NON-LINEAR REALIZATION OF SUPERSYMMETRY ALGEBRA FROM SUPERSYMMETRIC CONSTRAINT ☆

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We discuss spontaneous symmetry breaking of global supersymmetry for a single scalar superfield in an arbitrary Kähler manifold. We show that when the curvature of the manifold goes to infinity (or, equivalently, the masses of the scalar partners of the goldstino go to infinity) a non-linear realization of supersymmetry is obtained. The model can be described, in perfect analogy to the ordinary σ-models, by means of a supersymmetric constraint on the superfield Φ, of the form $\Phi^2=0$. The non-linear realization we obtain is different from that of Volkov and Akulov. The differences among the two realizations are discussed.

Planck 2018 and dS Supergravity from 10d

Renata Kallosh, Stanford

Remembering Raoul Gatto

September 28, 2018
GGI, Florence

Progress after Supergravity workshop in 2016 at GGI

For cosmology we need General Relativity

Supergravity is next natural step after General Relativity

Superstring theory is believed to be the most fundamental theory we know

However, string theory has an emergent concept of space-time. To use it in the context of the 4-dimensional General Relativity and cosmology requires many intermediate steps

If in these steps some amount of supersymmetry, maximal or minimal or intermediate, is preserved, one finds consequences for cosmology, potentially supportable or falsifiable by observations

What part of observations is important for fundamental physics?

Early Universe inflation and current Universe acceleration: dark energy

New in supersymmetry :

Fundamental role of a **non-linearly realized supersymmetry**

1972 Volkov, Akulov

1978-1979 Rocek, Lindstrom; Ivanov, Kapustnikov

1983 Samuel, Wess

1989, Casalbuoni, De Curtis, Dominici, Feruglio, Gatto

2009 Komargodski, Seiberg

2010 Kuzenko, Tyler

Recent: VA is suitable for inflation and dark energy with spontaneously broken supersymmetry

Antoniadis, Dudas, Ferrara, Sagnotti, Volkov-Akulov-Starobinsky supergravity 2014

Ferrara, RK, Linde, Cosmology with Nilpotent superfields, 2014

Zwirner, Dall'Agata, Bergshoeff, Van Proeyen, Freedman, Roest, Porrati, Sagnotti, Dudas, Antoniadis, Bandos, Tonin, Sorokin, Kuzenko, Kehagias, Riotto, Argurio, Quevedo, Uranga

Wrase, Vercnocke, Scalisi, Yamada,Farakos, McDonough, Carrasco, Martucci, Murli, Karlsson, Westphal, Delacretaz, Gorbenko, Senatore, Cribiori, Tournoy, Garcia del Moral, Parameswaran, Quiroz, Zavala

The discovery of dark energy and an observational success of early universe inflation, associated with de Sitter or nearly de Sitter spaces support Volkov-Akulov non-linearly realized supersymmetry from the sky

This is in a contrast to a non-discovery of a low-energy linearly realized supersymmetry at LHC

Cosmological Constant in Supergravity

Known to be negative in pure supergravity, without scalar fields (Townsend, 1977)

$$\Lambda < 0 \quad \textbf{AdS}$$

Supergravity with a positive cosmological constant without scalars was not known (till 2015)

$$\Lambda > 0 \quad \textbf{dS}$$

d=4

Standard linear $\mathcal{N}$=1 SUSY

1 Majorana fermion 1 complex scalar

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}(\partial_\mu B)^2 - \tfrac{1}{2} i\bar\psi\gamma^\mu\partial_\mu\psi - \tfrac{1}{2}m^2A^2 - \tfrac{1}{2}m^2B^2 - \tfrac{1}{2}im\bar\psi\psi - gmA(A^2+B^2) - \tfrac{1}{2}g^2(A^2+B^2)^2 - ig\bar\psi(A-\gamma_5 B)\psi.$$

Wess-Zumino, **1974**: minimal SUSY with a (Golfand,Likhtman, 1971) Majorana fermion and a complex scalar

Gravity NO-GO for de Sitter

AdS/CFT studies

$$\sqrt{g}\,\Lambda = -\sqrt{g}\,3\,e^K|W|^2 \leq 0$$

LHC, as of September 2018

No SUSY partners yet

Non-linear $\mathcal{N}$=1 SUSY

1 Majorana fermion 2 Majorana fermions

$$\mathcal{L} = -det\, E$$

$$\mathcal{L} = -f^2 + i\partial_\mu \overline{G}\overline{\sigma}^\mu G + \frac{1}{4f^2}\overline{G}^2\partial^2G^2 - \frac{1}{16f^6}G^2\overline{G}^2\partial^2G^2\partial^2\overline{G}^2$$

Volkov, Akulov, 1972 Non-linearly realized supersymmetry: only fermions are present

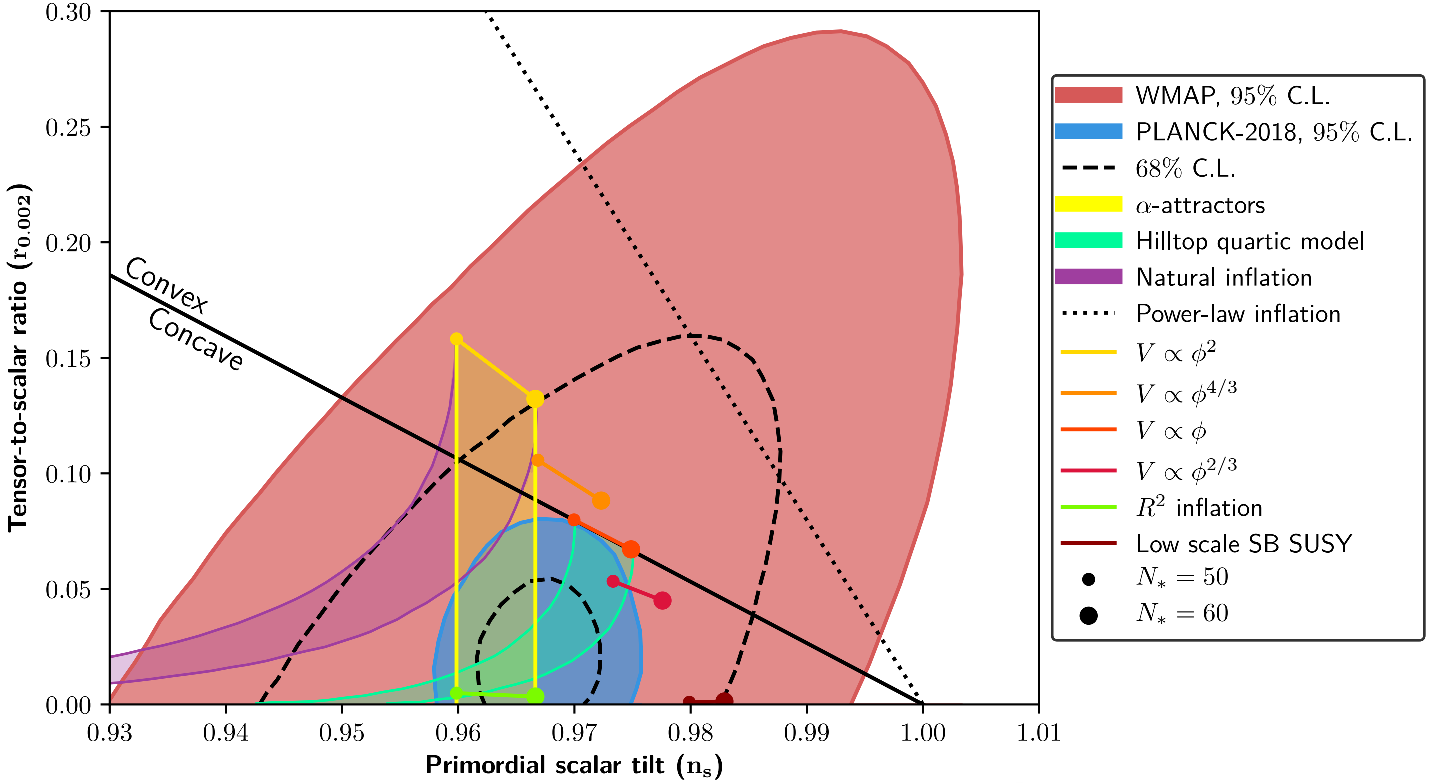
Local supergravity, with de Sitter vacua (without scalars) was constructed in **2015**

Bergshoeff, Freedman, RK, Van Proeyen; Hasegawa, Yamada

$$\sqrt{g}\,\Lambda = \sqrt{g}(f^2 - 3\,e^K|W|^2) > 0$$

Early Universe, CMB: PLANCK 2018 versus WMAP 2010

(after a billion dollars spent)



~~2004 Racetrack~~

~~Inflation in String theory~~

~~$n_s \leq 0.95$~~

~~1994 F-term, 1996 D-term Inflation~~

~~In supergravity, D3/D7 model in String theory,2007~~

~~$n_s \approx 0.98$~~

~~2003 KKLMMT~~

~~2010 inflection model~~

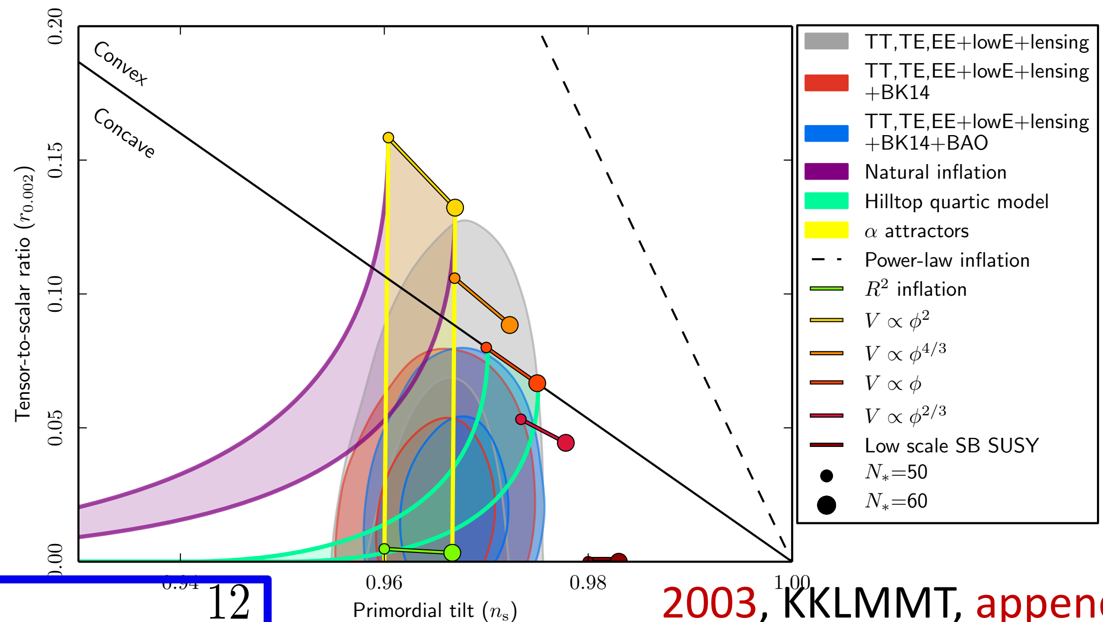
~~$n_s \leq 0.93$~~

With tiny exception

~~2005 N-flation~~

~~r=0.13~~

In Planck 2018 there are 2+ interesting models in the sweet spot of data



$$\boxed{n_s = 1 - \frac{2}{N}\,, \qquad r = \alpha \frac{12}{N^2}}$$

2003, KKLMMT, appendix C and Encyclopaedia Inflationaris

$$V = V_0\Big(1 - \frac{\mu^4}{\phi^4}\Big)$$

$$n_s = 1 - \frac{5}{3}\frac{1}{N}$$

$$r < 10^{-6}$$

2013, supergravity α-attractors,
RK, Linde, Roest; Ferrara, Porrati

Include Higgs and Starobinsky, α=1

Fibre inflation, Quevedo et al

α=2, ½, 2008

Stringy version?

Can be validated by the B-mode detection:
Detection means determination of the
Curvature of the moduli space from the sky

Cannot be invalidated:
Next 2 decades only, at best

$$r > 10^{-4}$$

Encyclopaedia Inflationaris, 2013
one of their 192 figures for 74 models

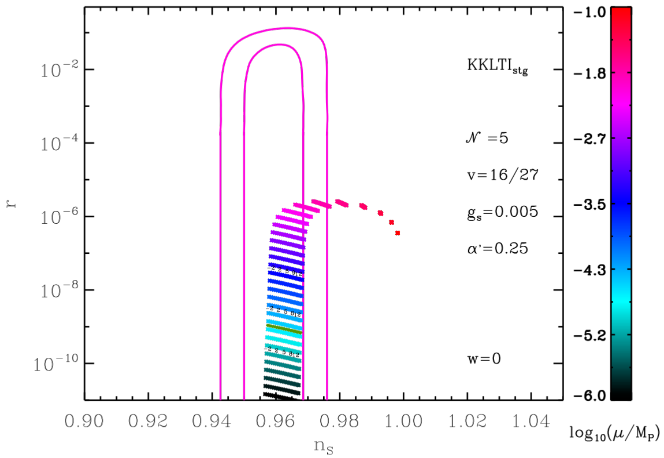
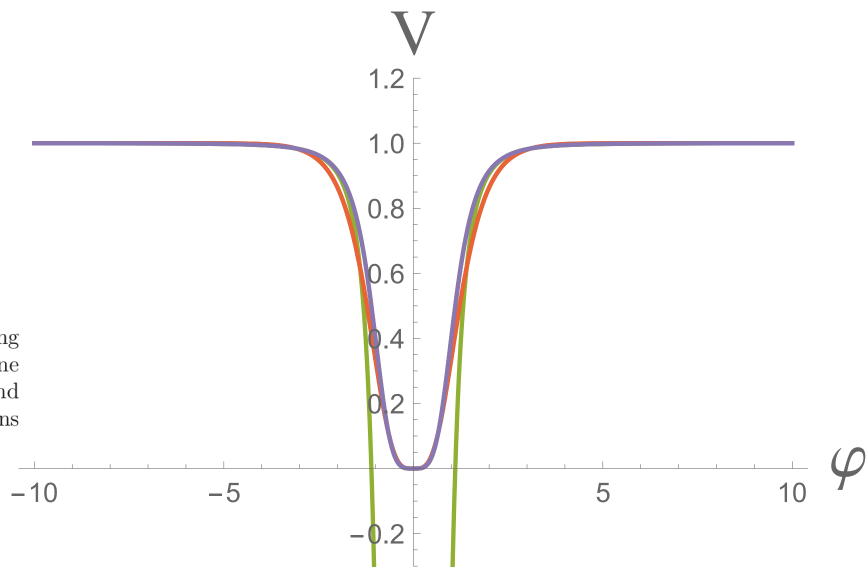


Figure 166. Reheating consistent slow-roll predictions for the KKLT inflation models in the string framework ($p = 4$, $\mu \ll M_{\rm Pl}$, for the fundamental parameters displayed in the figures), in the plane $(n_{\rm s}, r)$ (top panel) and the plane (ϵ_1, ϵ_2) (bottom panel). The two pink solid contours are the one and two-sigma Planck confidence intervals (marginalized over second order slow-roll). The annotations



Supersymmetric version of this model Including volume stabilization:
work in progress: RK, Linde, Yamada
using Geometric Inflation

KKLMMT, appendix C and
Encyclopaedia Inflationaris

Here is an alpha-attractor model (red line), a KKLMMT model (green) and a KKLMMT model with a cut-off near small phi (purple). Since inflation takes place at the shoulders of the plateau region close to the exit from the plateau region, it is not surprising that all these models fit Planck 2018 data very well.

- **Maximal supersymmetry and B-modes**
- M-theory in d=11
- Superstring theory in d=10
- $\mathcal{N}$=8 supergravity in d=4

2016, Ferrara and RK

Scalars are coordinates of the coset space in $\mathcal{N}$=8 supergravity in d=4 $\quad \frac{G}{H} = \frac{E_{7(7)}}{SU(8)}$

$$E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7$$

Geometries with discreet number of unit size Poincaré disks are possible when consistent reduction of supersymmetry is performed. Upon identification of their moduli one finds

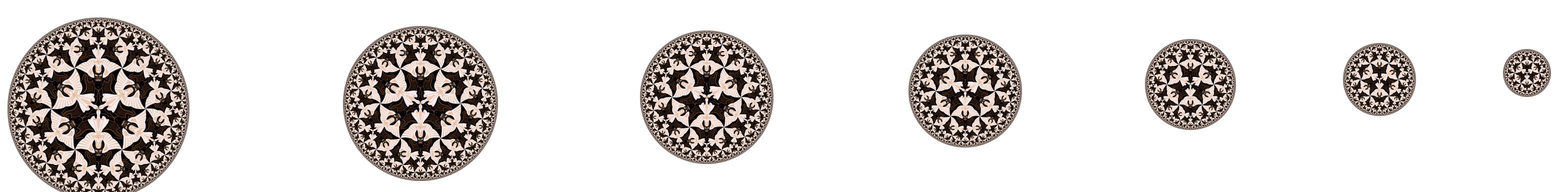
$$ds^2 = k\frac{dTd\bar{T}}{(T+\bar{T})^2}, \qquad k = 1,2,3,4,5,6,7 \quad = 3\,\alpha$$

At least one disk and no more than seven

N=55 e-foldings

$n_s \approx 0.963$

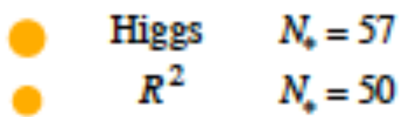
$$r \approx \{1.3,\ 2.6,\ 3.9,\ 5.2,\ 6.5,\ 7.8,\ 9.1\} \times 10^{-3}$$



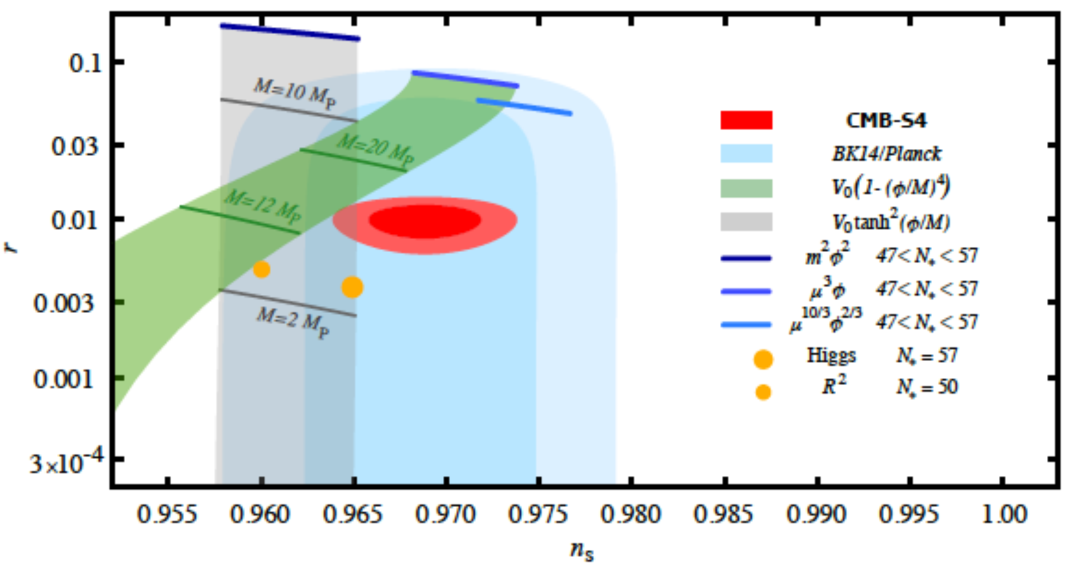
From Planck 2018

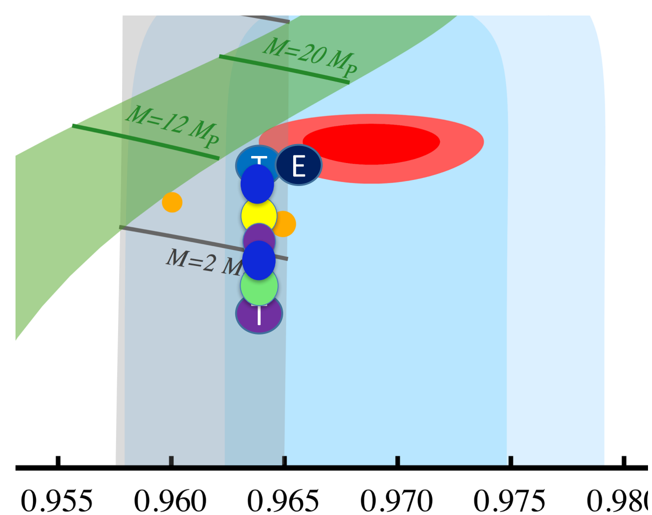
Planck and BK14 data set tight constraints on α attractors (Kallosh et al. 2013; Ferrara et al. 2013). We obtain $\alpha_{E1} < 32$ and $\alpha_{E2} < 16$ at 95% CL for the E- model. We obtain slightly tighter 95 % CL bounds for the T-model, i.e., $\alpha_{T1} < 12$ and $\alpha_{T2} < 10$.

Given the relation $|R_K| = 2/(3\alpha)$ between the curvature of the Kahler geometry R_K and α in some of the T-models motivated by supergravity, *Planck* and BK14 data imply a lower bound on $|R_K|$, which is still in the low-curvature regime. The discrete set of values $\alpha = i/3$ with an integer *i* in the range [1,2,3,4,5,6,7] motivated by maximal supersymmetry (Ferrara & Kallosh 2016; Kallosh, Linde, Wrase, & Yamada 2017) is compatible with the current data.



Seven new targets
For B-mode satellites





dS supergravity

No-go theorems on dS prohibit linearly realized supersymmetry.

New $\mathcal{N}$=1 dS supergravity has a non-linearly realized supersymmetry.

Bergshoeff, Freedman, RK, Van Proeyen
Hasegawa, Yamada

2015, pure dS supergravity

$$\boxed{\sqrt{g}\,\Lambda = \sqrt{g}(f^2 - 3\,e^K|W|^2) > 0}$$

dS supergravity, general: a nilpotent multiplet interacting with arbitrary chiral and vector multiplets with local non-linearly realized supersymmetry

RK, Wrase, Schillo, van der Woerd, Linde, Van Poeyen, Freedman, Roest dS supergravity in our observable d=4 has a non-linearly realized supersymmetry

Includes all chiral and vector standard unconstrained multiplets
+ a chiral nilpotent multiplet, equivalent to a presence of the anti-**D3** brane uplift

Simplest case: bosonic action

Complicated fermionic action

$$K = -3\log\left(T+\bar{T}\right) + S\bar{S}\,,$$

$$W = W_0 + A\exp(-aT) + \mu^2 S$$

$S^2=0$

Wrase, RK 2014

15 years after KKLT (2003) when we learned that anti-**D3** brane can uplift the minimum

RK, Wrase 2018, 1808.09427

anti-**Dp** brane uplift

Wrap any anti-Dp brane on a (p-3) cycle, whenever available,
each time there is an uplifter : a nilpotent superfield in dS supergravity action

The situation is like: d=4 supergravity with F-term potential has many versions, some of them are derived via flux compactification on calibrated manifolds from d=10 supergravity with local sources, Dp-branes and Op-planes.

Now we observe that general type dS supergravity has many versions, some of them are derived via flux compactification on calibrated manifolds from d=10 supergravity with local sources, Dp-branes and Op-planes as well as anti-Dp-branes.

Anti-D3 Brane Induced Geometric Inflation:

Model Building Paradise

RK, Linde, Roest, Yamada, 2017
McDonough, Scalisi 2016

$\mathcal{G}$

Kahler function

Cremmer, Ferrara, Girardello, Julia, Scherk, van Nieuwenhuizen, Van Proeyen, from 1978

We are interested in anti-D3 brane interaction with Calabi-Yau moduli T_i. In supergravity we expect some interaction between the nilpotent superfield S, **representing KKLT type anti-D3 brane**, and Calabi-Yau moduli T_i

$$\mathcal{G}(T^i, \bar{T}^i; S, \bar{S})$$

$$\mathcal{G} \equiv K + \log W + \log \bar{W}\,, \qquad \mathbf{V} = e^{\mathcal{G}}(\mathcal{G}^{\alpha\bar{\beta}}\mathcal{G}_\alpha \mathcal{G}_{\bar{\beta}} - 3)$$

simple relation between the potential and the nilpotent field geometry

$$\mathcal{G}^{S\bar{S}}(T_i, \bar{T}_i) = \frac{\mathbf{V}(T_i, \bar{T}_i) + 3|m_{3/2}|^2}{|m_{3/2}|^2}$$

From the sky to fundamental physics

$\mathcal{G}$

$$\mathcal{G} \equiv K + \log W + \log \bar{W}\,, \qquad \mathbf{V} = e^{\mathcal{G}}(\mathcal{G}^{\alpha\bar{\beta}}\mathcal{G}_\alpha \mathcal{G}_{\bar{\beta}} - 3)$$

Erich Kahler noticed in 1933 and Moroianu suggested in 2004, that once the **hermitian Kahler function** is introduced

“a long list of miracles occur then”

May 2017

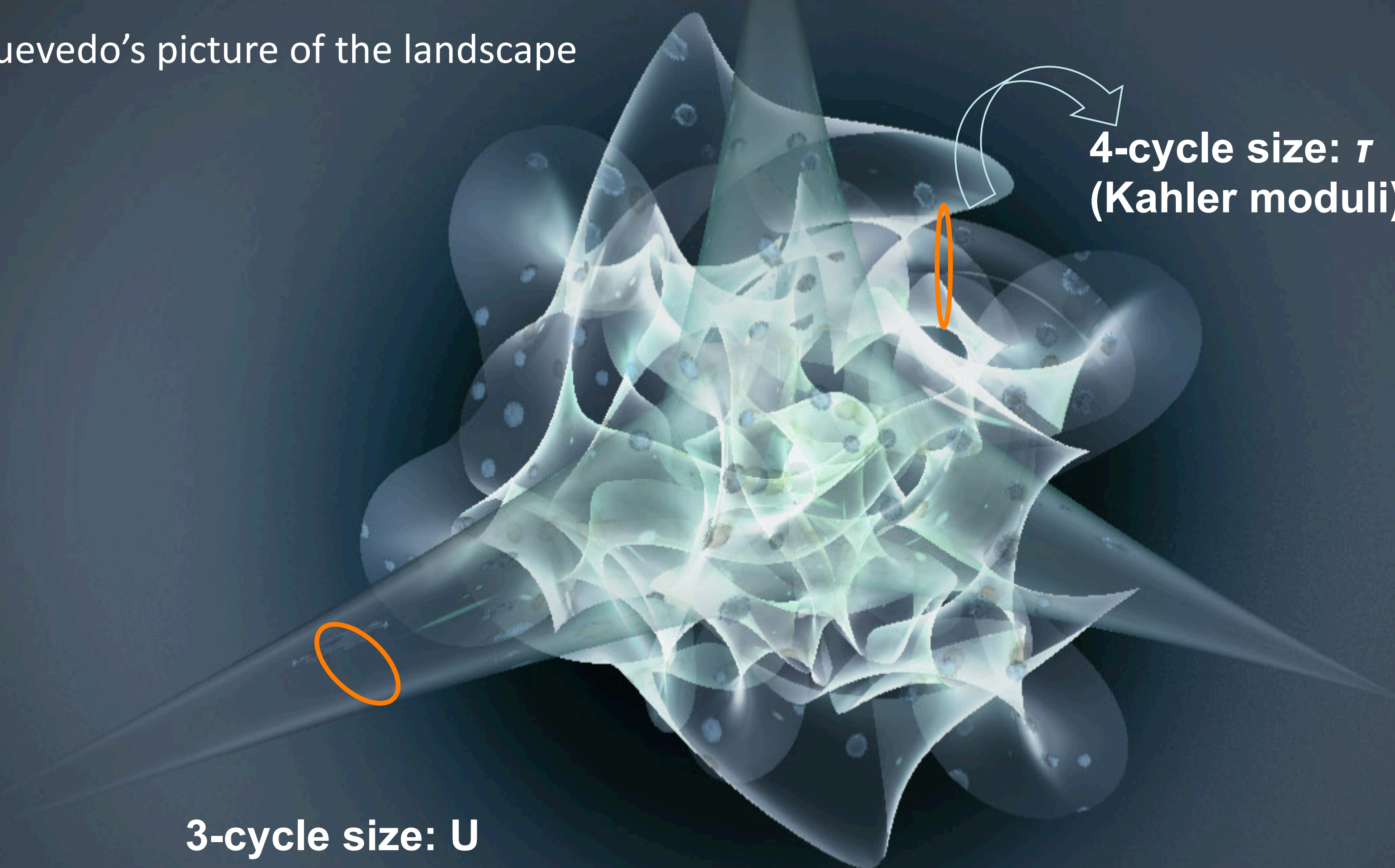
RK, Linde, Roest, Yamada

Model Building Paradise

now confirmed in the cosmological context

IIB MODULI STABILISATION

F. Quevedo's picture of the landscape



dS supergravity in d=4 from flux compactification of superstring theory in d=10

Flux Compactification from d=10 on SU(3)×SU(3)-structure **from d=10 supergravity** with fluxes and local sources, Dp-branes and Oq-planes

General case, consise notation

Koerber, Lectures on Generalized Complex Geometry for Physicists

The compactification background has to admit **calibrated** Dp-branes

The Kahler potential and the superpotential of the **$\mathcal{N}$ = 1 d=4 effective supergravity** theory are

$$\mathcal{K} = -\ln 4 \int_M e^{-4A}|C|^{-6} H(\mathrm{Re}\,\mathcal{Z}) - 2\ln 4 \int_M e^{2A} H(\mathrm{Re}\,\mathcal{T}) + 3\ln(8\kappa_{10}^2)\,,$$
$$\mathcal{W} = \frac{i}{4\kappa_{10}^2} \int_M \langle C^{-3}\mathcal{Z}, \mathrm{d}_H \mathcal{T}\rangle\ .$$

There is a dictionary

H(ReΨ) is the Hitchin function, related to the Mukai pairing $H(\mathrm{Re}\,\Psi) = \frac{i}{4}\langle\Psi,\bar{\Psi}\rangle\,,$

Pure spinor polyforms $\Psi_1 = \Psi^{\mp}$ and $\Psi_2 = \Psi^{\pm}$ for IIA/IIB

$$\underline{\Psi_1} = \underline{\Psi}^{\mp} = -\frac{8i}{|a|^2}\,\eta_+^{(1)}\,\eta_{\mp}^{(2)\dagger}\,, \qquad \underline{\Psi_2} = \underline{\Psi}^{\pm} = -\frac{8i}{|a|^2}\,\eta_+^{(1)}\,\eta_{\pm}^{(2)\dagger}\,,$$

globally defined nowhere-vanishing normalized internal Killing spinors η

$$V = e^K\left(K^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2\right) + \frac{1}{2}(\mathrm{Re} f)^{-1\alpha\beta} D_\alpha D_\beta\,.$$

Type IIA on CY$_3$

- Using fluxes F_0, F_2, F_4 and H_3 together with O6-planes, one can stabilize all moduli *classically* in AdS$_4$

DeWolfe, Giryavets, Kachru, Taylor hep-th/0505160

- However, it is impossible to have dS vacua

Hertzberg, Kachru, Taylor, Tegmark 0711.2512

$$\rho = (vol_6)^{\frac{1}{3}}, \qquad \tau = e^{-\phi}\sqrt{vol_6}$$

$$V(\rho,\tau) = \frac{A_H}{\rho^3\tau^2} + \sum_p \frac{A_{F_p}}{\rho^{p-3}\tau^4} - \frac{A_{O6}}{\tau^3}, \qquad \text{all } A_* > 0,$$

$$0 \neq \nabla V \sim -\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} \geq 9\,V > 0$$

To get dS need to switch from CY$_3$ to SU(3)-structure manifolds:

Negative curvature still tachyons

Now we can add anti-D6 and keep $\mathcal{N}$=1 non-linearly realized

Before presenting the **general new rules**, let us look at an example

The isotropic $S^3 \times S^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ example

S^2=0

$$K = -\ln\left[-\mathrm{i}(Z^1-\bar{Z}^1)\right] - 3\ln\left[-\mathrm{i}(Z^2-\bar{Z}^2)\right] - \ln\left[\mathrm{i}(t-\bar{t})^3 - \mathrm{i}\underline{\frac{S\bar{S}}{N_{\overline{D6},K}(Z^K-\bar{Z}^K)}}\right]$$

K=1,2

$$W = \frac{1}{2}(h+3t)Z^1 - \frac{3}{2}(h-t)Z^2 + 3f_2t^2 - f_0t^3 + \underline{\mu^2 S}$$

With S=0, no uplifting anti-D6, more than a **decade of efforts**

Villadoro, Zwirner, N=1 effective potential from dual type-IIA D6/O6 orientifolds with general fluxes, 2005

Camara, Font, Ibanez, Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold, 2005

Aldazabal, Font, A Second look at N=1 supersymmetric AdS(4) vacua of type IIA supergravity, 2007

Flauger, Paban, Robbins, Wrase, Searching for slow-roll moduli inflation in massive type IIA supergravity with metric fluxes, 2008

Danielsson, Haque, Shiu, Van Riet, Towards classical de Sitter solutions in string theory, 2009

Danielsson, Koerber, Van Riet, Universal de Sitter solutions at tree-level, 2010

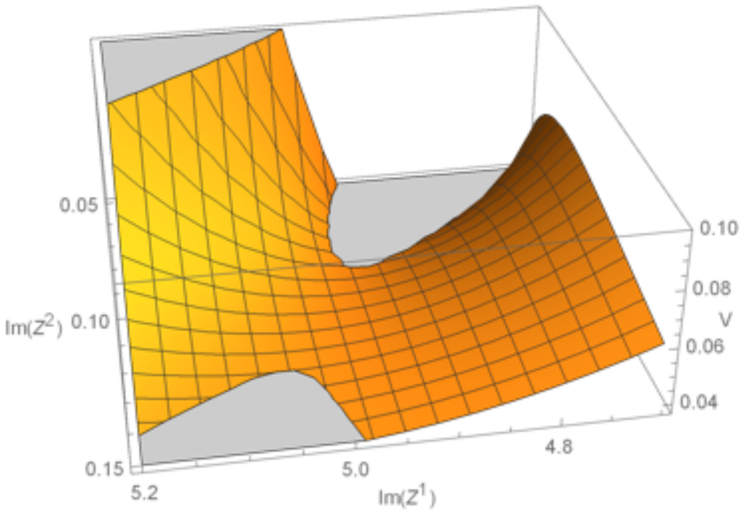
Blaback, Danielsson, Dibitetto, Accelerated Universes from type IIA Compactifications, 2013

Danielsson, Shiu, Van Riet, Wrase A note on **obstinate tachyons** in classical dS solutions, 2013

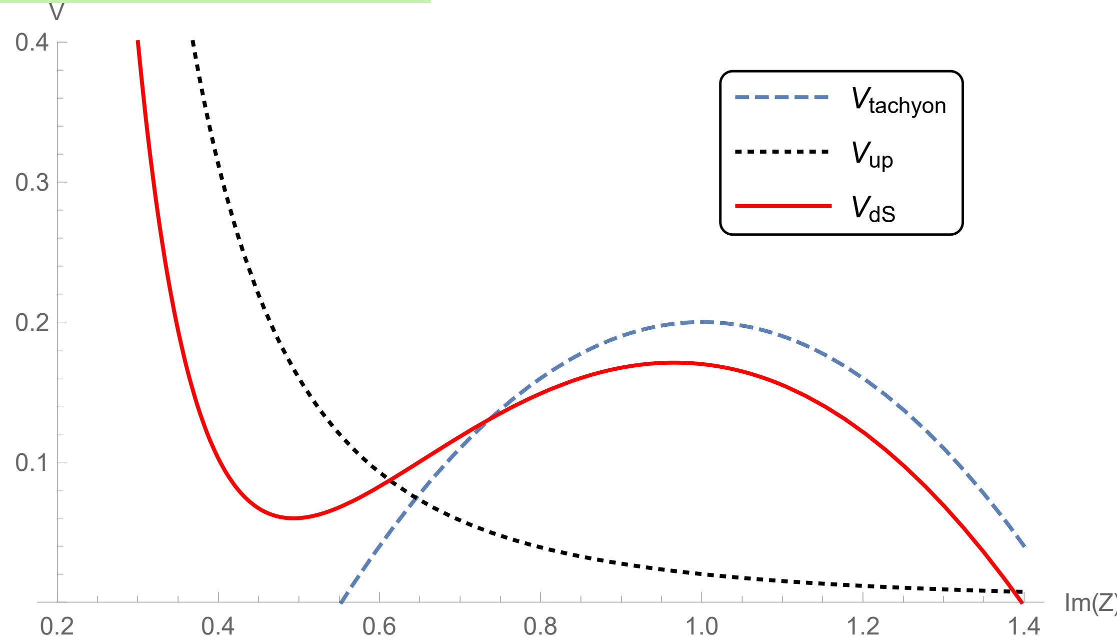
Junghans, Tachyons in Classical de Sitter Vacua, 2016

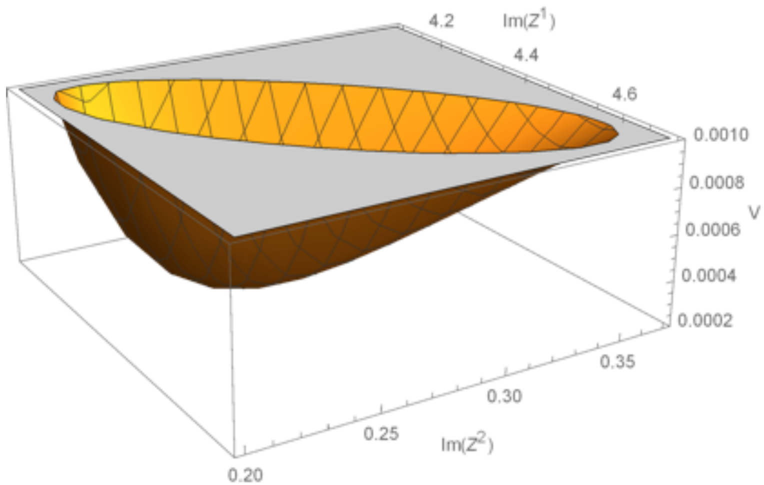
Junghans, Zagermann, A Universal Tachyon in Nearly No-scale de Sitter Compactifications, 2016

Anti-D6-uplift converts a dS saddle point into dS minimum

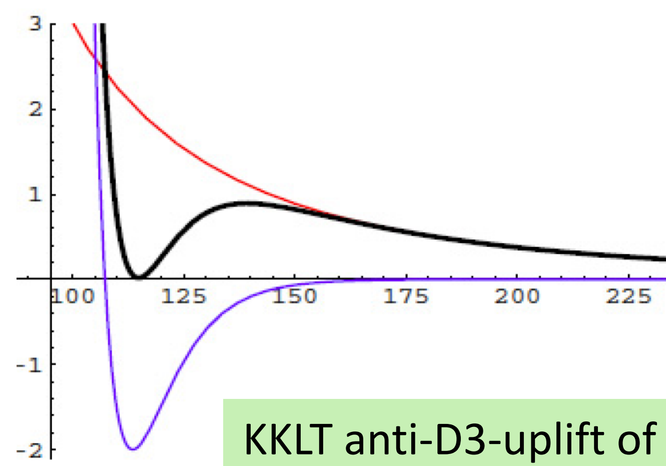


obstinate tachyon





dS minimum



KKLT anti-D3-uplift of AdS generates dS minimum

New rules

for dS supergravity in d=4 derived using flux compactification of d=10 string theory models with local sources, including calibrated Dp-branes/O-planes +
pseudo-calibrated anti-Dp-branes wrapped on supersymmetric cycles

$$\overline{D9} \text{ on a 6-cycle}\,, \qquad \overline{D7} \text{ on 4-cycles}\,, \qquad \overline{D5} \text{ on 2-cycles}\,, \qquad \overline{D3} \text{ on a 0-cycle}$$

$$\cancel{\overline{D8} \text{ on 5-cycles}}\,, \qquad \overline{D6} \text{ on 3-cycles}\,, \qquad \cancel{\overline{D4} \text{ on 1-cycles}}\,,$$

$$\boxed{\begin{aligned} K^{\text{new}}(z^i, \bar{z}^i; S, \bar{S}) &= K(z^i, \bar{z}^i) + \underbrace{K_{S\bar{S}}(z^i, \bar{z}^i) S\bar{S}}_{}\,, \\ W^{\text{new}}(z^i, S) &= W(z^i) + \underbrace{\mu^2 S}_{}\,. \end{aligned}}$$

Modifications of K and W due to the presence of the nilpotent multiplet. The new action has a non-linearly realized $\mathcal{N} = 1$ supersymmetry, which is a hallmark of dS supergravity.

additional nowhere vanishing positive term in the potential, associated with Volkov-Akulov non-linearly realized supersymmetry

$$D_S W(z) \neq 0$$

$$V^{\rm new}(z^i, \bar z^i) = V(z^i, \bar z^i) + \underbrace{e^{K(z^i, \bar z^i)} |D_S W|^2}_{} ,$$

$$|D_S W|^2 \equiv D_S W K^{S\bar S}(z^i, \bar z^i) \overline{D_S W} \, .$$

Fermion vertices have

$$\frac{1}{|D_S W|^2}$$

couplings

Type II compactifications with calibrated sources and pseudo-calibrated anti-Dp-branes

$$S = S_{II} + N_{Op} S_{Op} + N_{Dp} S_{Dp} + N_{\overline{Dp}} S_{\overline{Dp}}$$

Integrated tadpole cancellation condition

$$\int dF_{8-p} - H \wedge F_{6-p} = -2^{p-5} N_{Op} + N_{Dp} - N_{\overline{Dp}}$$

the new bosonic action

$$S_{\rm dS-SUGRA} = S_{\rm standard-SUGRA} + 2 N_{\overline{Dp}} S^{\rm DBI}_{\overline{Dp}}$$

Additional highly non-trivial fermionic terms for all anti-Dp-branes a new contribution to the scalar potential in four dimensions in string frame is of the form

$$V_{\overline{Dp}} = 2N_{\overline{Dp},\alpha} T_{Dp} \int_{\Sigma_\alpha} d^{p-3}\xi e^{-\phi} \sqrt{\det\left(G + B - 2\pi\alpha' F\right)}$$

α labels the different $(p-3)$-cycles Σ_α that are wrapped by the anti-Dp-branes and T_{Dp} denotes their tension

Pseudo-calibrated anti-Dp-branes in type IIB

$$V_{\overline{Dp}} = 2N_{\overline{Dp},\alpha} T_{Dp} \int_{\Sigma_\alpha} d^{p-3}\xi e^{-\phi} \mathrm{Re}\left(e^{J+iB}\right)$$

Pseudo-calibrated anti-Dp-branes in type IIA

$$V_{\overline{D6}} = \frac{2N_{\overline{D6},K} T_{D6}}{\mathcal{V}_6^2} \int_{\Sigma_K} e^{3\phi} \sqrt{\mathcal{V}_6} \mathrm{Re}\Omega = 2N_{\overline{D6},K} T_{D6} e^{4\phi_4} \mathrm{Im}(Z^K)$$

Answer IIB

$$
\begin{aligned}
K &= K_{\text{before}} + \mathrm{i} e^{K_{\text{before}}} \frac{e^{-4\phi_4}}{(\Phi - \bar{\Phi})} S\bar{S} \\
W &= W_{\text{before}} + \mu^2 S \,,
\end{aligned}
$$

$$
\mathrm{Im}\Phi = \{N_{\overline{D3}}\mathrm{Im}\tau, N_{\overline{D5},\alpha}\mathrm{Im}t^{\alpha}, -N_{\overline{D7},\alpha}\mathrm{Im}T^{\alpha}, -N_{\overline{D9}}\mathrm{Im}\mathcal{T}\}
$$

Answer IIA

$$
\begin{aligned}
K &= K_{\text{before}} + \mathrm{i} e^{K_{\text{before}}} \frac{e^{-4\phi_4}}{N_{\overline{D6},K}(Z^K - \bar{Z}^K)} S\bar{S} \,, \\
W &= W_{\text{before}} + \mu^2 S \,.
\end{aligned}
$$

$\mu^4 = T_{Dp}$

Specific example with anti-D6 was shown in Figures before where the presence of anti-D6 removed the obstinate tachyon.

Anti-D3 uplifter is not an exceptional case anymore, as it was during the last 15 years.

For dark energy/inflation we need de Sitter or near de Sitter vacua. Now we found many more opportunities in the Landscape based on any pseudo-calibrated anti-Dp-brane which upon wrapping supersymmetric cycles of dimension (p-3) becomes an uplifter.

String theory models resulting in de Sitter supergravity in d=4 with non-linearly realized supersymmetry use any of such pseudo-calibrated anti-Dp-branes.

Non-perturbative string theory has Volkov-Akulov non-linearly realized symmetry on the world-volume of its extended objects, M-branes and D-branes, but not at the world-sheet of the string.

Arguably, the discovery of dark energy and an observational success of early universe inflation, associated with de Sitter or nearly de Sitter spaces, may be viewed as an experimental discovery of the Volkov-Akulov non-linearly realized supersymmetry from the sky