On the Universality of the Chern-Simons Diffusion Rate

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Supersymmetric Quantum Field Theories
in the Non-perturbative Regime
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The Chern-Simons diffusion rate

Definition:

$$Q(x) = \frac{1}{32\pi^2} \mathrm{Tr} F \tilde{F}$$

Change in the Chern-Simons number

$$\Delta N_{CS} = \int d^4x \, Q(x)$$

Chern-Simons diffusion rate

$$\Gamma_{CS} = \frac{\langle (\Delta N_{CS})^2 \rangle}{Vt} = \int d^4x \langle Q(x) Q(0) \rangle$$

$$V = \text{volume}, \qquad t = \text{time}$$

Note: Minkowski correlator, real time physics.

The Chern-Simons diffusion rate

On a state with temperature T (e.g. Quark-Gluon Plasma):

- Kubo formula:

$$\Gamma_{CS} = -\lim_{\omega \to 0} \frac{2T}{\omega} \mathrm{Im}\, G_R(\omega, \vec{k} = 0)$$

  Thus: compute retarded correlator $G_R(\omega, \vec{k} = 0)$.

- Genesis: thermal fluctuations can excite sphalerons $\Rightarrow$ sphalerons decay $\Rightarrow$ $\Delta N_{CS} \neq 0$ (locally).

Why Γ_{CS} interesting

- Baryogenesis in Standard Model: sphaleron transitions cause $\Delta(B+L) \neq 0$.
  - Many studies at weak coupling.
- Chiral magnetic effect in QGP.

The Chern-Simons diffusion rate

Chiral magnetic effect

[Fukushima-Kharzeev-Warringa 2008]

Axial anomaly:

$$\partial_\mu J^\mu_A = -2Q$$

Then:

- ΔN_{CS} generates a $\Delta_{chirality} \Rightarrow \mu_A \neq 0$ (chemical potential).
- Non central collisions in QGP have large magnetic field $\vec{B}$.
- $\Delta_{chirality} + \vec{B}$ generate electric current $\vec{J}^{em} = \sigma_{CME}\vec{B}$, with $\sigma_{CME} = \frac{e^2}{2\pi}\mu_A$.
- Currently under experimental search at RHIC and LHC.

The Chern-Simons diffusion rate

Magnitude of Γ_{CS} in the QGP?

- Real time non-perturbative physics: no reliable computational methods in QCD.
- Effective theory result [Moore-Tassler 2010]:

$$\Gamma_{CS} \sim c \cdot \lambda^5 T^4$$

$$\lambda = g_{YM}^2 N_c \qquad \text{'t Hooft coupling}$$

Notes:

- c is non-perturbative; result valid at $\alpha_s \ll 1$.
- $\Gamma_{CS} \sim \mathcal{O}(N_c^0)$.

Holographic derivation

The Chern-Simons diffusion rate in $\mathcal{N} = 4$ SYM [Son-Starinets 2002]

- Background is $BH - AdS_5 \times S^5$, generated by N_c D3-branes.
- D3-brane action contains

$$\int d^4x \, C \, F\tilde{F}$$

$\Rightarrow$ gravity field dual to Q is RR-potential C.

Holographic derivation

The Chern-Simons diffusion rate in $\mathcal{N} = 4$ SYM [Son-Starinets 2002]

- Action for C in 5d:

$$\int d^5x\sqrt{-g_5}\left[-\frac{1}{2}\partial_M C\partial^M C\right]$$

- Solve eq of motion for $C \Rightarrow$ Retarded correlator G_R.
- Use Kubo, result

$$\Gamma_{CS} = \frac{\lambda^2}{256\pi^3}T^4$$

Holographic derivation

Comments:

- $\mathcal{N} = 4$ SYM "a bit different from QCD".
- Other holographic results look different, eg:
  - $\mathcal{N} = 4$ SYM with magnetic field B [Basar-Kharzeev 2012]:

    $\Gamma_{CS} = \Gamma_{CS}(B = 0) \cdot f(B)$
  - $\mathcal{N} = 4$ SYM with anisotropy a [Bu 2014]:

    $\Gamma_{CS} = \Gamma_{CS}(a = 0) \cdot g(a)$
  - Witten model of holographic Yang-Mills [Craps et al 2012]:

    $\Gamma_{CS} = \frac{1}{2\pi} \frac{\lambda^3}{3^6 \pi^2} \frac{1}{M_{KK}^2} T^6$
  - ...
- Situation different from universal $\frac{\eta}{s} = \frac{1}{4\pi}$ [Kovtun-Son-Starinets 2004].

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  - $\mathcal{N}=4$ SYM with anisotropy a [Bu 2014]:
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Or does it?!

"Universality" of the result

"Wrapped brane models"

Wrap Dp-brane on $(p-3)$-cycle Ω_{p-3}

$$\Downarrow$$

4d gauge theory in IR

- $\mathcal{N}=4$ SYM included.
- Some of the most interesting models included (Witten-Sakai-Sugimoto, Maldacena-Nuñez, ...).
- All computations of Γ_{CS} in the literature performed in this class of models.

Expanding DBI+WZ action at low energies get

$$\mathcal{L} = -\frac{1}{4g_{YM}^2}\mathrm{Tr}F^2 - \frac{\theta_{YM}}{32\pi^2}\mathrm{Tr}F\tilde{F}$$

with

$$\begin{aligned} \frac{1}{g_{YM}^2} &= \tau_p(2\pi\alpha')^2 \int_{\Omega_{p-3}} d^{p-3}x\, e^{\frac{p-7}{4}\phi}\sqrt{det(g_E)} \\ \theta_{YM} &= \tau_p(2\pi\alpha')^2 \int_{\Omega_{p-3}} C_{p-3} \end{aligned}$$

Thus: gravity field dual to Q is

$$C \equiv \tau_p(2\pi)^2(2\pi\alpha')^2 \int_{\Omega_{p-3}} C_{p-3}$$

“Universality” of the result

Derivation of the 5d action of C

Action of $F_{(p-2)} = dC_{(p-3)}$ in 10d

$$\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{10}}\, e^{\frac{7-p}{2}\phi}\left[-\frac{1}{2}F_{(p-2)}^2\right]$$

Reduction ansatz

$$ds_{10}^2 = e^f ds_5^2 + ds_{int}^2$$

Reduction of $F_{(p-2)}$

$$F_{(p-2)}^2 = \partial_M \tilde{C}\partial^M \tilde{C}[det(g_{\Omega'_{p-3}})]^{-1}e^{-f}$$

where
$C = \tau_p(2\pi)^2(2\pi\alpha')^2\mathrm{Vol}(\Omega_{p-3})\tilde{C}$
Ω'_{p-3} has unit volume.

"Universality" of the result

Final result:

$$\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\,H\left[-\frac{1}{2}\partial_M C\partial^M C\right]$$

with

$$H=\frac{1}{(2\pi)^4}\left(\frac{1}{\tau_p(2\pi\alpha')^2\int_{\Omega_{p-3}}e^{\frac{p-7}{4}\phi}\sqrt{\det g_E}}\right)^2=\frac{g_{YM}^4}{(8\pi^2)^2}$$

$\Downarrow$[1]

Chern-Simons diffusion rate has "universal" form

$$\Gamma_{CS}=\frac{\alpha_s^2(T)}{(2\pi)^3}sT$$

[1] [Son-Starinets 2002, Gursoy-Iatrakis-Kiritsis-Nitti-O'Bannon 2013]

Comments:

- Checked also in $\mathcal{N} = 4$ SYM with flavors and $\mathcal{N} = 1$ models.
- Can calculate first $1/\lambda$ correction: Γ_{CS} decreases [Bu 2014].
  Is holographic result an upper bound on Γ_{CS}?
- Problem in extending result to other models: identification of coupling λ and gravity field dual to Q.

Inclusion of the anomaly

Anomaly

$$\partial_\mu J_A^\mu = -qQ$$

holographically reproduced by Stuckelberg action [Klebanov et al 2002]

$$\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g_5}\left[-\frac{1}{2}\left(\partial_M C + qA_M\right)\left(\partial^M C + qA^M\right) - \frac{1}{4}F_{MN}F^{MN}\right]$$

A^M: gravity field dual to J_A^μ; q: anomaly coefficient.

From dimensional reduction of main holographic models: Klebanov-Strassler, $\mathcal{N}=4$ with flavors, Maldacena-Nuñez, Witten-Sakai-Sugimoto.

Inclusion of the anomaly

Define

$$B = (dC + qA) \qquad \Rightarrow \qquad dB = qF \equiv F_B$$

$$\Downarrow$$

action for a massive vector (mass $\sim q$)

$$\frac{1}{2\kappa_5^2 q^2}\int d^5x\sqrt{-g_5}\left[-\frac{1}{4}F_{B,MN}F_B^{MN} - \frac{1}{2}q^2 B^2\right]$$

Inclusion of the anomaly

Calculation of G_R on generic BH background (mild assumptions):

- Near horizon

$$B_t \sim \left(\frac{r}{r_h}\right)^{\Delta} \left(1 - \frac{r}{r_h}\right)^{-i\frac{\omega}{T}} \left[b_h^{(0)} + b_h^{(1)} \left(1 - \frac{r}{r_h}\right) + \cdots\right]$$
$$(4\Delta(\Delta - 1) = q^2)$$

- Get

$$\frac{1}{\omega}\mathrm{Im}G_R \sim \alpha \cdot |b_h^{(0)}|^2$$

with α independent of ω.

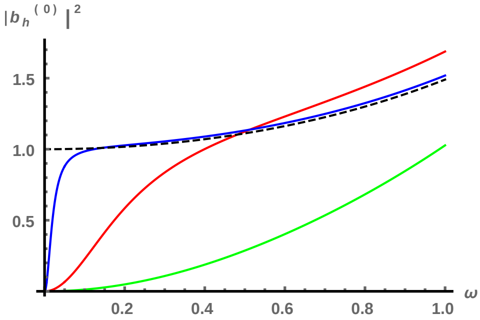
- For $\omega \to 0$

$$b_h^{(1)} = i\left(\frac{q^2}{\omega} + \text{regular in } \omega\right) \cdot b_h^{(0)}$$

$\Rightarrow$ Two possibilities:

- $q = 0$ (no anomaly), or
- $b_h^{(0)} \sim \omega^a$ with $a \geq 1$ for $\omega \to 0$ $\Rightarrow$ $\Gamma_{CS}(q \neq 0) = 0$.

Numeric result on *AdS*-BH:



Black: $q = 0$.
Blue: $q = 0.04$.
Red: $q = 0.44$.
Green: $q = 3$.

Inclusion of the anomaly

Why expected:

- Anomaly $(\partial_\mu J^\mu_A = -qQ) \quad \Rightarrow \quad \Gamma_{CS} \sim \langle QQ \rangle \sim \langle \partial J_A \partial J_A \rangle$
- $\mathcal{Q}_A = \int d^3x\, J^t_A$ not conserved (anomaly), thus

$$\Gamma_{CS} \sim \langle \mathcal{Q}_A(t \to \infty)\mathcal{Q}_A(0)\rangle_R = 0$$

- In fact, with only gapped modes expect

$$\langle \mathcal{Q}_A(t)\mathcal{Q}_A(0)\rangle_R \sim e^{-\frac{t}{\tau}}$$

τ: relaxation time.

Inclusion of the anomaly

Definition of Γ_{CS} makes sense if there is separation of time scales:

[Moore-Tassler 2010]

$$\Delta t < t_* \ll \tau$$

$$\Delta t = \text{(microscopic) time scale of CS number fluctuations}$$
$$t_* = \text{cut} - \text{off}$$
$$\tau = \text{relaxation time}$$

Thus can define

$$\Gamma_{CS} = \int^{t_*} dt \int d^3x \, \langle Q(t,x) Q(0) \rangle$$

Note:

- Can remove cut-off if $\tau \to \infty$.
- Large N_c: $\tau \sim N_c^2/T \gg 1/T \sim$ microscopic time scale.

Inclusion of the anomaly

Hydro model:

- Anomaly ($\partial_\mu J^\mu_A = -qQ$)) $\quad \Rightarrow \quad \langle(\Delta \mathcal{Q}_A)^2\rangle = q^2 \langle (\Delta N_{CS})^2\rangle$
- For $t \ll \tau$ [Iatrakis-Lin-Yin 2015]

$$q^2 \Gamma_{CS} = \frac{1}{Vt}\langle(\Delta \mathcal{Q}_A)^2\rangle \sim \frac{1}{t}\chi_A T\left[1 - e^{-\frac{2t}{\tau}}\right] \sim \frac{2\chi_A T}{\tau}$$

$\chi_A =$ axial susceptibility

so

$$\frac{1}{\tau} = \frac{q^2 \Gamma_{CS}}{2\chi_A T}$$

- Makes sense if

$$\Gamma_{CS} = \Gamma_{CS}(q = 0)$$

($\Rightarrow \tau = \infty$ for $q = 0$).

Inclusion of the anomaly

In holography:

- τ from quasi-normal modes of gravity field A^M dual to J_A^μ on black hole spacetime.
- General expected behavior at small q from A^M equations of motion

$$1/\tau \sim q^2$$

Conclusions:

- Holography seems to point towards a large coupling universality of Γ_{CS} in terms of s, T, α_s.
- Can use the result for estimates in the QGP, e.g. if
  - $\alpha_s(T_c) \sim 1/2$
  - $s(T_c) \sim 10\,T_c$ [Borsanyi et al 2013, Bazavov et al 2014]

$$\Downarrow$$

$$\Gamma_{CS}(T_c) \sim 10^{-2}\,T_c^4.$$

- Including anomaly:
  - Naive $\Gamma_{CS} = 0 \Rightarrow$ must use cut-off, or $\Gamma_{CS} = \Gamma_{CS}(q = 0)$.
  - Relaxation time goes as $1/\tau \sim q^2\Gamma_{CS}/\chi_A T$.

Thank you for your time!