HOLOGRAPHIC RG FLOWS
FOR FOUR-DIMENSIONAL N=2 SCFTs

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Based on: 1804.03276 with Nikolay Bobev & Hagen Triendl

Supersymmetric QFT in the Non-Perturbative Regime

Galileo Galilei Institute — 10 May 2018

Introduction

SCFTs with 16 supercharges appeared several times (and will still appear) in this conference.

Many different techniques :

- localization Nekrasov, Pestun, ...
- Gaiotto construction, AGT
- Indices
- Bootstrap
- Holography. AdS_5 : Maldacena, Nunez '00
  Maldacena, Gaiotto '09

What can holography say about d=4, N=2 SCFTs ?

Introduction

What can holography say about d=4, N=2 SCFTs ?

- should study string or M theory AdS_5 x M solutions with 16 supercharges and their deformations
- if there is a consistent truncation, can also use d=5 half-maximal supergravity

We take a different approach :

we work in d=5 sugra without committing to any specific higher-dimensional solution

What can d=5 supergravity say about d=4, N=2 SCFTs ?

- drawback: less control on the duality
- advantage: general results exploiting constrained structure of half-maximal sugra

Introduction

- Recently this approach has been taken to study :

marginal deformations of SCFTs in various dimensions

↕

moduli space of susy AdS solutions

Louis et al.

for d=4, N=2 : Louis, Triendl, Zagermann

➜ M. Bianchi's talk

- We will instead focus on **relevant** deformations

What can d=5 supergravity say about
susy RG flows of d=4, N=2 SCFTs ?

⚠ few other tools available

Outline

- Review:
  - Half-maximal supergravity in 5d
  - Holographic RG flows
- Holographic flows between N=2 SCFTs
- Holographic flows between N=2 and N=1 SCFTs
- Discussion & open questions

Half-Maximal Supergravity in 5D

- **Ungauged** theory specified by number of vector multiplets $\mathfrak{n}$

  Global symmetry $G_{\text{global}} = \mathrm{SO}(1,1) \times \mathrm{SO}(5,\mathfrak{n})$

  Bosonic fields:

  - metric
  - $6+\mathfrak{n}$ Abelian vectors $A^0,\ A^M$ $\qquad M = 1, \ldots, 5+\mathfrak{n}$
  - $1+5\mathfrak{n}$ scalars $\mathcal{M}_{\text{scal}} = \mathrm{SO}(1,1) \times \dfrac{\mathrm{SO}(5,\mathfrak{n})}{\mathrm{SO}(5) \times \mathrm{SO}(\mathfrak{n})}$

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  gravity multiplet

  $\mathcal{V} = e^{\phi^x t_x}$ coset representative

5D Half-Maximal Supergravity

- **Gauged** theory specified by **embedding tensor** Θ

deWit, Samtleben, Trigiante '04
Schon, Weidner '06

$$G_{\rm gauge} \xrightarrow{\Theta} G_{\rm global} = \mathrm{SO}(1,1) \times \mathrm{SO}(5,n)$$

$$\Theta = \{ f^{[MNP]}, \xi^{[MN]}, \xi^{N} \}$$

quadratic constraints: $f_{R[MN} f_{PQ]}{}^{R} = 0\,, \qquad \xi_M{}^Q f_{QNP} = 0$

(for $\xi^N = 0$)

covariant derivative $D = \nabla - A^M f_M{}^{NP} t_{NP} - A^0 \xi^{NP} t_{NP}$

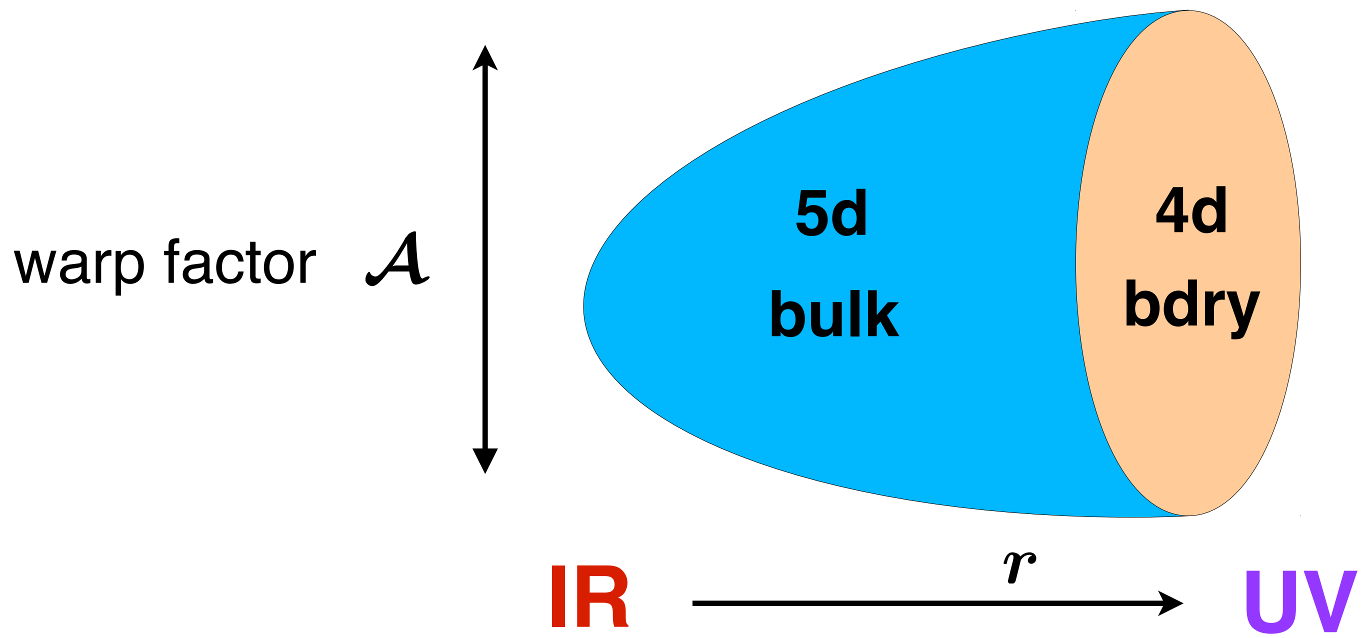
structure constants for subgroup of SO(5,n)

assigns charges under A^0

- charged non-adjoint vectors are eaten up by auxiliary tensor fields

➜ theory **fully** specified, including susy variations and scalar potential

Holographic RG flows



- metric $\quad ds^2 = e^{2\mathcal{A}(r)} ds^2(\mathbb{R}^{1,3}) + dr^2 \qquad \mathcal{A} \to \dfrac{r}{\ell_{\mathrm{UV}}}$ in the UV
- vectors $\quad A^0, A^M = 0$
- scalars $\quad \Sigma(r)\,,\ \phi^x(r) \qquad$ ➜ encode source & vevs of dual operators

Want to

- flow to another AdS_5 $\qquad \mathcal{A} \to \dfrac{r}{\ell_{\mathrm{IR}}}$ in the IR
- preserve 16 or 8 supercharges at fixed point

Holographic RG flows

vanishing of fermionic susy variations

$\downarrow$

solution controlled by a superpotential W — function of scalars & emb. tensor

its form depends on preserved susy

- Flow equations
$$\begin{cases} \mathcal{A}' = W \\ \Sigma' = -\Sigma^2 \partial_\Sigma W \\ \phi^{x\prime} = -3\, g^{xy} \partial_y W \end{cases}$$

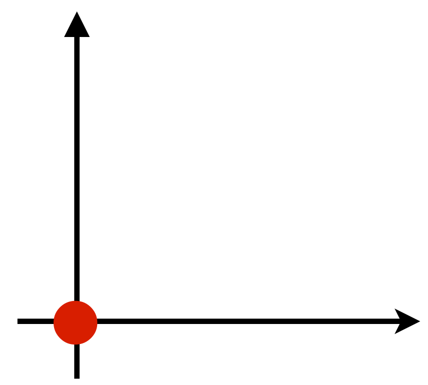
- Scalar potential $V = \frac{9}{2} g^{xy} \partial_x W \partial_y W + \frac{3}{2} \Sigma^2 (\partial_\Sigma W)^2 - 6 W^2$

- AdS fixed point: $\partial_\Sigma W = \partial_x W = 0 \qquad W_* = \frac{1}{\ell}$

⚠ additional constraints; equations above true after these are satisfied

Conditions for AdS5 vacuum with half-maximal susy

- due to homogeneity, we can assume this is at the origin of scalar manifold



$$\phi^x = 0$$

$$\Sigma = 1$$

Conditions for AdS$_5$ vacuum with half-maximal susy

Louis,Triendl,Zagermann '15

- due to homogeneity, we can assume this is at the origin of scalar manifold

- embedding tensor $\xi^N = 0$ ➜ forget from now on

$$\underbrace{f^{123} = g}_{\mathbf{SO(3)}}\ , \qquad \xi^{45} = -\frac{1}{\sqrt{2}}\, g$$

other non-vanishing components: $f^{1AB}, f^{2AB}, f^{3AB}, f^{ABC}, \xi^{AB}$

$$A, B, C = 6, 7, \ldots, \underbrace{\mathfrak{n}}_{\text{arbitrary}} + 5$$

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$f^{123} = g \;\longrightarrow\; \mathrm{SO}(3)$

other non-vanishing components: $f^{1AB}, f^{2AB}, f^{3AB}, f^{ABC}, \xi^{AB}$

$A, B, C = 6, 7, \ldots, \mathfrak{n} + 5$

$\mathfrak{n}$ arbitrary

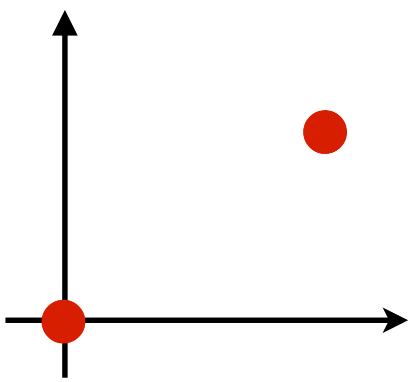
$$G_{\text{gauge}} = \mathrm{U}(1) \times H_{\text{nc}} \times H_F \;\subset\; \mathrm{SO}(5, \mathfrak{n})$$

H_{nc} non-compact, admits $\mathrm{SO}(3)$ as maximal compact subgroup

$H_F \subset \mathrm{SO}(\mathfrak{n})$ compact

In the vacuum broken to $\underbrace{\mathrm{U}(1) \times \mathrm{SO}(3)}_{\text{R-symmetry of dual SCFT}} \times \underbrace{H_F}_{\text{flavor symmetry}}$

Another AdS$_5$ vacuum with half-maximal susy



Another AdS$_5$ vacuum with half-maximal susy

$$f^{123} = g\ , \qquad \xi^{45} = -\frac{1}{\sqrt{2}}\, g$$

- If $H_F \subset \mathrm{SO}(\mathfrak{n})$ trivial (no flavor symmetries), then AdS$_5$ with 16 supercharges is **unique** (up to moduli) ➜ no holographic flows connecting two $\mathcal{N} = 2$ SCFTs

- For second AdS$_5$ need $f^{ABC} \neq 0$ $\qquad A, B, C = 6, 7, \ldots, \mathfrak{n}+5$

  Can take $f^{678} = g\,\lambda^{-1}$ ➘ parameter

Then $G_{\text{gauge}} = \mathrm{U}(1) \times H_{\text{nc}} \times H_F \subset \mathrm{SO}(5, \mathfrak{n})$

$$\begin{array}{ccc} H_{\text{nc}} & \times & H_F \\ \cup & & \cup \\ \mathrm{SO}(3) & \times & \mathrm{SO}(3)_F \\ & \cup & \\ & \mathrm{SO}(3)_{\text{IR}} & \end{array}$$

Another AdS₅ vacuum with half-maximal susy

$$f^{123} = g\,, \qquad \xi^{45} = -\tfrac{1}{\sqrt{2}}\,g$$

- If $H_F \subset \mathrm{SO}(\mathfrak{n})$ trivial (no flavor symmetries), then AdS₅ with 16 supercharges is **unique** (up to moduli) ➜ no holographic flows connecting two $\mathcal{N}=2$ SCFTs

- For second AdS₅ need $f^{ABC} \neq 0$ $\qquad A, B, C = 6, 7, \ldots, \mathfrak{n}+5$

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Then $$\begin{array}{ccccccc} G_{\mathrm{gauge}} = \mathrm{U}(1) & \times & H_{\mathrm{nc}} & \times & H_F & \subset & \mathrm{SO}(5,\mathfrak{n}) \\ & & \cup & & \cup & & \\ & & \mathrm{SO}(3) & \times & \mathrm{SO}(3)_F & & \\ & & & \cup & & & \\ & & & \mathrm{SO}(3)_{\mathrm{IR}} & & & \end{array}$$

R-symmetry second AdS₅

Another AdS$_5$ vacuum with half-maximal susy

- Ansatz

$$\mathcal{V} = e^{\phi(t_{16}+t_{27}+t_{38})} = \begin{array}{c} \overbrace{\hphantom{\cosh\phi \quad 0 \quad 0 \quad 0 \quad 0}}^{1,\ldots,5} \quad \overbrace{\hphantom{-\sinh\phi \quad 0 \quad 0}}^{6,7,8} \\ \begin{pmatrix} \cosh\phi & 0 & 0 & 0 & 0 & -\sinh\phi & 0 & 0 & \cdots \\ 0 & \cosh\phi & 0 & 0 & 0 & 0 & -\sinh\phi & 0 & \cdots \\ 0 & 0 & \cosh\phi & 0 & 0 & 0 & 0 & -\sinh\phi & \cdots \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\ -\sinh\phi & 0 & 0 & 0 & 0 & \cosh\phi & 0 & 0 & \cdots \\ 0 & -\sinh\phi & 0 & 0 & 0 & 0 & \cosh\phi & 0 & \cdots \\ 0 & 0 & -\sinh\phi & 0 & 0 & 0 & 0 & \cosh\phi & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array} .$$

- Non-trivial solution to susy conditions:

$$\tanh\phi = \lambda \qquad \rightarrow \qquad |\lambda| < 1$$

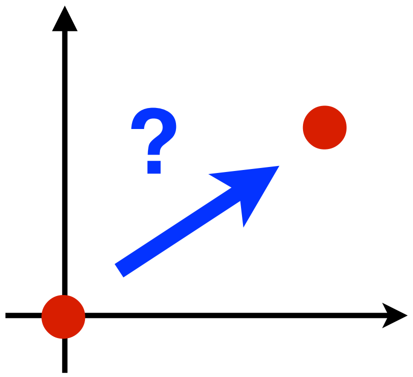
$$\Sigma = \left(1 - \lambda^2\right)^{-1/6}$$

- ratio of cosmological constants $\dfrac{V_{\mathrm{IR}}}{V_{\mathrm{UV}}} = \left(1 - \lambda^2\right)^{-2/3}$

- symmetry of UV vacuum $\mathrm{U}(1) \times \mathrm{SO}(3) \times H_F$

  of IR vacuum $\mathrm{U}(1) \times \mathrm{SO}(3)_{\mathrm{IR}} \times \mathcal{C}omm[\mathrm{SO}(3)_F, H_F]$

Flows between N=2 SCFTs

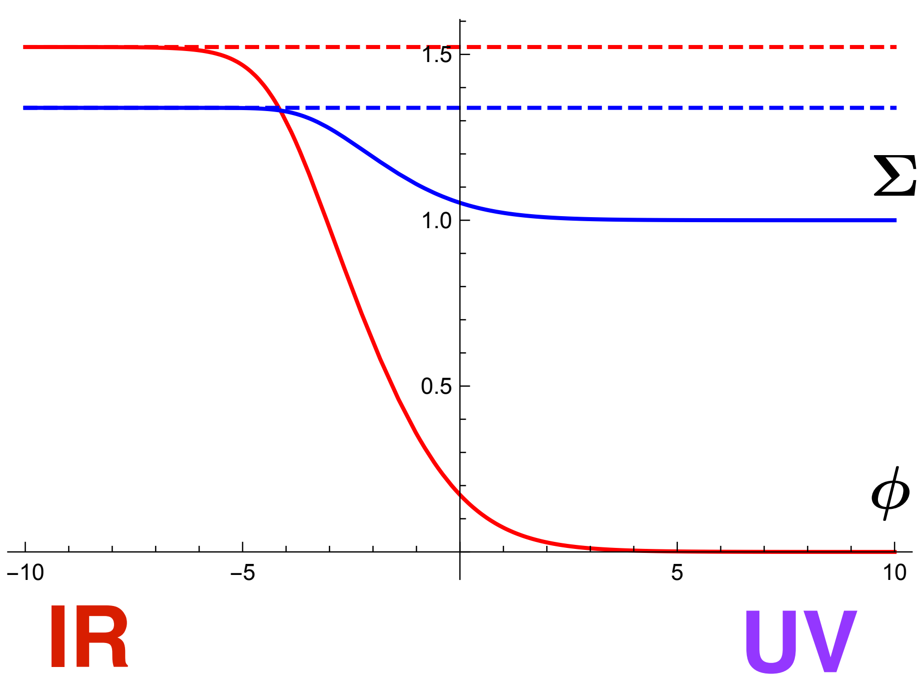


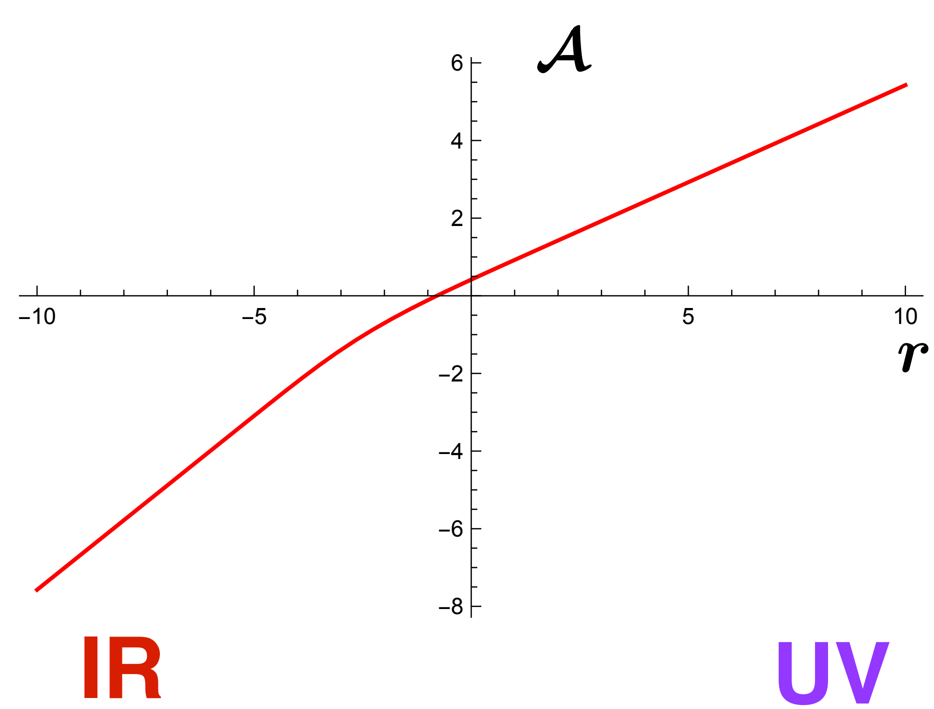
Flows between N=2 SCFTs

- superpotential $\quad W = \frac{1}{6} g\, \Sigma^2 + \frac{1}{3} g\, \Sigma^{-1} (\cosh^3 \phi - \lambda^{-1} \sinh^3 \phi)$

- $$\begin{cases} \mathcal{A}' = W \\ \Sigma' = -\Sigma^2 \partial_\Sigma W \\ \phi^{x\,\prime} = -3\, g^{xy} \partial_y W \end{cases}$$ flow equations solved analytically

$$\Sigma(\phi) = \frac{\left(\cosh\phi - \lambda^{-1} \sinh\phi\right)^{1/3}}{\left(\cosh(2\phi) - \frac{1}{2}(\lambda + \lambda^{-1}) \sinh(2\phi)\right)^{1/3}}$$





- Flow preserves 8 Poincaré supercharges

Flows between N=2 SCFTs

- Close to UV vacuum

$$\phi \approx v_\phi \, e^{-2r/\ell_{\rm UV}} \, , \qquad \Sigma \approx 1 + v_\Sigma \, e^{-2r/\ell_{\rm UV}} \, , \qquad \mathcal{A} \approx \frac{r}{\ell_{\rm UV}} \, , \qquad \ell_{\rm UV} = \frac{2}{g}$$

Flow triggered by vevs of operators with $\Delta = 2$ (in IR: $\Delta_\Sigma = 2 \, , \; \Delta_\phi = 6$)

$$v_\Sigma = \frac{\lambda}{3} v_\phi$$

- $\mathcal{O}_\Sigma$ in energy-momentum tensor multiplet
- $\mathcal{O}_\phi$ in SO(3) flavor current multiplet (moment map)

(3,3) of $SO(3)_R$ x $SO(3)_{\rm flavor}$, $\mathcal{O}_\phi$ invariant under diagonal subgroup

$$\boxed{\frac{a_{\rm IR}}{a_{\rm UV}} = \frac{c_{\rm IR}}{c_{\rm UV}} = \left(\frac{V_{\rm IR}}{V_{\rm UV}}\right)^{-3/2} = 1 - \lambda^2}$$

$\lambda^2 < 1$ ✔ a-theorem

SUPERGRAVITY

STANDBY

Field theory derivation c_{IR}/c_{UV}

$$\frac{a_{\rm IR}}{a_{\rm UV}} = \frac{c_{\rm IR}}{c_{\rm UV}} = 1 - \lambda^2$$

Does this make sense from a field theory perspective?

$$a = \frac{9}{32} Tr(R^3_{\mathcal{N}=1}) - \frac{3}{32} Tr(R_{\mathcal{N}=1})\,, \qquad c = \frac{9}{32} Tr(R^3_{\mathcal{N}=1}) - \frac{5}{32} Tr(R_{\mathcal{N}=1})$$

U(1) R-symmetry of N=1 superconformal subalgebra of N=2 superconformal algebra

- $R^{\rm UV}_{\mathcal{N}=1} = \frac{1}{3} R_{\mathcal{N}=2} + \frac{4}{3} I_3$

$$\mathrm{U(1)} \times \mathrm{SO(3)} \times \mathrm{SO(3)}_F$$

- $R^{\rm IR}_{\mathcal{N}=1} = \frac{1}{3} R_{\mathcal{N}=2} + \frac{4}{3} (I_3 + T_3)$

$$k_F\, \delta_{ab} = -2 Tr(R_{\mathcal{N}=2} T_a T_b)$$ flavor central charge

➜ $$\boxed{a_{\rm IR} = a_{\rm UV} - \frac{1}{4} k_F\,, \qquad c_{\rm IR} = c_{\rm UV} - \frac{1}{4} k_F}$$

Field theory derivation c_{IR}/c_{UV}

$$a_{\rm IR} = a_{\rm UV} - \tfrac{1}{4} k_F \ , \qquad\qquad c_{\rm IR} = c_{\rm UV} - \tfrac{1}{4} k_F$$

- unitary SCFT $k_F > 0$ ✔ a-theorem
- not assuming large N here
- Taking $a = c$ we can compare with sugra result:

$$\lambda^2 = \frac{k_F}{4 c_{\rm UV}}$$

> 0 ✔

when is this < 1 ?

- match to gravitational Chern-Simons term ✔

Summary of flows between N=2 SCFTs

Supergravity analysis + anomaly argument

- large N, $\mathcal{N}=2$ SCFT with SO(3) flavor symmetry such that $\lambda^2 = \dfrac{k_F}{4c} < 1$
- give vevs to scalar $\Delta = 2$ operators $\mathcal{O}_\Sigma$, $\mathcal{O}_\phi$ belonging to e.m. tensor and flavor current multiplets, so that $v_\Sigma = \dfrac{\lambda}{3} v_\phi$
- should flow to an $\mathcal{N}=2$ SCFT

$$a_{\mathrm{IR}} = a_{\mathrm{UV}} - \tfrac{1}{4} k_F \ , \qquad c_{\mathrm{IR}} = c_{\mathrm{UV}} - \tfrac{1}{4} k_F$$

seems quite universal

Explicit field theory realization?

Flows between N=2 and N=1 SCFTs

- One AdS5 vacuum with 16 supercharges

$$f^{123} = g \,, \qquad \xi^{45} = -\frac{1}{\sqrt{2}}\,g$$

other non-vanishing components: $f^{1AB}, f^{2AB}, f^{3AB}, f^{ABC}, \xi^{AB}$

$$A, B, C = 6, 7, \ldots, \mathfrak{n}+5$$

- When is there also an AdS5 vacuum with 8 supercharges ?
  - We worked out general conditions for such vacua
  - Here: simple model with $\mathfrak{n} = 2$

parameter

$$f^{123} = g \,, \qquad \xi^{45} = -\frac{g}{\sqrt{2}} \,, \qquad \xi^{67} = -\sqrt{2}\,g\rho^{-1}$$

For $\rho = 2$ uplifts to type IIB on S5,
describes flow of mass-deformed N=4 SYM to Leigh-Strassler fixed point
(and Z_k orbifolds)

Freedman, Gubser, Pilch, Warner '99

Flows between N=2 and N=1 SCFTs

$$f^{123} = g \ , \qquad \xi^{45} = -\frac{g}{\sqrt{2}} \ , \qquad \xi^{67} = -\sqrt{2}\, g \rho^{-1}$$

- Ansatz

$$\mathcal{V} = e^{-2\phi\, t_{16} - 2\phi\, t_{27}} = \begin{pmatrix} \cosh\phi & 0 & 0 & 0 & 0 & -\sinh\phi & 0 \\ 0 & \cosh\phi & 0 & 0 & 0 & 0 & -\sinh\phi \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\sinh\phi & 0 & 0 & 0 & 0 & \cosh\phi & 0 \\ 0 & -\sinh\phi & 0 & 0 & 0 & 0 & \cosh\phi \end{pmatrix}$$

- Susy conditions solved by

$$\sinh^2\phi = \frac{\rho - 1}{3} \qquad \rightarrow \qquad \rho > 1$$

$$\Sigma^3 = \rho$$

- ratio of cosmological constants

$$\frac{c_{\mathrm{IR}}}{c_{\mathrm{UV}}} = \frac{27\rho}{(2+\rho)^3}$$

Flows between N=2 and N=1 SCFTs

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$$\Sigma^3 = \rho$$

- ratio of cosmological constants

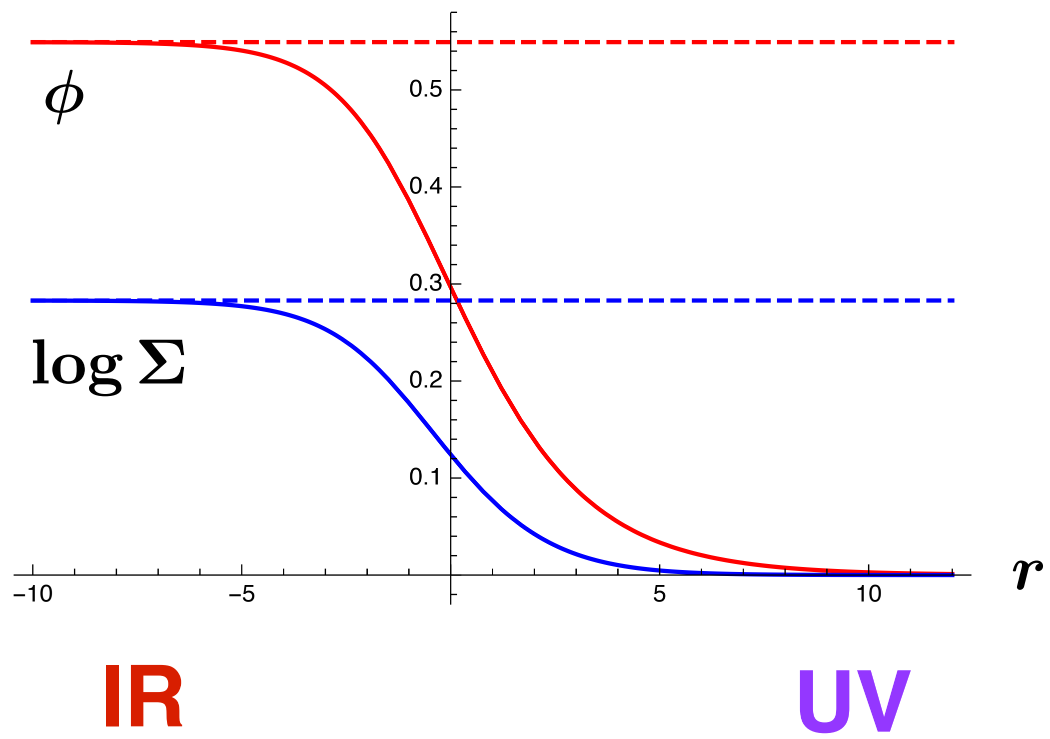
$$\frac{c_{\mathrm{IR}}}{c_{\mathrm{UV}}} = \frac{27\rho}{(2+\rho)^3} \overset{\rho = 2}{=} \frac{27}{32}$$

Tachikawa, Wecht '09

There is a marginal coupling

Flows between N=2 and N=1 SCFTs

- $$W = \frac{g}{3}\,\Sigma^{-1}\cosh^2\phi + \frac{g}{6}\,\Sigma^2\left(1 - 2\rho^{-1}\sinh^2\phi\right)$$ D.C., Dall'Agata, Faedo '12



- Flow preserves 4 Poincaré supercharges

- $$\Delta_{\mathcal{O}_\phi} = 2 + \frac{2}{\rho}\,, \qquad \Delta_{\mathcal{O}_\Sigma} = 2$$

Triggered by source for $\mathcal{O}_\phi$ (when $\rho = 2$ ➜ mass term)

Flows between N=2 and N=1 SCFTs

- anomaly argument, generalizing Tachikawa-Wecht

  studying the gauge symmetries in the IR AdS₅ vacuum, we find

$$R^{\mathrm{IR}}_{\mathcal{N}=1} = \frac{\rho}{\rho+2} R_{\mathcal{N}=2} + \frac{4}{\rho+2} I_3$$

  Plugging in $a = \frac{9}{32} Tr(R^3_{\mathcal{N}=1}) - \frac{3}{32} Tr(R_{\mathcal{N}=1}) \,, \qquad c = \frac{9}{32} Tr(R^3_{\mathcal{N}=1}) - \frac{5}{32} Tr(R_{\mathcal{N}=1})$

  ➜ obtain a linear map between $a_{\mathrm{IR}}, c_{\mathrm{IR}}$ and $a_{\mathrm{UV}}, c_{\mathrm{UV}}$

  for $a = c$ reproduces $\dfrac{c_{\mathrm{IR}}}{c_{\mathrm{UV}}} = \dfrac{27\rho}{(2+\rho)^3}$ ✔

- We can also construct models with one AdS₅ vacuum with 16 supercharges and multiple AdS₅ vacua preserving 8 supercharges

  ➜ study the flows between them

Discussion

- Message

half-maximal gauged supergravity is both general and constrained

↓

can treat all possible supergravity models in a unifying approach

↓

general results for holography without committing to specific models

in particular: holographic flows for d=4, N=2 SCFTs

- Holographic flow between N=2 SCFTs

similar flows known in examples of d=6 and d=7 half-maximal sugra

Karndumri '12, '14

most likely also possible in d=4 ➜ quite universal

- Embedding in higher dimension? Maldacena, Gaiotto ?
- Field theory realization? Finite N?

- Holographic flows from N=2 to N=1 SCFTs

could model MN2 ➜ MN1 of "27/32 type" Maldacena, Nunez '00

MN2 ➜ BBBW **not** of "27/32 type" Bah, Beem, Bobev, Wecht '12

SCTFs describing M5 branes on Riemann surfaces thanks for your attention !