$\mathcal{N} = 1$ Lagrangians for Argyres-Douglas theories

Prarit Agarwal
(Seoul National University)

Supersymmetric Quantum Field Theories in Non-perturbative Regime, 2018

- Based on

1. **N=1 Deformations and RG Flows of N=2 SCFTs - J. Song, K. Maruyoshi (arXiv: 1607.04281)**

2. **N=1 Deformations and RG Flows of N=2 SCFTs, Part II: Non-principal deformations – P. A. , J. Song, K. Maruyoshi (arXiv:1610.05311)**

3. **N=1 Lagrangians for generalized Argyres-Douglas theories – P. A. , J. Song, A. Sciarappa , (arXiv:1707.04751)**

Plan of the talk

- Brief review of Argyres-Douglas theories
- An $\mathcal{N} = 1$ Lagrangian for the H_0 theory
- Generalization of the Lagrangian for H_0 theory
- Summary

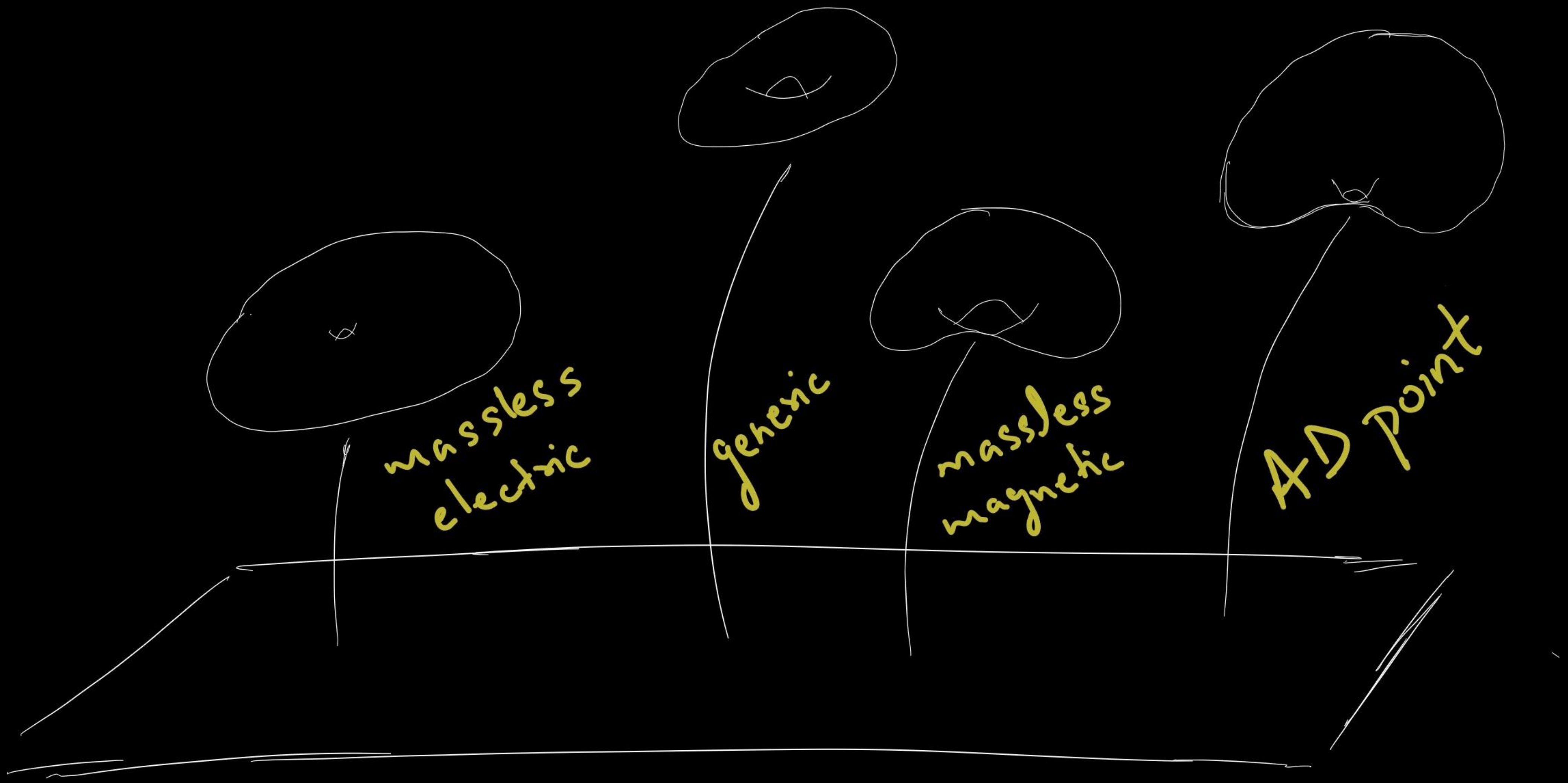
Review of Argyres-Douglas theories

Review of Argyres-Douglas theories

- $\mathcal{N} = 2$ Superconformal theories (SCFTs)

- Describe the low energy theory at special loci on the Coulomb branch of generic $\mathcal{N} = 2$ theories

- At these special loci, monopoles and electrically charged particles simultaneously become massless


massless electric
generic
massless magnetic
AD point

Simplest AD theory

- Supersymmetric U(1) gauge theory + electron + monopole/dyon
- AD point on the Coulomb branch of $\mathcal{N}=2$ SU(2) gauge theory with 1 doublet hyper [Argyres-Douglas `95] [Argyres-Plesser-Seiberg-Witten '95]
- Often called as the H_0 theory

- H_0 has a single Coulomb branch operator with scaling dimension

$$\Delta_{\mathcal{O}} = \frac{6}{5}$$

- central charges were computed by Shapere and Tachikawa in 2008

$$a = \frac{43}{120}, \quad c = \frac{11}{30}$$

Minimal 4d theory with $\mathcal{N}=2$ SUSY

- H_0 is believed to be the minimal 4d superconformal theory with 8 supercharges

- 4d $\mathcal{N}=2$ SCFTs obey an analytic lower bound on their central charge c

[Liendo-Ramirez-Seo `15]

$$c \geq \frac{11}{30}$$

- H_0 theory saturates this bound

AD theories from type IIB

- AD theories can be obtained by compactification of type IIB on CY_3 with an isolated singularity

$$CY_3 \subset \mathbf{C}^4 : W(x_i) = 0$$

$$dW = 0 \quad iff \quad x_i = 0$$

- Gives a (G, G') classification of AD theories [Cecotti-Neitzke-Vafa`10]

$$W(x, y, z, w) = W_G(x, y) + W_{G'}(z, w) = 0$$

- $W_G(x,y)$ is the superpotential defining ADE singularities

$$W_{A_n}(x,y) = x^{n+1} + y^2$$
$$W_{D_n}(x,y) = x^{n-1} + xy^2$$
$$W_{E_6}(x,y) = x^3 + y^4$$
$$W_{E_7}(x,y) = x^3 + xy^3$$
$$W_{E_8}(x,y) = x^3 + y^5$$

AD theories are Non-Lagrangian

- Impossible to write a manifestly Lorentz invariant Lagrangian with electrons as well as monopoles as elementary degrees of freedom
- Therefore AD theories are inherently non-perturbative
- Their Coulomb phase is well understood; much less is known about their conformal phase
- How to compute their partitions function on S^4, $S^3 \times S^1$ etc ?

$\mathcal{N}=1$ Lagrangian for H_0 theory

	q	q'	Φ	M
$SU(2)$	2	2	Adj	1

$$W = \mathrm{tr} q\Phi q + M \mathrm{tr}\Phi q' q'$$

- The term in red is irrelevant in the UV
- In reality it is dangerously irrelevant and therefore can not be ignored

[Maruyoshi-Song`16]

- This theory reproduces the central charges and coulomb branch of H_0

- Central charges of supersymmetric theories are exact functions of the R-charge [Anselmi-Freedman-Grisaru-Johansen`97]

$$a = \frac{9}{32}\mathrm{tr}R^3 - \frac{3}{32}\mathrm{tr}R$$

$$c = \frac{9}{32}\mathrm{tr}R^3 - \frac{5}{32}\mathrm{tr}R$$

- The scaling dimension of gauge invariant chiral operators can be obtained from their exact IR R-charges

$$\Delta_{\mathcal{O}} = \frac{3}{2}R_{\mathcal{O}}$$

- In $\mathcal{N}=1$ theories axial U(1) symmetries generically mix with the R-symmetry along the RG flow.

$$R_{IR} = R_{UV} + \sum \alpha_i A_i$$

- The mixing coefficients α_i have to maximize the central charge a

[Intriligator-Wecht`03]

- There is an axial U(1) flavor symmetry under which the various fields have charges given by

$$q : \frac{1}{2}, \;\; q' : \frac{7}{2}, \;\; \Phi : -1, \;\; M : -6$$

- Naïve a-maximization for the H_0 Lagrangian leads to

$$\Delta_{\mathrm{Tr}\Phi^2} < 1$$

- For gauge invariant operators, unitarity in conformal field theory requires

$$\Delta_{\mathcal{O}} \geq 1$$

- The inequality is saturated if and only if $\mathcal{O}$ is a free field

- It must be that $\mathrm{Tr}\Phi^2$ decoupled (as a free field) from the interacting CFT
- We have to remove this decoupled operator and re-maximize a

[Kutasov-Parnachev-Sahakyan'03]

- Finally, a gets maximized at a point where R-charge of M is given by

$$R_M = \frac{4}{5}$$

- Charges of other fields can be fixed by requiring IR R-symmetry to be non-anomalous and that each term in the superpotential should have R-charge 2

- At the fixed point of the above Lagrangian we therefore find that

$$a = \frac{43}{120}, c = \frac{11}{30}$$

- In particular, we find that

$$\Delta_M = \frac{3}{2} \times \frac{4}{5} = \frac{6}{5}$$

- Recall, that for H_0

$$\Delta_{\mathcal{O}} = \frac{6}{5},\ a = \frac{43}{120},\ c = \frac{11}{30}$$

- The central charges match with that of the H_0 theory
- Gauge singlet M plays the role of the Coulomb branch operator
- Claim: The above Lagrangian experiences SUSY enhancement and flows to the H_0 theory in the IR

- We can use the above Lagrangian to compute the full $\mathcal{N}=2$ superconformal index (SCI) of H_0

- The Schur and Macdonald limits of the SCI of various AD theories were previously obtained by using 4d/2d correspondence

[Buican - Nishinaka'15] [Song'15] [Cordova-Shao`15]

- These corresponding limits of the superconformal index computed from the Lagrangian match the above results

- What about other AD theories?

Lagrangians for generalized AD theories

- $\mathcal{N}=1$ deformation of an $\mathcal{N}=2$ SCFT

- $\mathcal{N}=2$ SCFT : SU(2) gauge theory with 8 fundamental half-hypers (Q)

- This has an SO(8) flavor symmetry

- Deform this by introducing gauge singlet chiral multiplets (M) in the adjoint representation of SO(8).

- Coupling is through a superpotential $W=\mathrm{tr}MQQ$

- Switch on a nilpotent vev for M : $\langle M \rangle = \rho(\sigma^+)$

[Gadde-Maruyoshi-Tachikawa-Yan`13] [PA-Song`13]
[PA-Bah-Maruyoshi-Song`14][PA-Intriligator-Song`15]

- ρ is the choice an $SU(2) \subset SO(8)$, such that

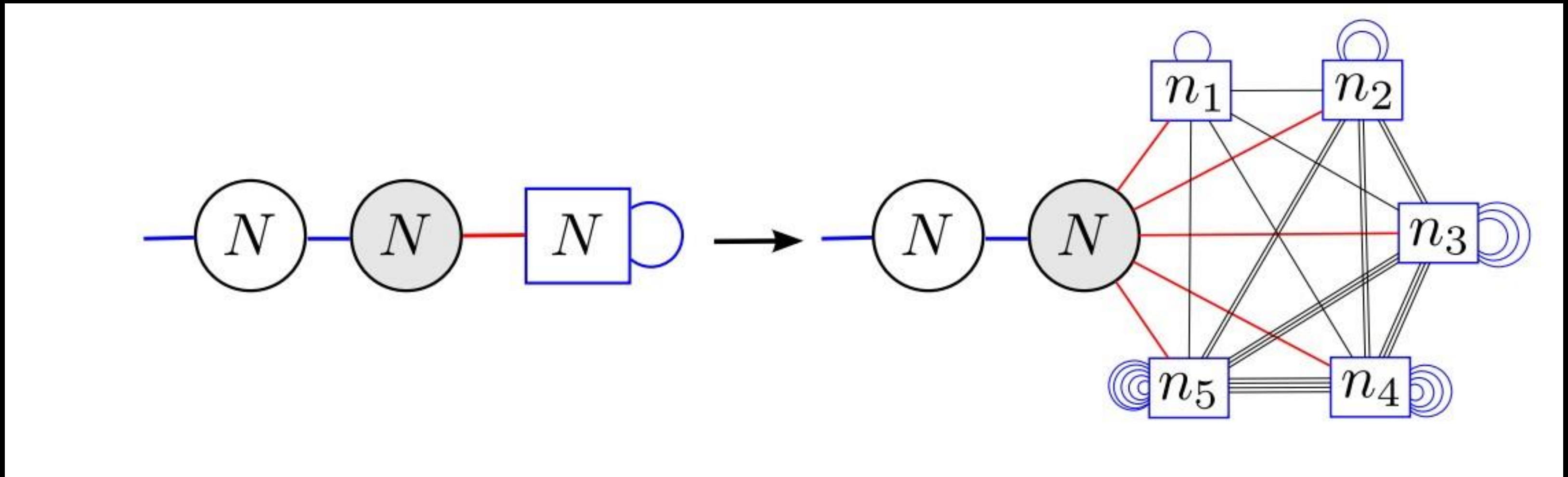
$$(\mathbf{8})_{SO(8)} \to (\mathbf{7} \oplus \mathbf{1})_{SU(2)}$$

- This is called the principal embedding in math literature

- Integrate out the quarks that get masses

- Decouple the multiplets containing Goldstone bosons

- The effective Lagrangian so obtained flows to the H_0 theory, after removing the operators that hit the unitarity bound



In general ….

- $\mathcal{N}=1$ deformations of the SU(N) gauge theory with 2N fundamental hypers
- This $\mathcal{N}=2$ SCFT has an SU(2N) flavor symmetry
- Introduce a gauge singlet chiral multiplet M in the adjoint representation of the flavor symmetry

- Consider the effective theory obtained after giving a nilpotent vev to M

$$\langle M \rangle = \rho(\sigma^+), \;\; \rho : SU(2) \hookrightarrow SU(2N)$$

- The SU(2) embeddings into Lie Algebras were classified by Dynkin

- For SU(2N), the SU(2) embeddings are in one-one correspondence with integer partitions of 2N

- The choice of integer partition tells us how to decompose the fundamental representation of SU(2N) into irreps of SU(2)

- Principal embedding : $(\mathbf{2N})_{SU(2N)} \to (\mathbf{2N})_{SU(2)}$

- The resulting theory flows to an IR fixed point that describes the so called (A_1, A_{2N-1}) type AD theories [Maruyoshi-Song`16]

- What about other SU(2) embedding?

- Other than sporadic occurrences, only one other SU(2) embedding gives AD theory at the fixed point [PA-Maruyoshi-Song`16]

$$(\mathbf{2N})_{SU(2N)} \to (\mathbf{2N-1} \oplus \mathbf{1})_{SU(2)}$$

- This flows to the (A_1, D_{2N}) type AD theory

- We can also consider similar deformation of the $\mathcal{N}=2$ SCFT based on Sp(N) gauge theory with (4N+4) half-hypers

- This has an SO(4N+4) flavor symmetry

- The deformations to be studied are therefore labelled by SU(2) embeddings of SO(4N+4)

- Principal embedding : $(\mathbf{4N+4})_{SO(4N+4)} \to (\mathbf{4N+3} \oplus \mathbf{1})_{SU(2)}$

- Gives an $\mathcal{N}=1$ Lagrangian that flows to the (A_1, A_{2N}) type AD theories

  [Maruyoshi-Song`16]

- When $\rho : (\mathbf{4N+4})_{SO(4N+4)} \to (\mathbf{4N+1} \oplus_{i=1}^{3} \mathbf{1})_{SU(2)}$

- The resulting Lagrangian describes (A_1, D_{2N+1}) AD theories

  [PA-Maruyoshi-Song`16]

- Other embeddings do not give anything interesting other than sporadically

Deformations of SU(N), Nf =2N

- Here we only list deformations that give rational central charges
- Other deformations give irrational central charges. These are necessarily fixed points with $\mathcal{N}=1$ SUSY

$SU(2N)$	$\rho: SU(2) \hookrightarrow SU(2N)$	a	c	4d $\mathcal{N}=2$ SUSY
$SU(4)$	$[1^4]$	$\frac{23}{24}$	$\frac{7}{6}$	Yes; $N_c=2,\ N_f=4$
	$[3,1]$	$\frac{7}{12}$	$\frac{2}{3}$	Yes; (A_1,D_4) AD th.
	$[4]$	$\frac{11}{24}$	$\frac{1}{2}$	Yes; (A_1,A_3) AD th.
$SU(6)$	$[1^6]$	$\frac{29}{12}$	$\frac{17}{6}$	Yes; $N_c=3,\ N_f=6$
	$[5,1]$	$\frac{13}{12}$	$\frac{7}{6}$	Yes; (A_1,D_6) AD th.
	$[6]$	$\frac{11}{12}$	$\frac{23}{24}$	Yes; (A_1,A_5) AD th.
$SU(8)$	$[1^8]$	$\frac{107}{24}$	$\frac{31}{6}$	Yes; $N_c=4,\ N_f=8$
	$[2,1^6]$	$\frac{73801}{17424}$	$\frac{43121}{8712}$	?
	$[4,4]$	$\frac{9097}{3888}$	$\frac{5129}{1944}$	?
	$[7,1]$	$\frac{19}{12}$	$\frac{5}{3}$	Yes; (A_1,D_8) AD th.
	$[8]$	$\frac{167}{120}$	$\frac{43}{30}$	Yes; (A_1,A_7) AD th.
$SU(10)$	$[1^{10}]$	$\frac{247}{24}$	$\frac{71}{6}$	Yes; $N_c=5,\ N_f=10$
	$[5,1^5]$	$\frac{5553943}{1383123}$	$\frac{6257387}{1383123}$	?
	$[5,3,1^2]$	$\frac{92540867}{24401712}$	$\frac{52091009}{12200856}$	?
	$[9,1]$	$\frac{25}{12}$	$\frac{13}{6}$	Yes; (A_1,D_{10}) AD th.
	$[10]$	$\frac{15}{8}$	$\frac{23}{12}$	Yes; (A_1,A_9) AD th.
$SU(12)$	$[1^{12}]$	$\frac{247}{24}$	$\frac{71}{6}$	Yes; $N_c=6,\ N_f=12$
	$[4^3]$	$\frac{754501}{138384}$	$\frac{424727}{69192}$	?
	$[11,1]$	$\frac{31}{12}$	$\frac{8}{3}$	Yes; (A_1,D_{12}) AD th.
	$[12]$	$\frac{397}{168}$	$\frac{101}{42}$	Yes; (A_1,A_{11})AD th.

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Deformations of USp(2N), Nf =4N+4 (half-hypers)

- Here we only list deformations that give rational central charges

- Other deformations give irrational central charges. These are necessarily fixed points with $\mathcal{N}=1$ SUSY

$SO(4N+4)$	$\rho : SU(2) \hookrightarrow SO(4N+4)$	a	c	4d $\mathcal{N}=2$ SUSY
$SO(8)$	$[1^8]$	$\frac{23}{24}$	$\frac{7}{6}$	Yes; $N_c=1,\ N_f=8$
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	$[4,4]\equiv[5,1^3]$	$\frac{11}{24}$	$\frac{1}{2}$	Yes; (A_1,D_3) AD th.
	$[5,3]$	$\frac{6349}{13872}$	$\frac{3523}{6936}$	?
	$[7,1]$	$\frac{43}{120}$	$\frac{11}{30}$	Yes; (A_1,A_2) AD th.
$SO(12)$	$[1^{12}]$	$\frac{37}{12}$	$\frac{11}{3}$	Yes; $N_c=2,\ N_f=12$
	$[4^2,2^2]$	$\frac{105027}{59536}$	$\frac{61145}{29768}$	?
	$[9,1^3]$	$\frac{19}{20}$		Yes; (A_1,D_5) AD th.
	$[11,1]$	$\frac{67}{84}$	$\frac{17}{21}$	Yes; (A_1,A_4) AD th.
$SO(16)$	$[1^{16}]$	$\frac{51}{8}$	$\frac{15}{2}$	Yes; $N_c=3,\ N_f=16$
	$[5,1^{11}]$	$\frac{109031}{27744}$	$\frac{123889}{27744}$	?
	$[5,3^3,1^2]$	$\frac{18250741}{5195568}$	$\frac{10440877}{2597784}$	?
	$[13,1^3]$	$\frac{81}{56}$	$\frac{3}{2}$	Yes; (A_1,D_7) AD th.
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	$[9, 1^3]$	$\frac{19}{20}$		Yes; (A_1, D_5) AD th.
	$[11, 1]$	$\frac{67}{84}$	$\frac{17}{21}$	Yes; (A_1, A_4) AD th.
$SO(16)$	$[1^{16}]$	$\frac{51}{8}$	$\frac{15}{2}$	Yes; $N_c = 3,\ N_f = 16$
	$[5, 1^{11}]$	$\frac{109031}{27744}$	$\frac{123889}{27744}$	?
	$[5, 3^3, 1^2]$	$\frac{18250741}{5195568}$	$\frac{10440877}{2597784}$	?
	$[13, 1^3]$	$\frac{81}{56}$	$\frac{3}{2}$	Yes; (A_1, D_7) AD th.
	$[15, 1]$	$\frac{91}{72}$	$\frac{23}{18}$	Yes; (A_1, A_6) AD th.

$SO(4N+4)$	$\rho : SU(2) \hookrightarrow SO(4N+4)$	a	c	4d $\mathcal{N}=2$ SUSY
$SO(8)$	$[1^8]$	$\frac{23}{24}$	$\frac{7}{6}$	Yes; $N_c=1,\ N_f=8$
	$[3^2,1^2]$	$\frac{7}{12}$	$\frac{2}{3}$	Yes;(A_1,D_4) AD th.
	$[4,4]\equiv[5,1^3]$	$\frac{11}{24}$	$\frac{1}{2}$	Yes; (A_1,D_3) AD th.
	$[5,3]$	$\frac{6349}{13872}$	$\frac{3523}{6936}$	?
	$[7,1]$	$\frac{43}{120}$	$\frac{11}{30}$	Yes; (A_1,A_2) AD th.
$SO(12)$	$[1^{12}]$	$\frac{37}{12}$	$\frac{11}{3}$	Yes; $N_c=2,\ N_f=12$
	$[4^2,2^2]$	$\frac{105027}{59536}$	$\frac{61145}{29768}$	?
	$[9,1^3]$	$\frac{19}{20}$		Yes; (A_1,D_5) AD th.
	$[11,1]$	$\frac{67}{84}$	$\frac{17}{21}$	Yes; (A_1,A_4) AD th.
$SO(16)$	$[1^{16}]$	$\frac{51}{8}$	$\frac{15}{2}$	Yes; $N_c=3,\ N_f=16$
	$[5,1^{11}]$	$\frac{109031}{27744}$	$\frac{123889}{27744}$	?
	$[5,3^3,1^2]$	$\frac{18250741}{5195568}$	$\frac{10440877}{2597784}$	?
	$[13,1^3]$	$\frac{81}{56}$	$\frac{3}{2}$	Yes; (A_1,D_7) AD th.
	$[15,1]$	$\frac{91}{72}$	$\frac{23}{18}$	Yes; (A_1,A_6) AD th.

Deformations of SO(N),Nf =2N-4 (half-hypers)

- No organizing principal behind the cases with rational central charges
- No correspondence with $\mathcal{N}=2$ SCFTs

$Sp(N-2)$	$\rho : SU(2) \hookrightarrow Sp(N-2)$	a	c	4d $\mathcal{N}=2$ SUSY
$Sp(2)$	$[1^4]$	$\frac{19}{12}$	$\frac{5}{3}$	Yes; $N_c=4,\ N_f=4$
	$[2,1^2]$	$\frac{10111}{7056}$	$\frac{5381}{3528}$	?
$Sp(3)$	$[1^6]$	$\frac{65}{24}$	$\frac{35}{12}$	Yes; $N_c=5,\ N_f=6$
	$[4,1^2]$	$\frac{325}{192}$	$\frac{341}{192}$	?
$Sp(4)$	$[1^8]$	$\frac{33}{8}$	$\frac{9}{2}$	Yes; $N_c=6,\ N_f=8$
$Sp(5)$	$[1^{10}]$	$\frac{35}{6}$	$\frac{77}{12}$	Yes; $N_c=7,\ N_f=10$
$Sp(6)$	$[1^{12}]$	$\frac{47}{6}$	$\frac{26}{3}$	Yes; $N_c=8,\ N_f=12$
	$[2^2,1^8]$	$\frac{589093}{80688}$	$\frac{329335}{40344}$	?
	$[4,1^8]$	$\frac{13065}{2312}$	$\frac{7085}{1156}$	?
$Sp(7)$	$[1^{14}]$	$\frac{81}{8}$	$\frac{45}{4}$	Yes; $N_c=9,\ N_f=14$
	$[5^2,1^4]$	$\frac{59094550}{10978707}$	$\frac{129141025}{21957414}$	?
	$[6,3^2,2]$	$\frac{375975613}{72745944}$	$\frac{406255085}{72745944}$	?
$Sp(8)$	$[1^{16}]$	$\frac{305}{24}$	$\frac{85}{6}$	Yes; $N_c=10,\ N_f=16$
	$[4^2,2^2,1^4]$	$\frac{389}{48}$	$\frac{53}{6}$	?
	$[5^2,3^2]$	$\frac{30593927}{4642608}$	$\frac{16735805}{2321304}$	?
	$[5^2,4,1^2]$	$\frac{28118905}{4348848}$	$\frac{3828919}{543606}$	?

- The deformations of $\mathcal{N}=2$ gauge theories based on SU(N) and Sp(N) gauge groups, together give all the AD theories of type (A_1, A_n) and (A_1, D_n)

- However, the AD theories have a much richer classification

- To begin with there are AD theories of type (A_k, A_n) and (A_k, D_n)

- Therefore an immediate question is to look for Lagrangians for these more general classes of AD theories

- A partial solution was reported in **arXiv:1707.04751** (in collaboration with A. Sciarappa and J. Song)

- Also see related work (**arXiv:1707.05113**) by Benvenuti and Giacomelli

- Consider $\mathcal{N}=1$ preserving "principal nilpotent deformations" of the following $\mathcal{N}=2$ quivers

$$(N) - (2N) - \cdots - (mN - N) - [mN]$$

- These flow to (A_{m-1}, A_{Nm-1}) type AD theories
- Deformations corresponding to other SU(2) embeddings of SU(mN) do not give anything interesting

- (A_{2m-1}, D_{2Nm+1}) type AD theories can be obtained from principal deformation of

$$[SO(2)]{-}Sp(N){-}SO(4N{+}2){-}Sp(3N){-}\cdots{-}Sp(2mN{-}N){-}[SO(4mN{+}2)]$$

- $(A_{2m}, D_{m(N-2)+\frac{N}{2}})$ type AD theory can be obtained from principal deformations of

$$SO(N) - Sp(N-2) - SO(3N-4) - Sp(2N-4) - \\ \cdots - Sp(mN-2m) - [SO(2mN+N-4m)]$$

- In addition to the above quivers, we also found that

$$\boxed{1} - (k+1) - (2k+1) - \ldots - (mk-k+1) - \boxed{mk+1} \rightsquigarrow (I_{m,mk}, S)$$

and

$$Sp(N) - SO(4N+4) - Sp(3N+2) - SO(8N+8) - \\ \ldots - Sp((m-1)(2N+2)+N) - \boxed{SO(4m(N+1))} \rightsquigarrow D_{m(2N+2)}^{m(2N+2)}[m]$$

- Quivers with USp(n) flavor symmetries usually do not give rational central charges

Deformations of general $\mathcal{N}=2$ SCFTs

- We can always consider $\mathcal{N}=1$ preserving deformations of any $\mathcal{N}=2$ SCFTs with global symmetry $\mathcal{G}$
- Couple to $\mathcal{G}$ - Adjoint chiral mutliplet M via $W = \mathrm{Tr}\mu M$
- μ is the scalar in the current multiplet for $\mathcal{G}$
- Turn on a nilpotent vev for M

Deformations of general $\mathcal{N}=2$ SCFTs

- Here we only list deformations that give rational central charges

$\mathcal{T}_{UV}$	ρ	Residual Flavor	$\mathcal{T}_{IR}[\mathcal{T}_{UV},\rho]$
$(I_{N,k},F)$	$[N]$	$\emptyset$	(A_{N-1},A_{N+k-1}) theory
$(I_{N,-N+2},F)$	$[N-1,1]$	$U(1)$	(A_1,D_N) theory
E_6 SCFT	E_6	$\emptyset$	H_0
	D_5	$SU(2)$	H_1
	D_4	$SU(3)$	H_2

H_2 is identical to (A_1,D_4)

[PA-Maruyoshi-Song`16]

- Interesting Duality:

$Sp(n)$ SQCD with $N_f=2n+2$ $\leftrightarrow$ $(I_{2n+1,-2n+1},F)$ AD theory

$\rho=[4n+1,1^3]\searrow$ $\quad\swarrow \rho=[2n,1]$

(A_1,D_{2n+1}) AD theory

$\mathcal{T}_{UV}$	ρ	Residual Flavor	$\mathcal{T}_{IR}[\mathcal{T}_{UV}, \rho]$
E_7 SCFT	E_7	$\emptyset$	H_0
	E_6	$SU(2)$	H_1
	$A_2 + 3A_1$	G_2	$a = \frac{12163}{8214}, \ c = \frac{121465}{65712}$
E_8 SCFT	E_8	$\emptyset$	H_0
	$E_7(a_4)$	$SU(2)$	$a = \frac{1691}{1452}, \ c = \frac{541}{363}$
	$E_7(a_5)$	$SU(2)$	$a = \frac{445}{324}, \ c = \frac{281}{162}$
	$A_3 + A_1$	$SU(2) \times SO(7)$	$a = \frac{139189}{60552}, \ c = \frac{91127}{30276}$
	A_3	$SO(11)$	$a = \frac{497803}{221952}, \ c = \frac{635435}{221952}$

[PA-Maruyoshi-Song *(unpublished)*]

$\mathcal{T}_{UV}$	ρ	Residual Flavor	$\mathcal{T}_{IR}[\mathcal{T}_{UV}, \rho]$
G_2 SCFT	$G_2(a_1)$	$\emptyset$	$a = \frac{29}{48},\ c = \frac{17}{24}$
F_4 SCFT	F_4	$\emptyset$	$a = \frac{83993}{262086},\ c = \frac{181571}{524172}$
	$F_4(a_1)$	$SU(2)$	$a = \frac{1691}{1452},\ c = \frac{541}{363}$

[PA-Maruyoshi-Song *(unpublished)*]

[Argyres-Martone`16]

- Deformations of theories in Bhardwaj-Tachikawa : Only sporadic occurrence of deformations with rational central charges [PA-Sciarappa-Song *(unpublished)*]

Summary

- Non- Lagrangianity of AD theories poses a major hurdle in our understanding of them
- We have been successful in constructing $\mathcal{N} = 1$ Lagrangians whose IR fixed points describe AD theories
- Can use these to compute RG protected quantities such as the superconformal index

- These lagrangians are interesting in their own regard.
- Rare examples of 4d QFTs with accidental SUSY
- The mechanism of SUSY enhancement is still not understood. This will be an interesting direction to pursue
- It will also be interesting to find string theory realization of the above Lagrangians and thereby understand the geometric settings that lead to theories with accidental SUSY

- These Lagrangians imply interesting integral identities

For e.g. the Schur limit of the SCI of the H_0 theory can be written as

[Buican - Nishinaka’15] [Song’15] [Cordova-Shao`15][Song-Xie-Yan`17]

$$I_{Schur,H_0} = \frac{1}{(q^2;q^5)_\infty (q^3;q^5)_\infty}$$

The Lagrangian description suggests that the Schur index should be

$$I_{Schur,H_0} = \lim_{t\to q} \kappa \frac{\Gamma((\frac{pq}{t})^{\frac{6}{5}})}{\Gamma((\frac{pq}{t})^{\frac{2}{5}})} \oint_C \frac{dz}{2\pi i z} \frac{\Gamma(z^{\pm}(pq)^{\frac{2}{5}} t^{\frac{1}{10}})\Gamma(z^{\pm}(pq)^{-\frac{1}{5}} t^{\frac{7}{10}})\Gamma(z^{\pm 2,0}(\frac{pq}{t})^{\frac{1}{5}})}{2\Gamma(z^{\pm 2})}$$

THANK YOU!

Superconformal Index

- The $\mathcal{N}=1$ superconformal index is defined as

$$\mathcal{I}_{\mathcal{N}=1} = \mathrm{tr}(-1)^F p^{j_1+j_2+\frac{R}{2}} q^{j_2-j_1+\frac{R}{2}} \prod_i a_i^{f_i}$$

- generically, a function of two fugacities p and q

- The $\mathcal{N}=2$ superconformal index is a function of 3 fugacities p, q and t

$$\mathcal{I}_{\mathcal{N}=2} = \mathrm{tr}(-1)^F p^{j_1+j_2+\frac{r}{2}} q^{j_2-j_1+\frac{r}{2}} t^{R-\frac{r}{2}} \prod_i a_i^{f_i}$$

- Recall, that all our Lagrangians necessarily have a U(1) axial symmetry

- A linear combination of this with the $\mathcal{N}=1$ R-symmetry becomes the cartan of the SU(2) R-symmetry of the $\mathcal{N}=2$ algebra

- A second independent linear combination becomes the $U(1)_r$

- Call the fugacity for axial U(1) as ξ
- $\mathcal{N}=1$ superconformal index can be transformed into $\mathcal{N}=2$ if

$$\xi = (t(pq)^{-\frac{2}{3}})^{\beta}$$

- β can be fixed by comparing the axial charge of the gauge singlets in the Lagrangian, to the $U(1)_r$ charge of the corresponding Coulomb branch operator in the AD theory

$$\mathcal{F}_{axial} = \frac{1}{\beta}(R - \frac{r}{2})$$