Phases of QCD3 and Duality

Adi Armoni

Swansea University

GGI, May 7, 2018

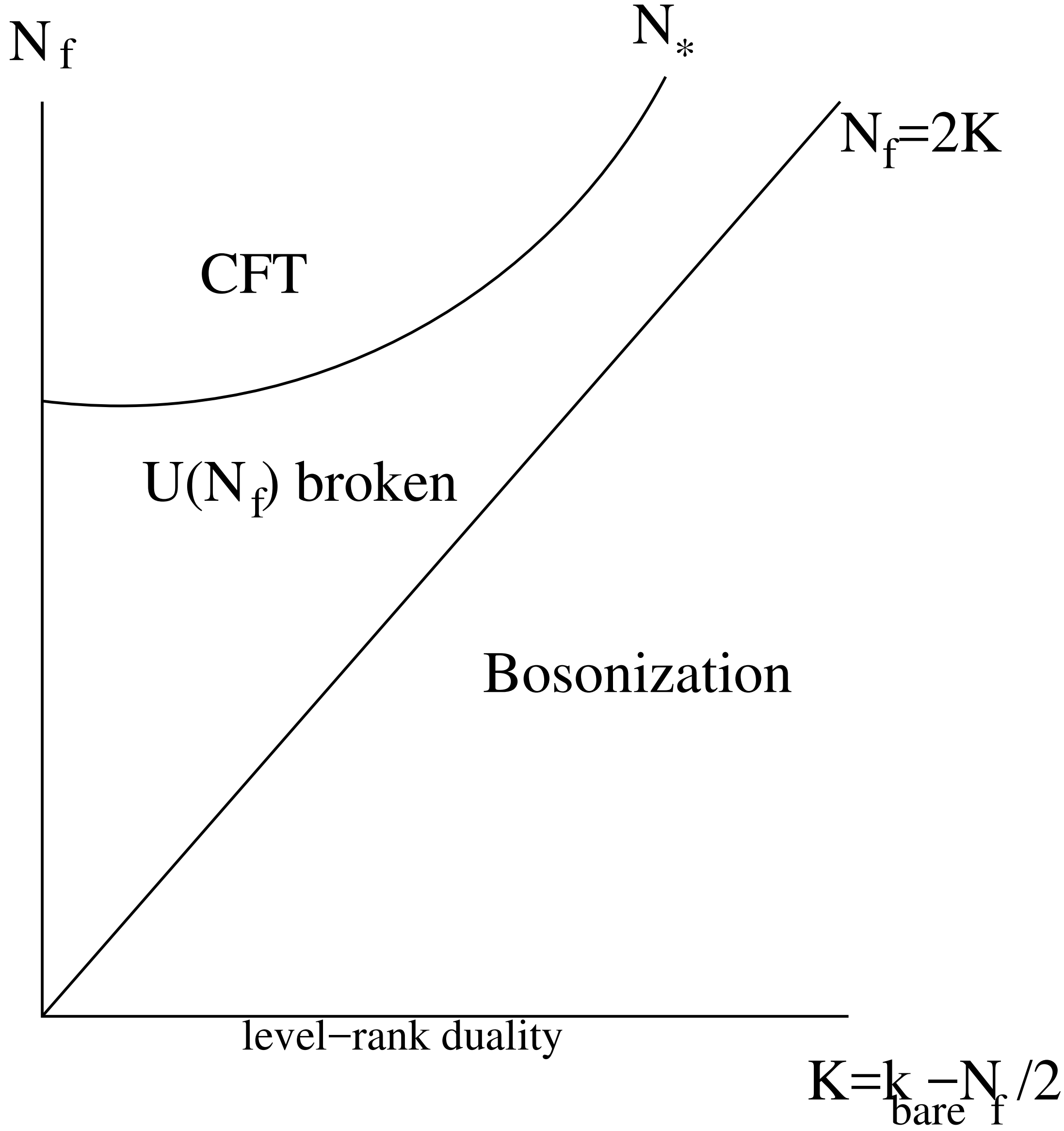
A. A. and Vasilis Niarchos, "Phases of QCD3 from Non-SUSY Seiberg Duality and Brane Dynamics", to appear in PRD

Introduction

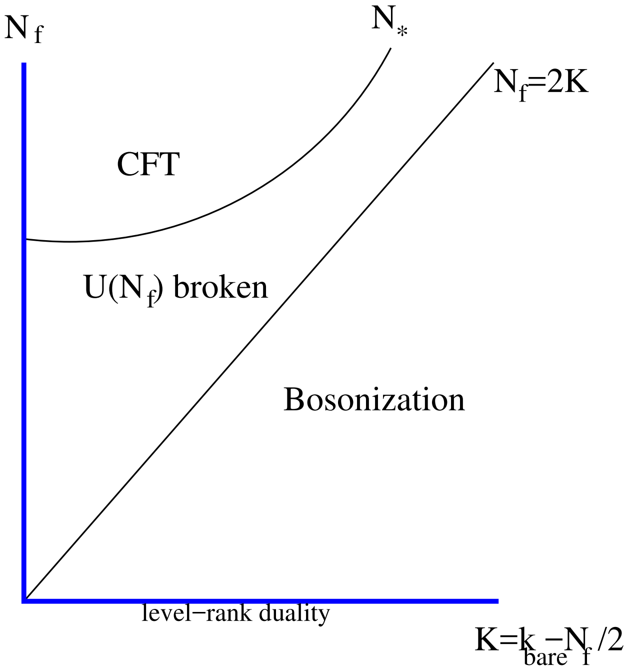
QCD3 with a Chern-Simons term

$$S_{U(1)} = \int d^3x\,(\bar{\Psi} i \not{D} \Psi - \frac{1}{4g^2} F_{\mu\nu}^2 + \frac{k_{bare}}{4\pi} \epsilon^{\mu\nu\rho} A_\mu F_{\nu\rho})$$

exhibits a rich phase diagram



“Old” Phases of the theory

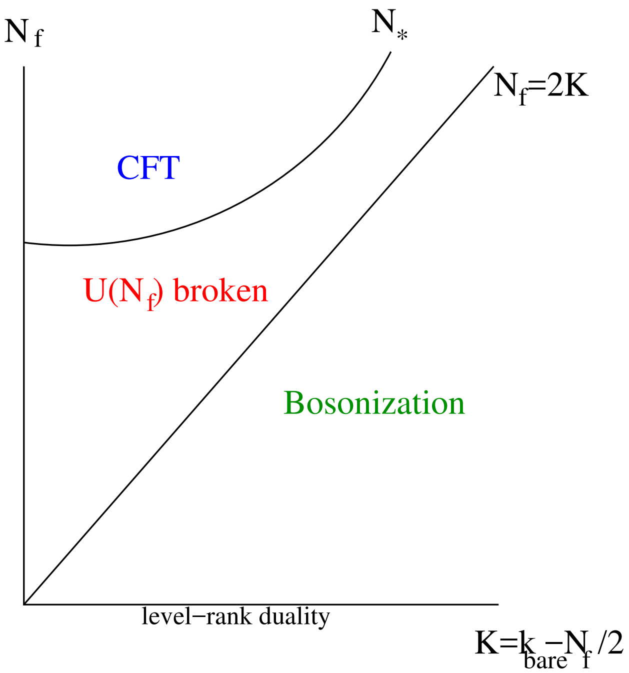


- $N_f = 0$, level-rank duality
  $U(N_c)_K \sim U(K)_{-N_c}$
- $k = 0$ (Appelquist and Nash) $SU(N_c) + N_f$ quarks $U(N_f) \to U(N_f/2) \times U(N_f/2)$ by a quark condensate $\langle \bar{\Psi}\Psi \rangle$, for $N_f < N_\star$, using a Schwinger-Dyson estimate.

  When $N_f > N_\star$ the IR is described by a non-trivial CFT

Note that the symmetry breaking pattern is dictated by Vafa-Witten theorem (parity cannot be broken).

“New” Phases of the theory



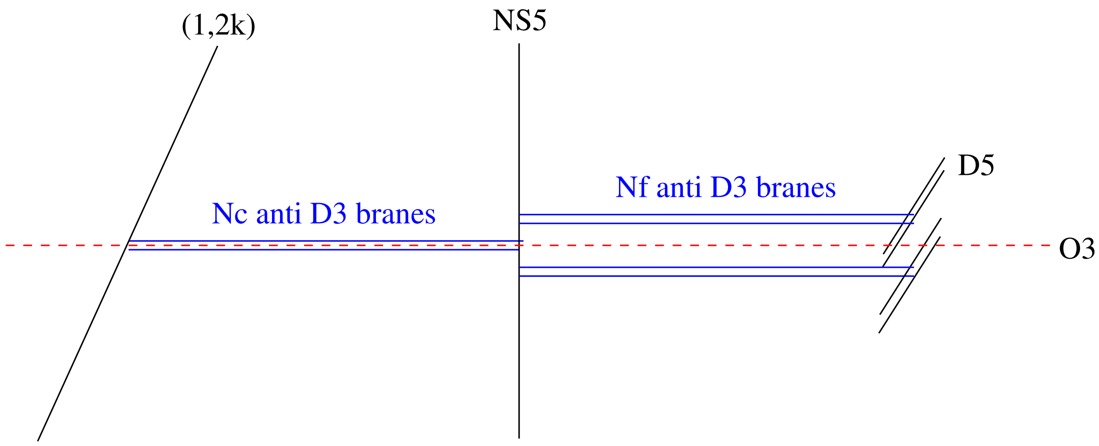
- $N_f < 2K$ Bosonization (Aharony, Minwalla, ...)
  $SU(N_c) + N_f\,\Psi \longleftrightarrow U(K + N_f/2) + N_f\,\phi$
- $2K \leq N_f < N_\star$ Symmetry breaking (Komargodski and Seiberg)
  $U(N_f) \to U(N_f/2 - K) \times U(N_f/2 + K)$
- $N_f > N_\star$ non-trivial CFT

Can we gain a better understanding of all these phases in terms of a dual magnetic theory? can string theory help?

Non-Supersymmetric Seiberg Duality

We wish to construct a non-supersymmetric electric/magnetic Seiberg duality.

Let us start with the following brane configuration ("electric theory")



In the above brane configuration branes were replaced by *antibranes*.

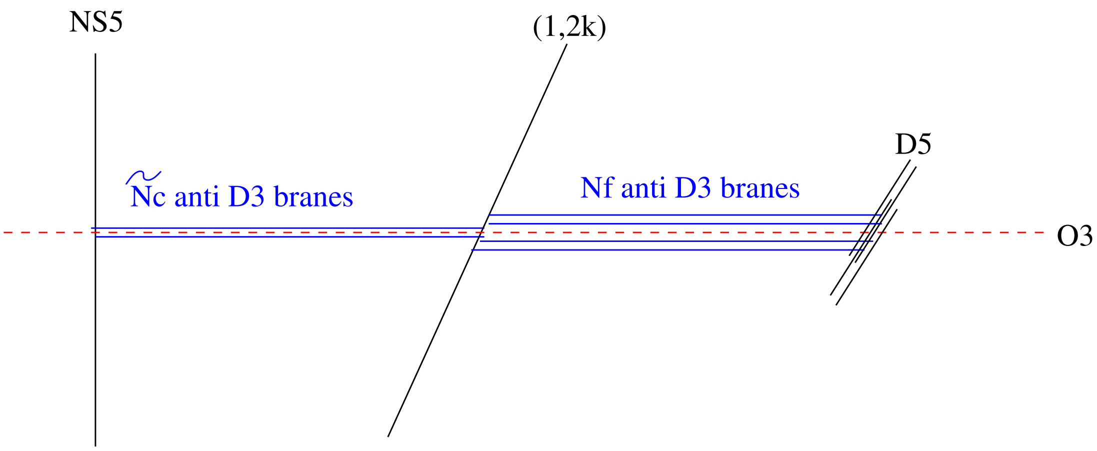
Matter Content of the Electric Theory

	$Sp(2N_c)$	$SO(2N_f)$
A_μ	$\square\!\square$ $N_c(2N_c+1)$	$\bullet$
σ	$\square\!\square$ $N_c(2N_c+1)$	$\bullet$
λ	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$ $N_c(2N_c-1)$	$\bullet$
Φ	$\square$ $2N_c$	$\square$ $2N_f$
Ψ	$\square$ $2N_c$	$\square$ $2N_f$

The theory is supersymmetric in the Veneziano limit.

Magnetic Theory

In order to obtain the magnetic theory we can either swap the fivebranes, or we can do something even better (Sugimoto): start with the SUSY electric/magnetic pair and add on both sides $N_f + k$ antibranes. After brane annihilation the electric SUSY theory becomes the magnetic non-SUSY theory and the magnetic SUSY theory becomes the electric non-SUSY theory.



With $\tilde{N}_c = N_f + k + 1 - N_c$

	$Sp(2\tilde{N}_c)$	$SO(2N_f)$
a_μ	$\Box\!\Box$ $\tilde{N}_c(2\tilde{N}_c+1)$	$\bullet$
s	$\Box\!\Box$ $\tilde{N}_c(2\tilde{N}_c+1)$	$\bullet$
l	$\begin{array}{c}\Box\\[-4pt]\Box\end{array}$ $\tilde{N}_c(2\tilde{N}_c-1)$	$\bullet$
ϕ	$\Box$ $2\tilde{N}_c$	$\Box$ $2N_f$
ψ	$\Box$ $2\tilde{N}_c$	$\Box$ $2N_f$
M	$\bullet$	$\begin{array}{c}\Box\\[-4pt]\Box\end{array}$ $N_f(2N_f-1)$
χ	$\bullet$	$\Box\!\Box$ $N_f(2N_f+1)$

Evidence for the Duality

How do we know that the duality holds? well, we don't ...

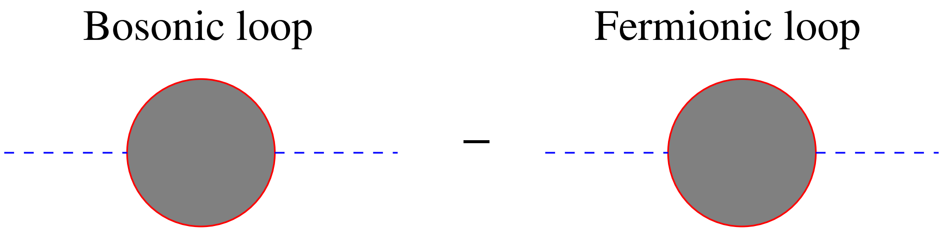
But there is some evidence

- We used the Elitzur-Giveon-Kutasov trick of swapping fivebranes (irrelevant operation?)
- We can derive the non-SUSY duality by adding $N_f + k$ antibranes to the SUSY electric and magnetic brane configurations
- The electric and magnetic pair become a SUSY pair in the Veneziano limit
- 3d global anomalies match
- Deformations (mass, vev, ...) and RG flows in both sides of the duality lead to known results

Let's see if the duality makes sense.

Dynamics of the electric theory

The non-SUSY electric and magnetic theories contain scalars.



In particular the scalar "gaugino" of the electric theory

$$M_\sigma^2 =$$

$$g^2(2N_c+2)\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{k^2} - g^2(2N_c-2)\int^\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{k^2}$$

$$\sim g^2\Lambda$$

becomes massive, namely $V(\sigma) \sim M^2\sigma^2$, hence $\langle\sigma\rangle = 0$, the color antibranes are attracted to the orientifold and sit on top of it.

Similarly the electric "squark" $M_\Phi^2 > 0$, $\langle\Phi\rangle = 0$.

The massive scalars decouple and we end with a QCD-like theory that contains a gauge field, a "gaugino" and quarks.

Dynamics of the magnetic theory

As in the electric theory the scalar gaugino becomes massive and decouple, $M_s^2 > 0$.

The squark is more subtle. It couples both to the vector multiplet and the meson multiplet. The one loop analysis indicates that $M_\phi^2 \sim (g^2 - y^2)\Lambda$, hence the squark may become either massive or tachyonic.

As we shall see, some parts of the QCD3 phase diagram are related to a tachyonic squark while other parts are due to a massive squark.

Tachyonic squark

Let us assume that the squark condenses, as follows

$$\langle \phi^a_j \rangle = v \delta^a_j$$

a phenomenon often called in QCD "color-flavor" locking.

In terms of brane dynamics it corresponds to reconnection of color and flavor branes.

Either the number of color branes is larger than the number of flavor branes or vice versa.

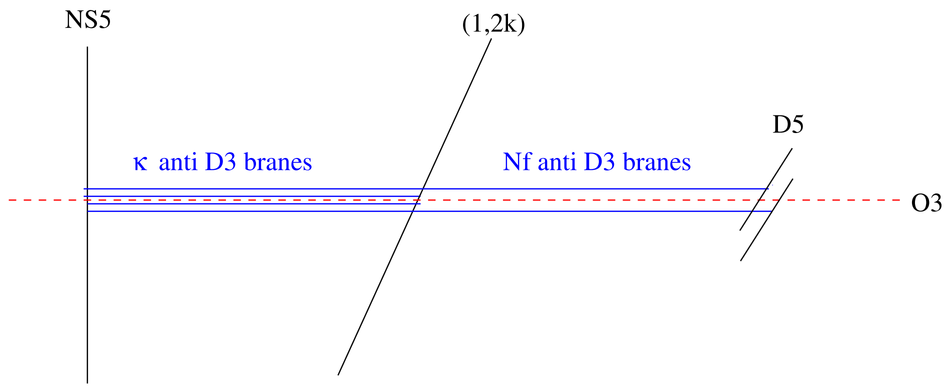
The two cases corresponds to different phases of the electric theory:

Bosonization and symmetry breaking.

Squark Condensation and Bosonization

Let us start with the case $N_f \leq \tilde{N}_c = k + 1 - N_c + N_f$, namely $0 \leq k + 1 - N_c \equiv \kappa$.

All the flavor branes are connected to color branes.



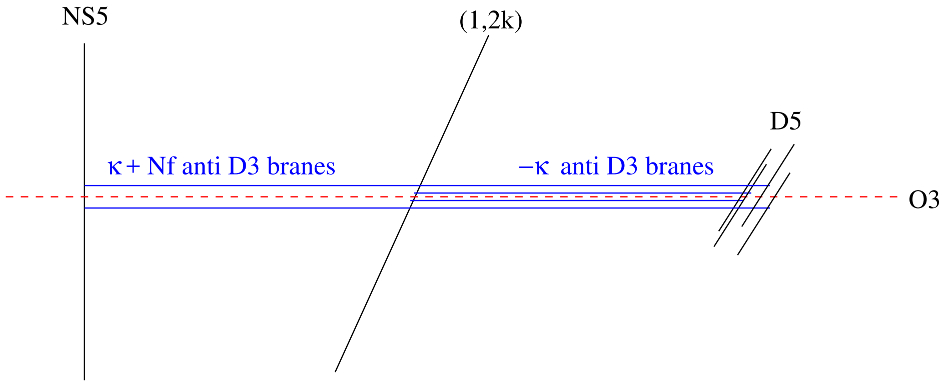
The gauge theory is $Sp(2\kappa) = Sp(2(K + N_f/2))$.

Yukawa couplings $\langle\phi\rangle\bar{\psi}\psi$ make the the fermions massive and they decouple.

We end up with a bosonic magnetic theory dual to the fermionic electric theory. The gauge group, matter content and ϕ^4 interactions are precisely as predicted by the bosonization conjecture.

Squark Condensation and Symmetry Breaking

Let us consider the case
$N_f > \tilde{N}_c = k + 1 - N_c + N_f$, namely
$0 > k + 1 - N_c \equiv \kappa$.



In this phase there is no gauge theory, instead we have two sets of flavor branes with manifest SO symmetry.

$$SO(2N_f) \to SO(2(N_f{+}k{+}1{-}N_c)){\times}SO(2|k{+}1{-}N_c|)$$

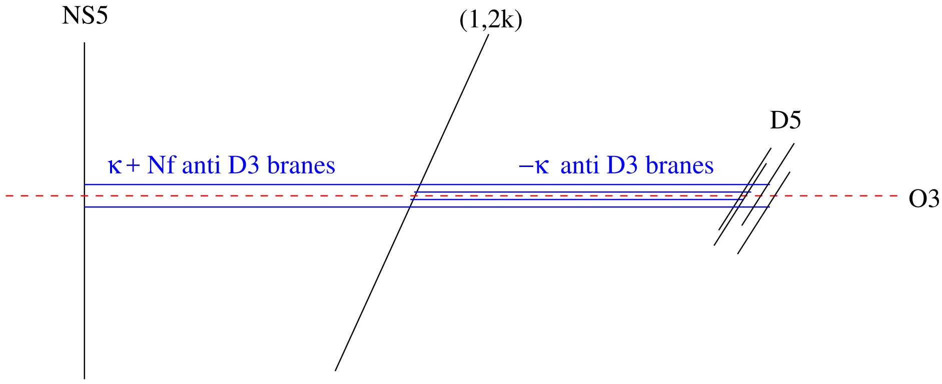
In the field theory limit the symmetry is enhanced to

$$Sp(2N_f) \to Sp(2(N_f{+}k{+}1{-}N_c)){\times}Sp(2|k{+}1{-}N_c|)$$

or

$$Sp(2N_f) \to Sp(N_f - 2K) \times Sp(N_f + 2K)$$

Symmetry Breaking and Nambu-Goldstone Bosons



A consistency check: the coset $Sp(2N_f)/Sp(N_f - 2K) \times Sp(N_f + 2K)$ should result in

$$(N_f - 2K)(N_f + 2K) = 4|k+1-N_c|(N_f + k + 1 - N_c)$$

Nambu-Goldstone bosons. These modes correspond to the open strings stretched between the two stacks of branes.

$k = 0$, parallel NS fivebranes

If the antibranes reconnect we would get

$$Sp(2N_f) \to Sp(2N_f - 2N_c + 2) \times Sp(2N_c - 2)$$

Such a pattern contradicts the Vafa-Witten theorem (it does not preserve parity).

$k = 0$ is a special case. The antisymmetric gluino in the electric theory does not acquire a Chern-Simons mass.

We therefore propose that for $k = 0$ the dynamics is such that the magnetic squark is not tachyonic, $\langle \phi \rangle = 0$, and there is no dynamical symmetry breaking of $Sp(2N_f)$ symmetry.

CFT

The magnetic theory contains a coupling between the meson and the squark of the form $M^2\phi^2$. When the Meson field acquires a vev the squark becomes massive, and $\langle\phi\rangle = 0$. We propose that this phase corresponds to $N_f > N_\star$.

Expanding around the vev of M we obtain a $U(N_f)$ field theory that contains a massless fermion and a massive meson of mass $1/N_f$

$$\mathcal{L} = \bar{\chi}_{[ij]} i \not{\partial} \chi_{[ij]} + (\partial M)^2 - \frac{\mu^2}{N_f} M^2 + \ldots$$

. We propose that this is the magnetic theory that describes the CFT at $N_f > N_\star$.

Indeed, in this phase anomalous dimensions should be controlled by a $1/N_f$ expansion. It would be interesting to compare the calculation in the electric side with the corresponding calculation in the electric side.

Conclusions

- We proposed a magnetic Seiberg dual to a non-supersymmetric QCD3 theory.
- Using the magnetic dual we related the phases of the electric theory with phenomena in the magnetic theory.
- In particular squark condensation (brane reconnection) describes both the bosonization phase and the symmetry breaking phase.
- The CFT phase is described by meson condensation.
- Finally, we proposed a new phase with $k = 0$ and no symmetry breaking.