Adj}_{\widetilde{SU(N)}}) = N^2 - 1 \qquad T(\mathrm{Adj}_{\widetilde{SU(N)}}) = 1$$

  - **Fundamental**: let us consider the (reducible) **N**+**<u>N</u>** of SU(N). The generators can be written as

$$\widetilde{T}^a = \begin{pmatrix} T^a & \\ & (T^a)^\star \end{pmatrix}$$

    The disconnected component (essentially complex conjugation) exchanges the **N** with the **<u>N</u>**. Hence the rep. becomes irreducible

$$\dim(\mathrm{Fund}_{\widetilde{SU(N)}}) = 2N \qquad T(\mathrm{Fund}_{\widetilde{SU(N)}}) = 1$$

- Mathematicians have worked out an integration formula over Principal Extensions Wendt'01

$$\int_{\widetilde{SU}(N)} d\eta_{\widetilde{SU}(N)}(X)\, f(X) = \frac{1}{2}\Big[\int d\mu_N^+(z)\, f(z) + \int d\mu_N^-(z)\, f(z) \Big]$$

- Here + represents the connected component (a copy of SU(N))

$$d\mu_N^+(z) = \prod_{j=1}^{N-1} \frac{\dot{z}_j}{2\pi i z_j} \prod_{\alpha \in R^+(\mathfrak{su}(N))} (1 - z(\alpha)) \ ,$$

- …and - the disconnected component. Its measure depends on N being odd or even

For $\widetilde{SU}(2N)$ the $-$ component involves a $SO(N+1)$ integration

$$N \text{ even:} \qquad d\mu_N^-(z) = \prod_{j=1}^{N/2} \frac{\underset{\cdot}{\mathrm{z}}_j}{2\pi i z_j} \prod_{\alpha \in R^+(B_{N/2})} (1 - z(\alpha)) \; .$$

$$N \text{ odd:} \qquad d\mu_N^-(z) = \prod_{j=1}^{(N-1)/2} \frac{\underset{\cdot}{\mathrm{z}}_j}{2\pi i z_j} \prod_{\alpha \in R^+(C_{(N-1)/2})} (1 - z(\alpha)) \; .$$

For $\widetilde{SU}(2N+1)$ the $-$ component involves a $Sp(2N)$ integration

4d $\mathcal{N} = 2$ based on $\widetilde{SU}(N)$

- One may imagine gauge theories based on Principal Extensions
- Since, at the end of the day, Principal Extensions are simply Lie groups, one can construct gauge theories following the textbook procedure
- This can be done in arbitrary dimensions. Today concentrate on N=2 in 4d as proof of concept

- As said, just import everything we know. In particular the ingredients will be **vector multiplets** (in the adjoint) and **hypermultiplets** (which we will assume in the fundamental)

- Using the group theory data above we can compute *e.g.* beta functions and consider CFT's etc.

- Representations are real, so no chiral anomalies

- Today concentrate on SQCD-like theories, with one vector multiplet and a bunch of fundamental matter
- There will be a moduli space with a Coulomb branch and a Higgs branch
- Just as for SU(N), the Higgs branch will be classical (non-renormalization).
- As for the Coulomb branch, we can consider SCFT’s so that they are easier

Coulomb branches

- One particularly powerful tool to study theories is to compute their index: information about the protected operators

$$I = \int d\eta_{\widetilde{SU(N)}}\, \mathrm{PE}[f]$$

"Single particle" contribution

- In particular, we have the integration formula over Principal Extensions, and so we can hope to extract protected useful information

- This is a complicated function. In particular limits it becomes much simpler. One such limit is sensitive only to the Coulomb branch

$$f^{\frac{1}{2}H} = 0 \qquad f^{V} = t$$

Gadde, Rastelli, Razamat& Yan'13

- The Coulomb branch index becomes

$$\mathcal{I}^{\text{Coulomb}}_{\widetilde{\text{SU}}(N)}(t) = \frac{1}{2}\left[\prod_{i=2}^{N}\frac{1}{1-t^i} + \prod_{i=2}^{N}\frac{1}{1-(-t)^i}\right] .$$

A.Bourget, A.Pini & D.R-G'18
Argyres & Martone'18
Bourton, Pini & Pomoni'18

- This can be re-written as

$$\mathcal{I}^{\text{Coulomb}}_{\widetilde{\text{SU}}(N)}(t) = \frac{\sum\limits_{k_1<\cdots<k_r \text{ odd}} t^{k_1+\cdots+k_r}}{\prod\limits_{i \text{ even}}(1-t^i)\prod\limits_{i \text{ odd}}(1-t^{2i})} ,$$

$\rightarrow$ **Non-freely generated Coulomb branch in general!!!!**

- Being more explicit

N	PL of $\mathcal{I}^{\text{Coulomb}}_{\widetilde{\text{SU}}(N)}(t)$
2	t^2
3	$t^2 + t^6$
4	$t^2 + t^4 + t^6$
5	$t^2 + t^4 + t^6 + t^8 + t^{10} - t^{16}$
6	$t^2 + t^4 + 2t^6 + t^8 + t^{10} - t^{16}$
7	$t^2 + t^4 + 2t^6 + t^8 + 2t^{10} + t^{12} + t^{14} - t^{16} - t^{18} + \ldots$ (infinite)

- Thus, from N=5 on we have a non-frely generated Coulomb branch. Note that N=4 is secretly O(6) (which should have a freely generated CB) and N=2 is trivial (and so should have a freely generated CB)

- This could have been foreseen…For an adjoint field

$$\phi = \phi_a\, T^a \qquad \mathcal{P}:\, \phi \to -A\, \phi^\star\, A^{-1}$$

- Hence

$$\mathcal{P}:\, \mathrm{Tr}\phi^k \to (-1)^k\, \mathrm{Tr}\phi^k$$

- So only even k’s are gauge-invariants. In fact, this is essentially what stands for the two terms in the Coulomb branch index!

Higgs branches

- We are considering SQCD-like theories, with one vector multiplet and F half-hyper fields (real representations)
- In the SU theory the flavor symmetry would be U(F). What about the Principal Extension?
- Again, we can use the index as a probe. This time we will compute the Hall-Littlewood limit of the index, a.k.a. Higgs branch Hilbert series

Gadde, Rastelli, Razamat& Yan'13

$$HS_{(N,\,N_f)} = \int_{G^\circ} d\eta_{G^\circ}(X)\,\frac{\det\left(1 - t^2\Phi_{\mathrm{Adj}}(X)\right)}{\det\left(1 - t\Phi_{\mathrm{F\bar{F}}}(X)\right)^{N_f}}\ ,$$

$$
\begin{aligned}
HS_{(3,\,6)} &= 1+[2,0,0]_{C_3}\,t^2+\Big([0,0,1]_{C_3}+[1,0,0]_{C_3}\Big)\,t^3 \\
&\quad +\Big(2\,[0,1,0]_{C_3}+2\,[0,2,0]_{C_3}+[4,0,0]_{C_3}+2\Big)\,t^4+O(t^5)\,. \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
HS_{(3,\,7)} &= 1+\Big([1,0,0]_{C_3}+[2,0,0]_{C_3}+1\Big)\,t^2+\Big([0,0,1]_{C_3}+[0,1,0]_{C_3}+[1,0,0]_{C_3}+1\Big)t^3 \\
&\quad +\Big([0,0,1]_{C_3}+2\,[0,1,0]_{C_3}+2\,[0,2,0]_{C_3}+3\,[1,0,0]_{C_3}+2\,[1,1,0]_{C_3} \\
&\quad +3\,[2,0,0]_{C_3}+[3,0,0]_{C_3}+[4,0,0]_{C_3}+3)\,t^4+O(t^5)\,.
\end{aligned}
$$

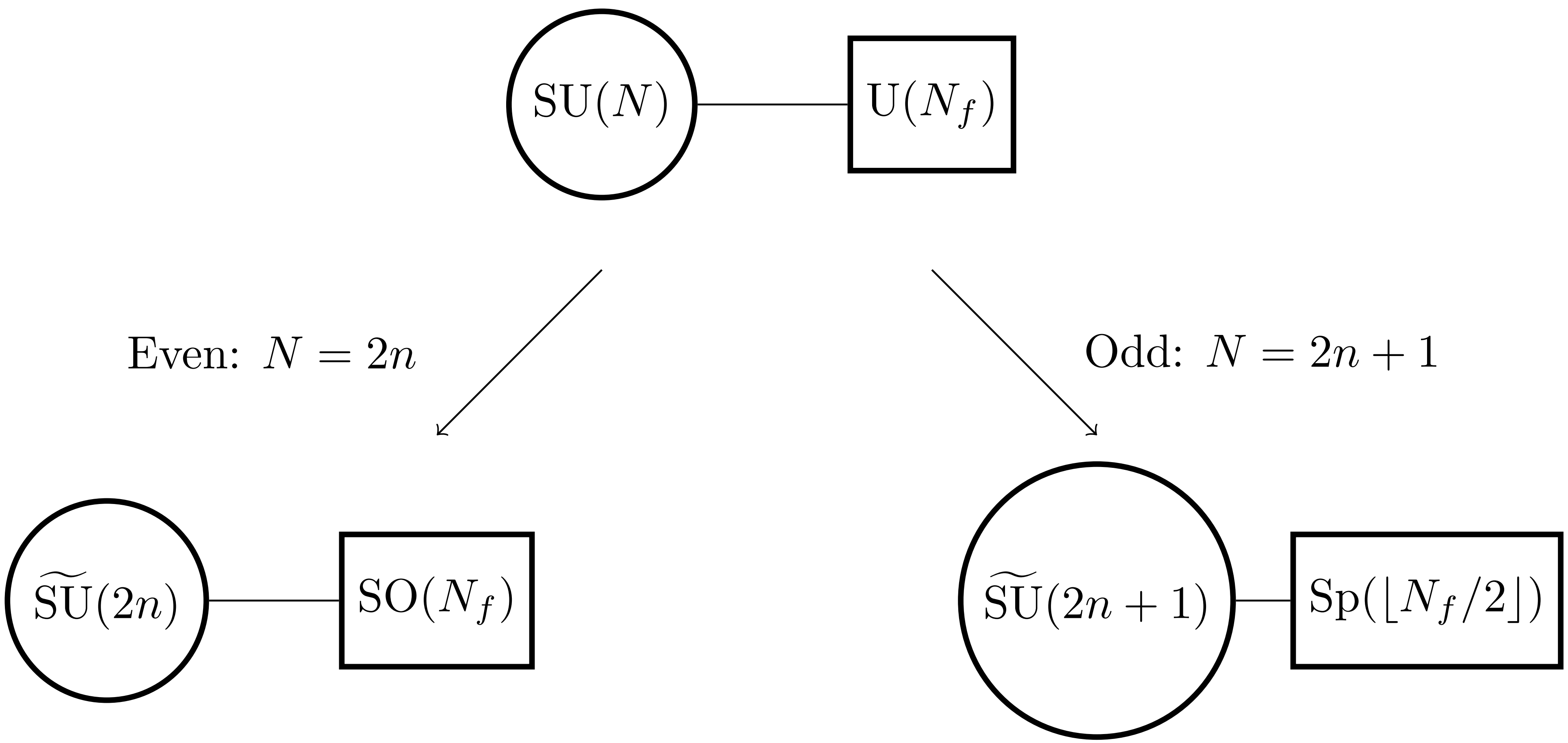
$$
\begin{aligned}
HS_{(3,\,8)} &= 1+[2,0,0,0]_{C_4}\,t^2+\Big([0,0,1,0]_{C_4}+[1,0,0,0]_{C_4}\Big)\,t^3 \\
&\quad +\Big([0,0,0,1]_{C_4}+2\,[0,1,0,0]_{C_4}+2\,[0,2,0,0]_{C_4}+[4,0,0,0]_{C_4}+2\Big)\,t^4+O(t^5)\,.
\end{aligned}
$$

$$
\begin{aligned}
HS_{(3,\,9)} &= 1+\Big([1,0,0,0]_{C_4}+[2,0,0,0]_{C_4}+1\Big)\,t^2 \\
&\quad +\Big([0,0,1,0]_{C_4}+[0,1,0,0]_{C_4}+[1,0,0,0]_{C_4}+1\Big)\,t^3 \\
&\quad +\Big([0,0,0,1]_{C_4}+[0,0,1,0]_{C_4}+2\,[0,1,0,0]_{C_4}+2\,[0,2,0,0]_{C_4}+3\,[1,0,0,0]_{C_4} \\
&\quad +2\,[1,1,0,0]_{C_4}+3\,[2,0,0,0]_{C_4}+[3,0,0,0]_{C_4}+[4,0,0,0]_{C_4}+3\Big)\,t^4+O(t^5)\,.
\end{aligned}
$$

$$
\begin{aligned}
HS_{(4,\,8)} &= 1+[0,1,0,0]_{D_4}\,t^2+\Big(2\,[0,0,0,2]_{D_4}+2\,[0,0,2,0]_{D_4}+2\,[0,2,0,0]_{D_4}+2\,[2,0,0,0]_{D_4} \\
&\quad +[4,0,0,0]_{D_4}+2\Big)\,t^4+O(t^5)\,.
\end{aligned}
$$

$$
\begin{aligned}
HS_{(4,\,9)} &= 1+[0,1,0,0]_{B_4}\,t^2+\Big(2\,[0,0,0,2]_{B_4}+2\,[0,2,0,0]_{B_4}+2\,[2,0,0,0]_{B_4} \\
&\quad +[4,0,0,0]_{B_4}+2\Big)\,t^4+O(t^5)\,.
\end{aligned}
$$

- Thus the Higgs branch HS groups itself in characters of the global symmetry algebra, which we can read off to be either SO(F) for even N or Sp(F) for odd N. Summarizing



Summary

- We have introduced a “new” family of gauge groups on which gauge theories can be based

- Actually one particular case is SO vs. O. Also the SU case made a modest appeareance

- The rules etc. to construct them are therefore just the usual ones. We could construct them in arbitrary dimensions. Today focused on 4d N=2.

- A powerful tool to explore the theories is the index (in particular because we have an integration formula). Using it we have seen that

    - The Coulomb brach is generically non-freely generated (first example of such a thing!)

    - The Higgs branch exhibits an “exotic” pattern of global symmetries

Open questions

- This only touches upon the tip of the iceberg… there are loads of things to explore. For instance, in random order

  - String embedding???
  - Global properties of the theories, spectrum of line operators…
  - Construction of quivers???
  - Versions in other dimensions (where perhaps other phenomena manifest)??
  - …

Thanks!