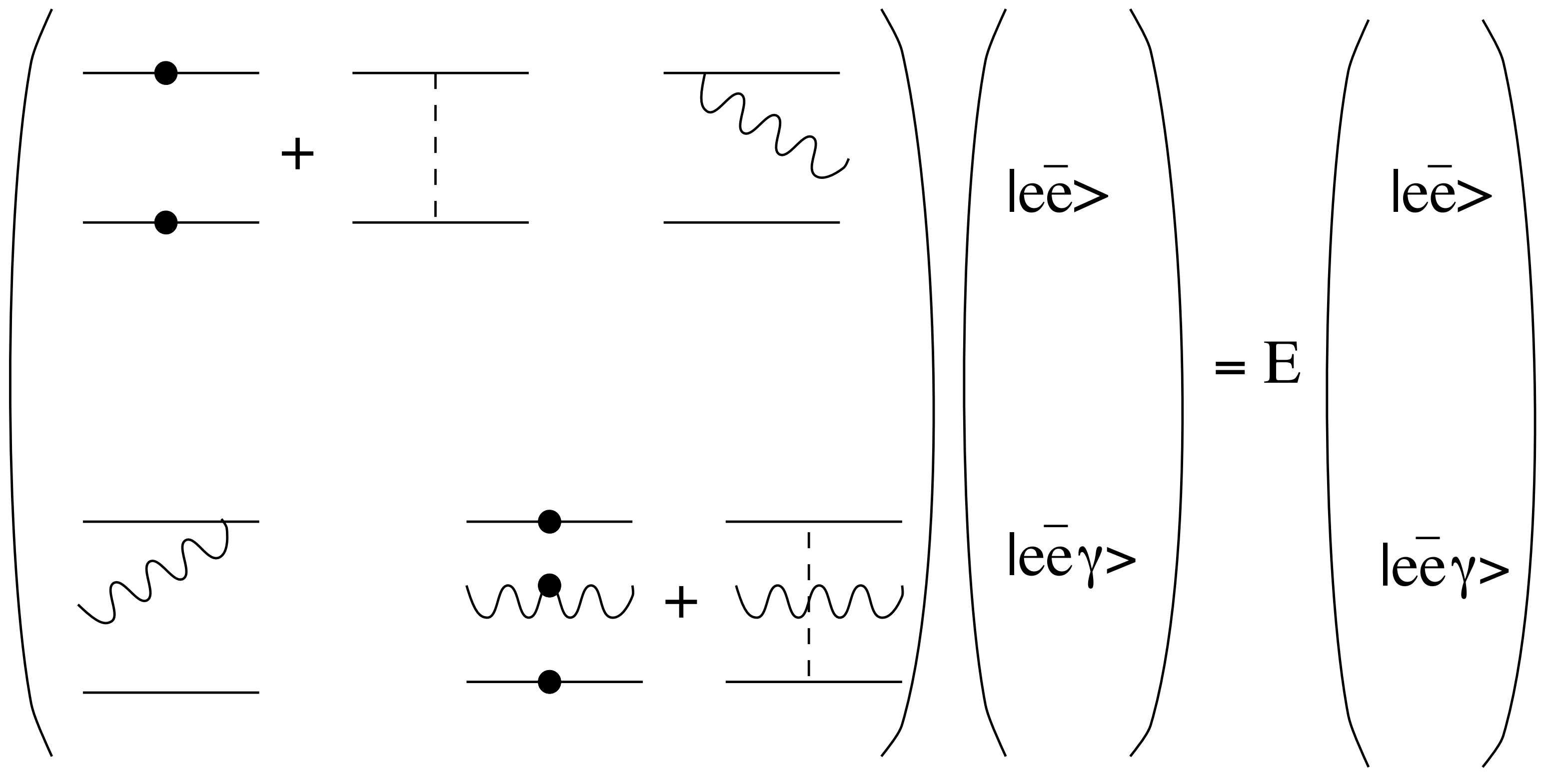
Gluonics

HYBRIDS

Hybrids

what is a hybrid?

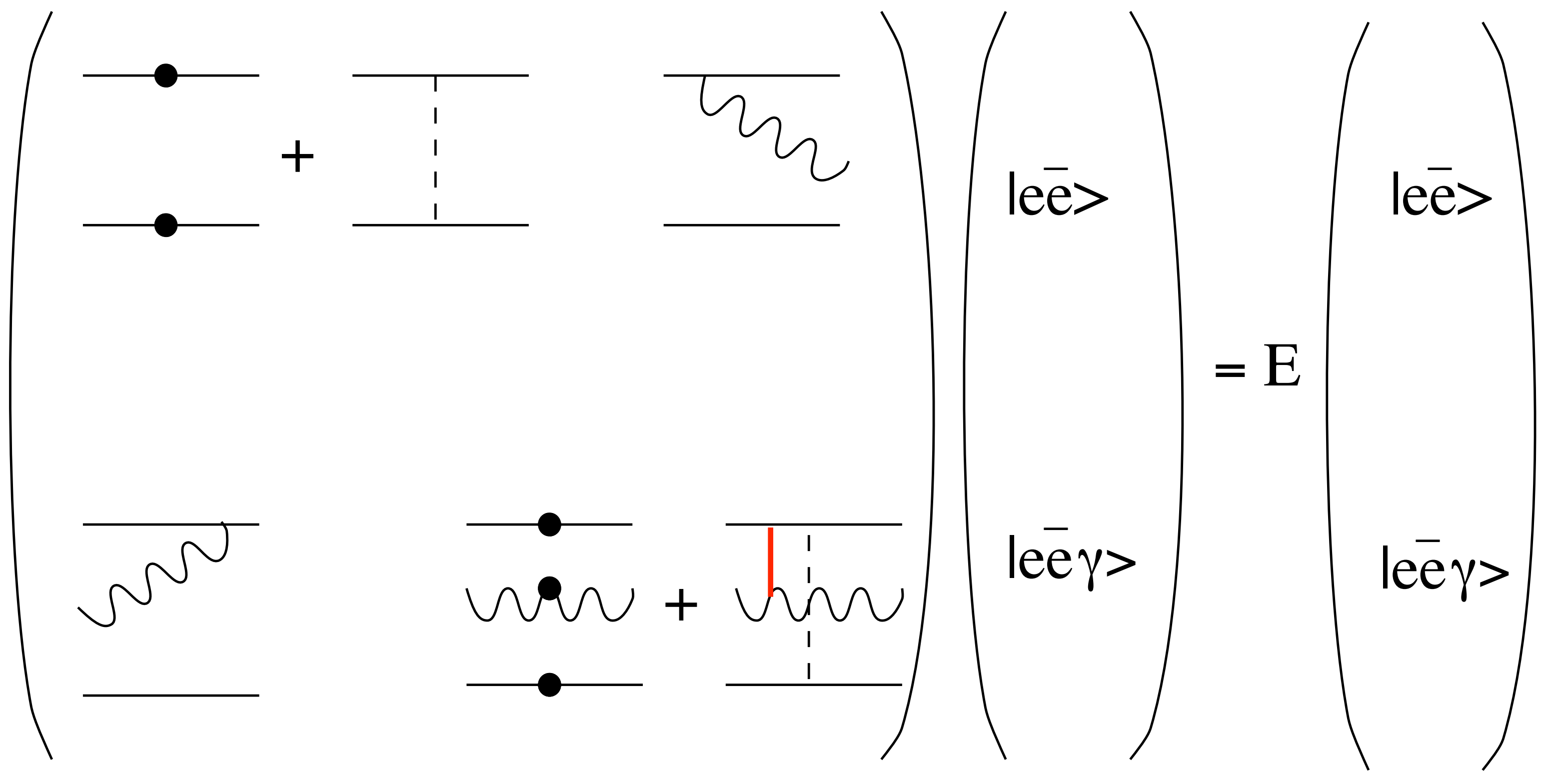


$$
\begin{aligned}
|0\rangle &= \sqrt{1-\epsilon^2}|e\bar{e}\rangle + \epsilon|e\bar{e}\gamma\rangle, \\
|1\rangle &= -\epsilon|e\bar{e}\rangle + \sqrt{1-\epsilon^2}|e\bar{e}\gamma\rangle.
\end{aligned}
$$

$\langle r|0\rangle \approx \phi(r)$
$\langle Rr|1\rangle \approx \phi(r)\psi(R)$

Hybrids

QCD



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\end{aligned}
$$

$\langle r|0\rangle \approx \phi(r)$
$\langle Rr|1\rangle \approx \phi(r)\psi(R)$

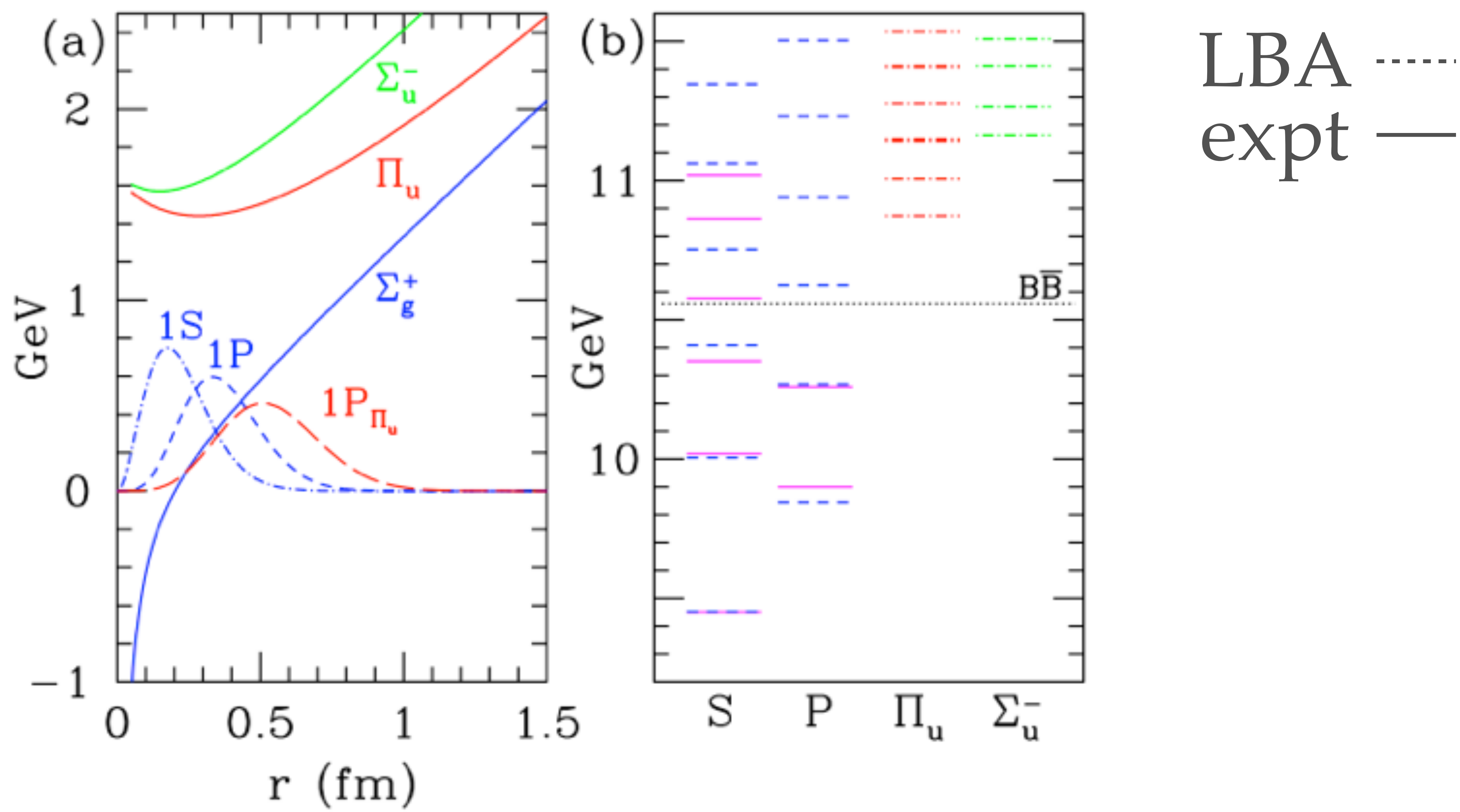
bound state models

Ingredients

- model quarks -> (constituent quarks, bag modes)
- model glue -> (constituent gluons, bag modes, flux tubes)
- model dynamics

Lattice Gauge Theory

JKM, nucl-th/0307116



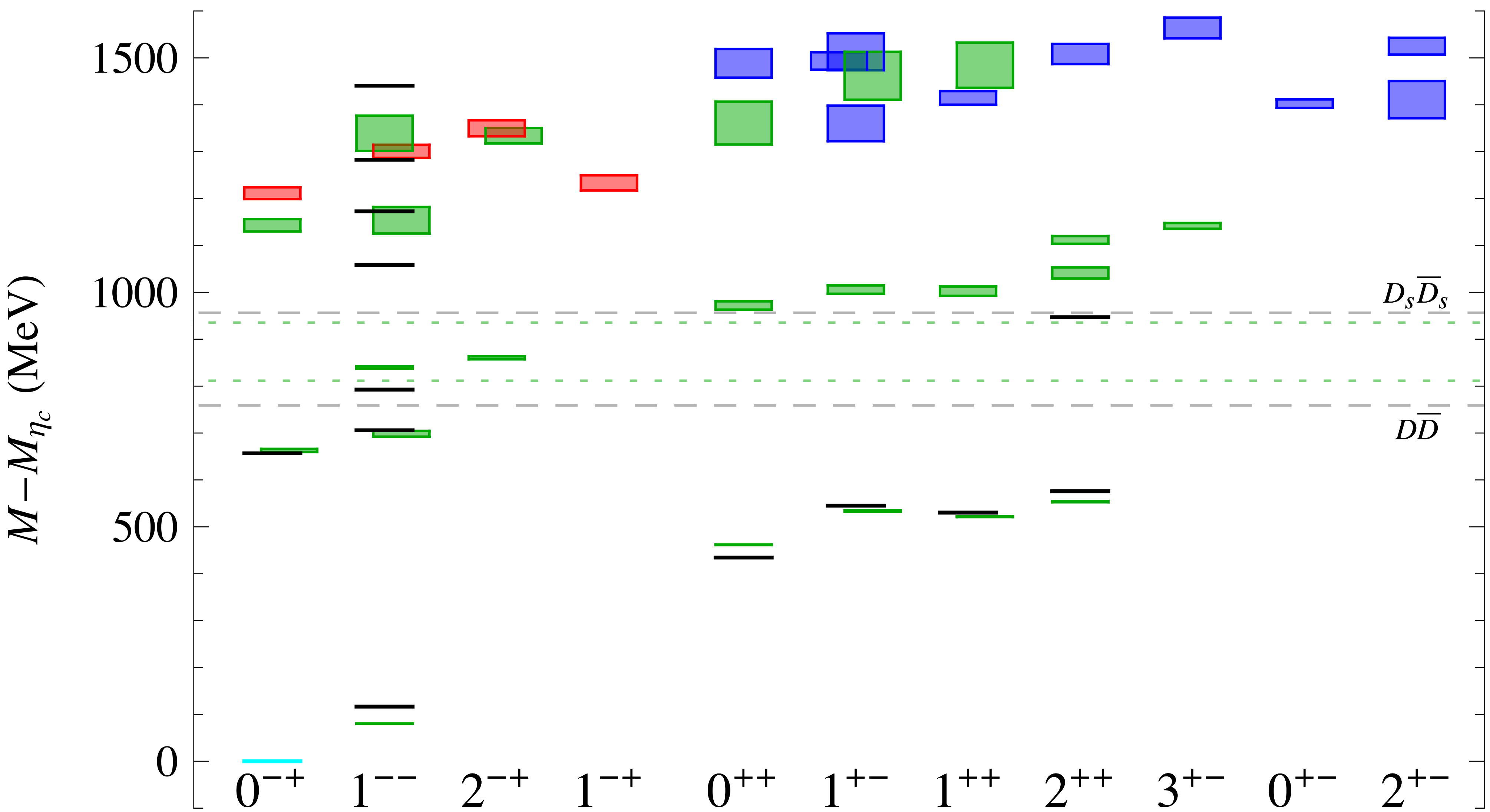
$$\Sigma_g^+(S) : 0^{-+}, 1^{--}$$

$$\Sigma_G^+(P) : 0^{++}, 1^{++}, 2^{++}, 1^{+-}$$

$$\Pi_u(P) : 0^{-+}, 0^{+-}, 1^{++}, 1^{--}, 1^{+-}, 1^{-+}, 2^{+-}, 2^{-+}$$

orange is hybrids ()
black = expt

$(J^{P\tilde{C}})_g = 1^{+-}$



S=0;L=0 =>0- + x 1+- = 1—; S=1;L=0 = 1— x1+- = (2,1,0)-+

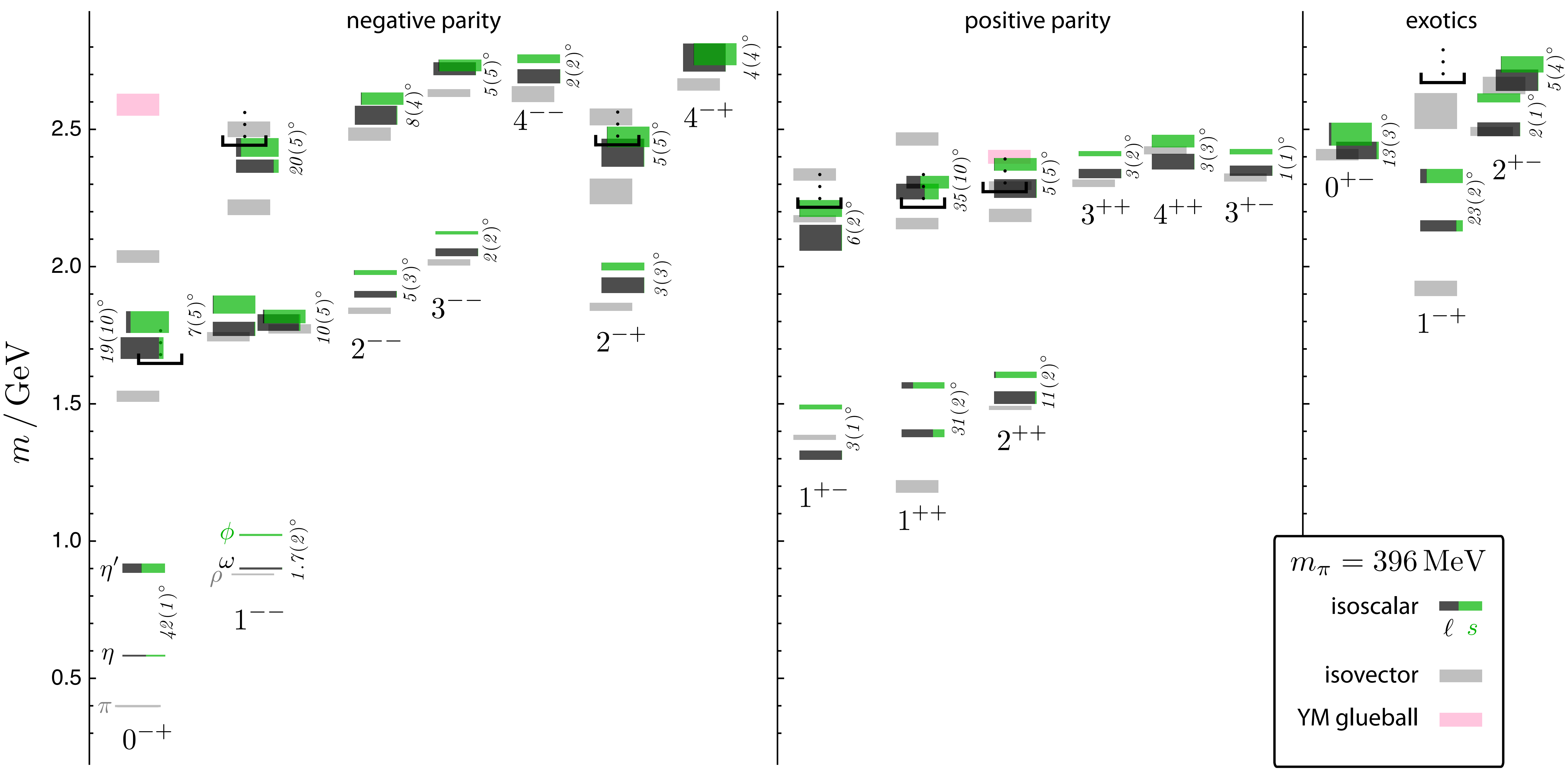
Exotic Theory: Lattice

light isos

Keh-Fei disagrees with the gluonic interpretation of the 1-+ 1202.2205

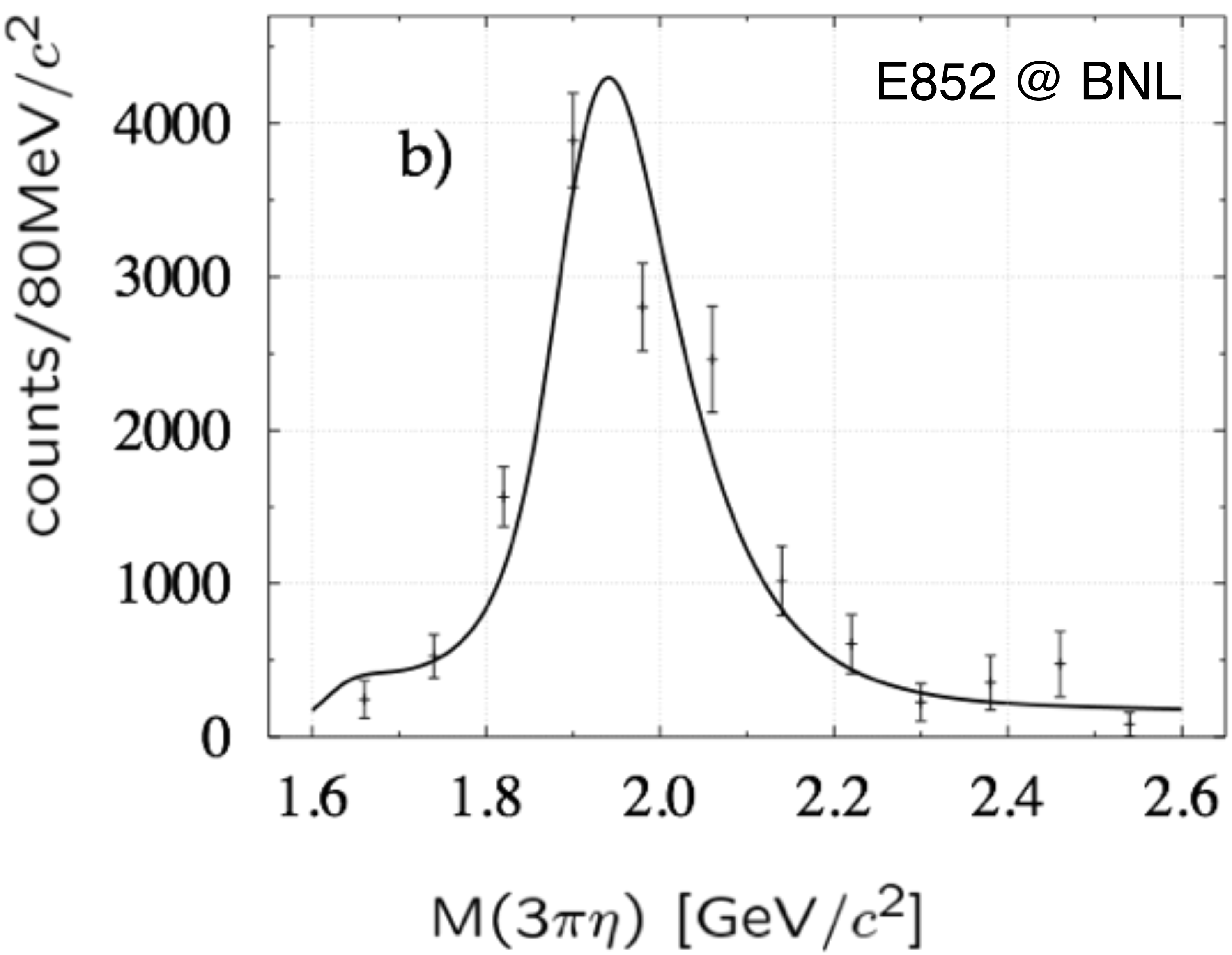
0++ missing!

Dudek et al. 1309.2608



Exotic Particles: Hybrids

$$\pi_2(1900) \to a_2(1320)\eta$$



Hybrids

Model	$u\bar{u}$	$s\bar{s}$	$c\bar{c}$	$b\bar{b}$
MIT Bag	1.3-1.8		$\sim$ 3.9	10.5
HHKR adiabatic bag			3.9	10.49(20)
QCD Sum Rules	2.1-2.5		4.1-5.3	10.6-11.2
Flux Tube	1.8-2.0		4.2-4.5	10.8-11.1
BCS	1.8-1.9	2.1-2.2	4.1-4.2	
lattice (UKQCD)		2.00(20)		
lattice (MILC)	1.97(9)(30)[a]	2.17(8)(20)	4.39(8)(20)	
lattice (adiabatic)			4.2	10.8
lattice (adiabatic)				10.8
lattice (NRQCD)				11.10(16)

tion.

if one used a massive gluon (with longitudinal dof) it would be possible to make a J=1 (gg) glueball w/ mass ~ 2 GeV. With transverse gluons this must be a (ggg) state with mass ~ 3 GeV.

Gluonic quasiparticles

- construct an efficient basis with the aid of a variational vacuum Ansatz.

$$\Psi_0[\mathbf{A}] = \langle \mathbf{A}|\omega\rangle = \exp\left[-\frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^3}\mathbf{A}^a(\mathbf{k})\omega(k)\mathbf{A}^a(-\mathbf{k})\right]$$

Schiff, Rosen,
Barnes and Ghandour

- this is equivalent to the Hartree Fock Bogoliubov transformation

$$\mathbf{A}^c(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{\sqrt{2\omega(k)}}\left[\boldsymbol{\epsilon}(\mathbf{k},\lambda)\alpha(\mathbf{k},\lambda,c) + \boldsymbol{\epsilon}^*(\mathbf{k},\lambda)\alpha^\dagger(-\mathbf{k},\lambda,c)\right]e^{i\mathbf{k}\cdot\mathbf{x}}$$

- determine ◗ via the gap equation

$$\frac{\delta}{\delta\omega}\langle\omega|H|\omega\rangle = 0$$

Gluonic quasiparticles

- a vital point:

THE QUASIPARTICLE INTERACTION DEPENDS ON

$$K^{ab}(\mathbf{x},\mathbf{y}) = \langle \mathbf{x}, a|(\boldsymbol{\nabla}\cdot\mathbf{D})^{-1}(-\boldsymbol{\nabla}^2)(\boldsymbol{\nabla}\cdot\mathbf{D})^{-1}|\mathbf{y}, b\rangle$$

- determine the gluonic quasiparticles and their interactions simultaneously: **COUPLED GAP EQUATIONS**

Szczepaniak and ES, hep-ph/0107078, Swift

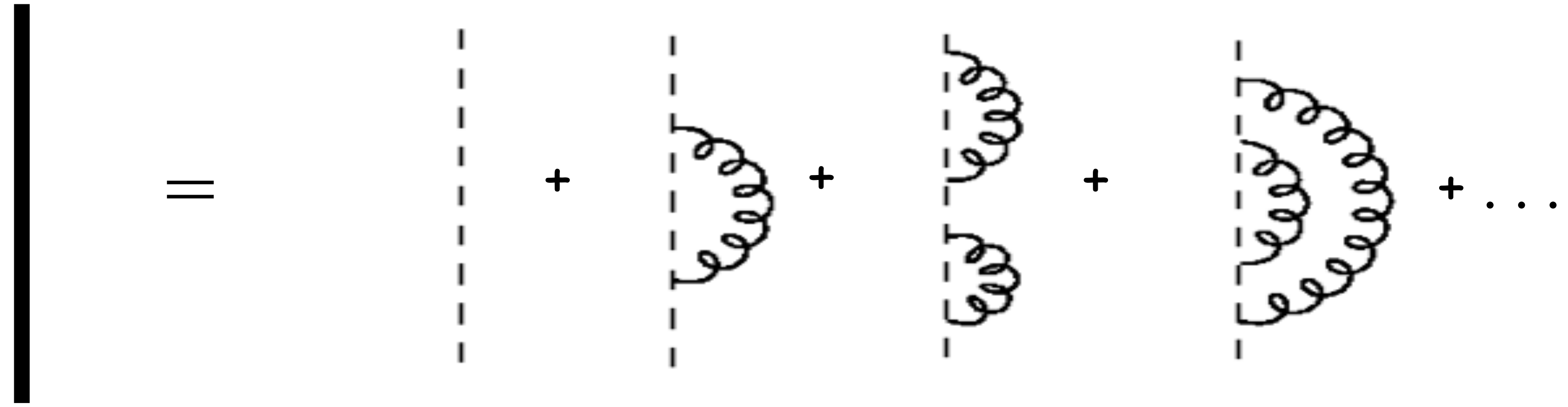
Szczepaniak and ES, Phys Rev D62, 094027 (2000)

Confinement in Coulomb gauge

- we need to evaluate <ω|K|ω>, we are aided by Swift's eqn:

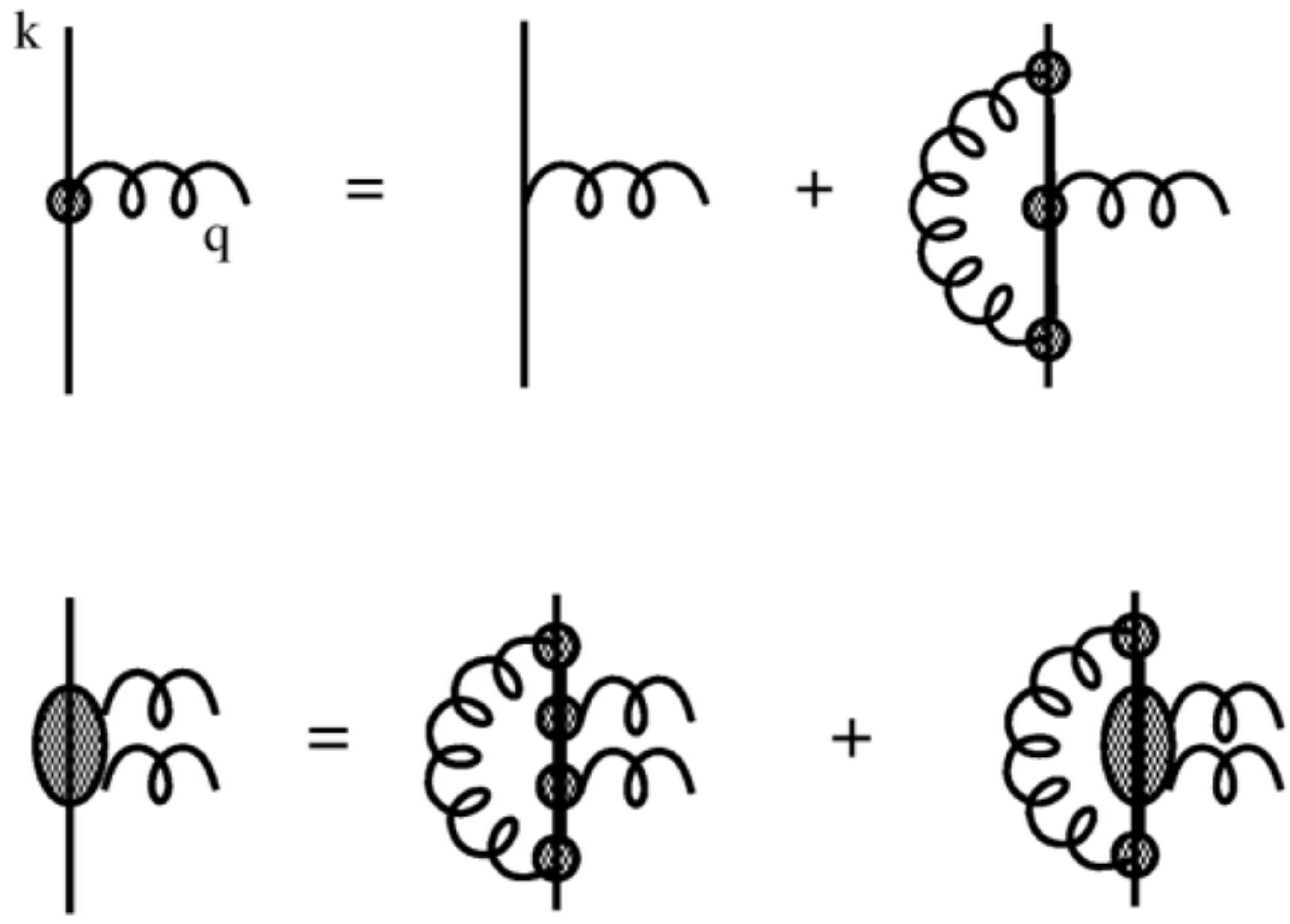
$$K_{ab}(\mathbf{x},\mathbf{y};\mathbf{A})_{\Lambda}, = g^2(\Lambda)\frac{d}{dg(\Lambda)}\langle \mathbf{x},a|\frac{g(\Lambda)}{\boldsymbol{\nabla}\cdot D}|\mathbf{x},b\rangle$$

- so we just need evaluate the vev of the Faddeev-Popov operator

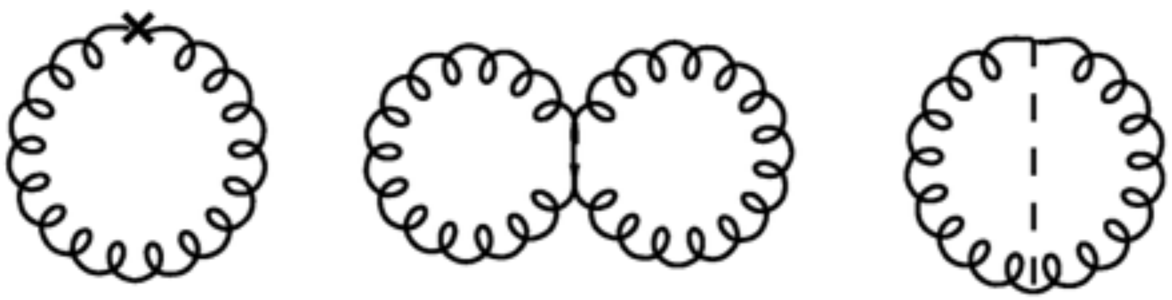


- we choose to sum in the rainbow ladder approximation. This appears to be justified in the IR limit (Swift)

vertex equations



Vacuum energy density

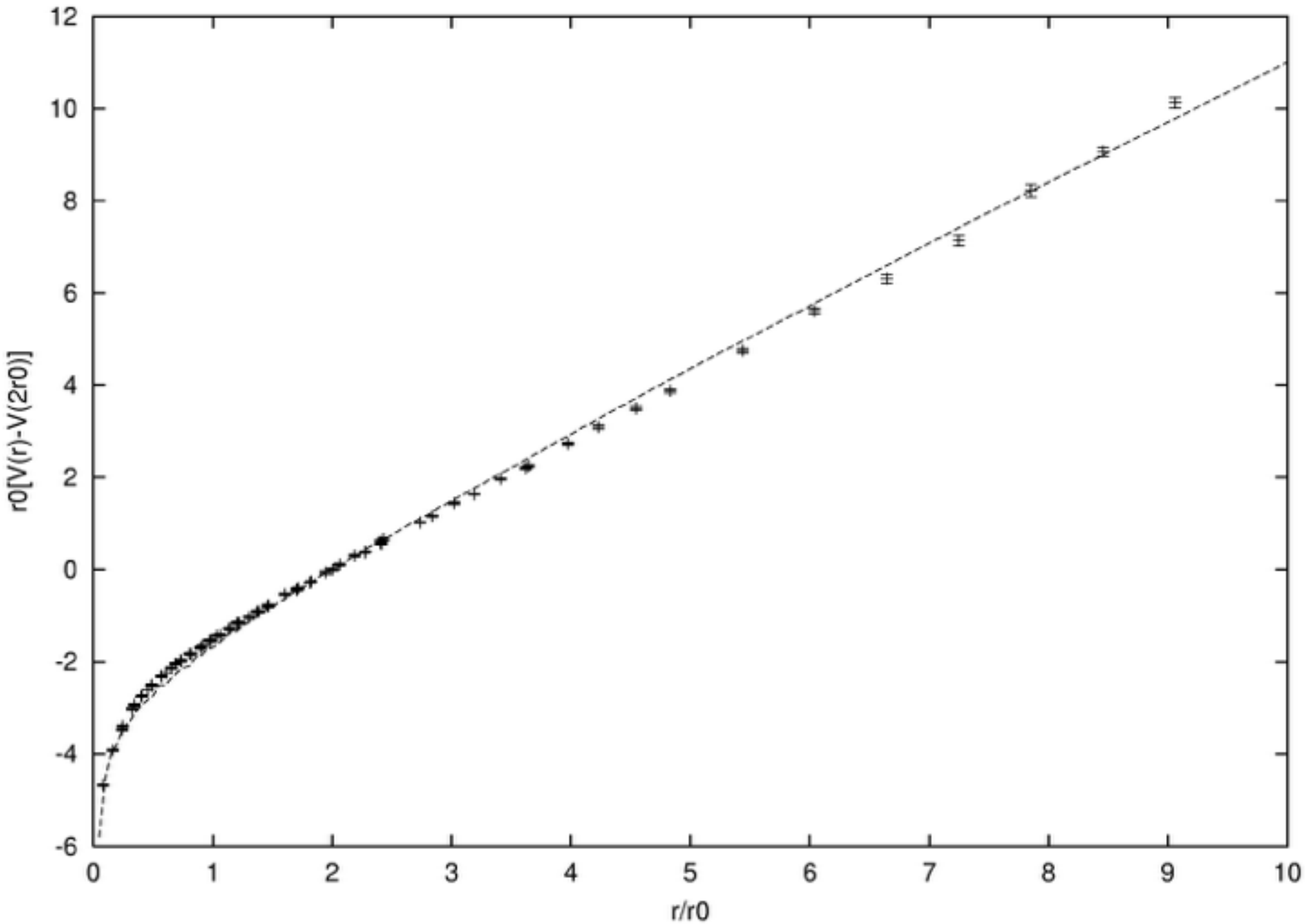


$$\boxed{Z_\Pi^2(\Lambda)}\,\omega^2(q;\Lambda) = \boxed{Z_A^2(\Lambda)}\,q^2 + \boxed{Z_m(\Lambda)\Lambda^2} + g^2(\Lambda)\frac{N_c}{4}\int^{\Lambda} \frac{d\mathbf{k}}{(2\pi)^3}\,\frac{(3-(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2)}{\omega(k;\Lambda)} +$$

$$+\frac{N_c}{4}\int^{\Lambda} \frac{d\mathbf{k}}{(2\pi)^3}\,\underbrace{\frac{f(\mathbf{k}+\mathbf{q};\Lambda)d^2(\mathbf{k}+\mathbf{q};\Lambda)}{(\mathbf{k}+\mathbf{q})^2}}_{\mathrm{K}^{(0)}}(1+(\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2)\,\frac{\omega^2(k;\Lambda)-\omega^2(q;\Lambda)}{\omega(k;\Lambda)}$$

$$\tilde{\mathrm{A}}_i^b(\mathbf{x}) = \int d^3y\,\frac{\Lambda^3}{(2\pi)^{3/2}}\mathrm{A}_i^b(\mathbf{y})\mathrm{e}^{-(\mathbf{x}-\mathbf{y})^2\frac{\Lambda^2}{2}} \qquad H(\Lambda) \to H(\Lambda) + \delta H(\Lambda) = H_{QCD}$$

cgQCD: quasiparticle potential



$m_g r_0 = 1.4$

Juge, Kuti, and Morningstar
Szczepaniak and ES, hep-ph/0107078

1. Bag Models

T. Barnes and F. E. Close, Phys. Lett. B 116, 365 (1982)
T. Barnes, F. E. Close and F. de Viron, Nucl. Phys. B 224, 241 (1983)
M. Chanowitz and S. Sharpe, Nucl. Phys. B 222, 211 (1983)

lowest mode is a 1+ TE gluon

the lightest multiplet is $0^{-+}, 1^{--}, 1^{-+}, 2^{-+}$

with mass of roughly 1.5 GeV

1. Bag Models

apply to gluelumps

Karl and Paton, PRD60, 034015 (99)

free gluon spectrum

$1^+, 2^-, 1^-, 3^+, 2^+, \ldots$

include Coulomb energy

$$
\begin{aligned}
1^+ &= 1.43 \text{ GeV} \\
2^- &= 1.97 \text{ GeV} \\
1^- &= 1.98 \text{ GeV} \\
3^+ &= 2.44 \text{ GeV} \\
2^+ &= 2.64 \text{ GeV}
\end{aligned}
$$

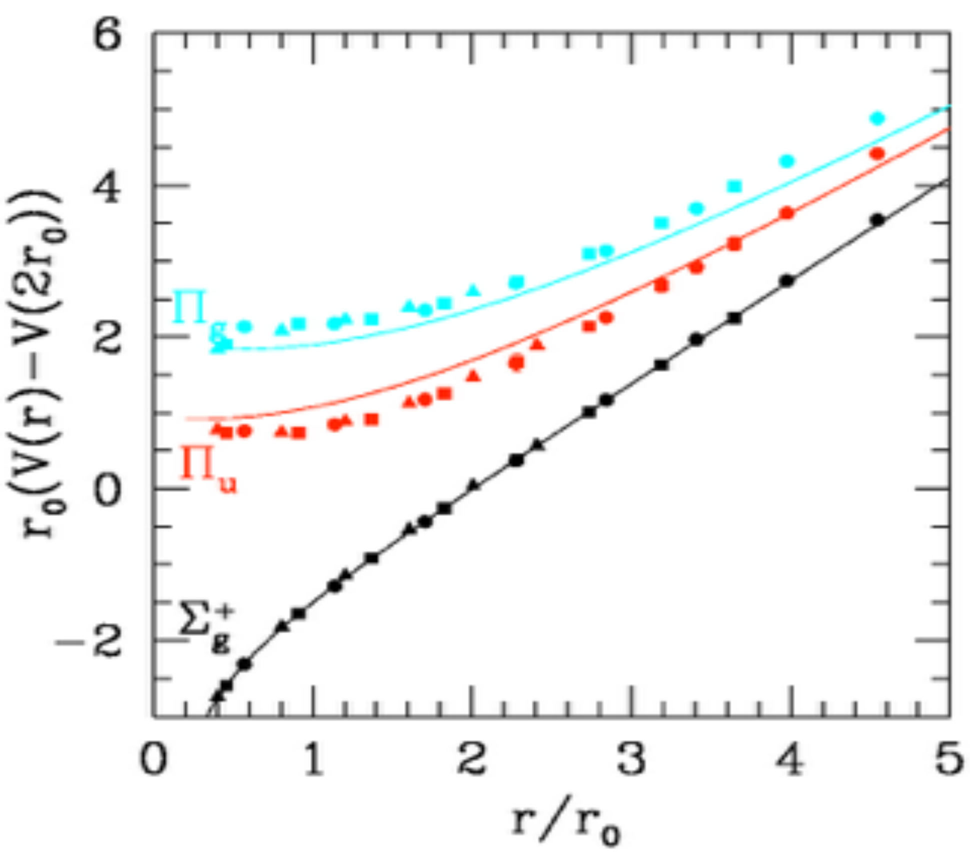
At small r, energies are shifted because a colour dipole develops:

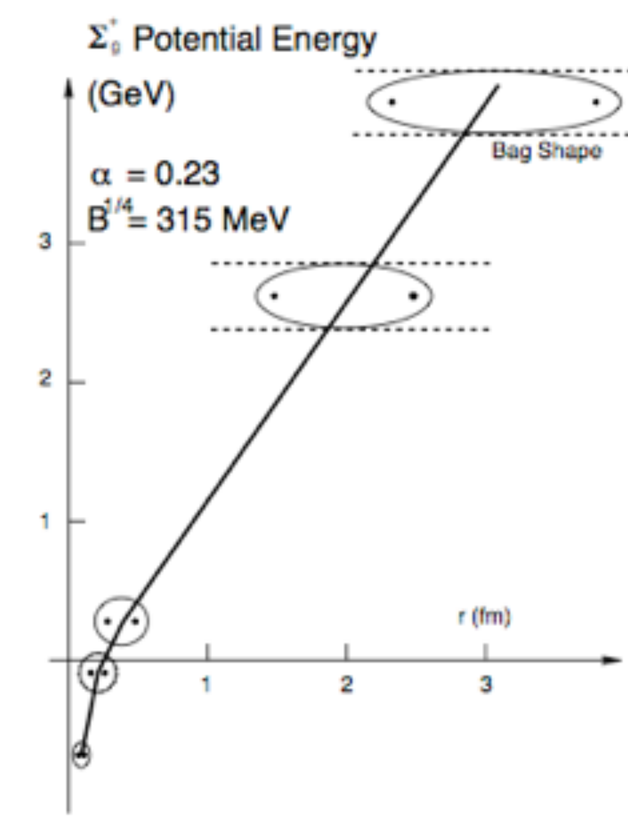
$$\delta E(r) = E(0) + \frac{9}{4}\alpha_s^2 \langle \frac{\cos^2\theta}{r^4} \rangle \frac{r2}{\bar{E}}$$

Bag Models

HHKR bag model computation

Juge, Kuti, and Morningstar, (98)



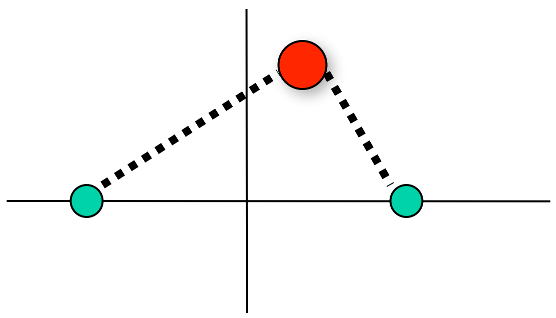


Constituent Glue Models

Constituent quarks plus pointlike, massless, spinless, colorless 'glue'.

Barnes, PhD thesis, Caltech, (77)

D. Horn and J. Mandula, PRD 17, 898 (78)



Neglect the repulsive short range colour octet interaction, $V_{\underline{8}} = +\frac{\alpha_s}{6r}$

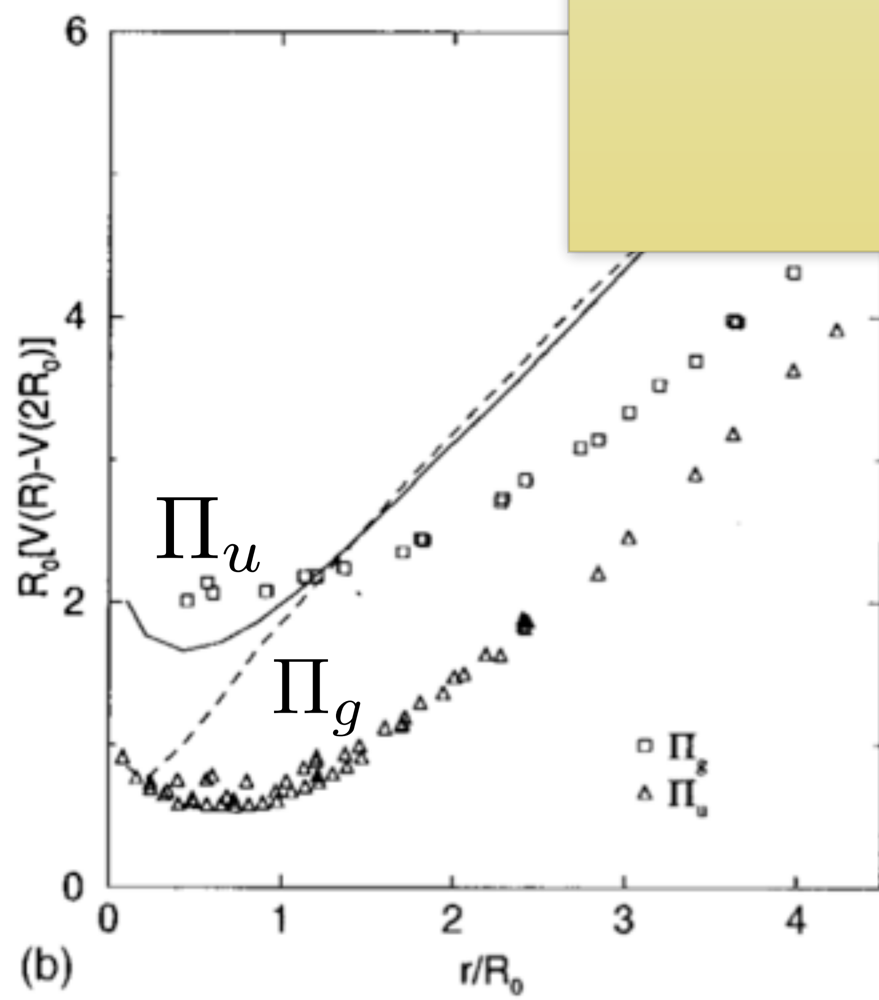
Constituent Glue Models

Swanson and Szczepaniak, PRD59, 014035 (98)

Coulomb gauge QCD, transverse gluons with colour, spin, and dynamically generated mass.

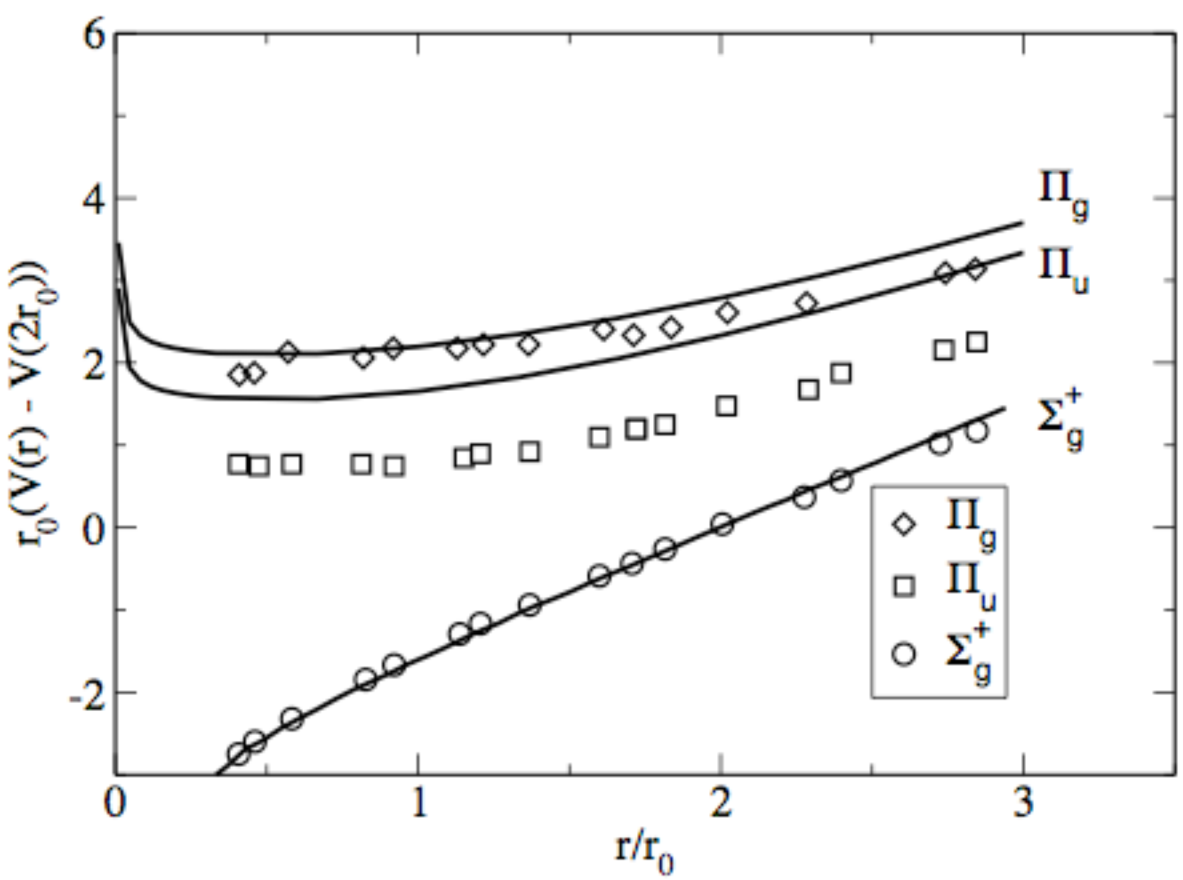
-- level ordering problem

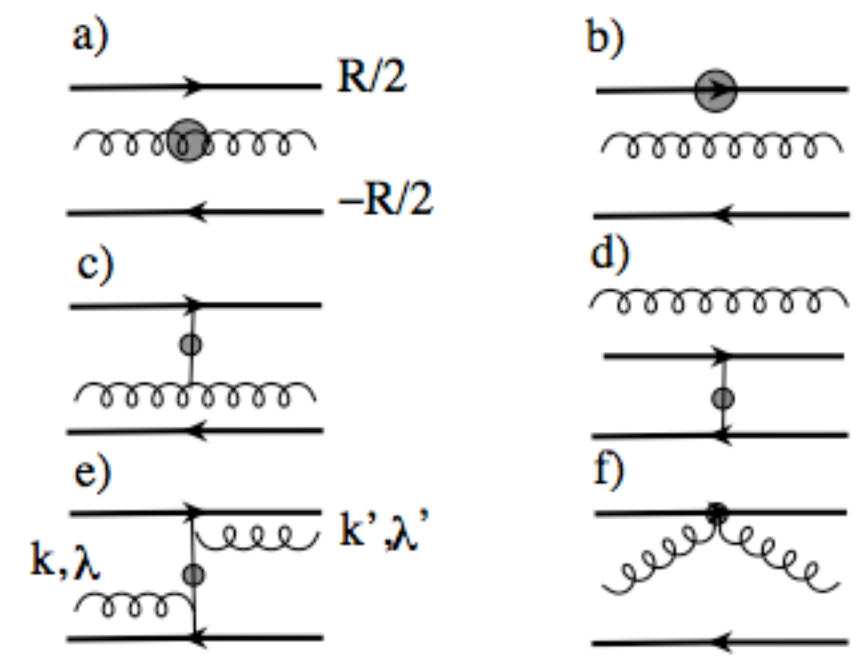
throw a gluon into a QQ: one expects this gives the first surface Pi_g quantum number (eg 1-- => PC=+ and L=1). But it is Pi_u



Constituent Glue Models

Szczepaniak and Krupinski, hep-ph/0604098





Hybrid State

$$|LM_L; s\bar{s}; \Lambda\{n_{m+}, n_{m-}\}\rangle \propto \int d^3r \phi(r) D^L_{M_L\Lambda}(\hat{r}) b^\dagger_{r/2,s} d^\dagger_{-r/2,\bar{s}} \prod_m (\alpha^\dagger_{m+})^{n_{m+}} (\alpha^\dagger_{m-})^{n_{m-}} |0\rangle$$

$$\Lambda = \sum_m^\infty (n_{m+} - n_{m-})$$

$$E = E_0 + N\frac{\pi}{R}$$

$$N = \sum_{m=1}^\infty m(n_{m+} + n_{m-})$$

Hybrid Quantum Numbers

$$P|LM_L; SM_S; \Lambda\{n_{m+}, n_{m-}\}\rangle = (-)^{L+\Lambda+1} |LM_L; SM_S; -\Lambda\{n_{m-}, n_{m+}\}\rangle$$

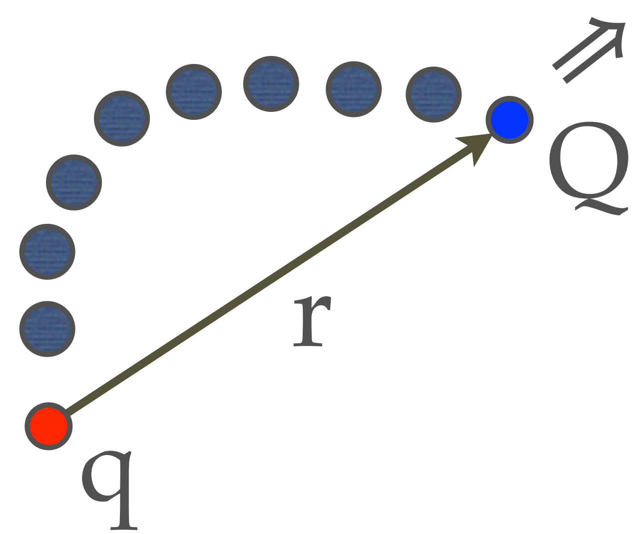
$$C|LM_L; SM_S; \Lambda\{n_{m+}, n_{m-}\}\rangle = (-)^{L+S+\Lambda+N} |LM_L; SM_S; -\Lambda\{n_{m-}, n_{m+}\}\rangle$$

$\Lambda + \zeta \; (-\Lambda)$

Adiabatic Surface Quantum Numbers

the diatomic molecule

$\Lambda = \Sigma, \Pi, \Delta, \ldots$

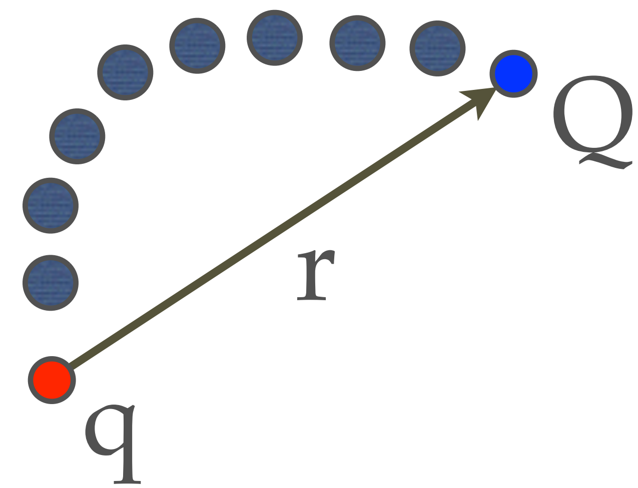


projection of the string angular momentum onto the $q\bar{q}$ axis

Adiabatic Surface Quantum Numbers

the diatomic molecule

η = u/g

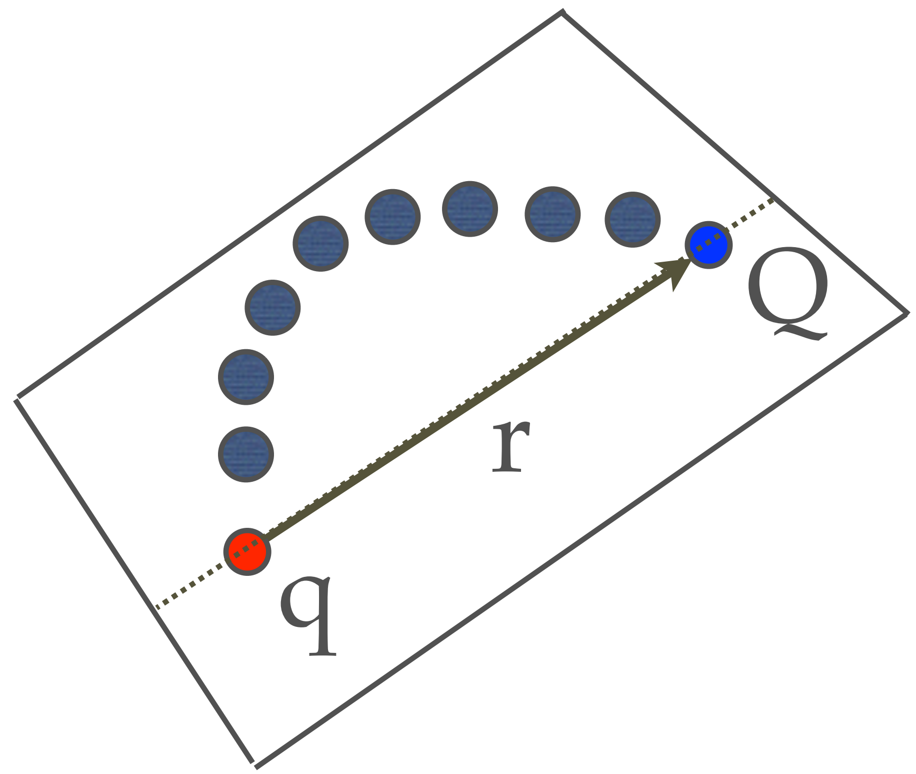


PC acting on the glue

Adiabatic Surface Quantum Numbers

the diatomic molecule

$Y = +/-$



reflection in a plane containing the qq axis

ex: $\Lambda^{Y}_{\eta} = \Sigma^{+}_{g}$ = ground state

FTM Model Hamiltonian

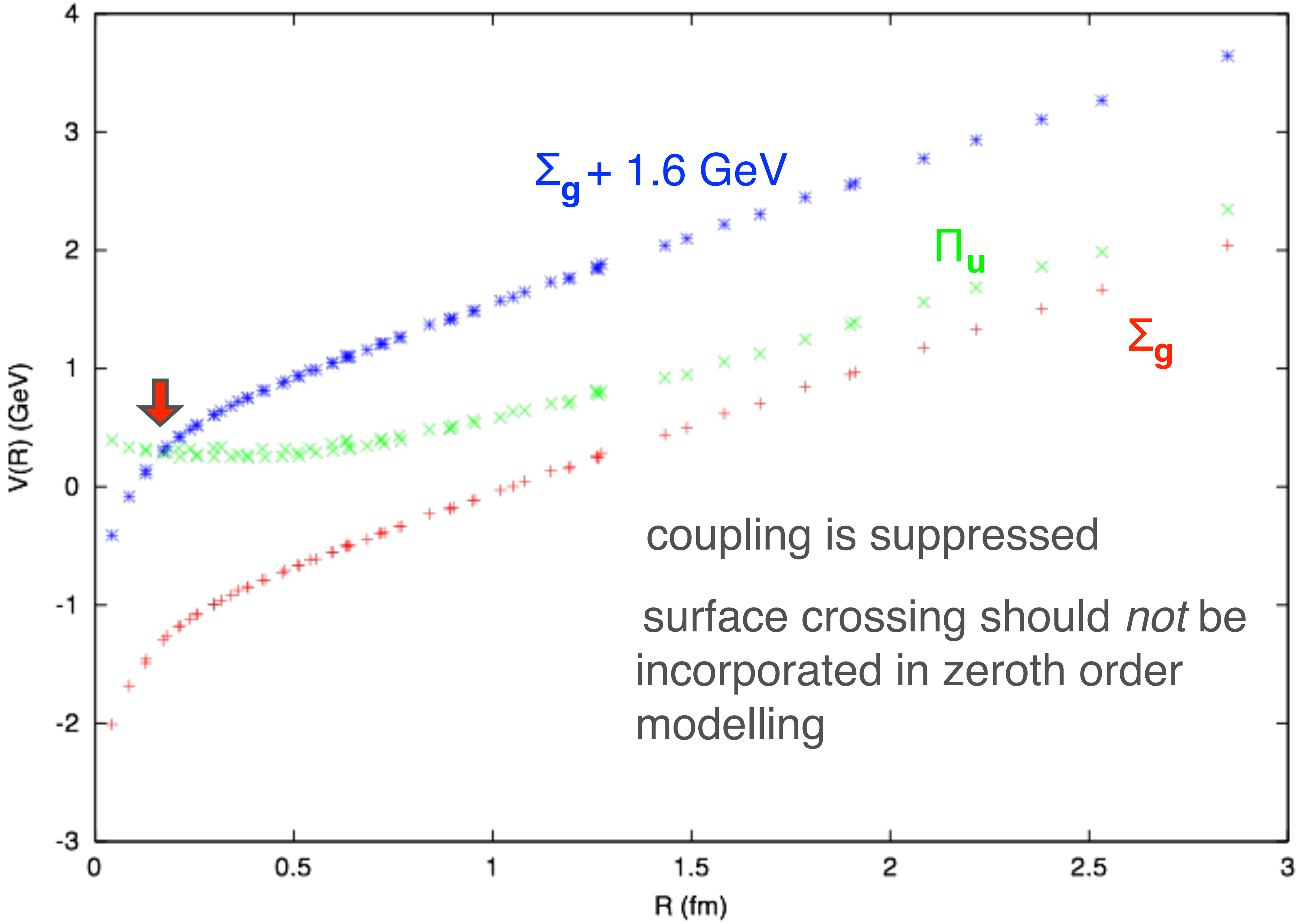
$$H_{FTM} = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L(L+1) - \Lambda^2 + \cancel{\langle L^2_{S_\perp}\rangle}}{2\mu r^2} - \frac{4\alpha_s}{3r} + br + \frac{\pi}{r}(1 - e^{-f\sqrt{br}})$$

I&P Hybrid Masses

flavour	m	m′
I=1	1.67	1.9
I=0	1.67	1.9
$s\bar{s}$	1.91	2.1
$c\bar{c}$	4.19	4.3
$b\bar{b}$	10.79	10.8

$2^{\pm\mp}, 1^{\pm\mp}, 0^{\pm\mp}, 1^{\pm\pm}$

Should the Coulomb potential be there?



Born-Oppenheimer + lattice potentials

$$H_{LGT} = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L(L+1) - 2\Lambda^2 + \cancelto{2}{\langle J_g^2 \rangle}}{2\mu r^2} + V_{\text{lat}}$$

flavour	m	m′	lat
I=1	1.67	1.9	1.85
I=0	1.67	1.9	1.85
$s\bar{s}$	1.91	2.1	2.07
$c\bar{c}$	4.19	4.3	4.34
$b\bar{b}$	10.79	10.8	10.85

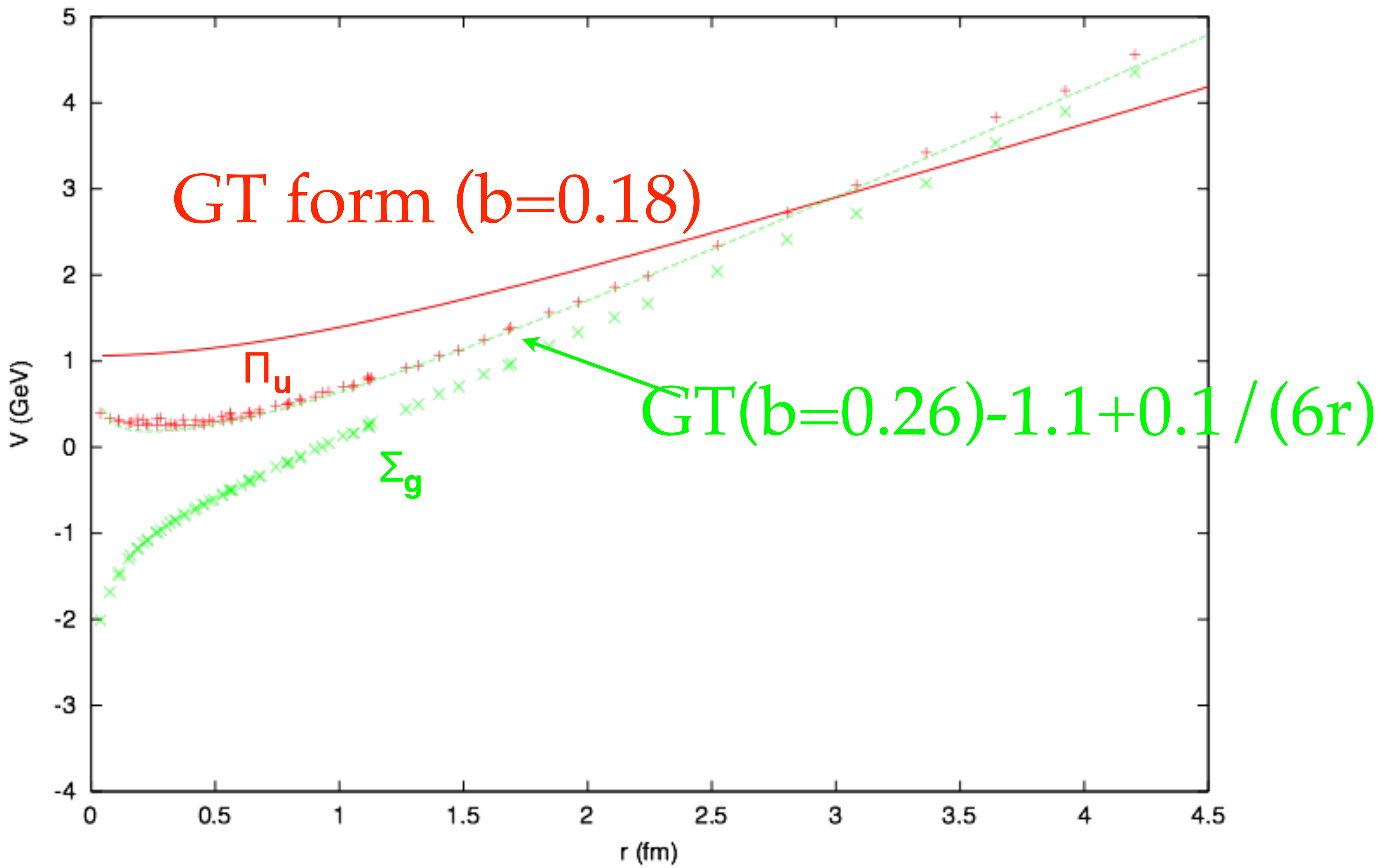
K. Waidelich

$$V_N = \sigma r \left(1 + \frac{2N\pi}{\sigma r^2}\right)^{1/2}$$

GT

$$V_{NG} = \sigma r \left(1 - \frac{D-2}{12\sigma r^2} + \frac{2N\pi}{\sigma r^2}\right)^{1/2}$$

J.F. Arvis, PLB127, 106 (83); Luescher



GT also computed finite-mass corrections, and spin-orbit splittings.

16 APPLICATION OF THE QUARK-C

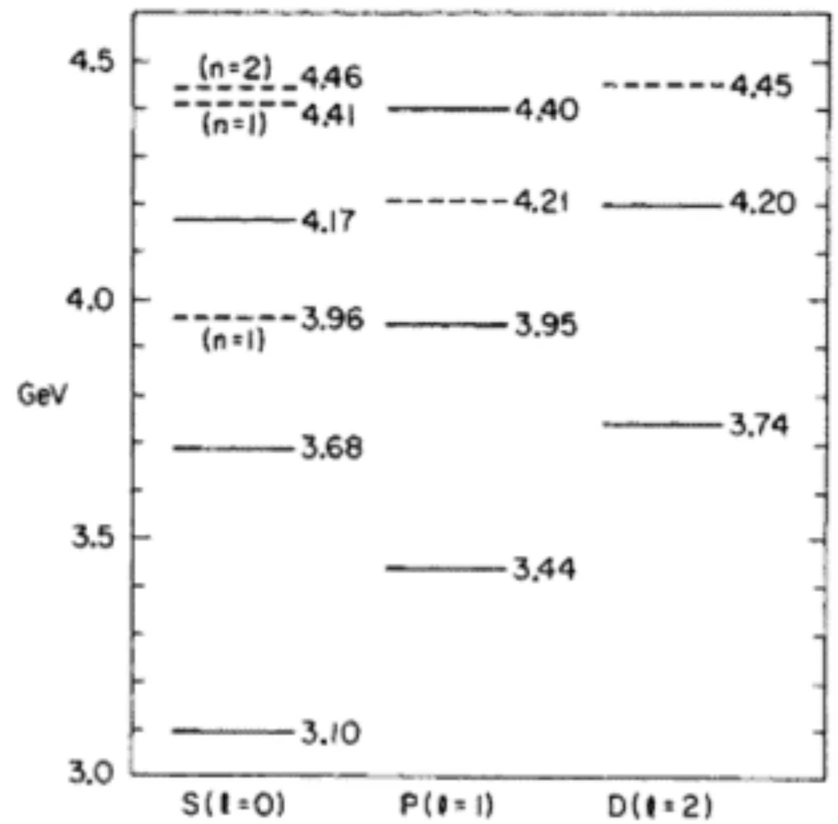
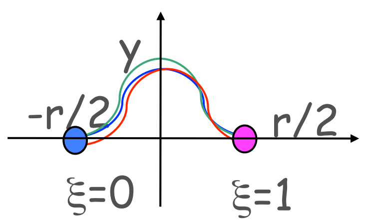


FIG. 4. The nonrelativistic spectroscopy of the charm string. $\psi(3.10)$ and $\psi(3.68)$ are fitted to obtain $M = 1.154$ GeV and $k = 0.21$ GeV^2. The dashed lines are the vibrational levels absent in the charmonium model. Levels with $E > 4.5$ GeV or $l > 2$ are not shown.

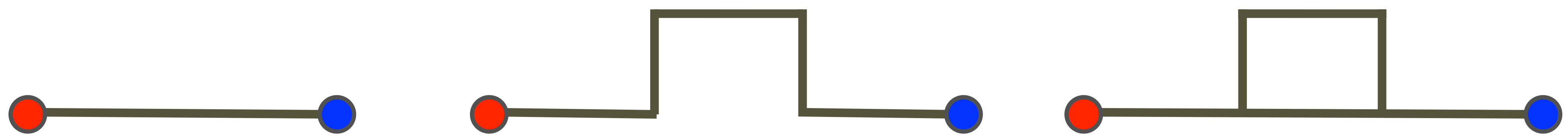
Flux Tube Model

Isgur and Paton, PRD31, 2910 (85).

strong coupling Hamiltonian lattice gauge theory



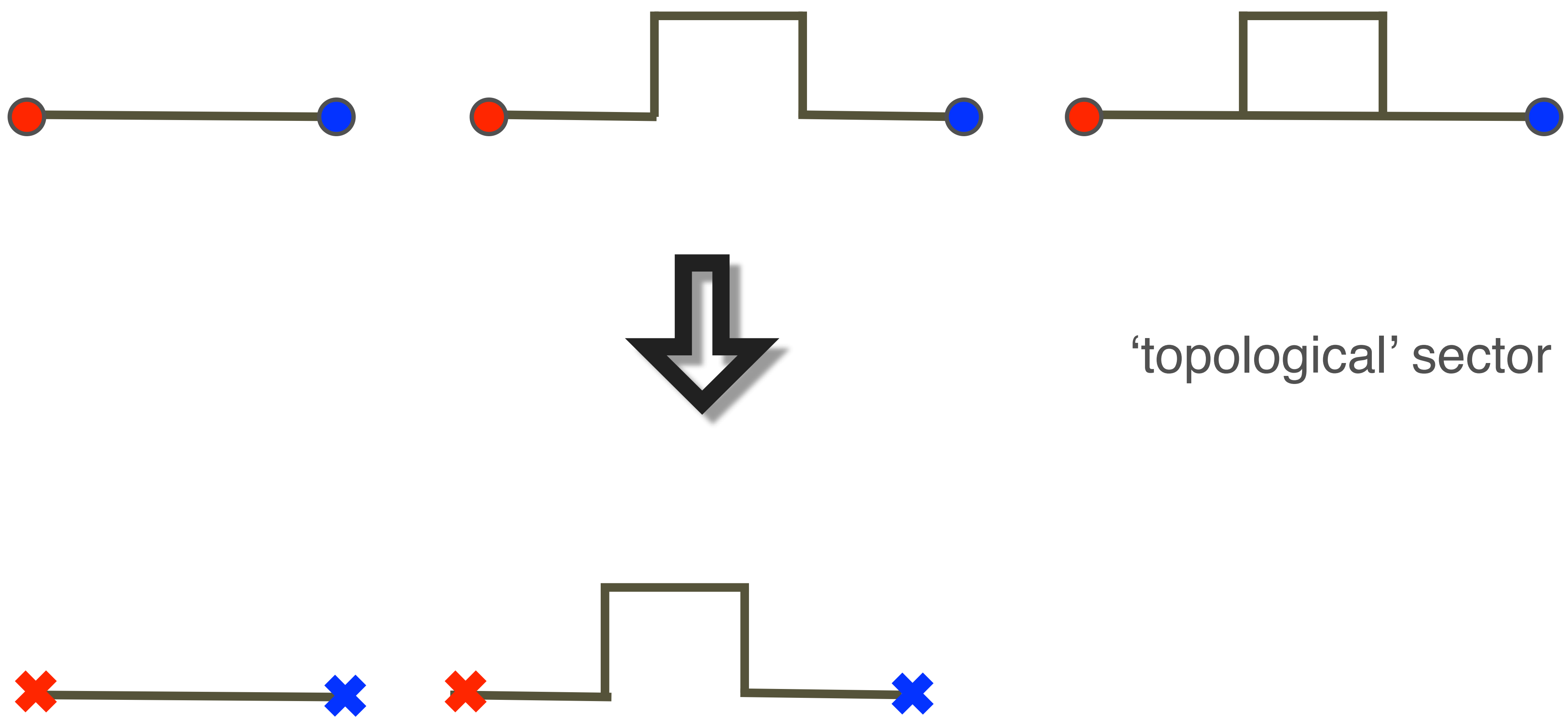
$$H = \frac{g^2}{2a}\sum_\ell E_\ell^a E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi_n^\dagger\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U_P^\dagger)$$



Flux Tube Model

Isgur and Paton, PRD31, 2910 (85).

$$H = \frac{g^2}{2a}\sum_{\ell} E^a_\ell E_{a\ell} + \sum_n m\bar{\psi}_n\psi_n + \frac{1}{a}\sum_{n,\mu}\psi^\dagger_n\alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U^\dagger_P)$$

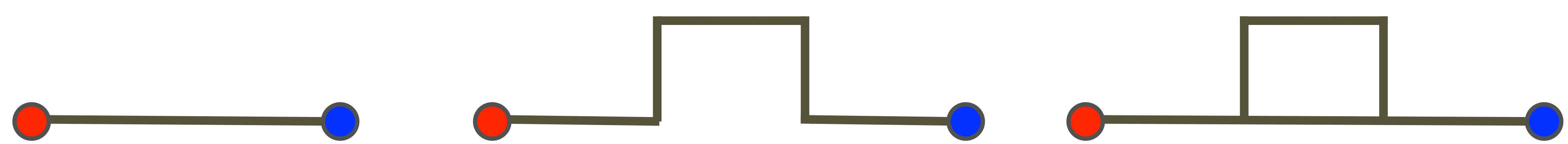


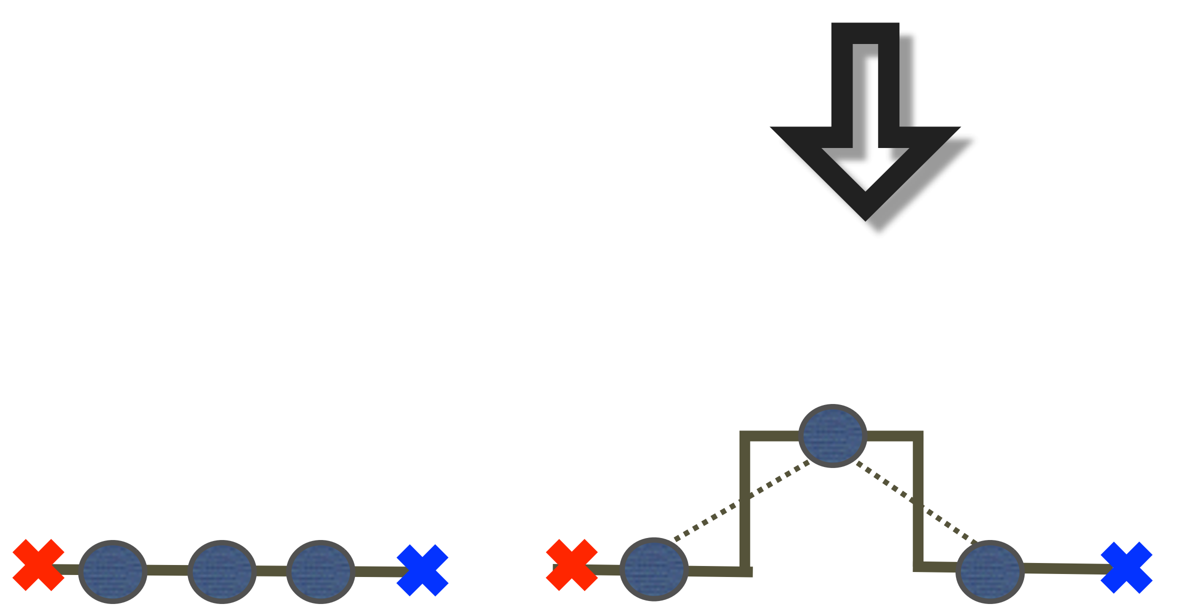
'topological' sector

Flux Tube Model

Isgur and Paton, PRD31, 2910 (85).

$$H = \frac{g^2}{2a}\sum_{\ell} E^a_{\ell} E_{a\ell} + \sum_n m\bar{\psi}_n \psi_n + \frac{1}{a}\sum_{n,\mu} \psi^\dagger_n \alpha_\mu U_\mu(n)\psi_{n+\mu} + \frac{1}{ag^2}\sum_P \mathrm{tr}(N - U_P - U_P^\dagger)$$





adiabatic

small oscillation

nonrelativistic beads

$m_b = b\,a$

Flux Tube Model

string Hamiltonian

$$H = b_0 R + \sum_n \left[\frac{p_n^2}{2ba} + \frac{b}{2a}(y_n - y_{n+1})^2 \right]$$

$$s_{m\lambda} = \sum_n^N y_n(\lambda) \sqrt{\frac{2}{N+1}} \sin \frac{nm\pi}{N+1}$$

$$y_n(\lambda) = \sum_m s_{m\lambda} \sqrt{\frac{2}{N+1}} \sin \frac{nm\pi}{N+1}$$

$$H = b_0 R + \sum_{n\lambda} \left[\frac{p_n^2}{2ba} + \frac{ba}{2} \omega_n^2 s_{n\lambda}^2 \right]$$

$$\omega_n = \frac{2}{a} \sin \frac{\pi n}{2(N+1)}$$

$$\omega_1 \to \frac{\pi}{R}$$

$$\alpha_{n\lambda} = \sqrt{\frac{b_0 \omega_n}{2}} s_{n\lambda} + i \frac{p_{n\lambda}}{\sqrt{b_0 a \omega_n}}$$

$$H = b_0 R + \sum_{n\lambda} \omega_n \left(\alpha_{n\lambda}^\dagger \alpha_{n\lambda} + \frac{1}{2} \right)$$

$$\boxed{H = b_0 R + \left(\frac{4}{\pi a^2} R - \frac{1}{a} - \frac{\pi}{12 R} \right) + \sum_{n\lambda} \omega_n \alpha_{n\lambda}^\dagger \alpha_{n\lambda}}$$

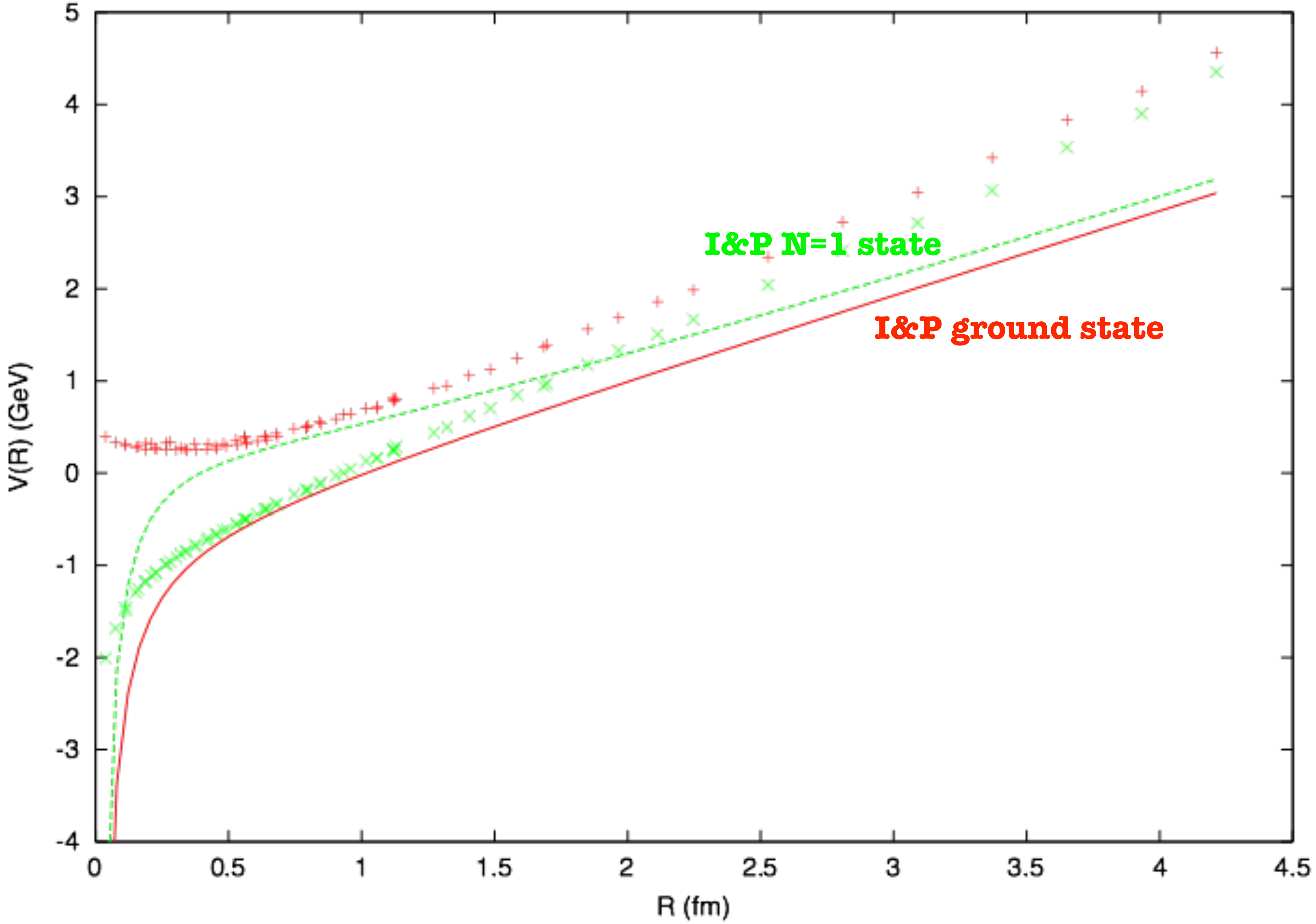
Flux Tube Model

$$H = b_0 R + \left(\frac{4}{\pi a^2} R - \frac{1}{a} - \frac{\pi}{12R} \right) + \sum_{n\lambda} \omega_n \alpha^\dagger_{n\lambda} \alpha_{n\lambda}$$

	L	S	J^{PC}
	1	0	1^{++}
	1	1	$(2,1,0)^{+-}$
$\zeta = +$	2	0	2^{--}
	2	1	$(3,2,1)^{-+}$
	1	0	1^{--}
$\zeta = -$	1	1	$(2,1,0)^{-+}$
	2	0	2^{++}
	2	1	$(3,2,1)^{+-}$

Flux Tube Model

Comparison to the lattice



Flux Tube Model

$$|LM_L; s\bar{s}; \Lambda\{n_{m+}, n_{m-}\}\rangle \propto \int d^3r\phi(r)D^L_{M_L\Lambda}(\hat{r})b^\dagger_{r/2,s}d^\dagger_{-r/2,\bar{s}}\prod_m(\alpha^\dagger_{m+})^{n_{m+}}(\alpha^\dagger_{m-})^{n_{m-}}|0\rangle$$

Hybrid State

$$\Lambda = \sum_m^\infty (n_{m+} - n_{m-})$$

$$E = E_0 + N\frac{\pi}{R}$$

$$N = \sum_{m=1}^\infty m(n_{m+} + n_{m-})$$

Hybrid Quantum Numbers

$$P|LM_L; SM_S; \Lambda\{n_{m+}, n_{m-}\}\rangle = (-)^{L+\Lambda+1}|LM_L; SM_S; -\Lambda\{n_{m-}, n_{m+}\}\rangle$$

$$C|LM_L; SM_S; \Lambda\{n_{m+}, n_{m-}\}\rangle = (-)^{L+S+\Lambda+N}|LM_L; SM_S; -\Lambda\{n_{m-}, n_{m+}\}\rangle$$

$\Lambda + \zeta \; (-\Lambda)$

4. Flux Tube (& String)Models

Giles and Tye, PRL37, 1175 (76).

Coupled quarks to a relativistic 2d sheet... the "Quark Confining String Model".

"The presence of vibrational levels gives ... extra states in quantum mechanics. ... that are absent in the charmonium model."

$$V_N = \sigma r \left(1 + \frac{2N\pi}{\sigma r^2}\right)^{1/2}$$ GT

$$V_{NG} = \sigma r \left(1 - \frac{D-2}{12\sigma r^2} + \frac{2N\pi}{\sigma r^2}\right)^{1/2}$$ J.F. Arvis, PLB127, 106 (83); Luescher

16 APPLICATION OF THE QUARK-C

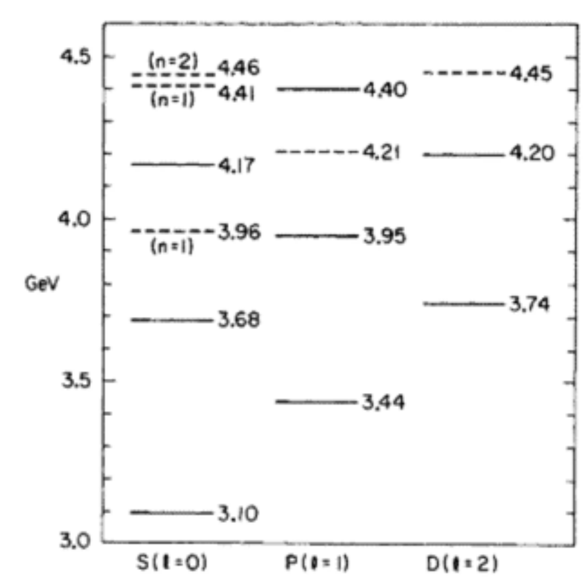
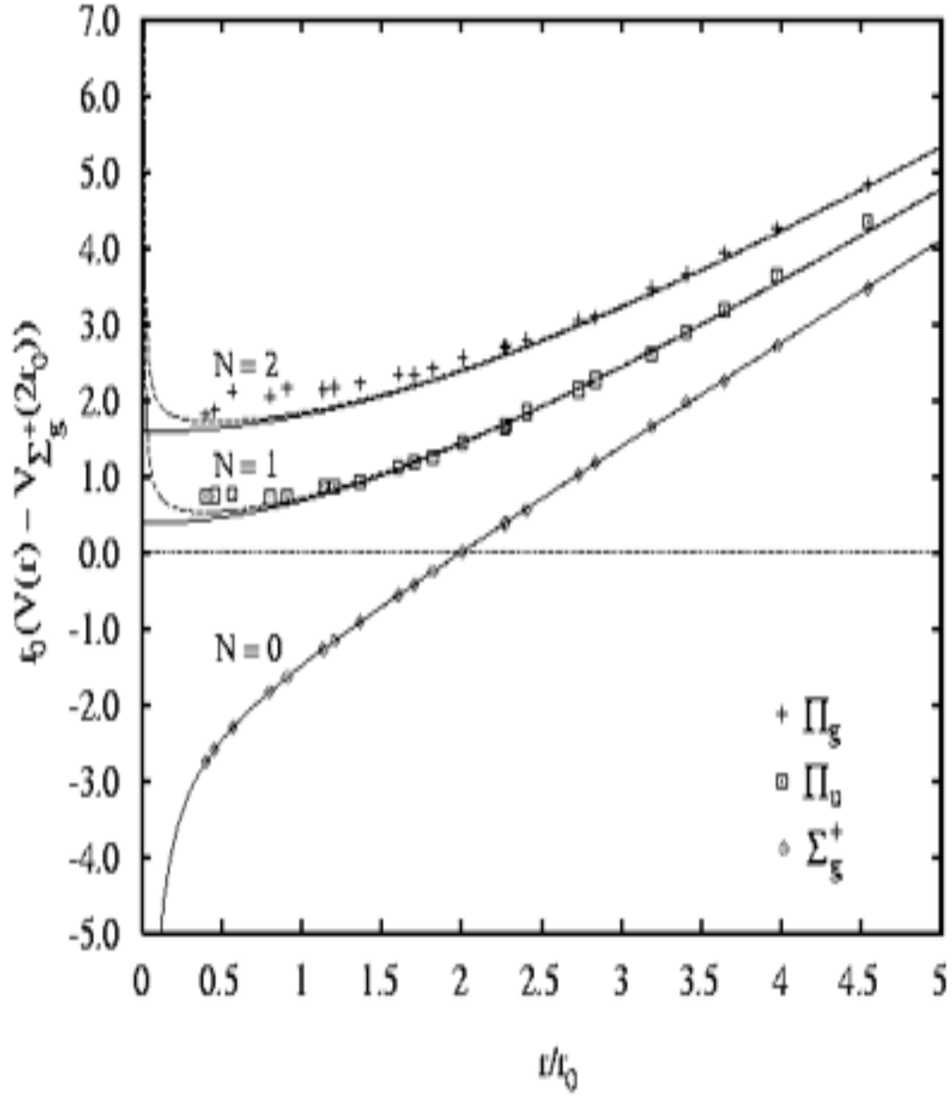


FIG. 4. The nonrelativistic spectroscopy of the charm string. $\psi(3.10)$ and $\psi(3.68)$ are fitted to obtain $M = 1.154$ GeV and $k = 0.21$ GeV^2. The dashed lines are the vibrational levels absent in the charmonium model. Levels with $E > 4.5$ GeV or $l > 2$ are not shown.

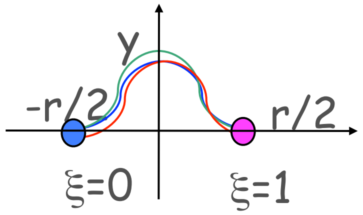
4. Flux Tube (& String)Models

T. Allen and M.G. Olsson, PLB434, 110 (98)



4. Flux Tube (& String)Models

Isgur and Paton, PRD31, 2910 (85).



Lowest multiplet:

0+-,0-+,1+-,1-+,2+-,2-+,1++,1--

1.9(2) GeV

string Hamiltonian

$$H = b_0 R + \sum_n \left[\frac{p_n^2}{2ba} + \frac{b}{2a}(y_n - y_{n+1})^2 \right]$$

$$s_{m\lambda} = \sum_n^N y_n(\lambda) \sqrt{\frac{2}{N+1}} \sin \frac{nm\pi}{N+1}$$

$$y_n(\lambda) = \sum_m s_{m\lambda} \sqrt{\frac{2}{N+1}} \sin \frac{nm\pi}{N+1}$$

$$H = b_0 R + \sum_{n\lambda} \left[\frac{p_n^2}{2ba} + \frac{ba}{2}\omega_n^2 s_{n\lambda}^2 \right]$$

$$\omega_n = \frac{2}{a} \sin \frac{\pi n}{2(N+1)}$$

$$\omega_1 \to \frac{\pi}{R}$$

$$\alpha_{n\lambda} = \sqrt{\frac{b_0 \omega_n}{2}} s_{n\lambda} + i \frac{p_{n\lambda}}{\sqrt{b_0 a \omega_n}}$$

$$H = b_0 R + \sum_{n\lambda} \omega_n \left(\alpha_{n\lambda}^\dagger \alpha_{n\lambda} + \frac{1}{2} \right)$$

$$H = b_0 R + \left(\frac{4}{\pi a^2} R - \frac{1}{a} - \frac{\pi}{12R} \right) + \sum_{n\lambda} \omega_n \alpha_{n\lambda}^\dagger \alpha_{n\lambda}$$

5. Schwinger-Dyson Formalism

C.J. Burden et al, PRC55, 2649 (97).

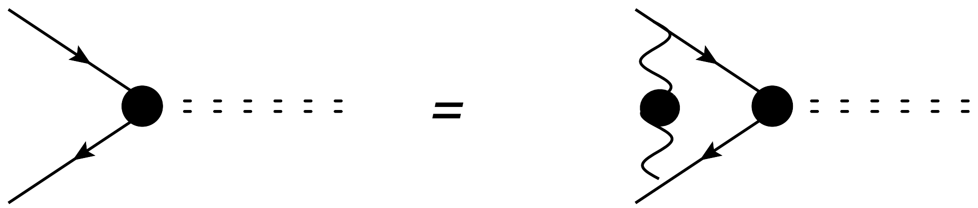
C.J. Burden & M.A. Pichowsky, Few Body Sys. 32, 119 (02).

separable Ansatz for the scattering kernel:

$$D_{\mu\nu}(p-q) = \delta_{\mu\nu}[G(p^2)G(q^2) + p\cdot q F(p^2)F(q^2)]$$

presumably 0+- NOT 0-+

$$F \propto A - 1$$
$$G \propto B - m$$
$$S = \frac{i}{A\not{p} + B}$$



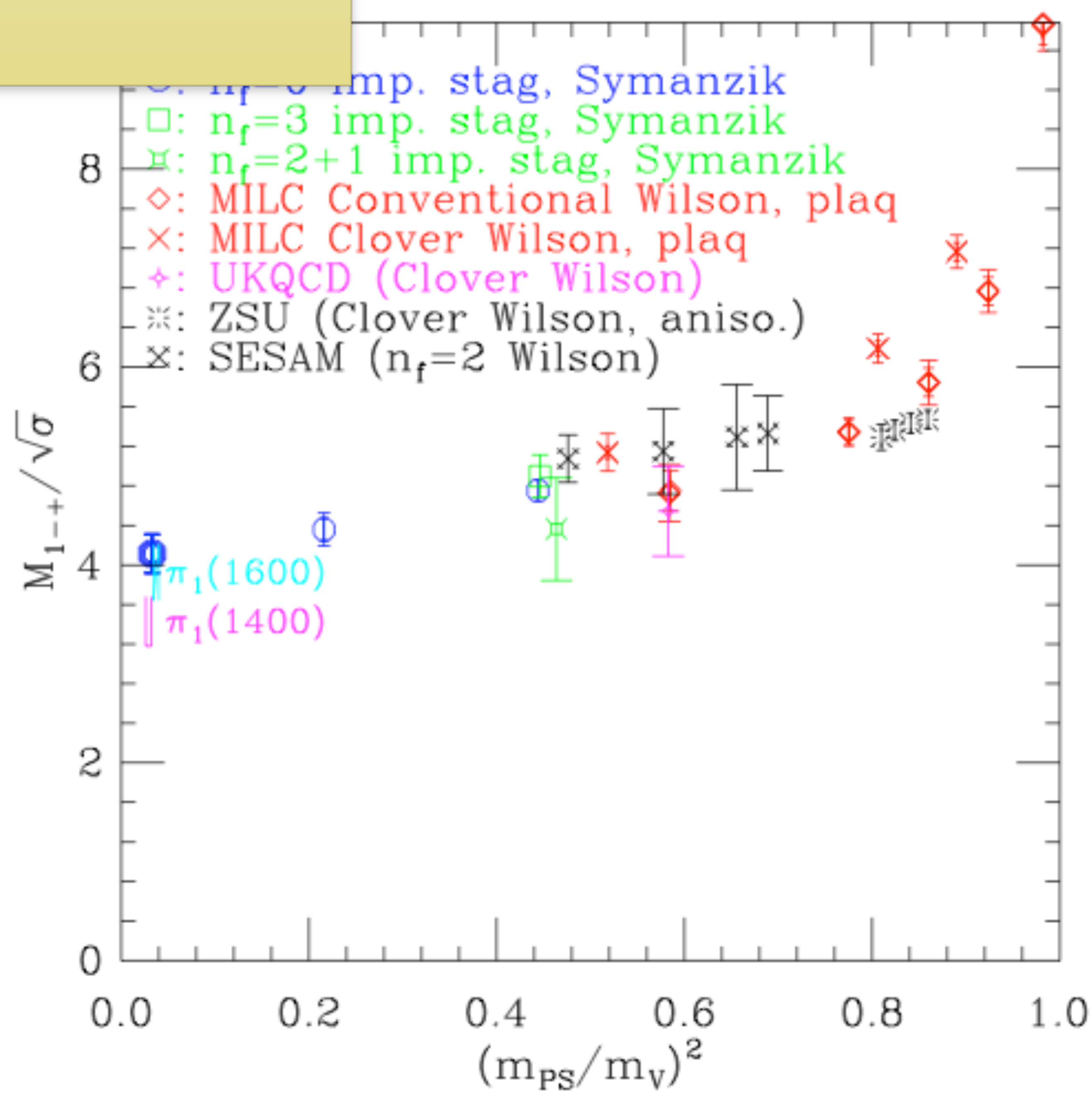
J^{PC}	mass (MeV)	state
0^{++}	749	$\sigma(540)$, $a_0(980)$
0^{-+}	1082	exotic
0^{--}	1319	exotic
1^{--}	730	$\rho(770)$
1^{+-}	1244	$h_1(1170)$
1^{++}	1337	$a_1(1260)$
1^{-+}	1439, 1498	$\pi_1(1400)$, $\pi_1(1600)$

6. Lattice Gauge Theory

JJD is probably near the green square (M/sqrt(s) ~ 5.1)

JKM, nucl-th/0307116

C. Bernard et al., hep-lat/0301024.



J.J. Dudek et al. PRL103, 262001 (09)

isovector, dynamical lattice, m_u=m_s, m_π=700 MeV

$$M(1^{-+}) \approx 2.17$$

$$M(0^{+-}) \approx M(2^{+-}) \approx 2.51$$

2. VECTOR DECAY MODEL

Szczepaniak & Swanson, PRD55, 3987 (97)
Page, Szczepaniak & Swanson, PRD59, 014035 (99)

map chromofields to phonon degrees of freedom

$$E^a_\lambda(n) = \frac{\kappa}{a^3}\left(y^a_\lambda(n+1) - y^a_\lambda(n)\right)$$

$$B^a_\lambda(n) = \frac{-i}{\kappa a}\frac{\partial}{\partial y^a_\lambda(n)} \qquad\qquad \kappa = a\sqrt{b_0}$$

$$B^a_\lambda(n) = \frac{-i}{\kappa}\sqrt{\frac{b_0}{r}}\sum_m \sin\frac{m\pi}{N+1} n\sqrt{\omega_m}\left(\alpha^a_{m\lambda}\mathrm{e}^{-i\omega_m t} - \alpha^{a\dagger}_{m\lambda}\mathrm{e}^{i\omega_m t}\right)$$

2. VECTOR DECAY MODEL

use the same mapping to obtain $\bar{\psi}\alpha \cdot A\psi$

$$H_{int} = \frac{iga^2}{\sqrt{\pi}} \sum_{m,\lambda} \int_0^1 d\xi \cos(\pi\xi) T^a_{ij}\, h^\dagger_i(\xi \mathbf{r}_{Q\bar{Q}}) \sigma \cdot \hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}}_{Q\bar{Q}}) \left(\alpha^a_{m\lambda} - \alpha^{a\dagger}_{m\lambda}\right) \chi_j(\xi \mathbf{r}_{Q\bar{Q}})$$

$$\langle H|H_{int}|AB\rangle = i\frac{ga^2}{\sqrt{\pi}}\frac{2}{3}\int_0^1 d\xi \int d\mathbf{r} \cos(\pi\xi)\sqrt{\frac{2L_H+1}{4\pi}}\mathrm{e}^{\frac{i\mathbf{p}\cdot\mathbf{r}}{2}}\varphi_H(r)\varphi^*_A(\xi\mathbf{r})\varphi^*_B((1-\xi)\mathbf{r})\cdot \left[\mathcal{D}^{L_H *}_{M_L\Lambda}(\phi,\theta,-\phi)\chi^{PC}_{\Lambda,\lambda}\hat{\mathbf{e}}_\lambda(\hat{\mathbf{r}})\cdot\langle\sigma\rangle\right]$$

both models obtain the "S+P" decay selection rule: hybrids cannot decay to two S-wave states with identical spatial wavefunctions.

P.R. Page, PLB401, 313 (97)

3. Lattice Decay

C. McNeile, C. Michael, and P. Pennanen [UKQCD], PRD 65, 094505 (02).

$$1^{-+}(b\bar{b}) \to \eta_b \eta(s\bar{s}) \sim 1 \text{ MeV}$$

$$1^{-+}(b\bar{b}) \to \chi_b \sigma(s\bar{s}) \sim 60 \text{ MeV}$$

			alt	10.9 GeV hybrid	standard	IKP	reduced
2^{-+}	B^*B	P	.1	0	.5	3	44
1^{-+}	B^*B	P	.1	0	.5	3	44
0^{-+}	B^*B	P	.5	0	2	13	177
1^{--}	B^*B	P	.2	0	1.2	7	88
2^{+-}	B^*B	D	.08	.05	.25	1	22
1^{+-}	B^*B	S	.02	.1	.2	5	13
	B^*B	D	.02	.02	.15	.6	12
1^{++}	B^*B	S	.01	.05	.25	2	7
	B^*B	D	.1	.05	.5	1	24

5. Hybrid-Nonexotic Mixing

T. Burch and D. Toussaint [MILC], PRD68, 094504 (03)

Compute vector meson - vector hybrid meson mixing in NRQCD on the lattice.

This mixing occurs via $\mathcal{O} = g\dfrac{\sigma \cdot B}{2M}$

Obtain

$$\Upsilon(H) \approx 0.4\% \qquad \eta_b(H) \approx 1\%$$

$$J/\psi(H) \approx 2.3\% \qquad \eta_c(H) \approx 6\%$$

Conclusions

- modelling hybrids requires much guesswork. The lattice helps.
- decay selection rule will help identify states (assuming open flavour decays)
- newer lattice results (mass, photocoupling, mixing) support older guesses

Adiabatic and small oscillation approximations

Barnes, Close, & ES, PRD52, 5242 (95)

$$H_{flux\ tube} = -\frac{1}{2m_b}\sum_{i=1}^{N}\left(\sum_{\hat\eta_T}(\hat\eta_T\cdot\vec\nabla_i)^2\right) + \sum_{i=1}^{N+1} V\left(|\vec r_i - \vec r_{i-1}|\right) .$$

m_b = 0.2 GeV

linear potential

Adiabatic limit (check small osc)

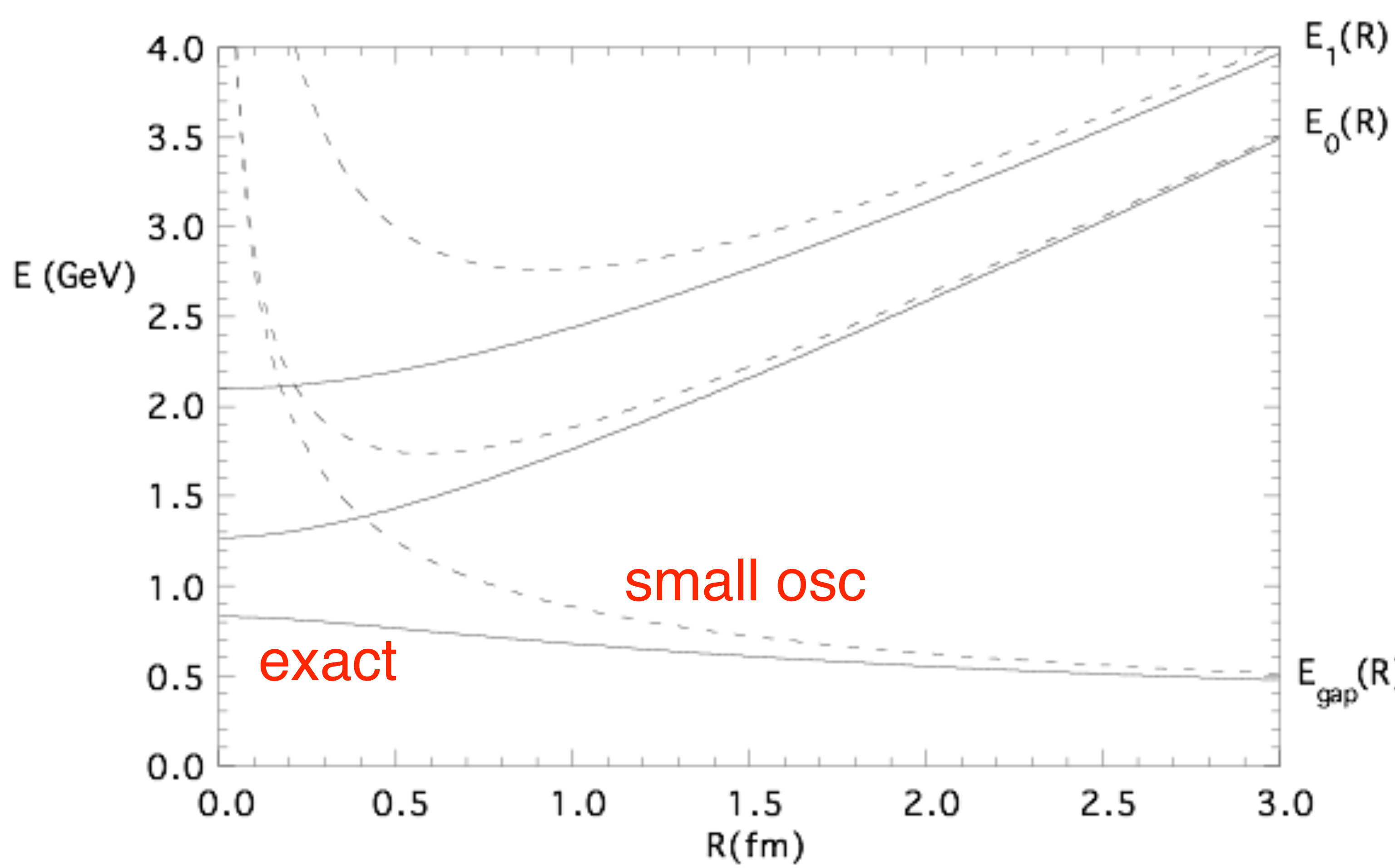
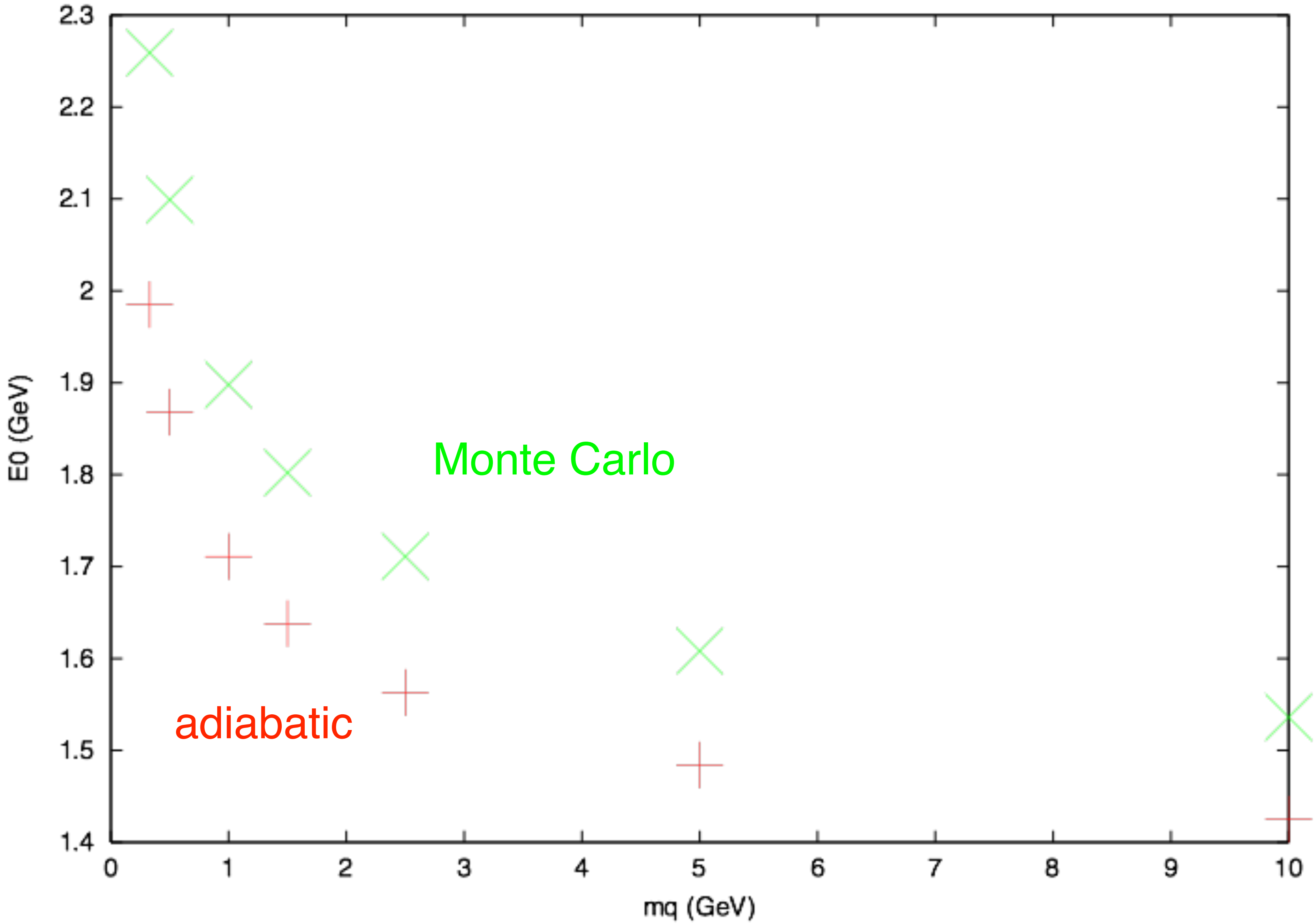


Fig.1. Ground state and first hybrid adiabatic potentials and their difference, for N=
Solid lines are exact and dashed lines are the small oscillation approximation.
String tension a=1.0GeV/fm, bead mass m_b=0.2GeV.

Adiabatic approximation (ground state meson)



Adiabatic approximation (hybrid gap)

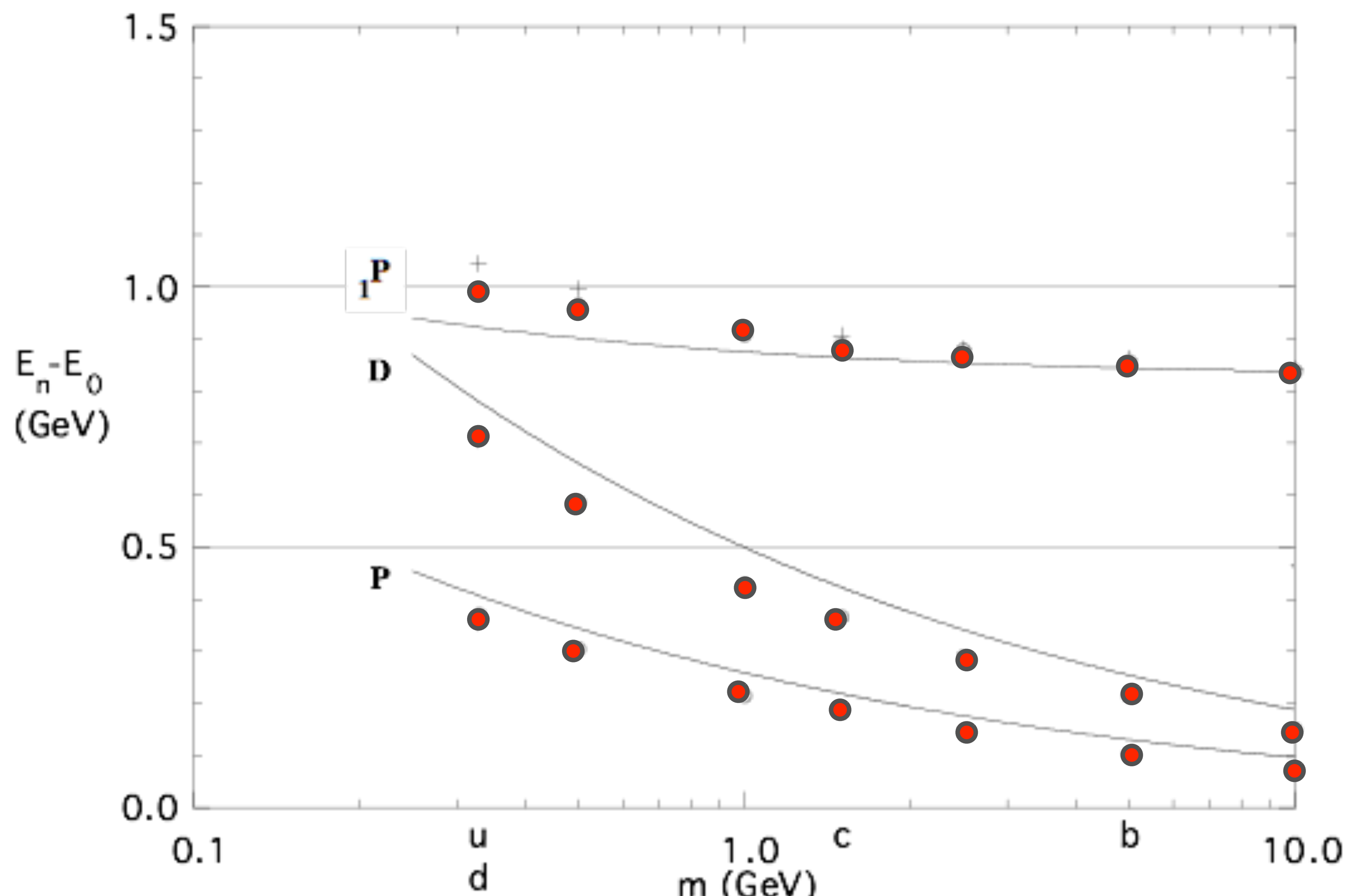


Fig.4. Energies of the lightest L=1,2 $q\bar{q}$ and $_\Lambda L=_1P$ hybrid states relative to $E_0=E_S$ for N=1. Lines show the adiabatic approximation and the points are Monte Carlo, M=0 (open) and M=L (plus) m_b=0.2 GeV, a=1.0 GeV/fm, α_s=0.

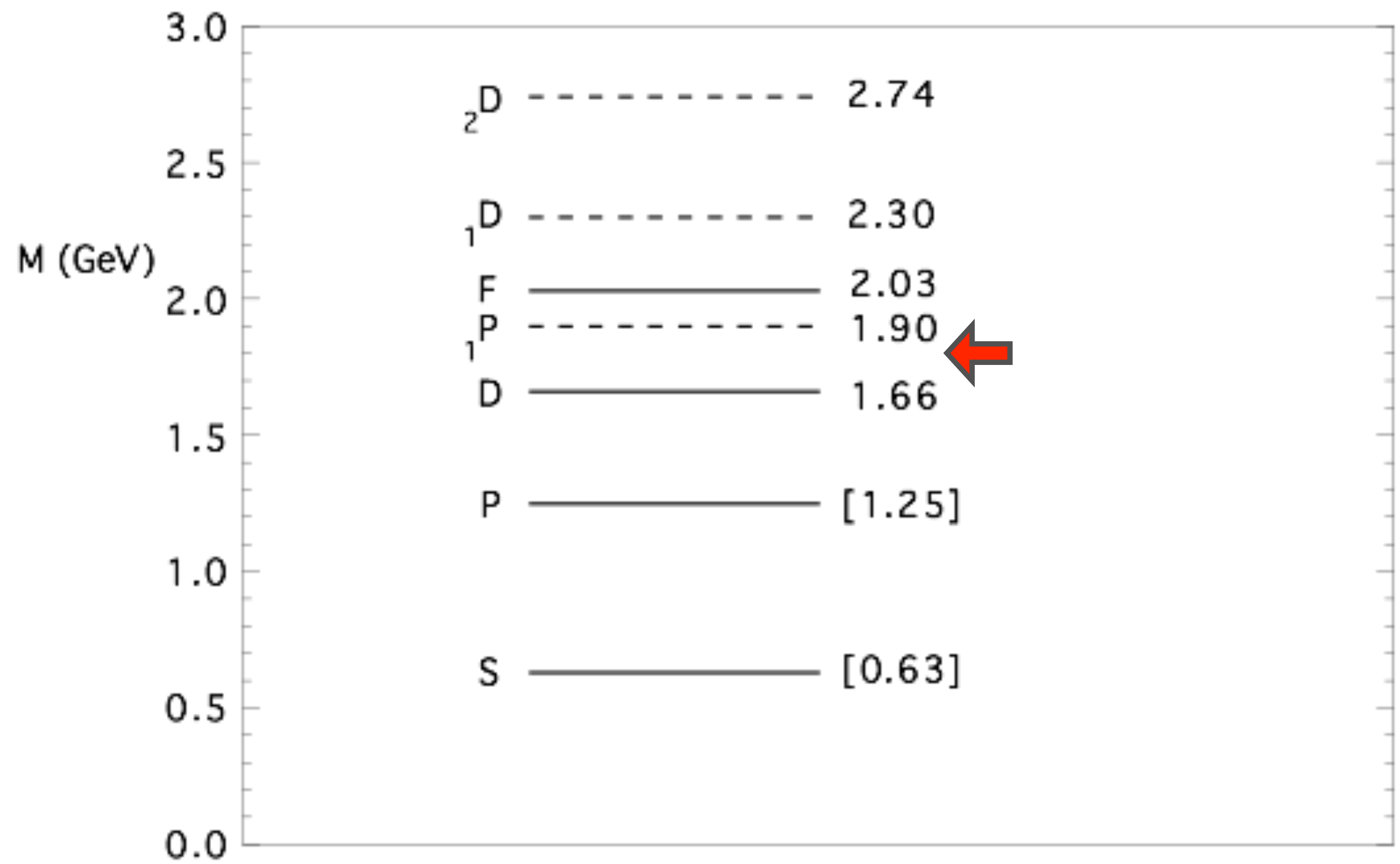
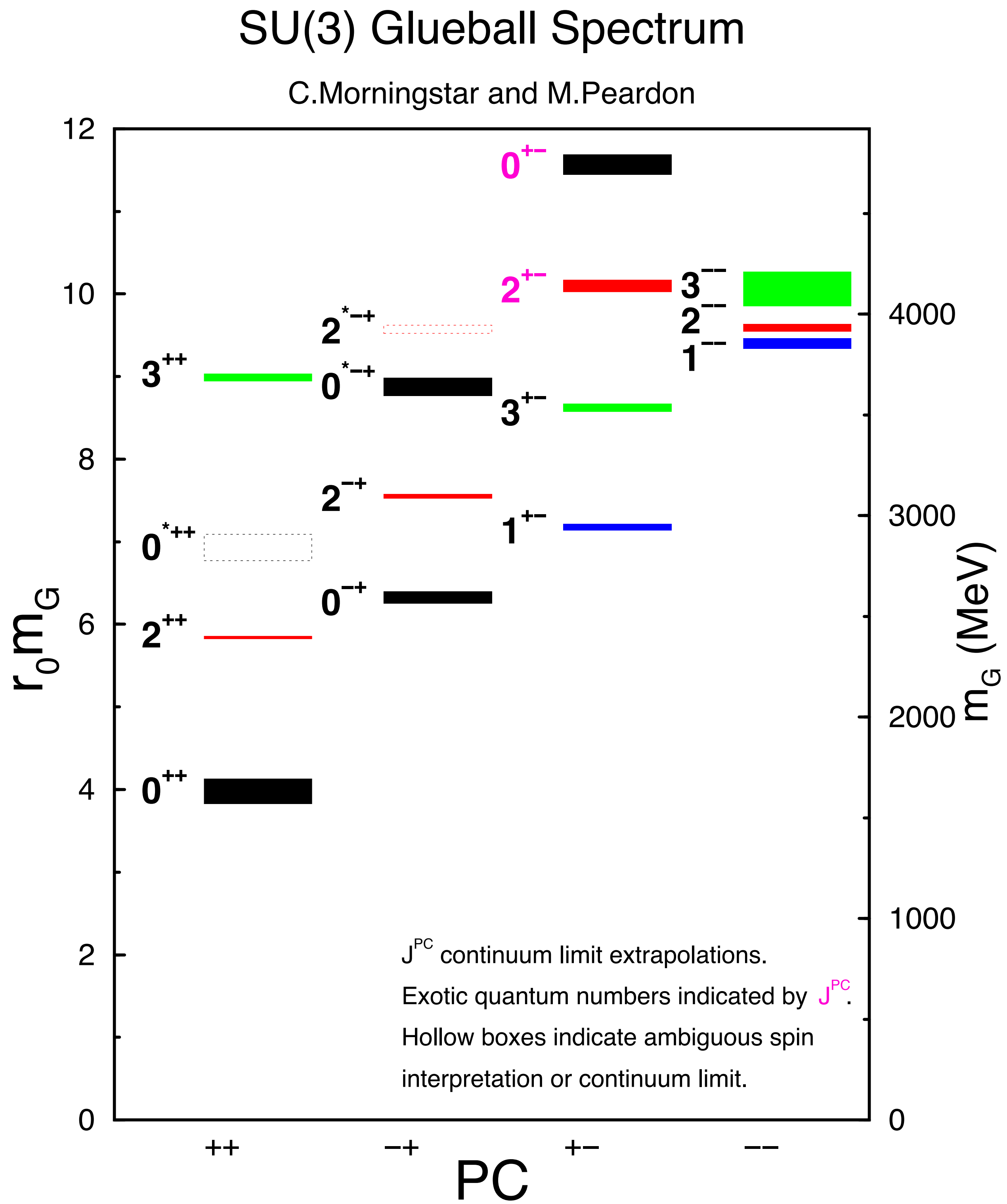


Fig.5. The lightest L=0-3 $q\bar{q}$ (q=u,d) and $_\Lambda$L= $_1$P, $_1$D and $_2$D hybrid masses from Monte Carlo with physical parameters, m_q=0.33GeV, m_b=0.2GeV, a=1.0GeV/fm, α_s^{ft}=1.3. Square brackets denote masses used as input.

Glueballs

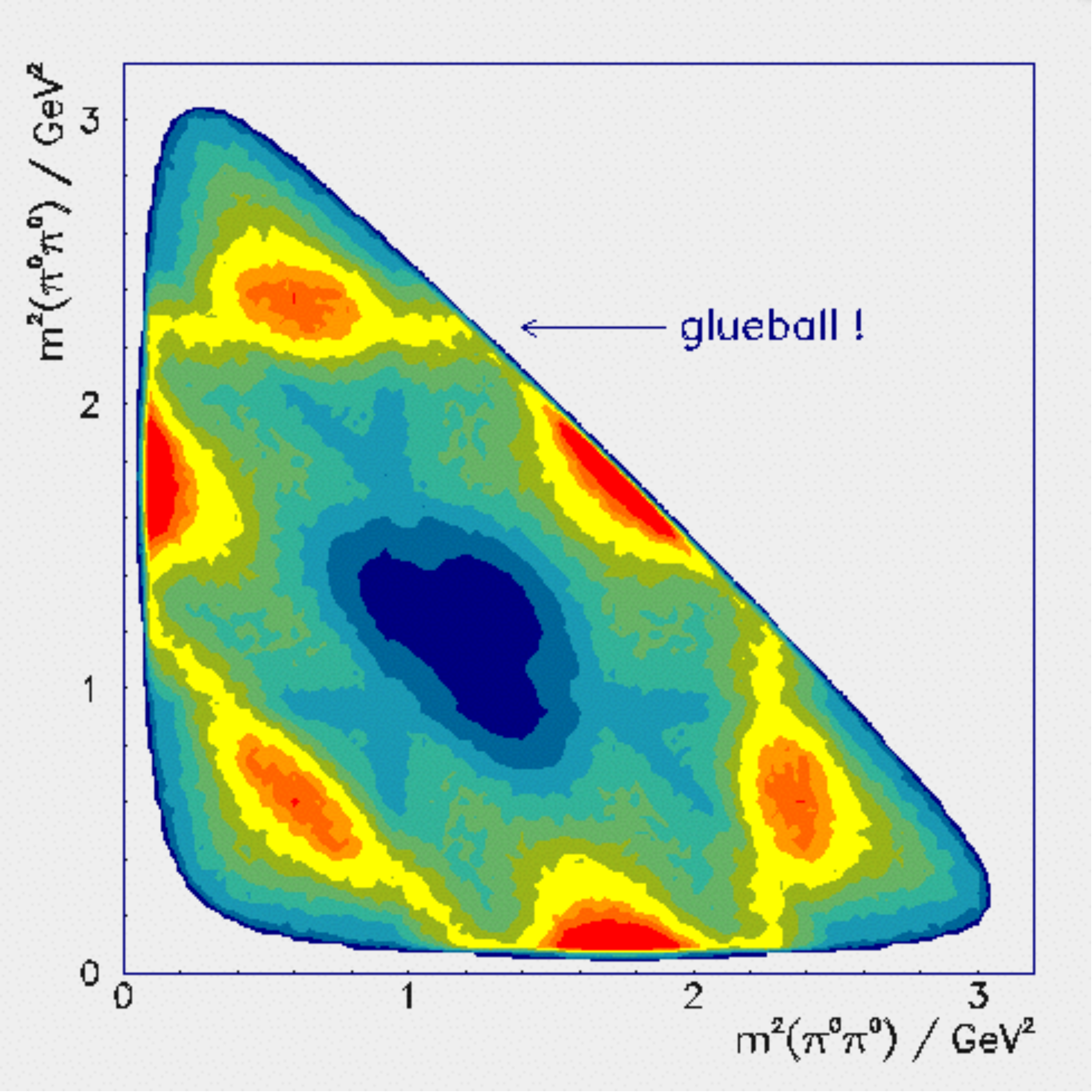
Exotic Particles: Glueballs

Exotic Particles: Glueballs



Exotic Particles: Glueballs

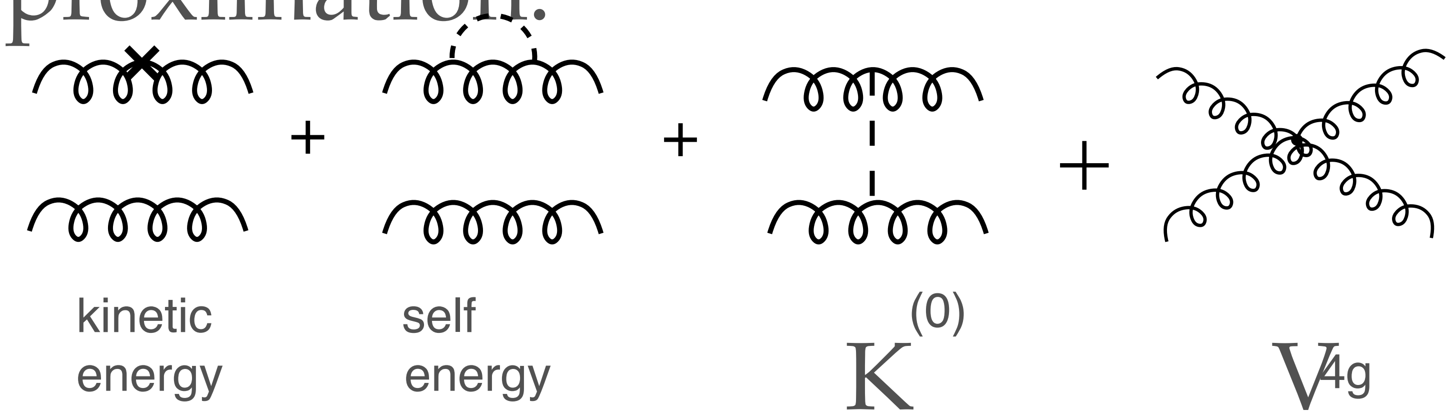
ppbar collision at CERN giving 3 pi0 which resonate at certain frequencies = energies



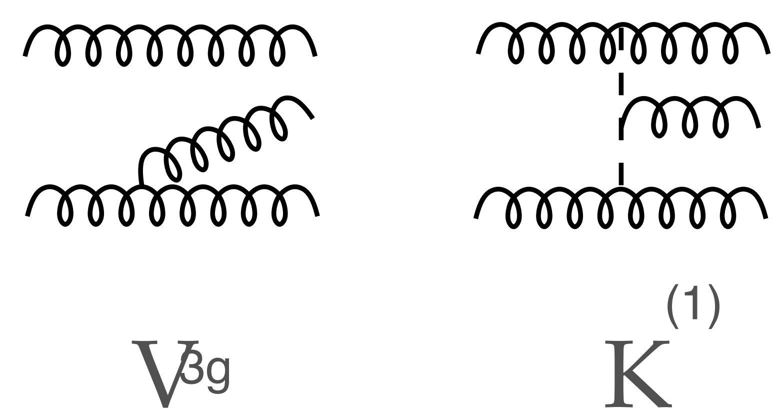
Glueballs

no free parameters!

Tamm-Dancoff approximation:



Fock sector mixing

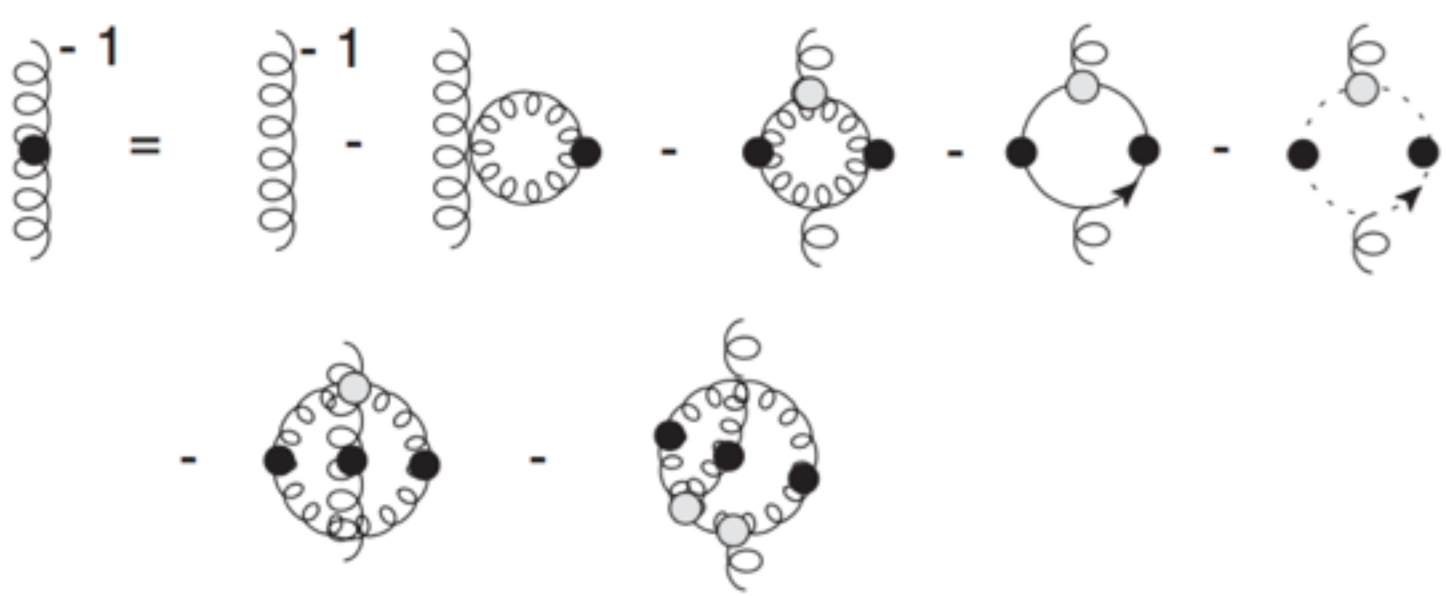


Szczepaniak & ES, to appear

Szczepaniak, et al., PR**L76**, 2001 (96)

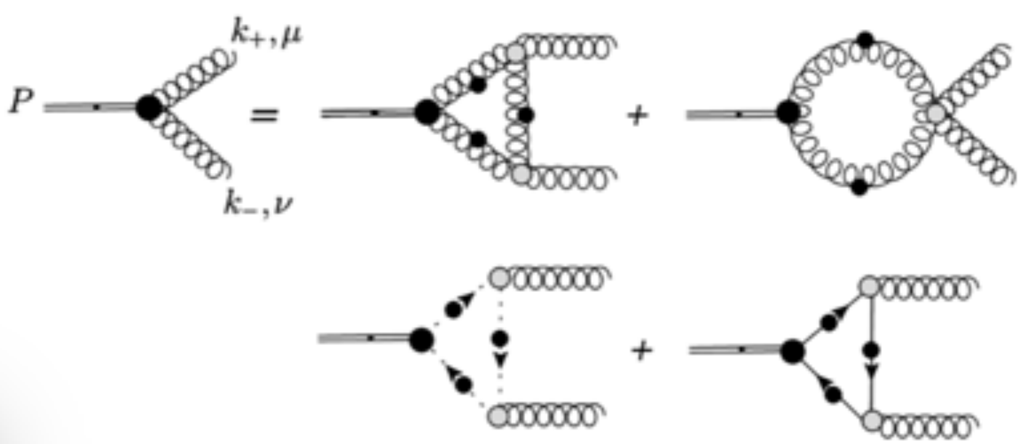
Glueballs

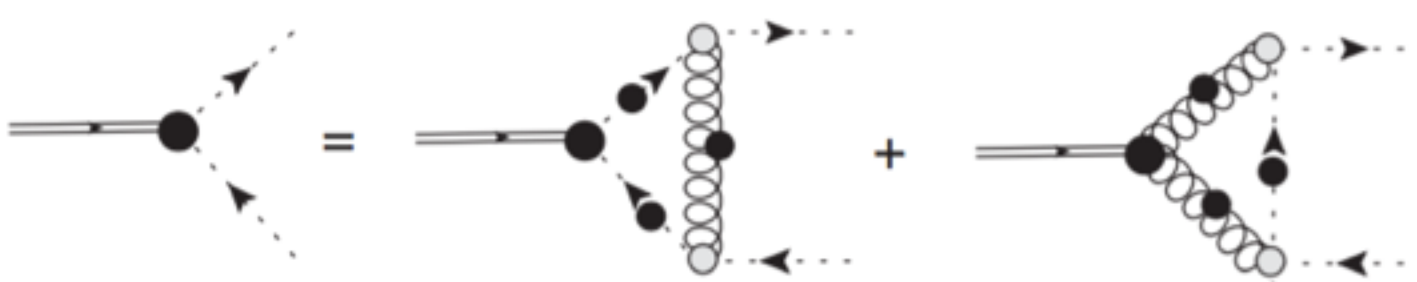
Bethe-Salpeter:



Glueballs

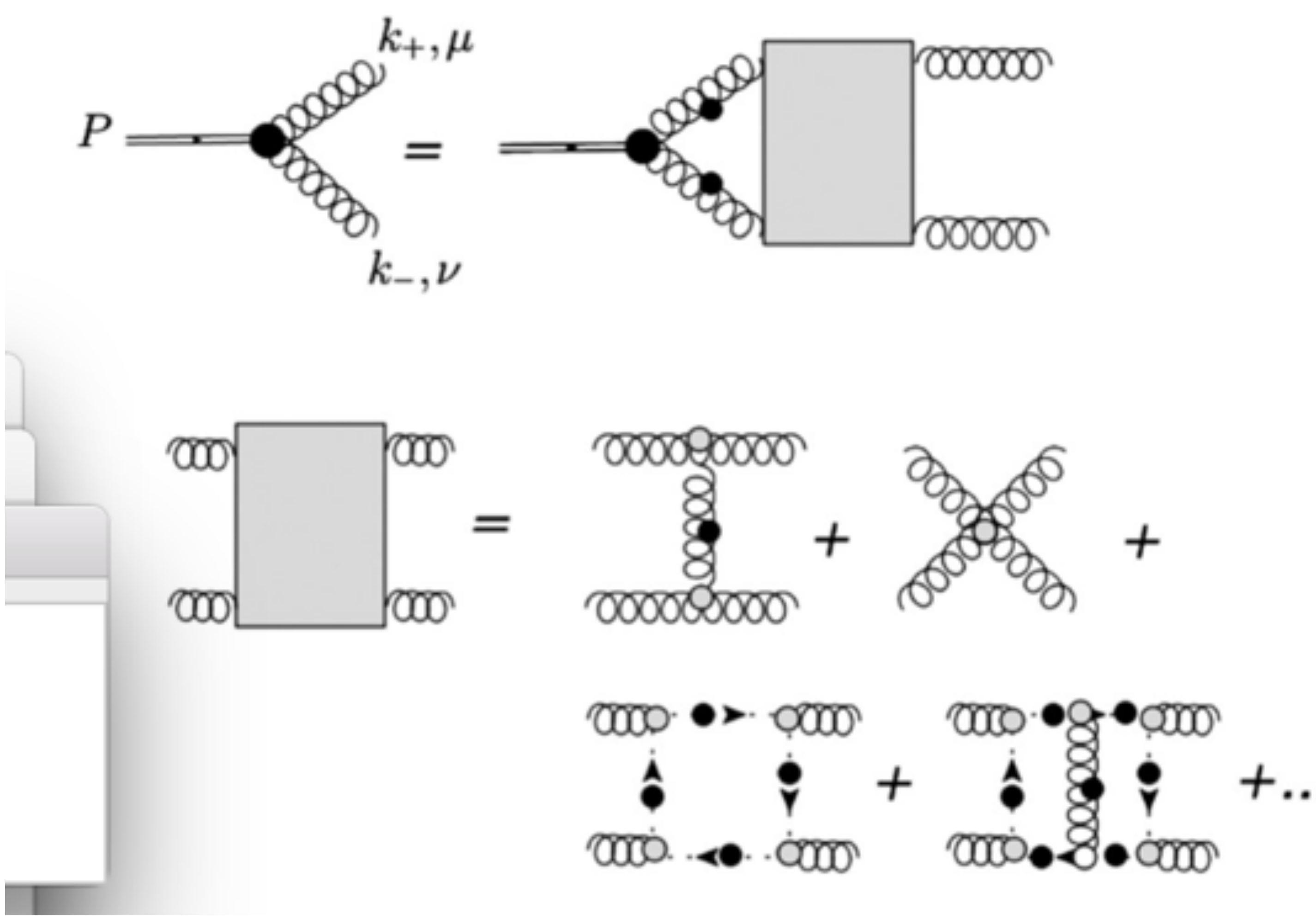
Bethe-Salpeter:



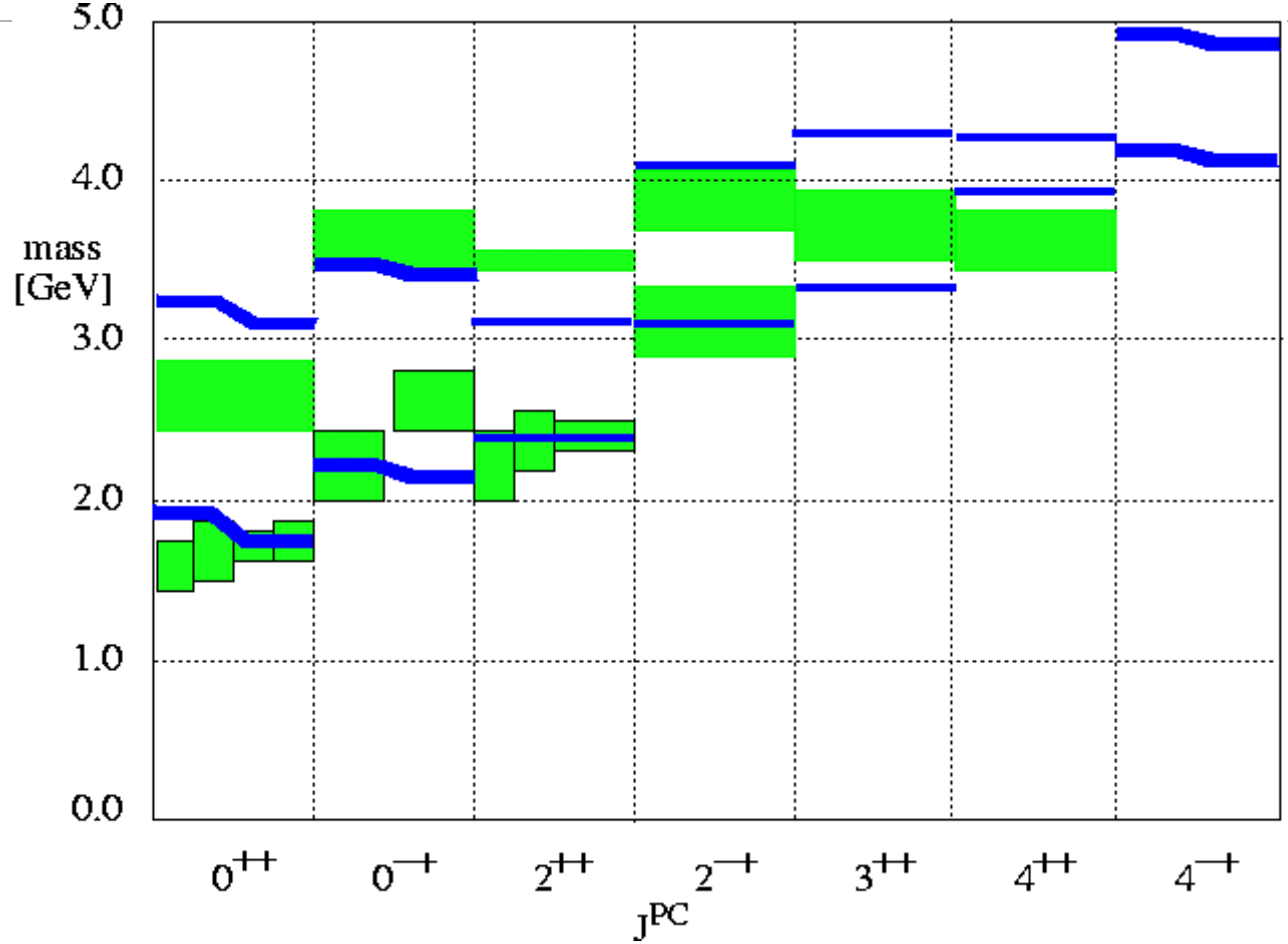


Glueballs

combining the previous B-S eqns:



Glueballs



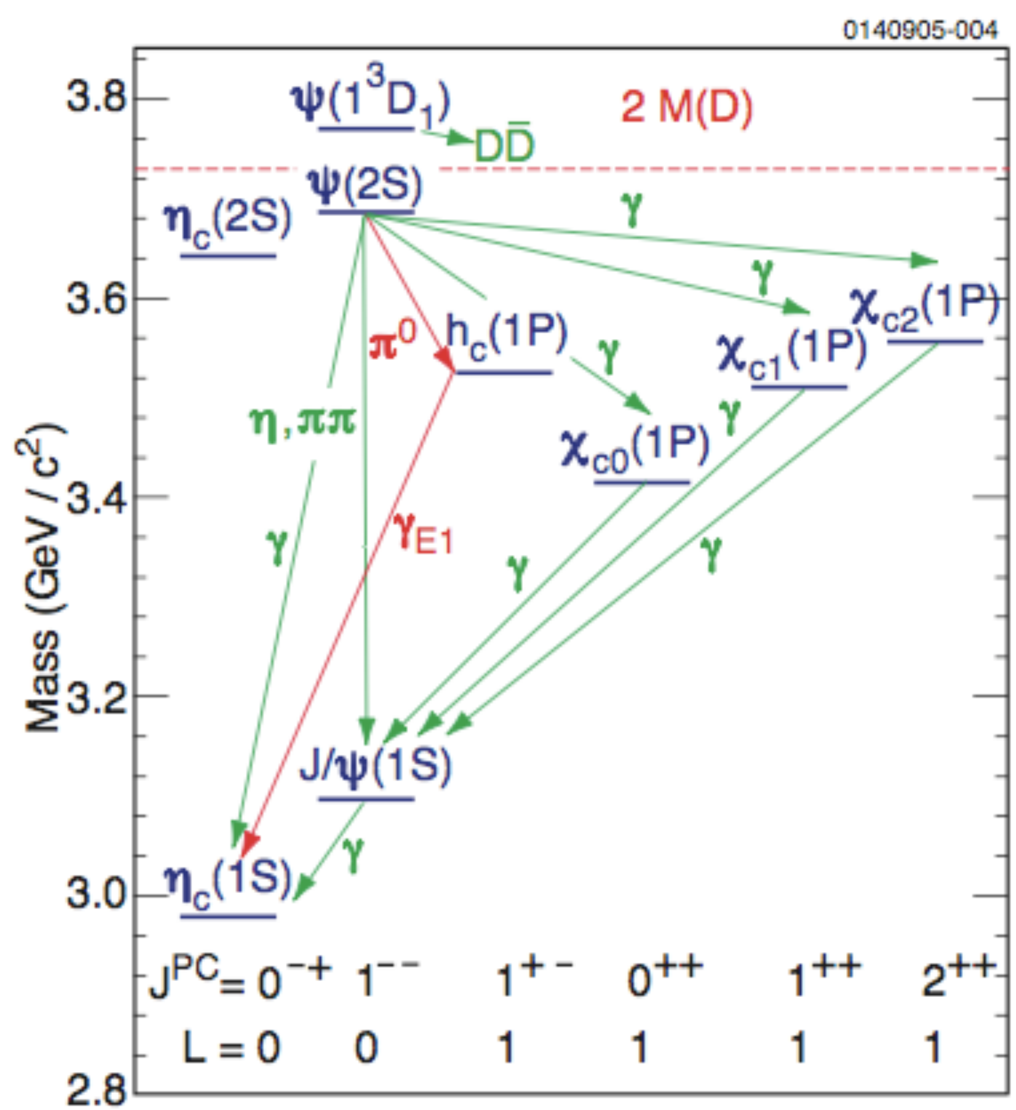
Glueballs

Model	0^{++}	$(0^{++})'$	0^{-+}	2^{++}	$(2^{++})'$	2^{-+}
lattice (anisotropic)	1.63(6)(8)			2.40(1)(12)	3.32(2)(16)	
lattice (UKQCD)	1.55(5)			2.27(10)		
lattice (GF11)	1.74(7)			2.36(13)		
lattice (Teper)	1.57(9)	2.87(34)	2.16(27)	2.22(12)		3.06(26)
lattice (SESAM)	1.66(5)			2.32(25)		
QCDSR (SVZ)	$\sim$1.2		2-2.5	$\sim$1.2		
QCDSR (Narison)	1.5(2)		2.05(20)	2.0(1)		
bag (MIT)	$\sim$1		$\sim$1.2	$\sim$1		$\sim$1.2
bag (BCM)	$\sim$1		$\sim$1.5	$\sim$1.5		$\sim$2.1
flux tube	1.52	2.75	2.79	2.84		2.84
const glue (Barnes)	$\sim$1.5	$\sim$2.1	$\sim$1.5	$\sim$1.8	$\sim$2.1	$\sim$2.1
const glue (Cornwall+Soni)[a]	1.5		1.76	2.08		
const glue (NCSU)	1.60	2.64	2.03	2.05	2.83	2.82

[a] I have taken $m_g = 650$ to fit the 0^{++}.

Exotics

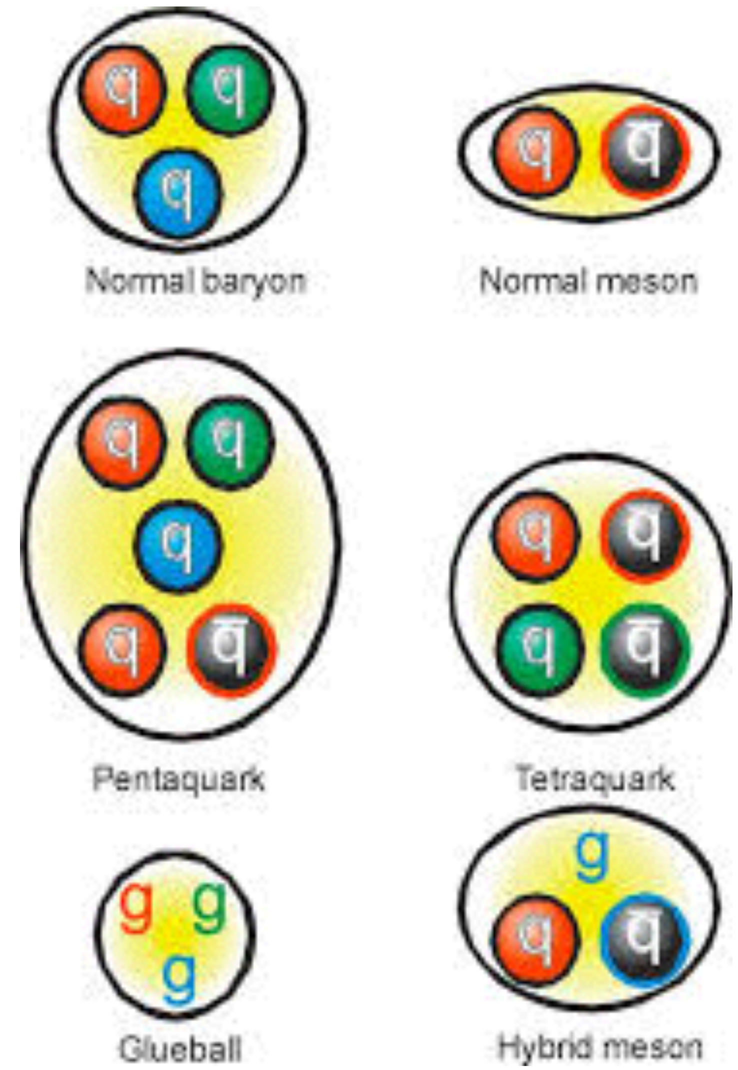
OUR FORMERLY COMFORTABLE WORLD (CF. 1932 E,P,N)

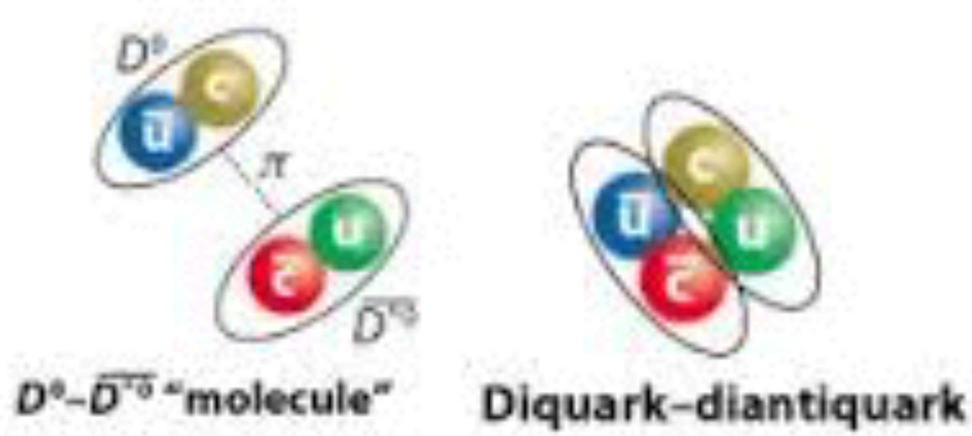


New Forms of Matter

- molecular bound states
- deeply bound states/ tetraquarks
- deeply bound states/ diquarks

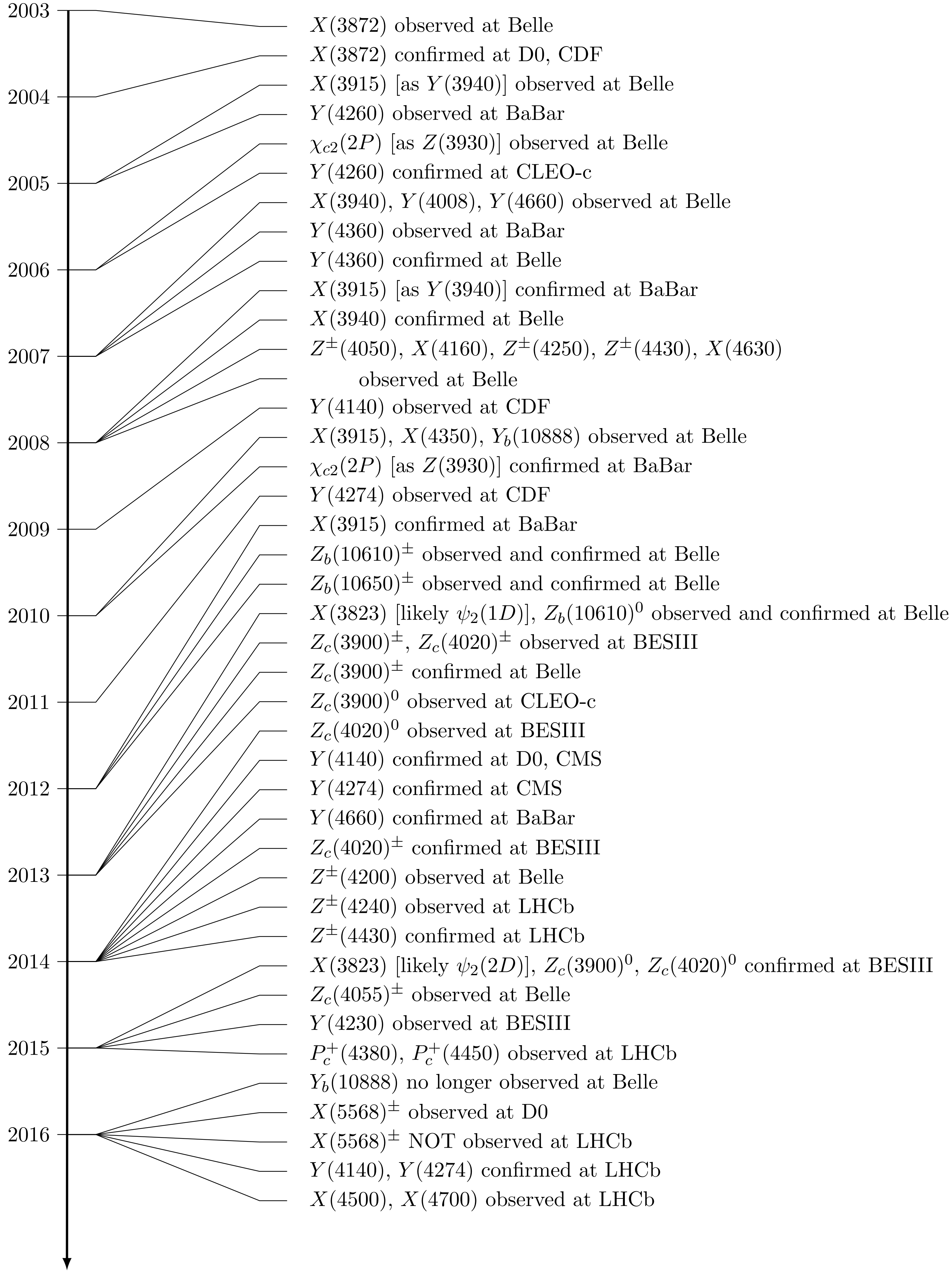
- “kinematical”effects

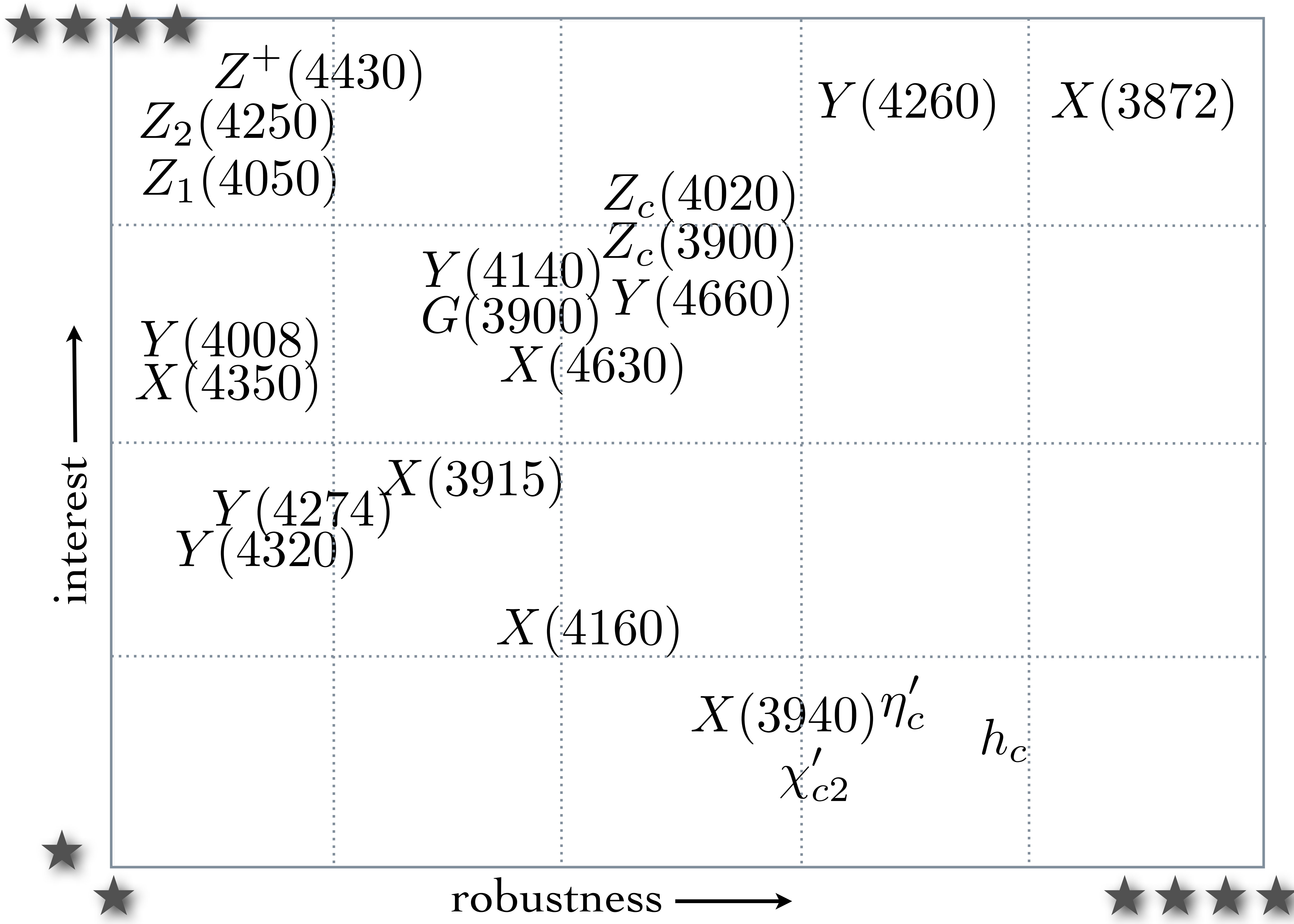




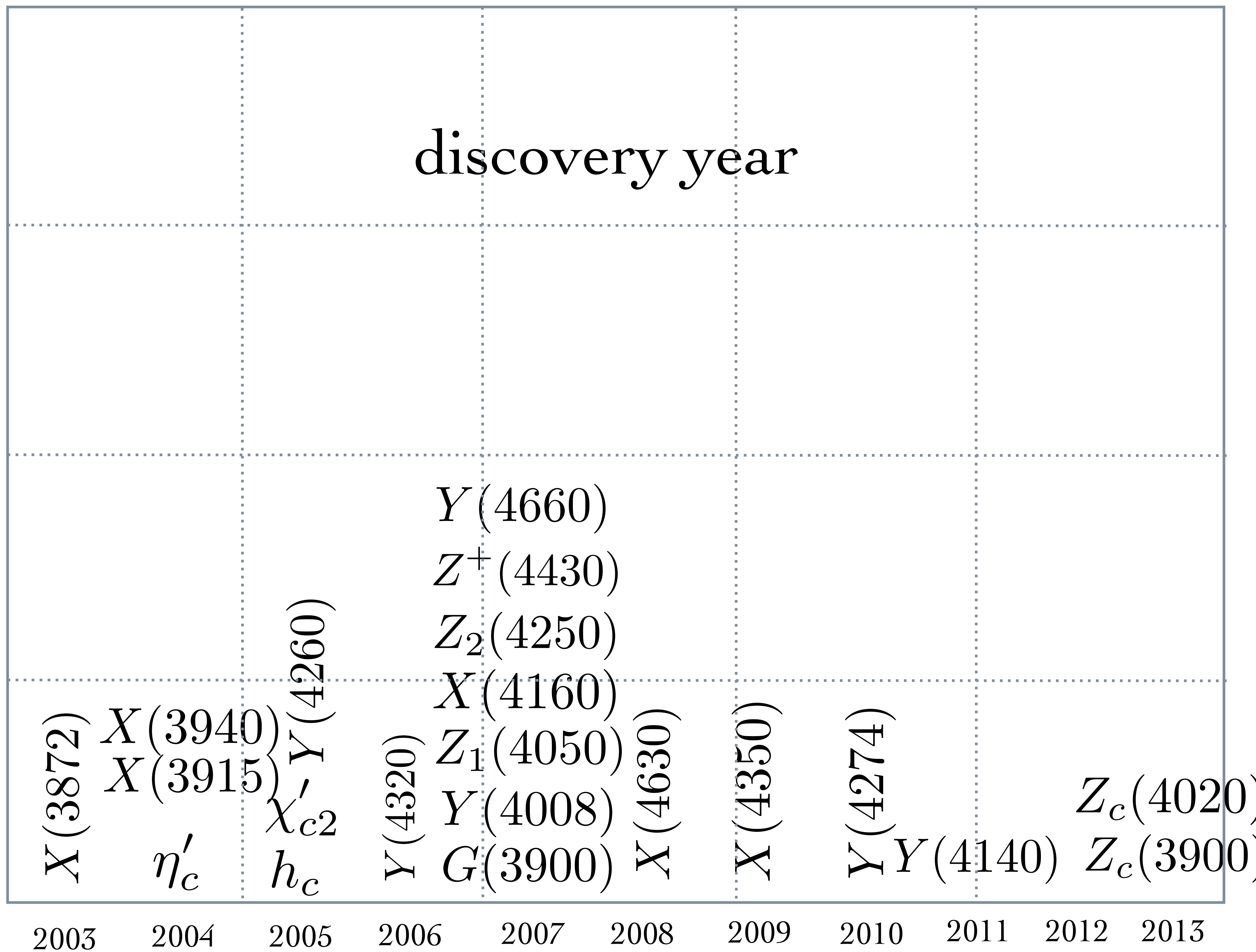
New Forms of Matter

Particles as Resonances


2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
$X(3872)$ observed at Belle
$X(3872)$ confirmed at D0, CDF
$X(3915)$ [as $Y(3940)$] observed at Belle
$Y(4260)$ observed at BaBar
$\chi_{c2}(2P)$ [as $Z(3930)$] observed at Belle
$Y(4260)$ confirmed at CLEO-c
$X(3940)$, $Y(4008)$, $Y(4660)$ observed at Belle
$Y(4360)$ observed at BaBar
$Y(4360)$ confirmed at Belle
$X(3915)$ [as $Y(3940)$] confirmed at BaBar
$X(3940)$ confirmed at Belle
$Z^{\pm}(4050)$, $X(4160)$, $Z^{\pm}(4250)$, $Z^{\pm}(4430)$, $X(4630)$ observed at Belle
$Y(4140)$ observed at CDF
$X(3915)$, $X(4350)$, $Y_b(10888)$ observed at Belle
$\chi_{c2}(2P)$ [as $Z(3930)$] confirmed at BaBar
$Y(4274)$ observed at CDF
$X(3915)$ confirmed at BaBar
$Z_b(10610)^{\pm}$ observed and confirmed at Belle
$Z_b(10650)^{\pm}$ observed and confirmed at Belle
$X(3823)$ [likely $\psi_2(1D)$], $Z_b(10610)^0$ observed and confirmed at Belle
$Z_c(3900)^{\pm}$, $Z_c(4020)^{\pm}$ observed at BESIII
$Z_c(3900)^{\pm}$ confirmed at Belle
$Z_c(3900)^0$ observed at CLEO-c
$Z_c(4020)^0$ observed at BESIII
$Y(4140)$ confirmed at D0, CMS
$Y(4274)$ confirmed at CMS
$Y(4660)$ confirmed at BaBar
$Z_c(4020)^{\pm}$ confirmed at BESIII
$Z^{\pm}(4200)$ observed at Belle
$Z^{\pm}(4240)$ observed at LHCb
$Z^{\pm}(4430)$ confirmed at LHCb
$X(3823)$ [likely $\psi_2(2D)$], $Z_c(3900)^0$, $Z_c(4020)^0$ confirmed at BESIII
$Z_c(4055)^{\pm}$ observed at Belle
$Y(4230)$ observed at BESIII
$P_c^+(4380)$, $P_c^+(4450)$ observed at LHCb
$Y_b(10888)$ no longer observed at Belle
$X(5568)^{\pm}$ observed at D0
$X(5568)^{\pm}$ NOT observed at LHCb
$Y(4140)$, $Y(4274)$ confirmed at LHCb
$X(4500)$, $X(4700)$ observed at LHCb

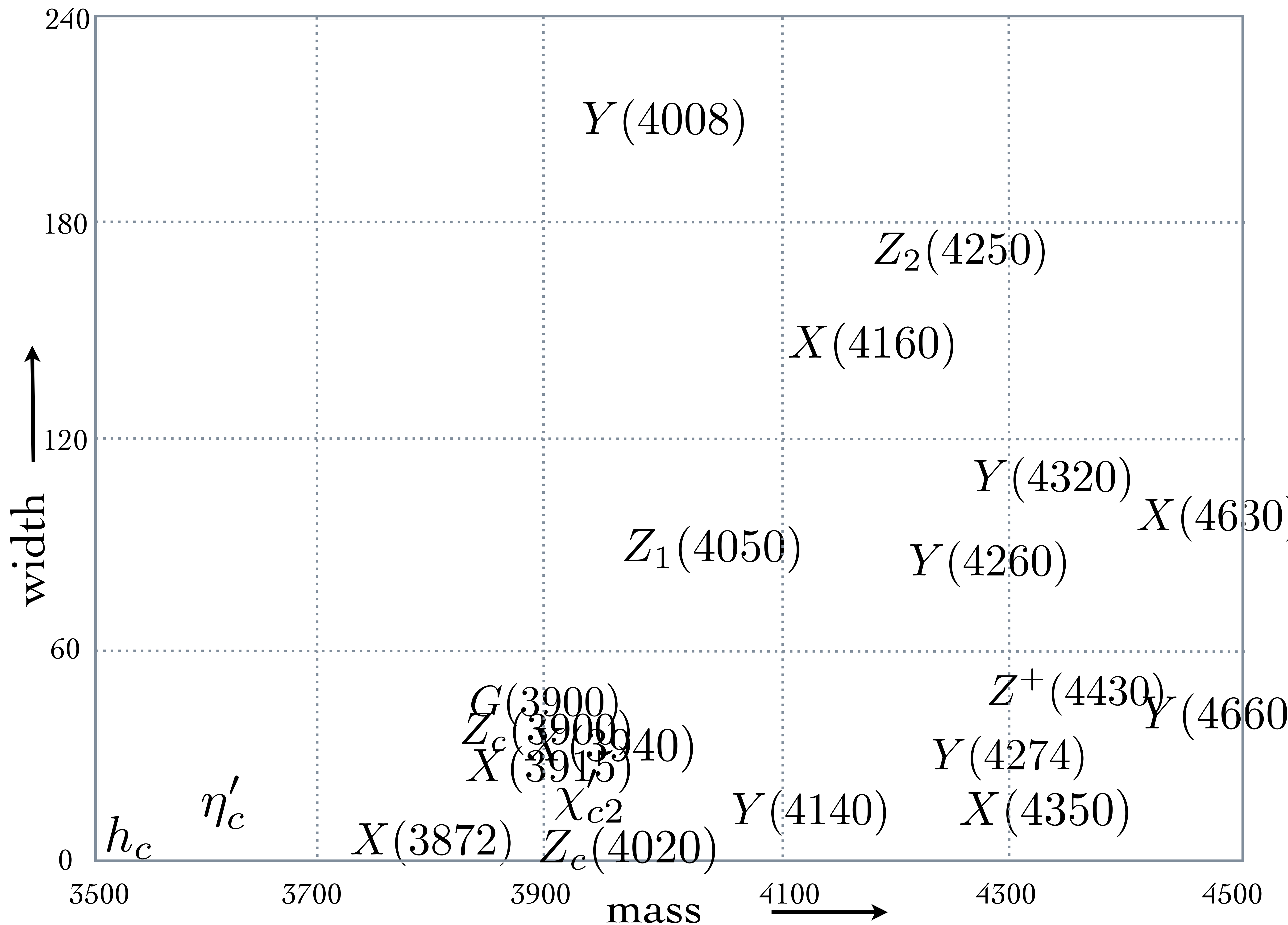


$Z^+(4430)$
$Z_2(4250)$
$Z_1(4050)$
$Y(4260)$
$X(3872)$
$Z_c(4020)$
$Z_c(3900)$
$Y(4140)$
$G(3900)$
$Y(4660)$
$Y(4008)$
$X(4350)$
$X(4630)$
$X(3915)$
$Y(4274)$
$Y(4320)$
$X(4160)$
$X(3940)$
η_c'
h_c
χ_{c2}'
interest
robustness


discovery year
X(3872)
X(3940)
X(3915)
η'_c
Y(4260)
χ'_c2
h_c
Y(4320)
Y(4660)
Z^+(4430)
Z_2(4250)
X(4160)
Z_1(4050)
Y(4008)
G(3900)
X(4630)
X(4350)
Y(4274)
Y(4140)
Z_c(4020)
Z_c(3900)
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013

discovery experiment

ATLAS EXPERIMENT	BABAR	BELLE		CDF	CESR CLEO	DØ	LHCb
	$Y(4320)$	$Y(4660)$	$Z_c(4020)$	$Y(4274)$	η_c'		
	$Y(4260)$	$X(4630)$	$Z_c(3900)$	$Y(4140)$	h_c		
	$G(3900)$	$Z^+(4430)$					
		$X(4350)$					
		$Z_2(4250)$					
		$X(4160)$					
		$Z_1(4050)$					
		$Y(4008)$					
		$X(3940)$					
		$X(3915)$					
		$X(3872)$					
		χ_{c2}'					



width
mass
0
60
120
180
240
3500
3700
3900
4100
4300
4500
$Y(4008)$
$Z_2(4250)$
$X(4160)$
$Y(4320)$
$X(4630)$
$Z_1(4050)$
$Y(4260)$
$Z^+(4430)$
$Y(4660)$
$G(3900)$
$Z_c(3900)$
$X(3940)$
$X(3915)$
$Y(4274)$
η_c'
χ_{c2}
$Y(4140)$
$X(4350)$
h_c
$X(3872)$
$Z_c(4020)$

production mode

$Z^+(4430)$ η_c' $Y(4274)$ $Z_2(4250)$ $Y(4140)$ $Z_1(4050)$ $X(3915)$ $X(3872)$	$Y(4660)$ $X(4630)$ $Y(4320)$ $Y(4260)$ $Y(4008)$ $G(3900)$	$Z_c(4020)$ $e^+e^- \to \pi X$	h_c $\psi' \to KX$ $X \to \gamma\eta_c$
		χ_{c2}' $X(4350)$	$Z_c(3900)$ $X(4160)$ $X(3940)$
$B \to KX$	$e^+e^- \to \gamma X$	$e^+e^- \to e^+e^- X$	$e^+e^- \to J/\psi X$
$X \to \phi J/\psi$ $X \to \pi\chi_{c1}$ $X \to \pi\pi J/\psi$ $X \to \omega J/\psi$ $X \to KK\pi$ $X \to \pi^+\psi'$	$X \to \pi\pi J/\psi$ $X \to \pi\pi\psi'$ $X \to \Lambda\bar{\Lambda}$ $X \to \bar{D}D$	$X \to \phi J/\psi$ $X \to D\bar{D}$	$X \to \bar{D}D^*$ $X \to \pi\pi$

Multiquarks

Multiquarks

qqqqqq model of the deuteron

property	theory	experiment
E_d (MeV)	-2.9(3)	-2.33
$\langle r_d^2 \rangle^{1/2}$ (fm)	2.2(50	1.95
Q_d (mb)	2.1(50	2.86
μ_d (μ_N)	0.859(3)	0.857
$E(^1S_0)$ (MeV)	-0.4(3)	unbound

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図書室

HUTP-7

MOLECULAR CHARMONIUM: A NEW SPECTROSCOPY ?*

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ABSTRACT

Recent data compel us to interpret several peaks in the e^-e^+ annihilation cross section into hadrons as being due to the production of four-quark molecules, i.e., resonances between two charmed mesons. A rich spectroscopy of such states is predicted, and may be studied in e^-e^+ annihilation.

Multi-electron States

1946: Wheeler suggests that Ps_2 might be bound

Wheeler, J. A. Polyelectrons. Ann. NY Acad. Sci. 48, 219–238 (1946).

1946: Ore proves it is unbound

1947: Hylleraas & Ore prove it is bound

Hylleraas, E. A. & Ore, A. Binding energy of the positronium molecule. Phys. Rev.71, 493–496 (1947).

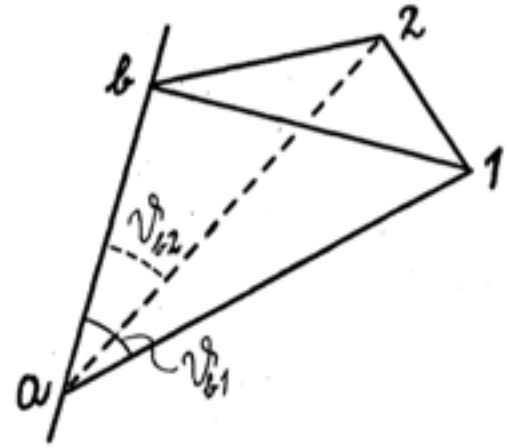
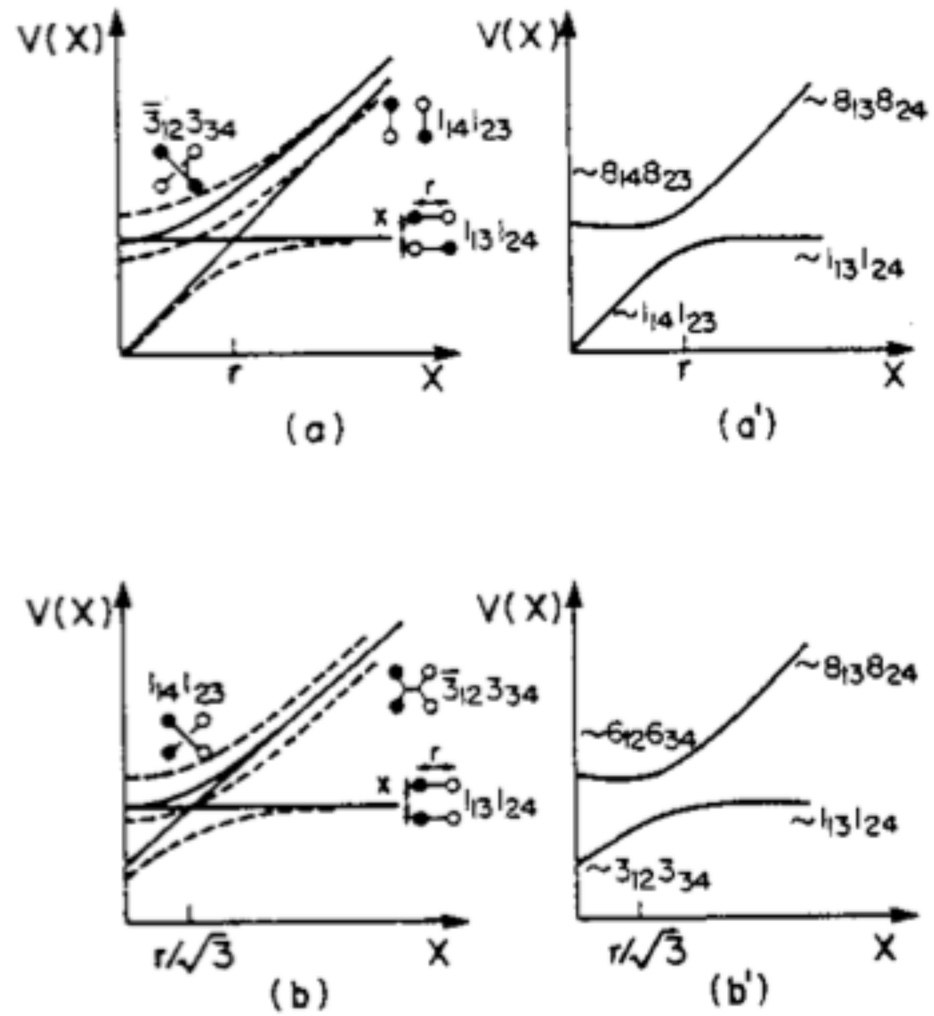


FIG. 1. Coordinate system for the positronium molecule.

2007: Ps_2 is observed

Cassidy, D.B.; Mills, A.P. (Jr.) (2007). "The production of molecular positronium". Nature 449 (7159): 195–197

Multiquarks

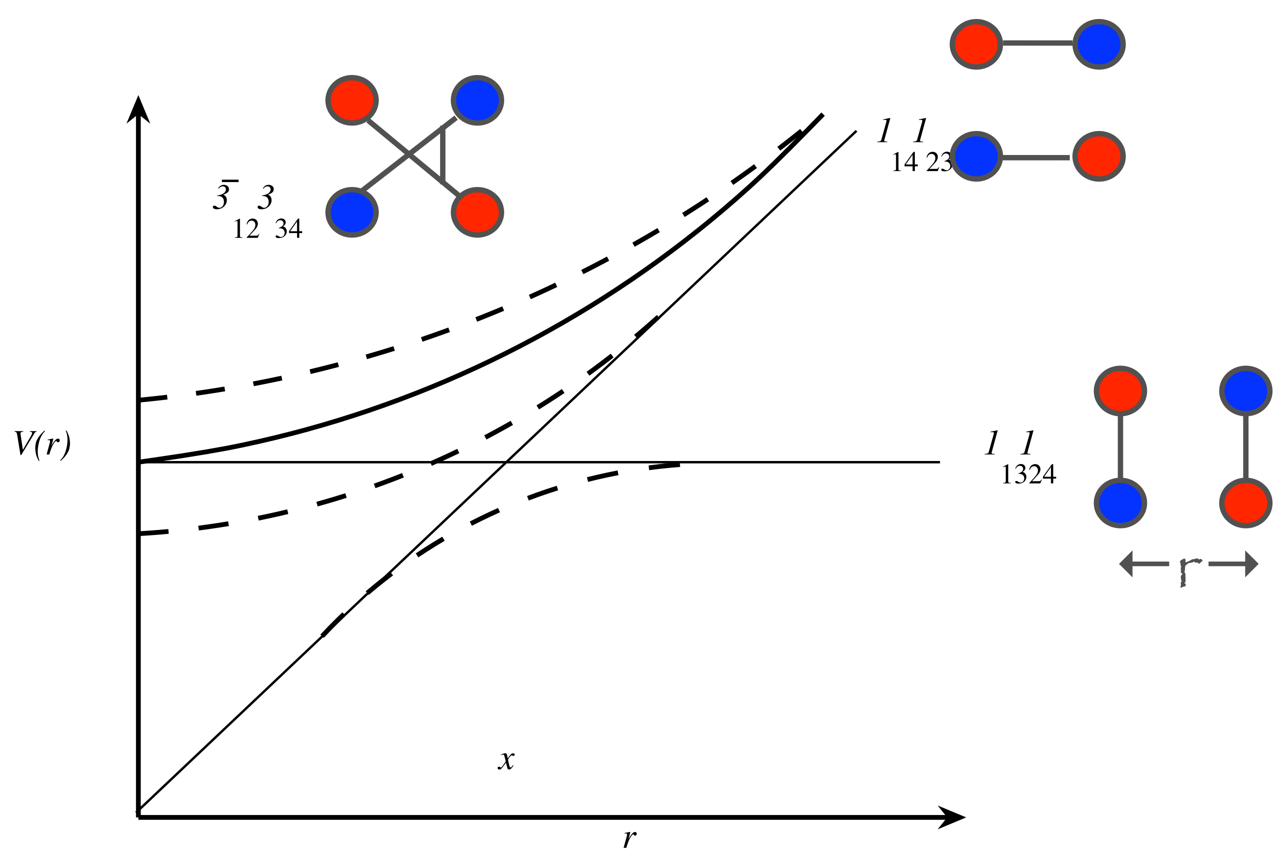


N. Isgur, “Hadron Spectroscopy: an Overview with Strings Attached”

Multiquarks

for the first time, the colour structure becomes important.

ex:

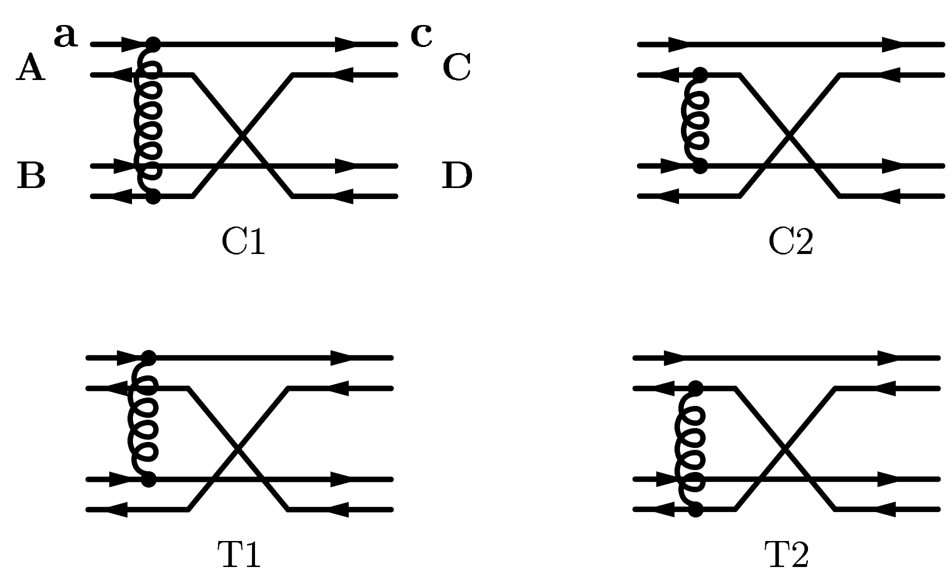


Multiquarks

model Hamiltonian

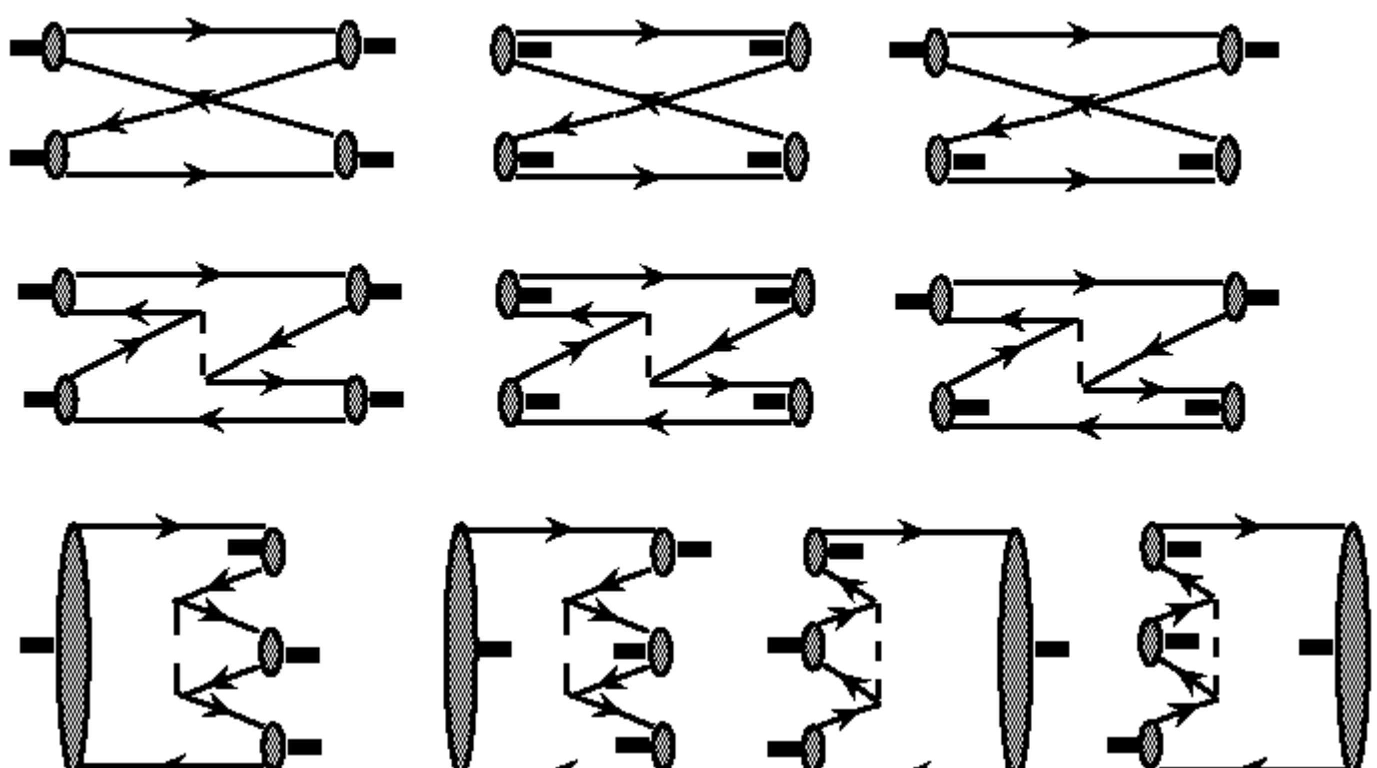
$$H = \sum_{i=1}^{4} \left(m_i + \frac{p_i^2}{2m_i} \right) - \sum_{i<j} \left(-\frac{\alpha_s}{r_{ij}} + \frac{3}{4} b r_{ij} + \frac{8\pi\alpha}{3m^2} \, \frac{\sigma^3}{\pi^{3/2}} \mathrm{e}^{-\sigma^2 r_{ij}^2} \, \vec{S}_i \cdot \vec{S}_j \right) \vec{F}_i \cdot \vec{F}_j$$

Born order amplitude

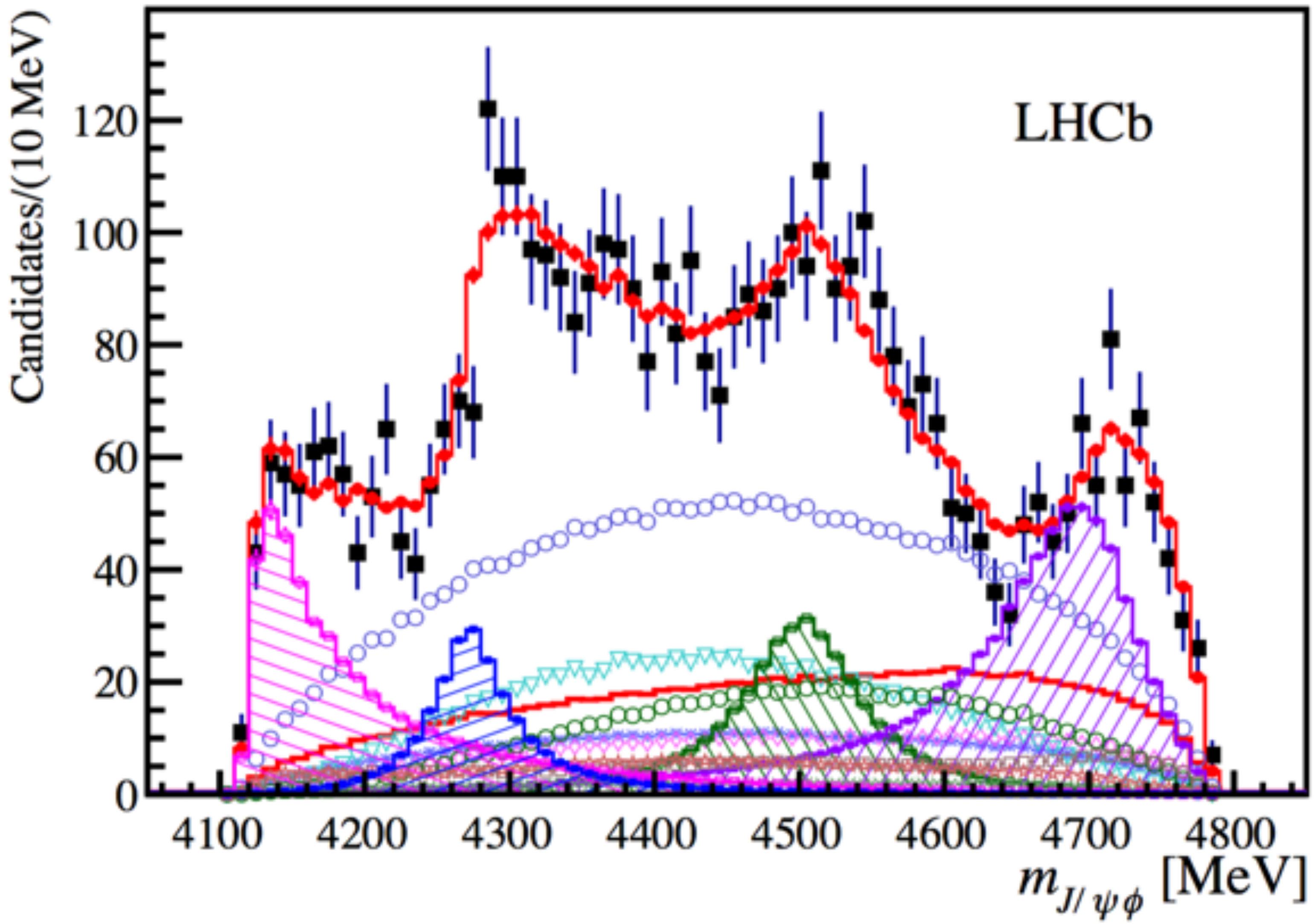


Multiquarks

Born diagrams with RPA mesons & annihilation



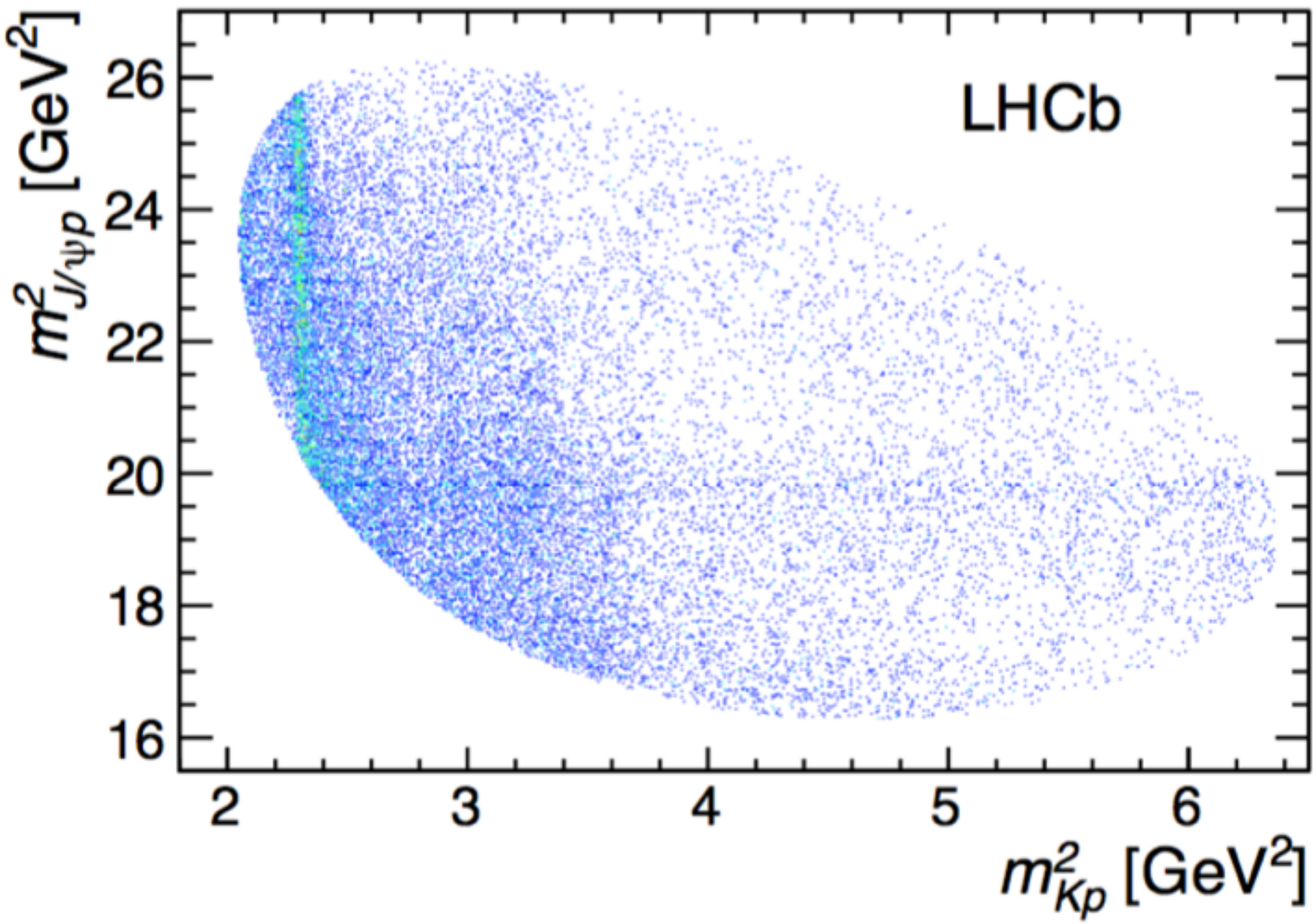
$$B \to K J/\psi\phi$$



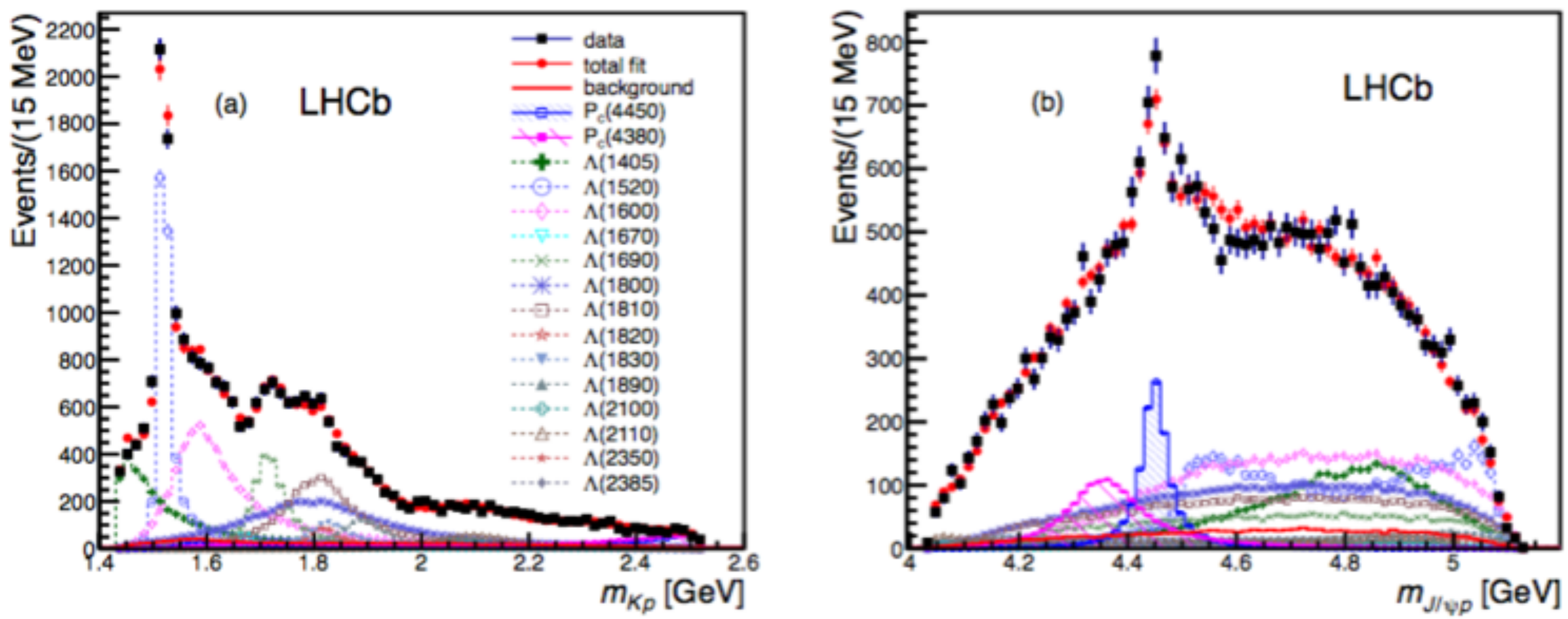
State	Mass (unct.) [MeV]	Width (unct.) [MeV]	J^{PC}
$Y(4140)$	4165.5(5,3)	83(21,16)	1^{++}
$Y(4274)$	4273.3(8,11)	56(11,10)	1^{++}
$X(4500)$	4506(11,13)	92(21,21)	0^{++}
$X(4700)$	4704(10,19)	120(31,35)	0^{++}

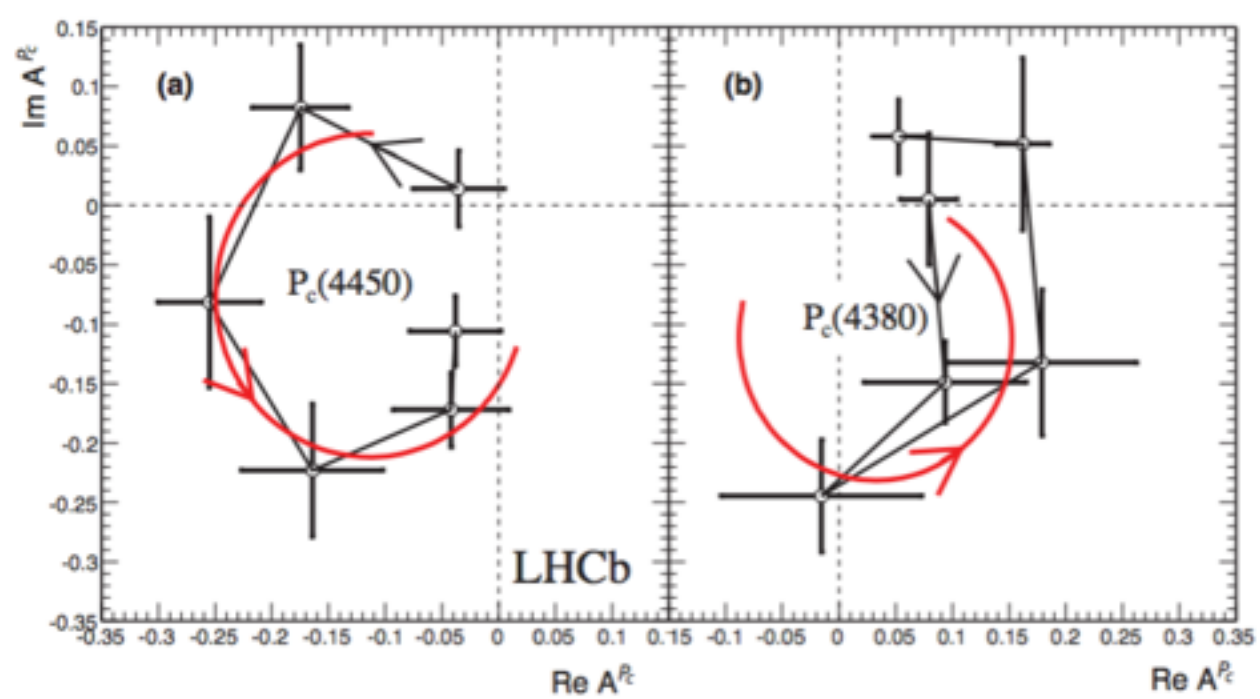
arXiv:1606.07895v1

$P_c(4370)$ & $P_c(4450)$



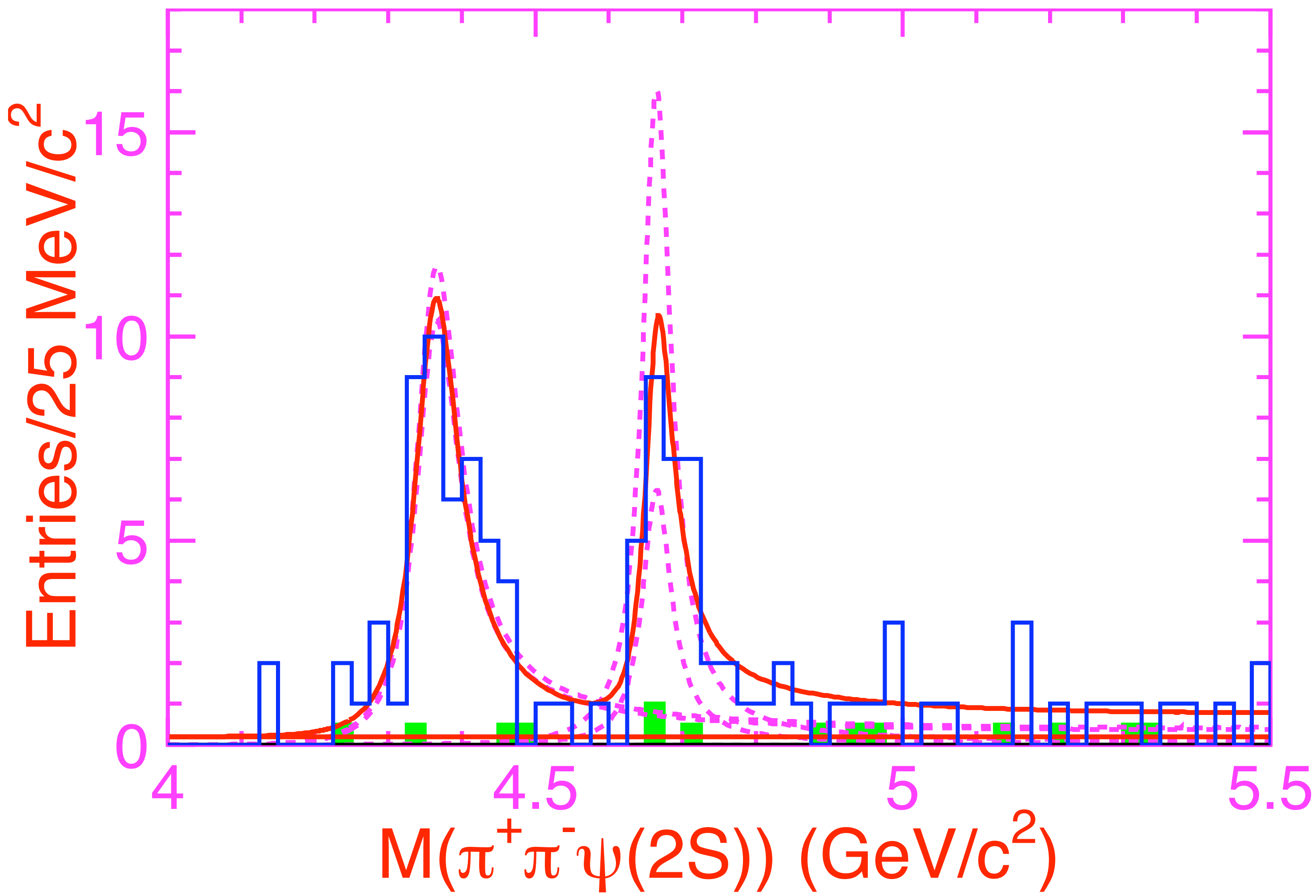
$P_c(4370)$ & $P_c(4450)$





$Y(4660)$

X.-L. Wang et al. [Belle] PRL99, 142002 (2007)



$e^+e^- \to \pi^+\pi^-\psi'$ (ISR)

$M = 4664 \pm 12$

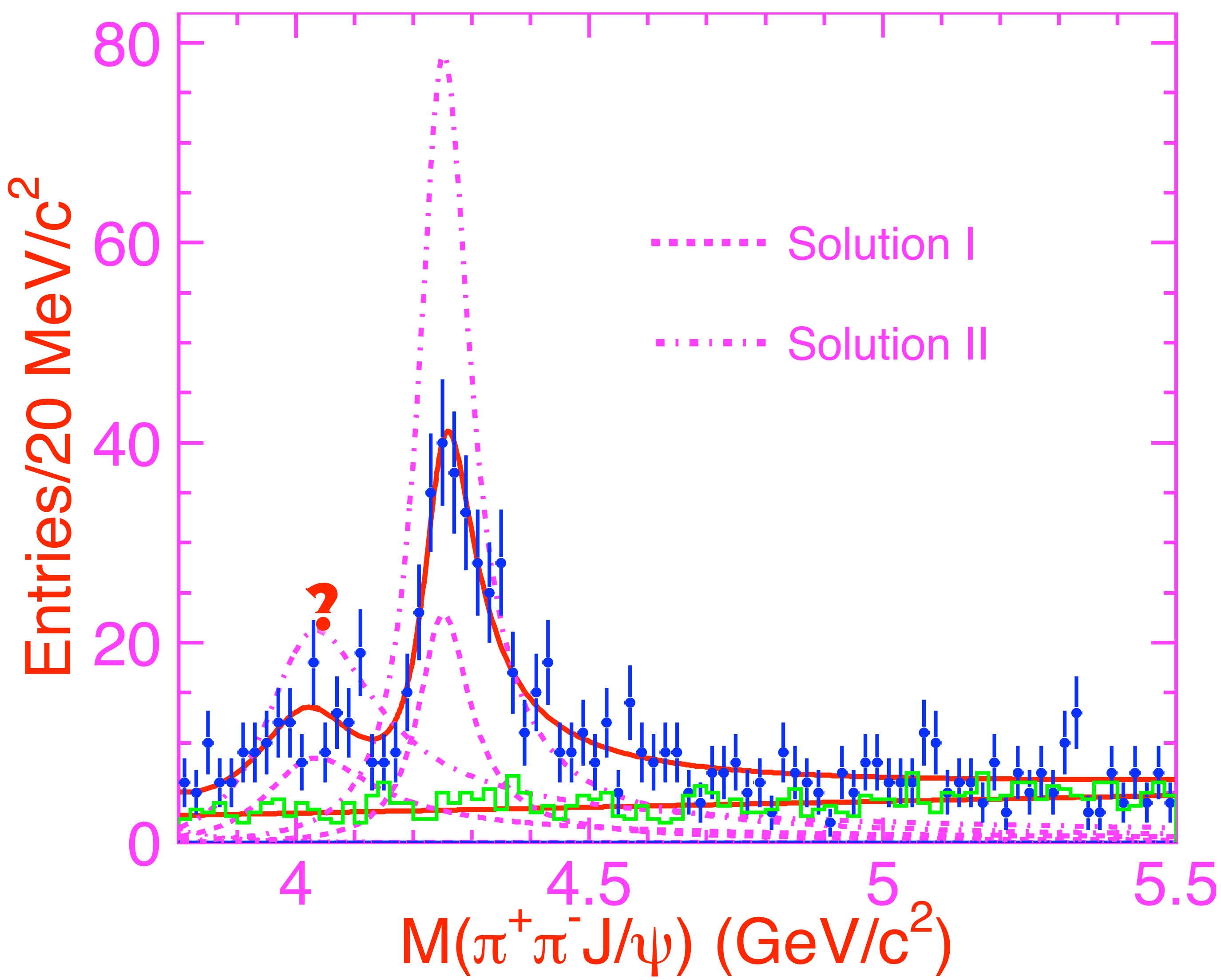
$\Gamma = 48 \pm 15$

not seen in
$e^+e^- \to \pi^+\pi^- J/\psi$

Y(4008)

$Y(4008)$

C.-Z. Yuan et al. [Belle] PRL99, 182004 (2007)



$$e^+e^- \to \pi^+\pi^- J/\psi$$

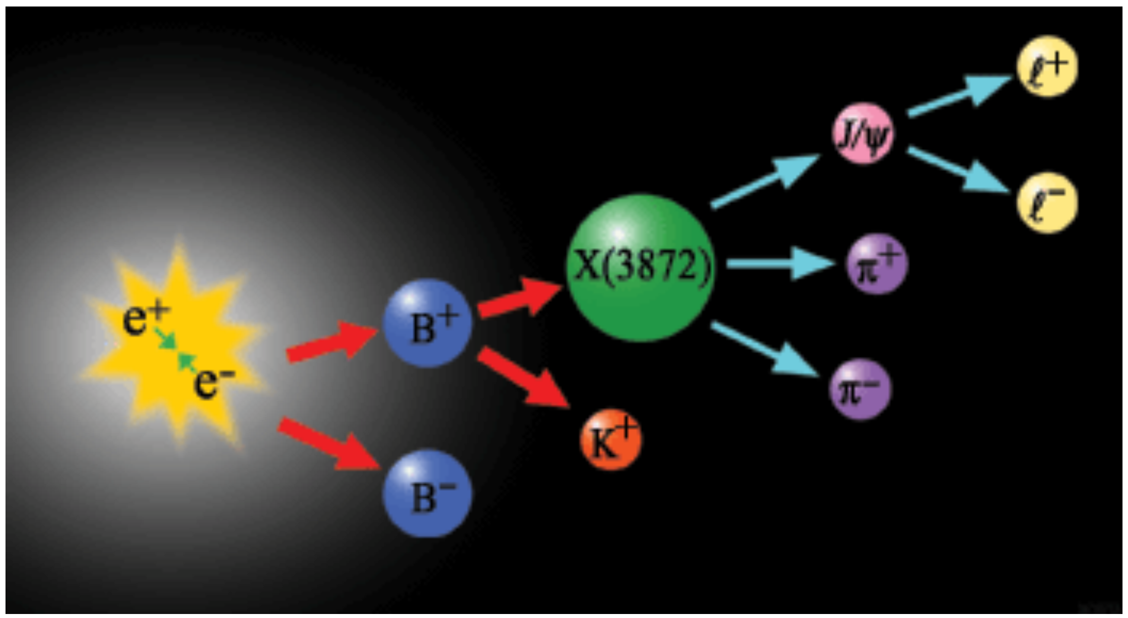
$$M = 4008 \pm 40^{+114}_{-28}$$

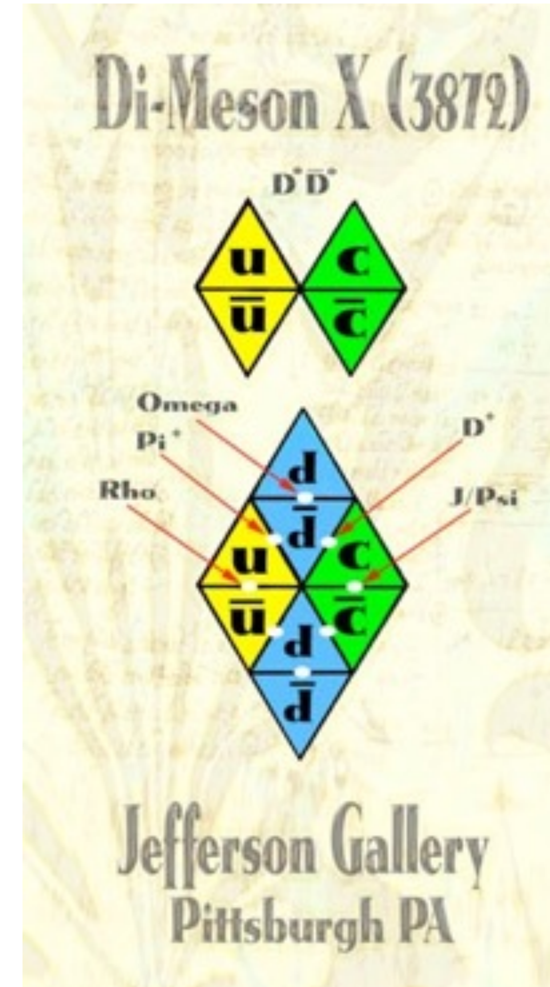
$$\Gamma = 226 \pm 44 \pm 87$$

BaBar claim no signal

Mokhtar, 0810.1073

X(3872)



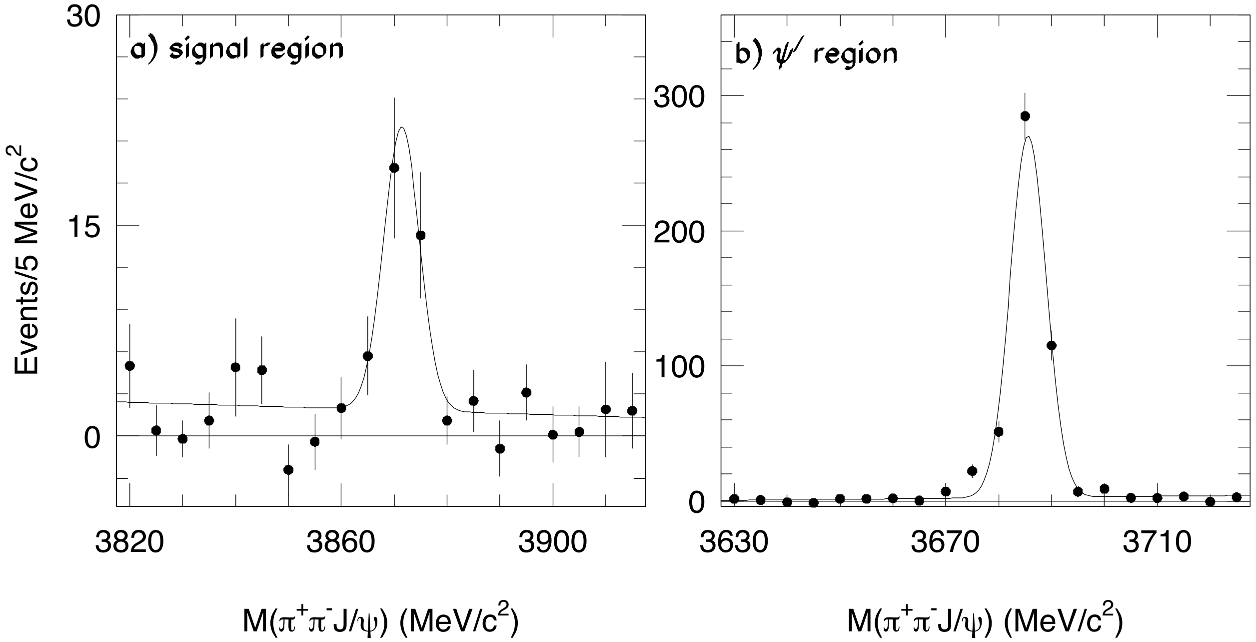


model the X(3872) as a $D\bar{D}^*$ bound state

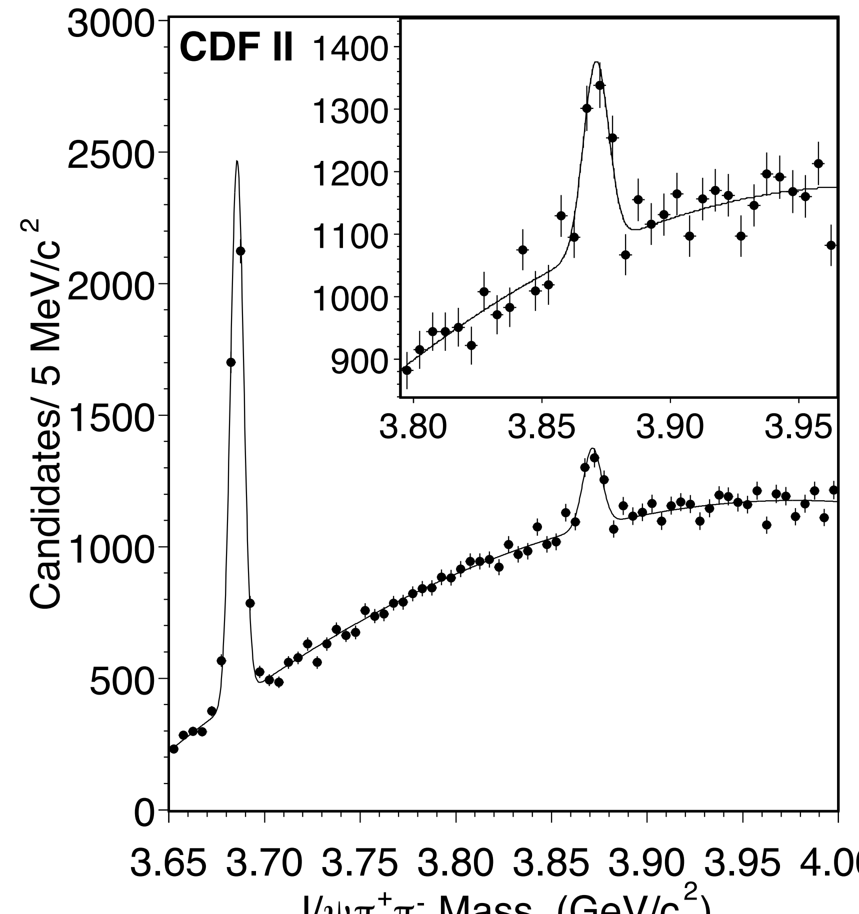
with $\omega J/\psi$ and $\rho J/\psi$ components.

X(3872)

$$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$$



S.-K. Choi (Belle), hep-ex/0309032



D. Acosta (CDF) hep-ex/0312021

B. Aubert (Babar) hep-ex/0402025

The X(3872) in 1992

190

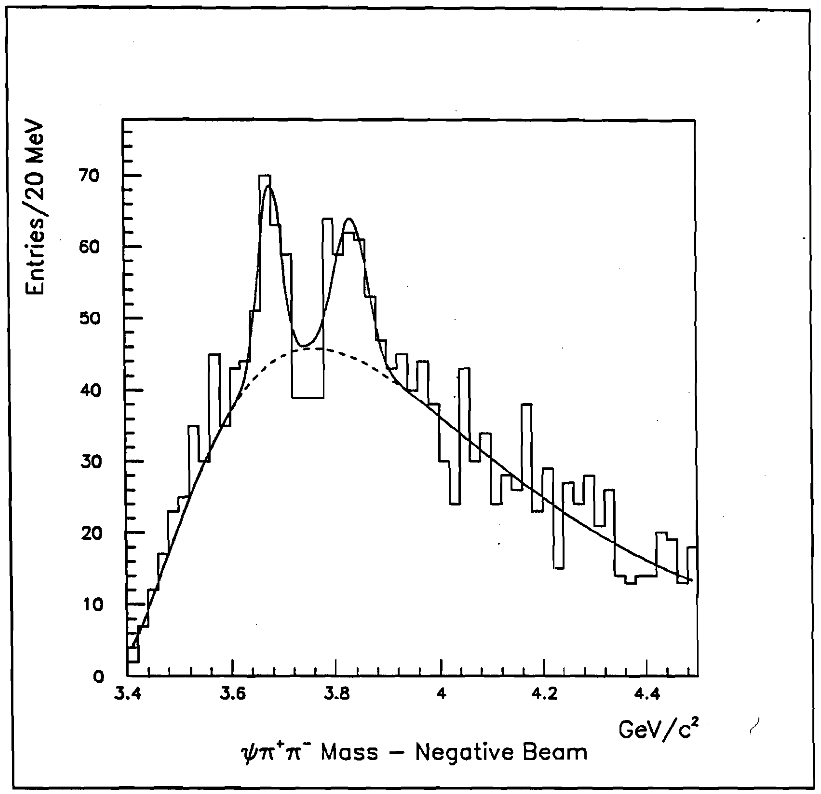


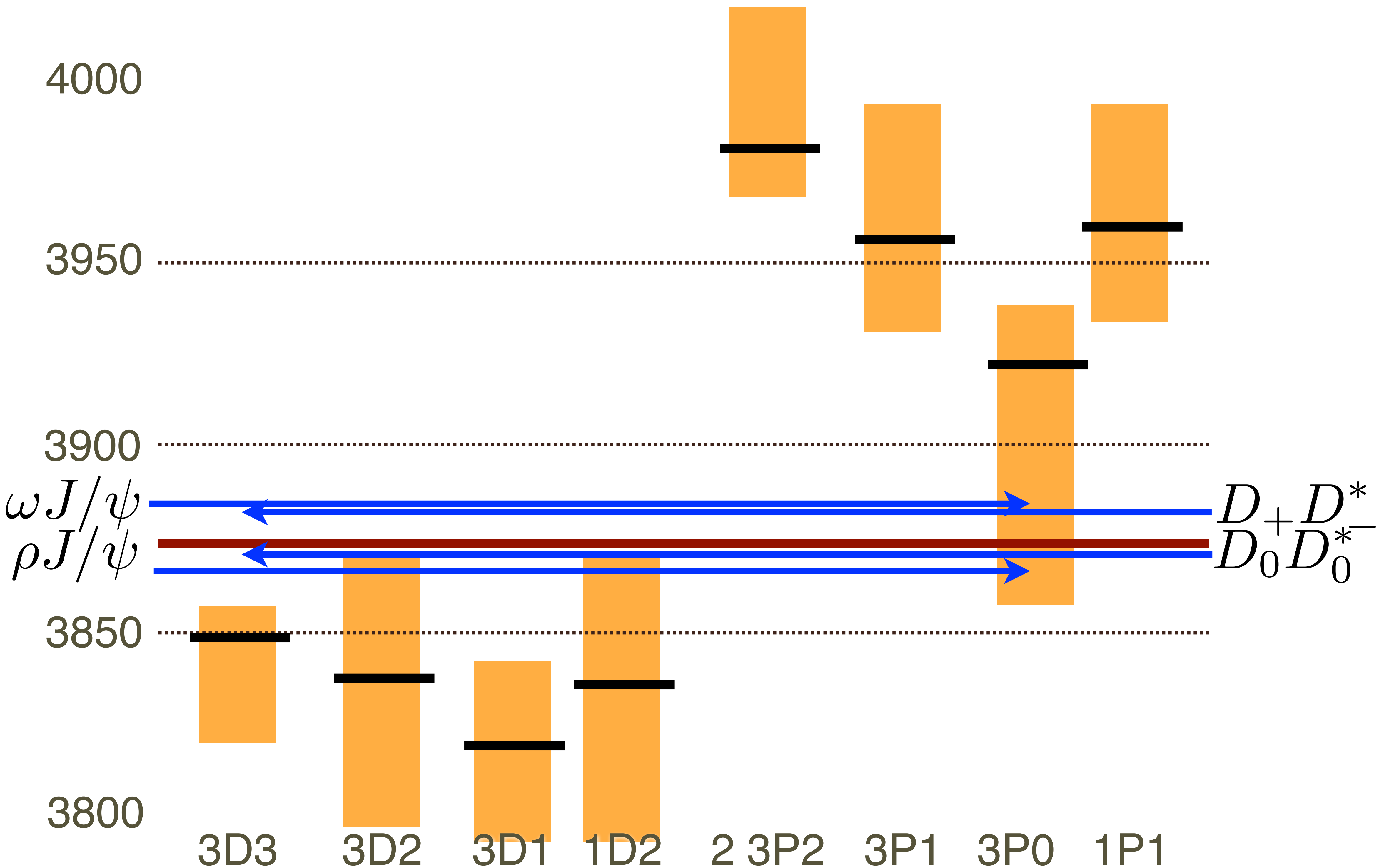
Figure 6.12 $\psi\pi\pi$ mass spectrum, standard cuts, negative beam.

A single peak above background does not fit the observed signal well. A second peak above the ψ' was added to the fit to improve this. The fit parameters are shown on the following page:

E-705

Tom LeCompte,
Northwestern thesis
E705 at FNAL

X(3872) ...


4000
3950
3900
3850
3800
$\omega J/\psi$
$\rho J/\psi$
$D_+ D^*_-$
$D_0 D^*_0$
3D3
3D2
3D1
1D2
2 3P2
3P1
3P0
1P1

Tornqvist, 0308277

Close & Page, PLB578, 119 (04)

ESS, PLB588, 189 (2004)

Molecular State (fast version)

- Model X as a $D\bar{D}^*$ state with admixture of $\omega\,\psi$ and $\rho\,\psi$
- microscopic model = ope + quark dynamics; project onto continuum channels.
- find only one bound state with $J^{PC} = 1^{++}$
- X decays via $\rho\,\psi$ to $\pi\,\pi\,\psi$ and $\omega\,\psi$ to $\pi\,\pi\,\pi\,\psi$ with comparable strength (isospin violation is natural in this model)

nonrelativistic effective field theory

$$\hat{H} = -\int d^3x \hat{\psi}_f^\dagger \tau_3 \left(m_f - \frac{\nabla^2}{2m_f} \right) \hat{\psi}_f +$$

$$+\frac{1}{2}\int d^3x d^3y \hat{\psi}_{f'}^\dagger(x) \hat{\psi}_{f'}(x) V(x-y) \hat{\psi}_f^\dagger(y) \hat{\psi}_f(y)$$

$$+\frac{1}{2}\int [\partial_\mu \pi^a \partial^\mu \pi^a + \frac{1}{2}\pi^2 m_\pi] + \frac{g}{\sqrt{2} f_\pi} \int \bar{\psi} \gamma^\mu \gamma_5 \tau^a \psi \partial_\mu \pi^a$$

project onto channels of interest

$$
\begin{aligned}
|\Psi\rangle = & \int d^3r_1 d^3r_2 \phi_{QQ}(r_1 - r_2)\varphi(\frac{r_1 + r_2}{2}) b_Q^\dagger(r_1) d_Q^\dagger(r_2)|0\rangle \\
+ & \int d^3r_1 d^3r_2 d^3r_3 d^3r_4 \varphi\left(\frac{u}{2}(r_1 + r_4) + \frac{v}{2}(r_2 + r_3)\right) \\
& \psi\left(u(r_1 - r_4) + v(r_3 - r_2)\right) \phi_{Qq}(r_1 - r_3)\phi_{qQ}(r_2 - r_4) \\
& b_Q^\dagger(r_1) d_q^\dagger(r_3) b_q^\dagger(r_2) d_Q^\dagger(r_4)|0\rangle .
\end{aligned}
$$

pion exchange

nonrelativistic PsV interaction:

$$V_\pi = -\gamma V_0 \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} C(r) + \begin{array}{c} \begin{array}{cc} {}^3S_1 & {}^3D_1 \end{array} \\ \begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} \end{array} T(r) \right]$$

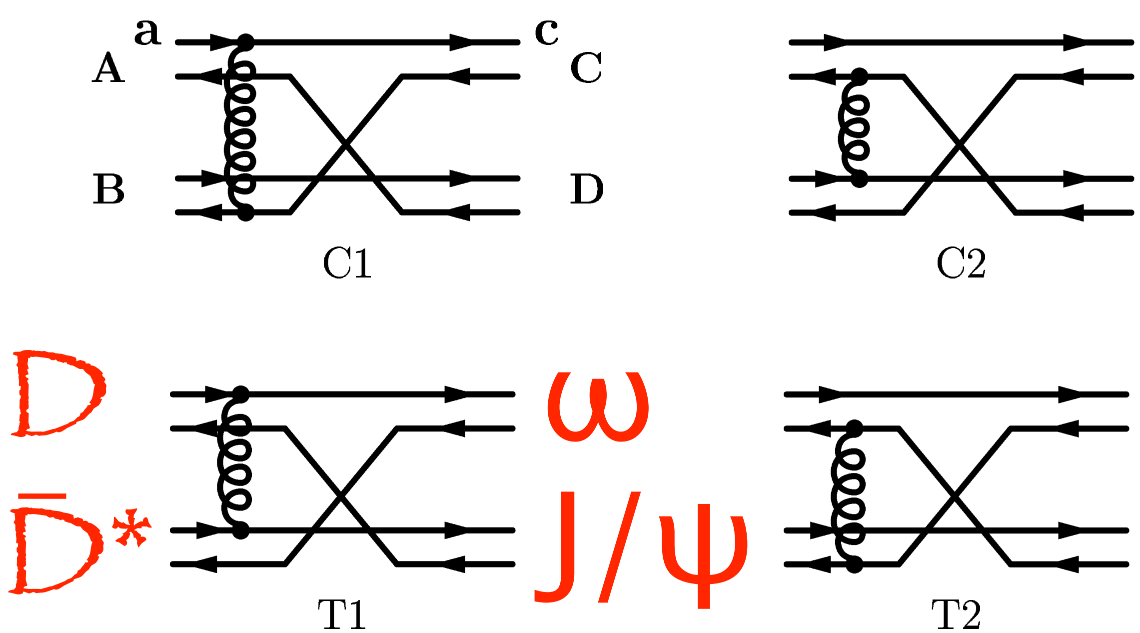
$$C(r) = \frac{\mu^2}{m_\pi^2} \frac{\mathrm{e}^{-\mu r}}{m_\pi r}$$

$$T(r) = C(r) \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2} \right)$$

I	C	γ	
0	+	3	1^{++}
1	-	1	
1	+	-1	
0	-	-3	

N. Tornqvist, Z. Phys. C61, 525 (1994)

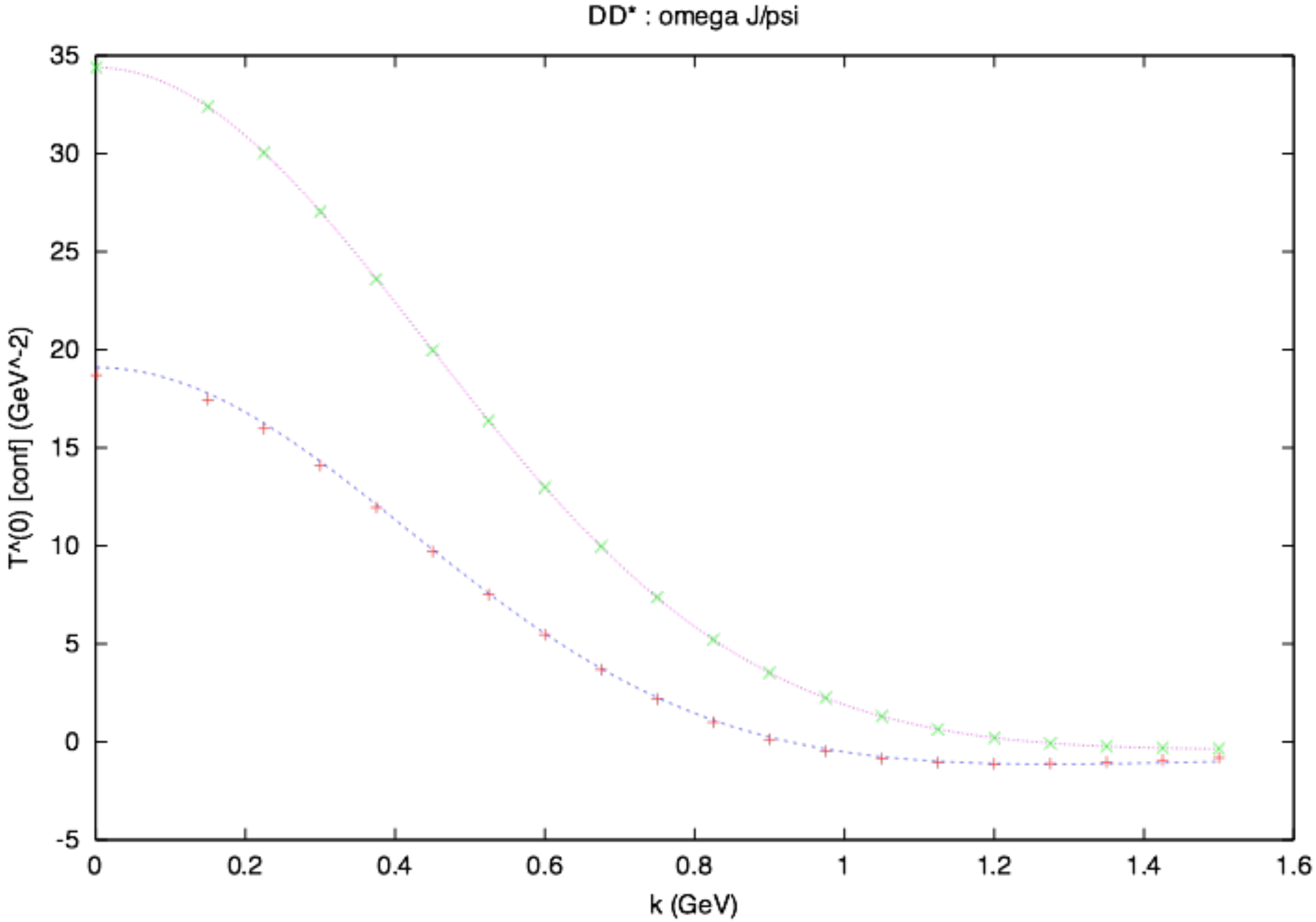
quark-level interactions



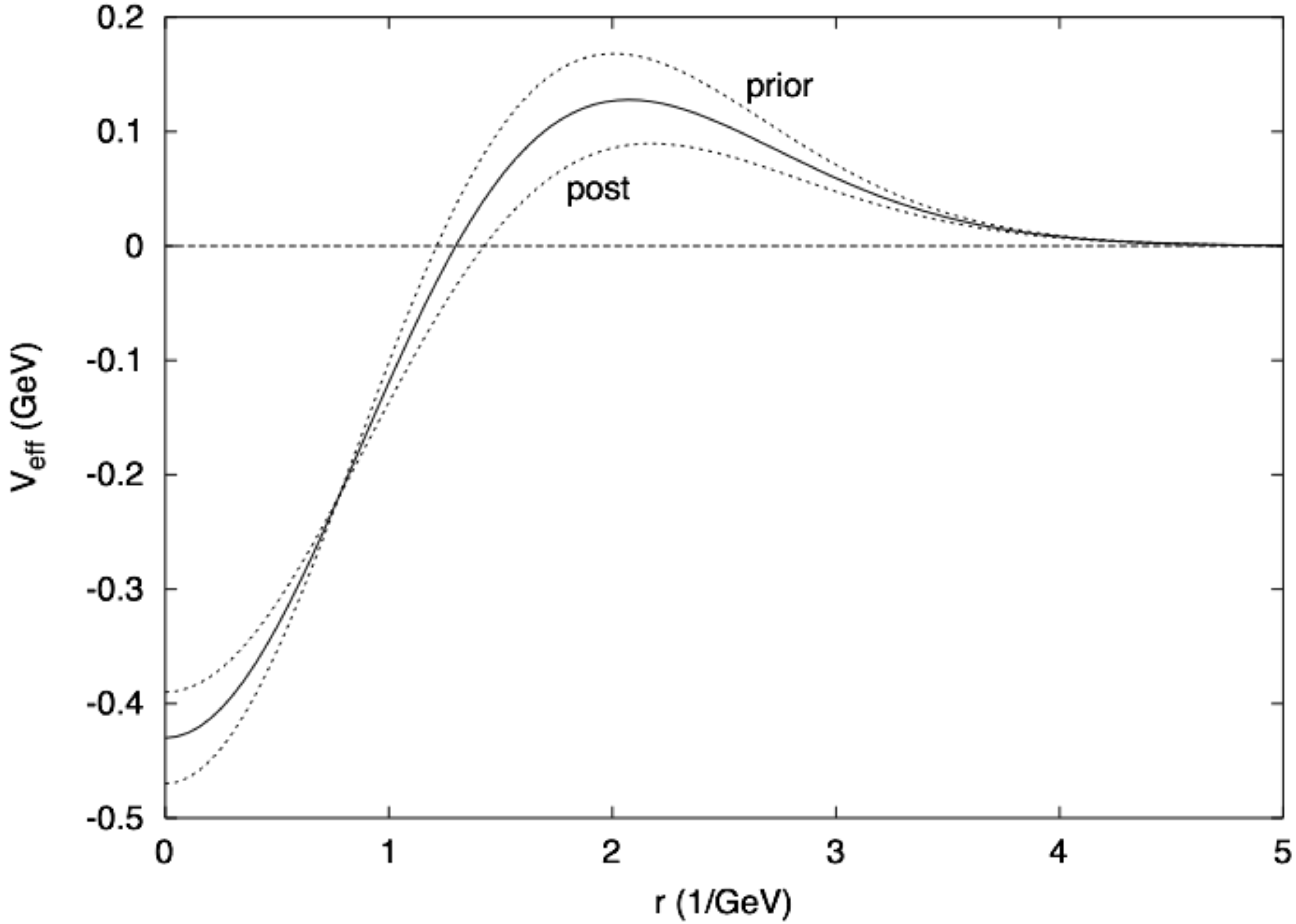
T. Barnes & ESS, PRD46, 131 (1992)

ESS, Ann Phys 220, 73 (1992)

DD* →ω J/ψ T-Matrix



Effective Potential



channels

- $\rho J/\psi$ at 3.8679 GeV
- $\frac{1}{\sqrt{2}}(D^0\bar{D}^{0*} + \bar{D}^0 D^{0*})_S$ at 3.8712 GeV
- $\frac{1}{\sqrt{2}}(D^0\bar{D}^{0*} + \bar{D}^0 D^{0*})_D$ at 3.8712 GeV

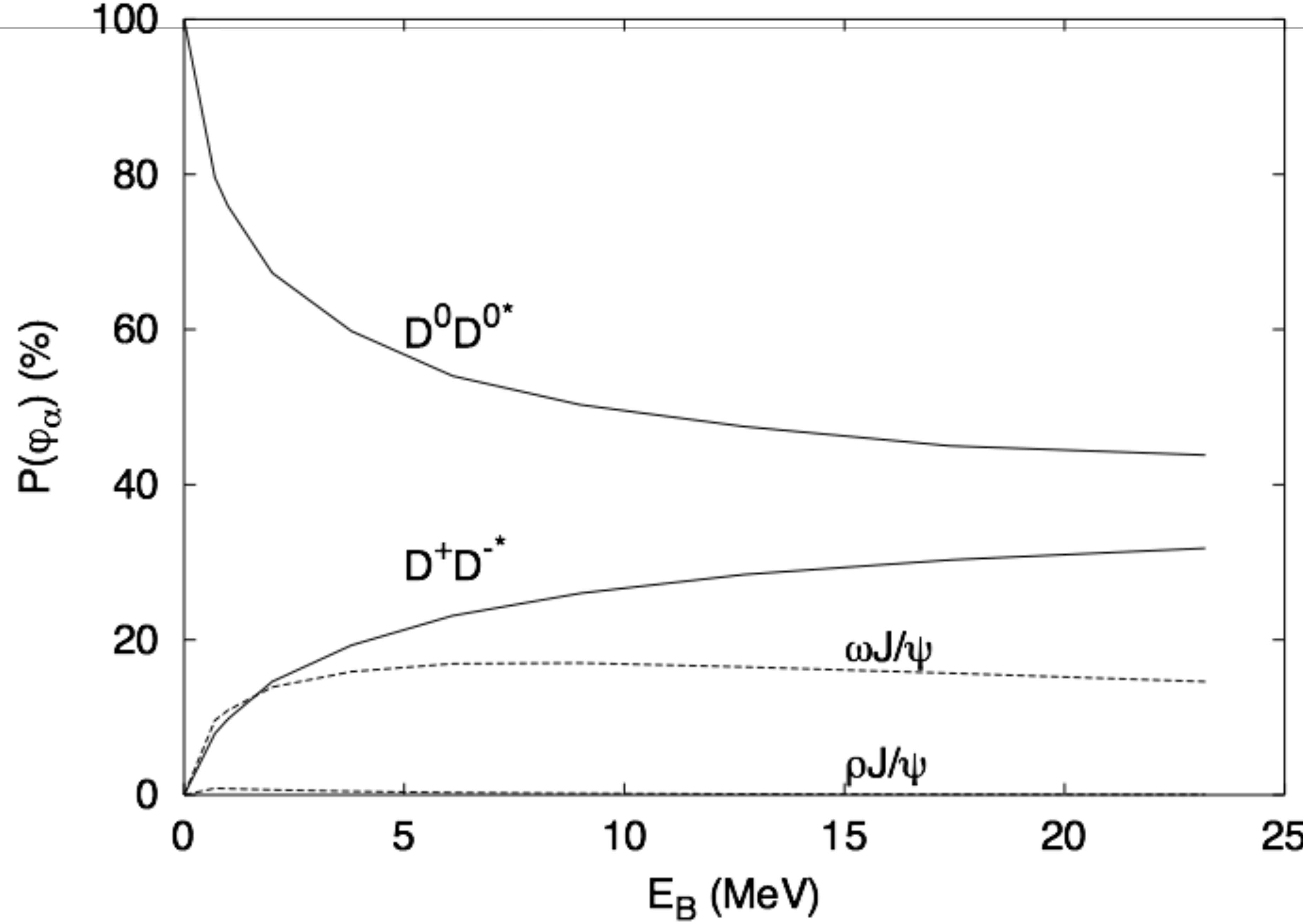
isospin breaking

- $\frac{1}{\sqrt{2}}(D^+ D^{-*} + D^- D^{+*})_S$ at 3.8793 GeV
- $\frac{1}{\sqrt{2}}(D^+ D^{-*} + D^- D^{+*})_D$ at 3.8793 GeV
- $\omega J/\psi$ at 3.8795 GeV.

channels

V	$\rho\psi$	$D^0\bar{D}^{0*}$	D^+D^{-*}	$\omega\psi$
$\rho\psi$	$-$	V_q	V_q	$-$
$D^0\bar{D}^{0*}$		V_π	V_π	V_q
D^+D^{-*}			V_π	V_q
$\omega\psi$				$-$

$\hat{\chi}_{c1}$ components



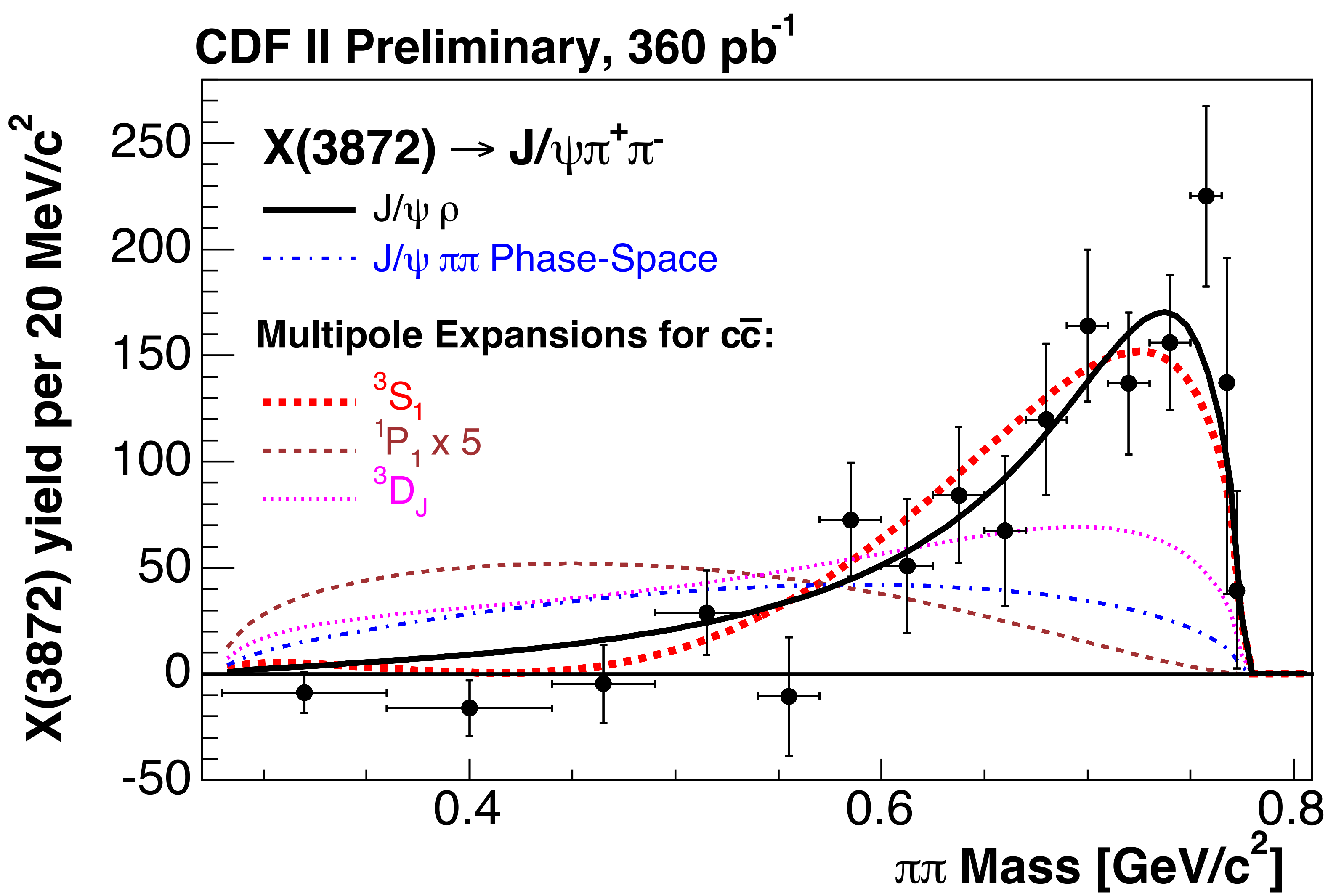
$\hat{\chi}_{c1}$ decay widths

weak binding $\rightarrow$ use free space decay widths to estimate dissociation decay modes

	D^{0*}	D^{0*}	D^{-*}	D^{-*}	D^{-*}	ρ	ρ/ω	ω	ρ/ω
B_E (MeV)	$D^0\bar{D}^0\pi^0$	$D^0\bar{D}^0\gamma$	$D^+D^-\pi^0$	$(D^+\bar{D}^0\pi^-+\text{c.c})/\sqrt{2}$	$D^+D^-\gamma$	$\pi^+\pi^-J/\psi$	$\pi^+\pi^-\gamma J/\psi$	$\pi^+\pi^-\pi^0J/\psi$	$\pi^0\gamma J/\psi$
0.7	67	38	5.1	4.7	0.2	1290	12.9	720	70
1.0	66	36	6.4	5.8	0.3	1215	12.1	820	80
2.0	57	32	9.5	8.6	0.4	975	9.8	1040	100
3.8	52	28	12.5	11.4	0.6	690	6.9	1190	115
6.1	46	26	15.0	13.6	0.7	450	4.5	1270	120
9.0	43	24	16.9	15.3	0.8	285	2.9	1280	125
12.7	38	22	18.5	16.7	0.9	180	1.8	1240	120

$$\frac{\Gamma(\hat{\chi} \longrightarrow \pi\pi\pi J/\psi)}{\Gamma(\hat{\chi} \longrightarrow \pi\pi J/\psi)} = 0.56$$

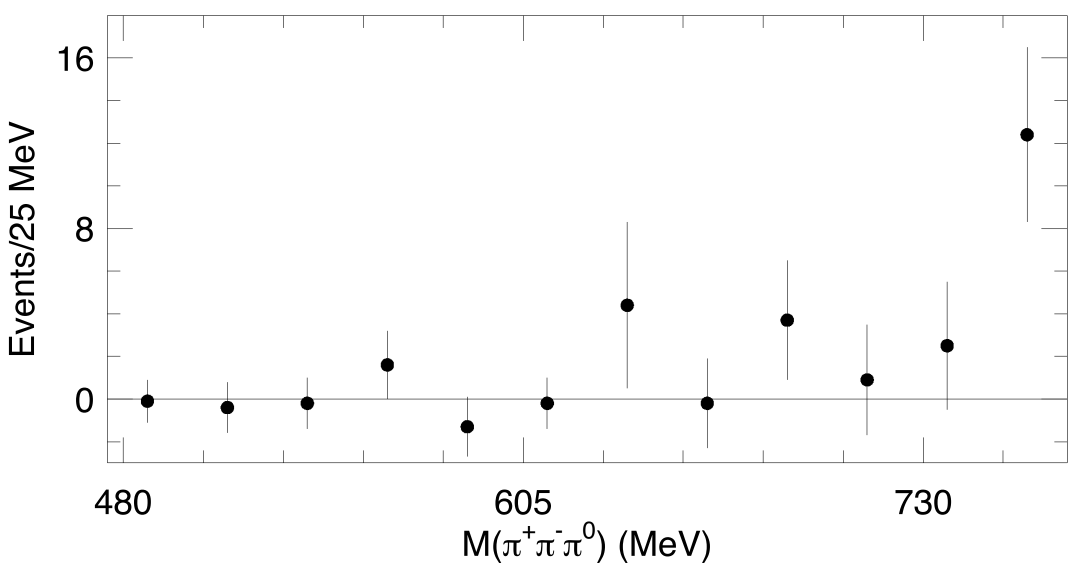
dipion spectrum



CDF note 05-03-24

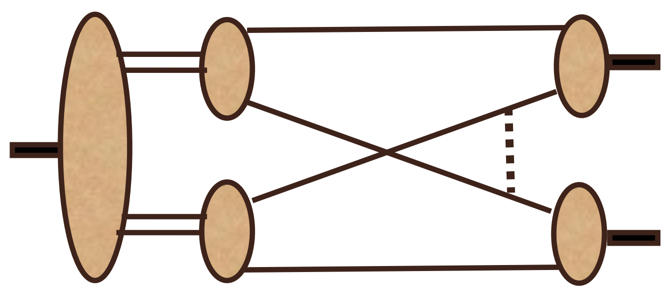
Exotic Theory: Models

$$X \to 3\pi J/\psi$$

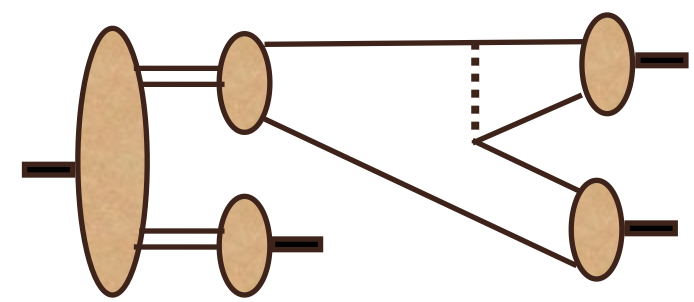


$$\frac{\Gamma(X \to \omega J/\psi)}{\Gamma(X \to \pi^+\pi^- J/\psi)} = 1.0(4)(3)$$

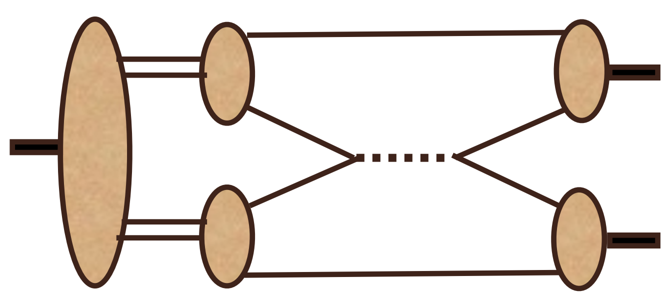
Decay Mechanisms



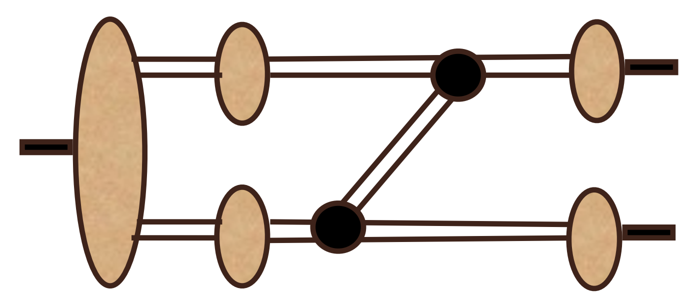
rearrangement



dissociation

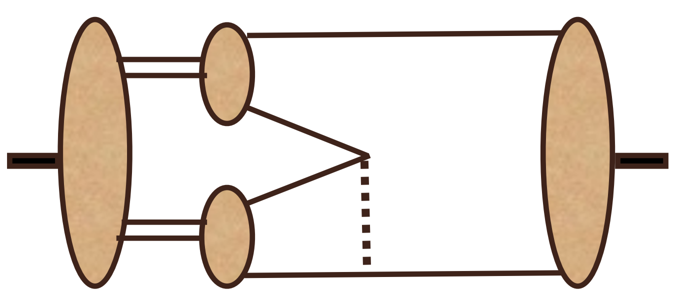


annihilation

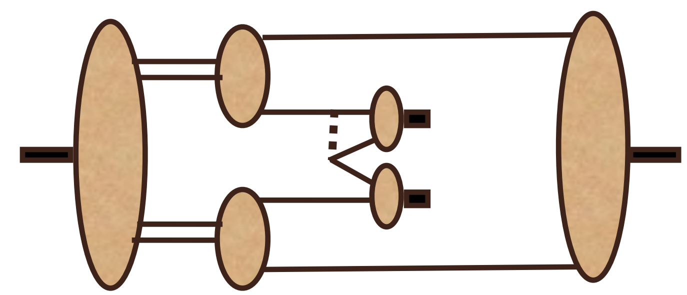


pion induced

Decay Mechanisms



mixing

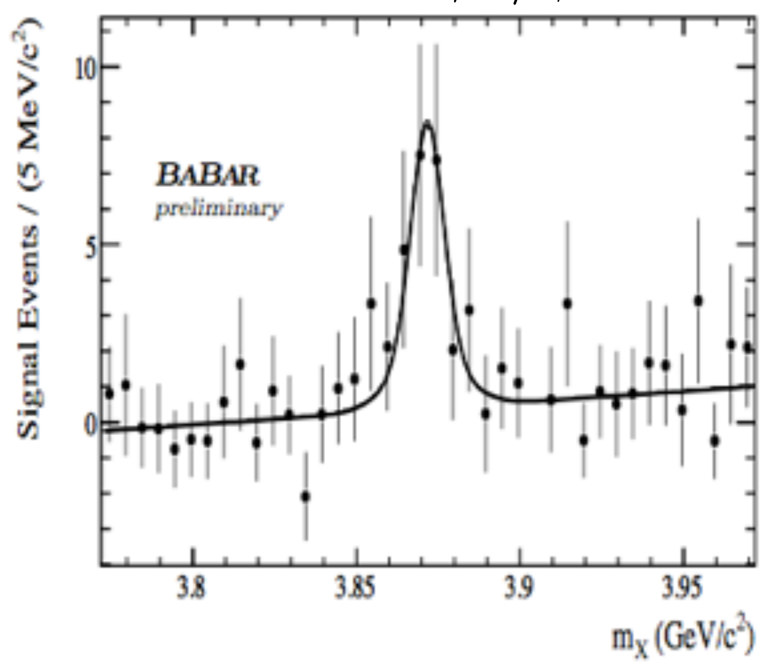


ππ production

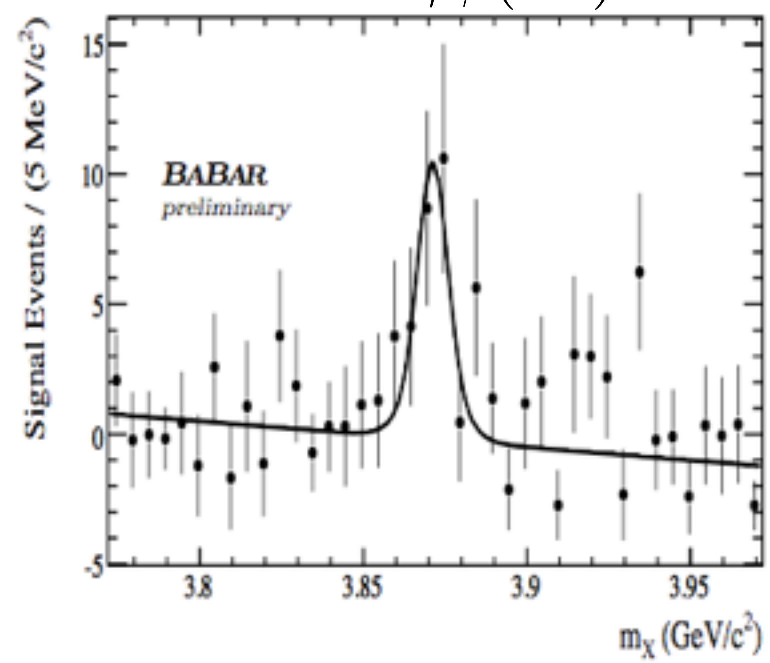
BaBar...

Aubert et al., 0809.0042

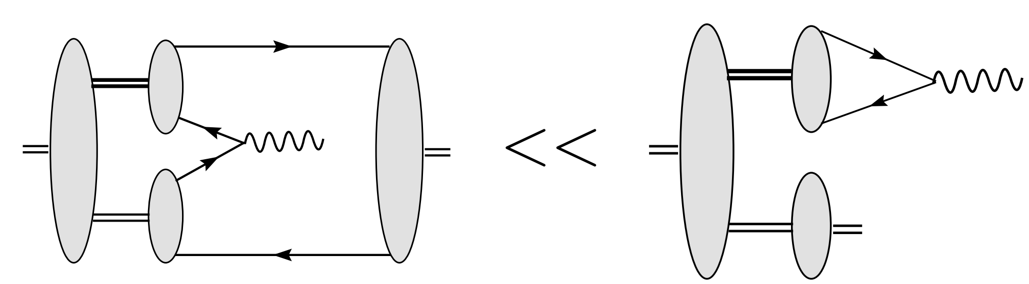
$X \to \gamma J/\psi$



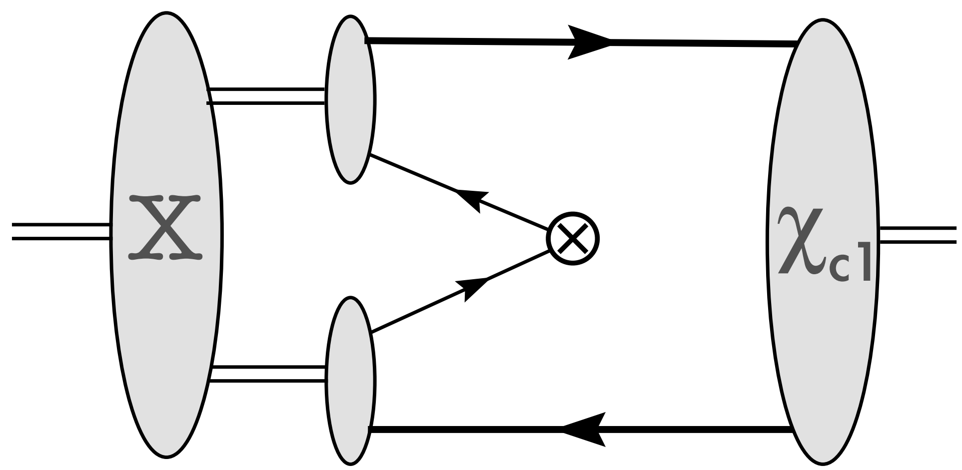
$X \to \gamma \psi(2S)$



$$\frac{Bf(X \to \gamma\psi(2S))}{Bf(X \to \gamma J/\psi)} = 3.5(1.0)$$



Mixing



$$a_\chi = \sqrt{2} Z_{00}^{1/2} \int d^3k\, \psi_X(k) \mathcal{A}(-k)$$

state	E_B (MeV)	a (fm)	Z_{00}	a_χ (MeV)	prob
χ_{c1}	0.1	14.4	93%	94	5%
	0.5	6.4	83%	120	10%
χ'_{c1}	0.1	14.4	93%	60	100%
	0.5	6.4	83%	80	$>100\%$

ESS, probably will never complete....

Other Molecules

I=0 $D^*\bar{D}^*$ states

no MM mixtures

state	J^{PC}	channels	mass (MeV)	E_B
$D^*\bar{D}^*$	0^{++}	${}^1S_0,\ {}^5D_0$	4019	1.0
$B\bar{B}^*$	0^{-+}	${}^3P_0$	10543	61
$B\bar{B}^*$	1^{++}	${}^3S_1,\ {}^3D_1$	10561	43
$B^*\bar{B}^*$	0^{++}	${}^1S_0,\ {}^5D_0$	10579	71
$B^*\bar{B}^*$	0^{-+}	${}^3P_0$	10588	62
$B^*\bar{B}^*$	1^{+-}	${}^3S_1,\ {}^3D_1$	10606	44
$B^*\bar{B}^*$	2^{++}	${}^1D_2,\ {}^5S_2,\ {}^5D_2,\ {}^5G_2$	10600	50

Diquarks and the New Charmonia

Maiani, Piccinini,Polosa, Riquer; PRD71, 014028 (2005)

Bigi, Maiani, Piccinini,Polosa, Riquer; PRD72, 114016 (2005)

Maiani, Riquer, Piccinini, Polosa; PRD72, 031502 (2005)

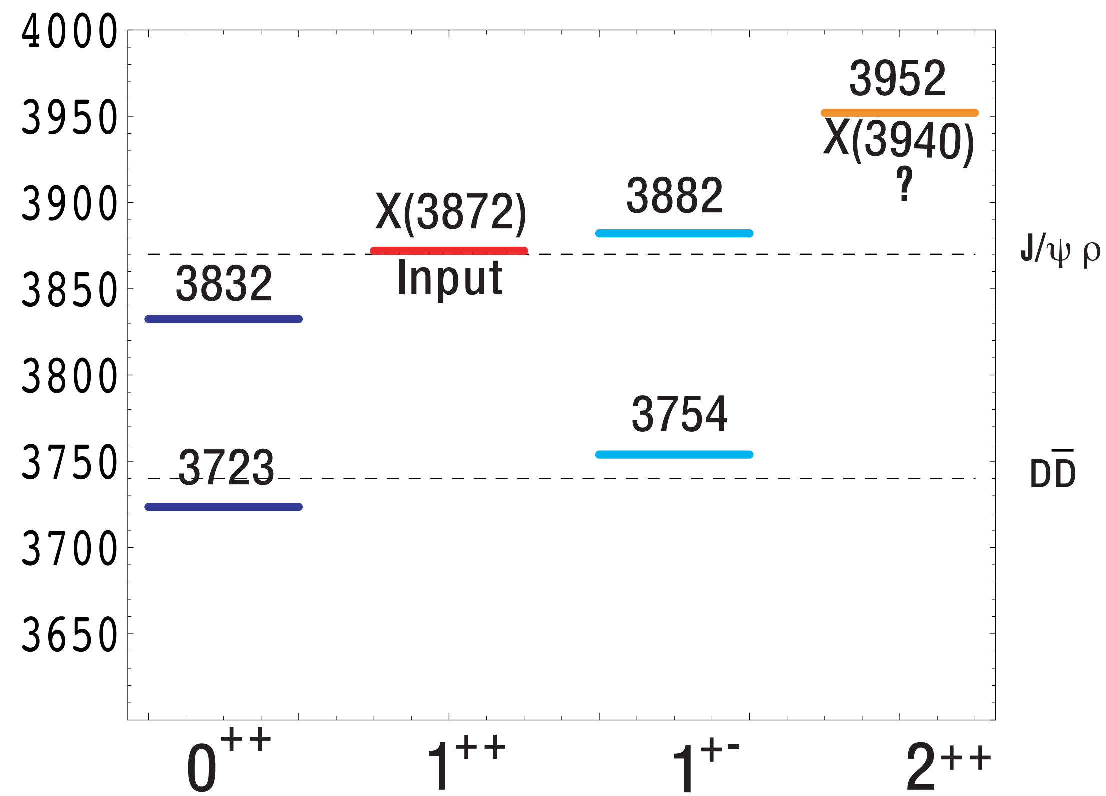
Maiani, Polosa, Riquer; PRL99, 182003 (2007)

Maiani, Polosa, Riquer; arXiv:0708.3997

$$M([cq]_S) = 1933$$
$$M([cq]_V) = 1933$$

Assume a spin-spin interaction

$$|0^{++}\rangle = |[cq]_S[\bar{c}\bar{q}]_S; J=0\rangle \quad (1)$$
$$|0^{++\prime}\rangle = |[cq]_V[\bar{c}\bar{q}]_V; J=0\rangle \quad (2)$$
$$|1^{++}\rangle = \frac{1}{\sqrt{2}}\left(|[cq]_S[\bar{c}\bar{q}]_V; J=1\rangle + |[cq]_V[\bar{c}\bar{q}]_S; J=1\rangle\right) \quad (3)$$
$$|1^{+-}\rangle = \frac{1}{\sqrt{2}}\left(|[cq]_S[\bar{c}\bar{q}]_V; J=1\rangle - |[cq]_V[\bar{c}\bar{q}]_S; J=1\rangle\right) \quad (4)$$
$$|1^{+-\prime}\rangle = |[cq]_V[\bar{c}\bar{q}]_V; J=1\rangle \quad (5)$$
$$|2^{++}\rangle = |[cq]_V[\bar{c}\bar{q}]_V; J=2\rangle \quad (6)$$


4000
3950
3900
3850
3800
3750
3700
3650
3952
X(3940)
?
X(3872)
Input
3882
3832
3754
3723
J/ψ ρ
DD̄
0++
1++
1+-
2++