ELECTROWEAK PHASE TRANSITIONS AND HIGGS COUPLINGS

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Based mostly on
PM, H. Ramani 1807.07578
PM. S. Homiller 1808/9.xxxxx
David Curtin, PM, H. Ramani 1612.00466
Why do I care about the early universe and the Higgs and not DM for instance?
MOTIVATION...

Why we are so keen to study DM?

![Graph showing the number of papers over the years for SUSY, Higgs, Top, DM, EXD](image)

- Number of papers on a logarithmic scale.
- Year range from 1970 to 2020.
MOTIVATION…

“It doesn’t seem like there’s anything interesting in pheno lately. Maybe Neutrinos?” L. Alvarez-Gaume (Simons Center for Geometry & Physics director at Stony Brook)

my atavistic pheno impulse is to give a panglossian view of our field:

“X,Y, and Z are being done, amazing new possibilities”
No clear sign of any deviations, or where to even test!!
By and large we still have all the same problems we’ve had for decades:

- Hierarchy Problem
- Dark Matter
- Matter anti-Matter asymmetry
- Neutrino Mass origin
- Strong CP problem
- Flavor
- Number of generations
- Apparent Unification of Coupling Constants
- Inflation
- Reheating
- Unification with Gravity
- Cosmological Constant Problem
EXPERIMENT TO THE RESCUE?
THERE IS PROGRESS EXPERIMENTALLY!

FIG. 4: Ideas to probe low-mass DM via scattering off, or absorption by, nuclei (NR) or electrons (ER).

“Dark Sectors” abound... just ask Howie
UNFORTUNATELY WITH EXPERIMENTAL PROGRESS THERE IS ALSO THEORETICAL PROGRESS...

Wise professor tells entering graduate student 2002:

WIMPs are very motivated so it’s likely: \[ m_{DM} \sim m_{\text{weak}} \]
UNFORTUNATELY WITH EXPERIMENTAL PROGRESS THERE IS ALSO THEORETICAL PROGRESS...

Wise professor tells entering graduate student 2002:

WIMPs are very motivated so it's likely: \( m_{DM} \sim m_{weak} \)

Wise professor tells entering graduate student 2018:

could be \( m_{DM} \sim 10^{-22} \text{ eV} \)

but... also could be 50 to 90 orders of magnitude heavier depending on assumptions of course
LAMPPOST EFFECT

Not just limited to Dark Matter of course
WHY NO NOBEL FOR INFLATION?

HISTORY OF THE UNIVERSE

Inflation!
WHY NO NOBEL FOR INFLATION?

We don’t know *when* it happened to better than ~15 orders of magnitude!!!
I'm not a wise professor nor lucky, so...
Is there a confluence of a lamp post and a theory motivation?
HIGGS LAMPPPOST

• Naturalness
• Higgs Potential
• Higgs Portal to other sectors
• Cosmological History
• ...
Figure 4: Excluded parameter space and expected sensitivities at the 2\(\times\)CL of current and future data for spin-0 (left), spin-1/2 (middle), and spin-1 (right) top-partners. We assume that the two spin-0 top partners are degenerate in mass, \(m_{\tilde{t}_1} = m_{\tilde{t}_2}\). We assume that top partners contribute only in the \(hgg\) and \(h\) loops, there are no modifications of the Higgs couplings to other SM particles, and there are no exotic or invisible Higgs decays. The parameter space excluded by current LHC and Tevatron data is shown in dark gray, while the expected sensitivity of the current data is shown in light gray. Future LHC runs and the proposed future colliders (ILC, CEPC, and FCC-ee/hh) are shown in various colors.

6.1.2 Comparison of Constraints between Spin-0, Spin-1/2, and Spin-1

To compare constraints on spin-0 particles with constraints on spin-1/2 and spin-1, we focus on the degenerate direction for spin-0, \(m_{\tilde{t}_1} = m_{\tilde{t}_2}\), because our canonical spin-1/2 and spin-1 models only have a single top partner. Recall that along the high-mass spin-0 degenerate direction, the contributions from the left-handed sbottom and from stop \(D\)-terms only matter at a few-percent level. For the remainder of Section 6, we set \(g_{h\tilde{b}_1\tilde{b}_1} = 0\), which requires that the choice of stop-sector masses and mixing allow the left-handed sbottom to be real, see Section 5.1 (note that we include \(D\)-term contributions in the stop-sector, i.e., large \(\tan\beta\)).

In Fig. 4 we show the current constraints and expected sensitivities for degenerate spin-0 (left), spin-1/2 (middle), and spin-1 (right) top-partners. The current constraints from Tevatron and LHC data for these different spin-states are about 350 GeV, 700 GeV, and 2.2 TeV, respectively. The LHC Run 4 is expected to improve on these by a few hundred GeV.

Example: Colored Naturalness

Essig, PM, Ramani, Zhong 1707.03399

Have to get “lucky” or...
We certainly learn something, but **what** it is telling us isn’t as clear.
Here there is a chance where quantitative measurements can yield qualitative differences!
\[ \phi \]

**HIGGS POTENTIAL**

**Higgs self interactions**

\[
\begin{align*}
\frac{\partial V(\phi)}{\partial \phi} \bigg|_{\phi=v} &= 0 \\
\frac{\partial^2 V(\phi)}{\partial \phi^2} \bigg|_{\phi=v} &= m_h^2
\end{align*}
\]

VEV and mass are sufficient for Higgs potential, but BSM?
NEXT UP IS THE TRIPLE HIGGS COUPLING IN THE SM…

Unfortunately it’s very difficult and it interferes with itself where the SM Lagrangian corresponds to $c_V = c^2_V = 1$. Generally, the terms of interest to all Higgs production modes include

$$L = \sum m^2_W W^\mu W^\mu + m^2_Z Z^\mu Z^\mu + \sum m^2_h h^3 v^2 + c_V h v + c^2_V h^2 v^2.$$ 

(2.3)

$$m_t \bar{t} t + c_t h v + c^2_t h^2 v^2 + g^2 s^4 \phi^2 + c_g h v + c^2_g h^2 v^2 G_{\mu} \phi G_{\mu}.$$ 

(2.4)

where the SM corresponds to $c_V = c^2_V = 1$, $c_t = 1$, $s^4 = 1$ and $c^2_t = c^2_g = 0$. Note in particular that modifications to the top yukawa coupling, as well as potential new contact interactions with gluons ($c_g, c^2_g$), can arise from integrating out the avys states.

2.1 Di-Higgs Production and the Trilinear Coupling

Figure 1: Diagrams contributing to di-Higgs production via gluon-gluon fusion. An additional diagram that comes from crossing the top quark box is not shown.

Di-Higgs production can proceed through a number of different production modes. Essentially, any single-Higgs production mode can be modified by taking the Higgs off-shell and inserting a trilinear interaction, $3$. The dominant production mode is via gluon-gluon fusion, shown in Fig. 2. In addition to the triangle diagram proportional to $3$, there is another diagram resulting from a box of top quarks, which has the same final state and interferes destructively due to the extra fermion line. The two diagrams scale roughly as

$$A_{\phi} \sim s^4 \phi^2 + c_g h v + c^2_g h^2 v^2 + G_{\mu} \phi G_{\mu}.$$ 

(2.5)

Despite this interference, the gluon-gluon fusion cross section is still roughly an order of magnitude greater than the subleading VBF mode, as shown in Fig. 3. The interference also leads to an interesting shape in the $gg \rightarrow hh$ vs. $3$ curve, as shown in Fig. 4.

In Fig. 5, we show the $gg \rightarrow hh$ cross section as a function of $3$ at several different orders of the computation (LO, NLO(heft), NLO FTapprox, and the full NLO). Here we can see that while the overall behavior remains the same going from LO to NLO, the shape of the curve changes slightly, and the minimum value of $3$ even shifts slightly. Perhaps most importantly, the NLO cross section seems to depend more sensitively on $3$ than the LO computation that we usually work with. The slope

$$|\frac{d\sigma}{d\phi}|_{SM} / |\frac{d\sigma}{d\phi}|_{SM}|$$

increases from 0.

However, just measuring the SM value would be seeing something qualitatively new! To go beyond though, is it just another lamppost? Can be huge deviations… How precisely do we need to measure it?
WHAT IS OUR QUALITATIVE PICTURE OF THE COSMOLOGICAL HISTORY OF EWSB??

HISTORY OF THE UNIVERSE

**Key**
- $\gamma$ rays
- Cosmic rays
- Protons
- Quarks
- Electrons
- Photons
- Neutrons
- Quasars
- Black holes
- Black holes

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Cosmology

stuck here

Need particle physics to go further!
ELECTROWEAK PHASE TRANSITION

This heuristic picture of the cosmological history comes from analyzing a scalar potential at finite temperature which has been around for awhile...
FINITE TEMPERATURE FIELD THEORY STARTS WITH SOME RUSSIANS IN 1972...
IF YOU HEAT UP A SYSTEM WITH A BROKEN SYMMETRY DOES THE SYMMETRY GET RESTORED?

WHAT’S THE CURIE TEMPERATURE OF THE UNIVERSE?
IF YOU HEAT UP A SYSTEM WITH A BROKEN SYMMETRY DOES THE SYMMETRY GET RESTORED?

WHAT’S THE CURIE TEMPERATURE OF THE UNIVERSE?

ANSWERS: YES, HMM...
There's no place like home,
There's no place like home,
There's no place like home…
“A recent paper by Kirzhnits and Linde suggests that this is indeed the case. **However, although their title refers to a gauge theory, their analysis deals only with ordinary theories with broken global symmetries. Also, they estimate but do not actually calculate the critical temperature at which a broken symmetry is restored.”**
IF THERE’S AN EWPT HOW DO WE QUALITATIVELY DISTINGUISH?

SM like

Second order

First order

EWPT
2ND ORDER PHASE TRANSITION

\[ V(\phi, T) = D(T^2 - T_o^2)\phi^2 + \frac{\lambda(T)}{4} \phi^4 \]
The curvature of the finite temperature potential (219) is no longer dependent, \( m^2(\phi, T) = 3\lambda\phi^2 + 2D(T^2 - T_0^2) \) and its stationary points, i.e. solutions to \( dV(\phi, T)/d\phi = 0 \), give

\[
\phi(0) = 0 \quad \text{and} \quad \phi(T_0) = \sqrt{2D(T^2_0 - T^2)} \lambda(T).
\]

Therefore the critical temperature is given by \( T_0 \). At \( T > T_0 \), \( m^2(0, T) > 0 \) and the origin \( \phi = 0 \) is a minimum. At \( T = T_0 \), \( m^2(0, T_0) = 0 \) and the solution \( \phi(0) \) does exist. At \( T = T_0 \), \( m^2(0, T_0) = 0 \) and this solution disappears. The potential (219) becomes

\[
V(\phi, T_0) = \lambda(T_0)\phi^4(222)
\]

At \( T < T_0 \), \( m^2(0, T) < 0 \) and the origin becomes a maximum. Simultaneously, the solution \( \phi(T) \neq 0 \) does appear in (221). This phase transition is of second order, because there is no barrier between the symmetric and broken phases. Actually, when the broken phase is formed, the origin (symmetric phase) becomes a maximum. The phase transition may be achieved by a thermal fluctuation for a field located at the origin. However, in many interesting theories there is a barrier between the symmetric and broken phases. This is characteristic of first order phase transitions. A typical example is provided by the potential

\[
V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda(T)}{4}\phi^4
\]

where, as before, \( D \), \( T_0 \) and \( E \) are \( T \)-independent coefficients, and \( \lambda \) is a slowly varying \( T \)-dependent function. Notice that the difference between (223) and (219) is the cubic term with coefficient \( E \). This term can be provided by the contribution to the effective potential of bosonic fields (174). The behaviour of (223) for the different temperatures is reviewed in Refs. 12, 31. At \( T > T_1 \) the only minimum is at \( \phi = 0 \). At \( T = T_1 \)

\[
T_1 = \frac{8\lambda(T_1)D}{9E^2} - \frac{\lambda(T_1)}{2D}T_2
\]

A second minimum separated by a barrier!
IF THERE’S AN EWPT HOW DO WE QUALITATIVELY DISTINGUISH?

The qualitative difference is an effective cubic at finite temperature!

Why is this so useful? Thermal Decoupling!

\[ e^{-\frac{m}{T}} \]
Triple Higgs + EWPT

There could be a very large contribution and a FO EWPT - get lucky

There also could be a minimum contribution within experimental reach
HIGGS POTENTIAL AND COSMOLOGICAL HISTORY

**Triple Higgs + EWPT**

There *could* be a very large contribution and a FO EWPT - get lucky

There also *could* be a minimum contribution *within* experimental reach

**WHY?**

- Has to couple to the Higgs strongly enough to affect potential
- Mass must not be too far away from EW scale!

*“No-Lose”*

Can make this even sharper if you connect to Electroweak Baryogenesis
THEORETICAL MINIMUM

SM + SINGLET
This model has been studied numerous times for a variety of reasons.

If the Singlet mixes with the Higgs you can see it easily via Higgs properties and has been studied quite a bit

If the Singlet DOESN’T mix but its mass is less than half the Higgs mass you can see it in decays easily...

What if the singlet doesn’t mix with the Higgs and is heavy?
$$V_0 = -\frac{1}{2} \mu^2 h^2 + \frac{1}{4} \lambda h^4 + \frac{1}{2} \mu_S^2 S^2 + \frac{1}{2} \lambda_{HS} h^2 S^2 + \frac{1}{4} \lambda_S S^4$$

$$m_S^2 = \mu_S^2 + \lambda_{HS} v^2 > 0$$

Phenomenological parameter space depends only on 

$$(m_S, \lambda_{HS})$$

There are two qualitatively different regions depending on the sign of $\mu_S^2$.
WHERE IN THE PARAMETER SPACE IS THERE A GOOD PHASE TRANSITION?

Figure 3. Regions in the \((m_S, \lambda_{HS})\) plane with viable EWBG. Red shaded region: for \(\mu_S^2 < 0\) it is possible to choose \(\lambda_S\) such that EWBG proceeds via a tree-induced strong two-step electroweak phase transition. Orange contours: value of \(v_c/T_c\) for \(\mu_S^2 > 0\). The orange shaded region indicates \(v_c/T_c > 0.6\), where EWBG occurs via a loop-induced strong one-step phase transition. Above the green dashed line, singlet loop corrections generate a barrier between \(h = 0\) and \(h = v\) even at \(T = 0\), but results in the dark shaded region might not be reliable, see Section 3.1.3.
A “NO-LOSE” THEOREM

Nonperturbative $\lambda_S$ required
for $V(v,0) < V(0,w)$
(tree-level)

Two-step EWPT

One-step EWPT

One-Loop Analysis of EWPT breaks down

$\mu_S^2 > 0$

$\mu_S^2 < 0$

Nonperturbative $\lambda_S$ required to avoid negative runaways (tree-level)

Collider

$\frac{S}{\sqrt{B}} \geq 2$

for 100 TeV 30/ab

TLEP exclusion with Zh shift

2 Sigma Exclusion with triple Higgs at 100 TeV

Figure 10. Summary of the nightmare scenario’s parameter space.
TRIPLE HIGGS

• Experimentally there are a number of different probes but the triple Higgs coupling does the heavy lifting

• In that study we used an assumed sensitivity of 10% on triple Higgs with 30/ab @ 100 TeV

• Strong FO EWPT typically naively has a 20-30% shift in this scenario

• A 100 TeV collider is a lot of money and a lot of time, what are the other possibilities?
TRIPLE HIGGS MEASUREMENTS

2010 - 2035

Graph showing the peak luminosity and integrated luminosity over the years 2010 to 2035, with distinct luminosity periods labeled as LS1 to LS5.
TRIPLE HIGGS MEASUREMENTS

• By 2035 we won’t be able to tell the triple Higgs coupling compared to the SM better than

\[-0.8 < \lambda_3 < 7.7\] at 95% C.L.

Ugh…
Chinese can’t be faster?? although maybe politically more feasible…
TRIPLE HIGGS MEASUREMENTS

• Refined studies show that FCC-hh could get to 1.6%-few% precision, but these are missing some backgrounds and need some work

• What about HE-LHC?
HE-TRIPLE HIGGS MEASUREMENTS

• Han & Plehn et al claim you can get 5 sigma significance and a precision of 30%

Nevertheless HE-LHC does have a lot to say!

Homiller, PM to appear: a little more conservative results
Most non-experimental studies have left out key backgrounds
NICE SIMPLE STORY... MEASURE TRIPLE HIGGS WELL ENOUGH YOU KNOW THE HISTORY OF THE UNIVERSE TO AN EARLIER TIME...
NICE SIMPLE STORY…
MEASURE TRIPLE HIGGS WELL ENOUGH
YOU KNOW THE HISTORY OF THE
UNIVERSE TO AN EARLIER TIME…

NO, NOT THAT SIMPLE!
THEORETICAL PROGRESS!

Sometimes... it feels safer to live in the dark.

Theory

Blow it off so we won't see anything scary!
EXPERIMENTAL TESTS OF COSMOLOGICAL HISTORY

- **Second order**
  - SM like
- **EWPT**
- **First order**

**third possibility**

The EWSB was never restored or it was delayed, or there were multiple EW phase transitions!!

Symmetry Non-Restoration

SNR phase

PM, Ramani 1807.07578
SYMMETRY-NON RESTORATION

• Weinberg in his original finite-T paper noted counter examples
  • Rochelle salts
  • $O(N) \times O(M)$ model
  • Since been verified on the lattice and with various other methods!
VERY SIMPLE TO SEE WHERE IT COMES FROM...

\[ V \sim (T^2 - \mu^2)\phi^2 + \lambda\phi^4 \]

This comes from a term

\[ V \supset \Pi_\phi\phi^2 \quad \Pi_\phi \sim \lambda T^2 \]

In a more general theory, e.g. for the Higgs

\[ \Pi_h = T^2 \left( \frac{\lambda_t^2}{4} + \frac{3g^2}{16} + \frac{g'^2}{16} + \frac{\lambda}{2} \right) \]
NOW LET’S TAKE OUR SIMPLE SINGLET MODEL...

\[ V_0 = -\frac{1}{2} \mu^2 h^2 + \frac{1}{4} \lambda h^4 + \frac{1}{2} \mu_S^2 S^2 + \frac{1}{2} \lambda_{HS} h^2 S^2 + \frac{1}{4} \lambda_S S^4 \]

\[ \Pi_h = T^2 \left( \frac{\lambda_t^2}{4} + \frac{3g^2}{16} + \frac{g'^2}{16} + \frac{\lambda}{2} + \frac{\lambda_{HS}}{12} \right) \]

and flip a sign...

\[ \Pi_h = T^2 \left( \frac{\lambda_t^2}{4} + \frac{3g^2}{16} + \frac{g'^2}{16} + \frac{\lambda}{2} - \frac{\lambda_{HS}}{12} \right) \]
IF THE SINGLET DOMINATES WE HAVE A QUALITATIVELY DIFFERENT PICTURE...

\[ V \sim -(\mu^2 + T^2)h^2 + \lambda h^4 \]

The VEV *increases* with temperature!

\[ \langle h \rangle \sim T \]

The EW symmetry is *never* restored in the early universe.
HOW WAS THIS MISSED?

- It’s not quite so trivial, as you still have to make sure your calculation is under control and you have a good vacuum

\[ \lambda_{HS}^2 \leq \lambda_s \lambda \]

So to satisfy this and dominate the thermal mass you run into non-perturbativity very quickly with the s quartic

\[ \Pi_h = T^2 \left( \frac{\lambda_t^2}{4} + \frac{3g^2}{16} + \frac{g'^2}{16} + \frac{\lambda}{2} - \frac{\lambda_{HS}}{12} \right) \]
SIMPLE TRICK -
SWITCH TO O(N) SINGLET

\[
\Pi_h = T^2 \left( \frac{\lambda_t^2}{4} + \frac{3g^2}{16} + \frac{g'^2}{16} + \frac{\lambda}{2} - N_s \frac{\lambda_{HS}}{12} \right)
\]

\[
\Pi_s = T^2 \left( (N_s + 2) \frac{\lambda_s}{12} - \frac{\lambda_{HS}}{3} \right)
\]

Now it can dominate the thermal mass but keep the potential stable for small \( \lambda_{HS} \)

A rough estimate yields \( \lambda_{HS} N_s \geq 4.8 \)

\( \lambda_{HS}^c \equiv \lambda_{HS} N_s \)
UNDER CONTROL…

You don’t run into issues with the pure singlet quartic

\[ \lambda_s \geq \left( \frac{\lambda_{HS}^c}{N_s} \right)^2 \frac{1}{\lambda} \]

\[ \beta \] function running of the couplings stable to high scales
DOING THIS MORE CAREFULLY

- Must take into account resummation and finite mass effects correctly - Optimized Partial Dressing

D. Curtin, PM, H. Ramani 1612.00466

“Unfortunately, despite the fact that one is dealing with a weakly coupled theory, many aspects of the phase transition are surprisingly complicated. Indeed, the literature contains contradictory claims and statements on almost every important question.”

’92 Dine, Leigh, Huet, Linde, Linde
Higher order effects

\[ \Pi_{\eta}[\text{GeV}^2] \]

\[ -N_s \lambda_{hs} \]

\[ m_s = 200 \text{ GeV}, \ T = 500 \text{ GeV} \]

- Naive
- Daisy
- Superdaisy

\[ \frac{\nu(T)}{T} = 0.6 \]

\[ \frac{\nu(T)}{T} = 1 \]
VERY COOL EARLY UNIVERSE POSSIBILITIES

Depending on the Singlet Mass you can get SNR-R-SNR.

\[
\lambda_{hs}^c = 6
\]
COSMOLOGICAL CHANGES

• Sphalerons are controlled by $\frac{\nu(T)}{T} \equiv \kappa$

• for $\kappa \sim 1$ sphalerons are turned off

• “GUT” Baryogenesis can work- Maximons

• Models that use sphalerons would be dead (EWBG, some Leptogenesis) - can look SM like at low energies

Can also just postpone EWBG: see Baldes, Servant and Rattazzi, Vecchio
COSMOLOGICAL CHANGES

• Avoid defects if you avoid phase transitions…

• Is decoupling any different? In principle yes

\[ m(T) \sim g\kappa T \]

• For very large kappa, particles are non-relativistic instead of relativistic

\[ \kappa \sim \frac{\sqrt{\Pi h / \lambda}}{T} \]

Can enhance with running \( \lambda \)

Even more interesting you can get exotic equations of state!
LARGE N SCALING CHANGES
COLLIDER OBSERVABLES AS WELL

However, let’s look at the scaling for collider observables…

\[ \delta_{Zh} \sim N_s \lambda_{HS}^2 \quad \sigma_{h^* \rightarrow ss} \sim N_s \lambda_{HS}^2 \quad \delta_{h^3} \sim N_s \lambda_{HS}^3 \]

If we fix \[ \lambda_{HS} N_s = \lambda_{HS}^c \]

\[ \delta_{Zh} \sim \frac{(\lambda_{HS}^c)^2}{N_s} \quad \sigma_{h^* \rightarrow ss} \sim \frac{(\lambda_{HS}^c)^2}{N_s} \quad \delta_{h^3} \sim \frac{(\lambda_{HS}^c)^3}{N_s^2} \]

In the scaling limit the effects disappear!

Can we tell whether or not the early universe was in a SNR phase?
ANOTHER INTERESTING COMPLICATION

Can you confuse SNR with a strong FOPT? yes, up to triple higgs
One would have a strong gravitational wave signal, the other wouldn’t
CONCLUSIONS

• Lots of interesting physics under the Higgs lamppost

• Need a new flowchart for thinking about triple Higgs couplings, but it is likely the most important measurement for understanding qualitative differences about our universe from particle perspective