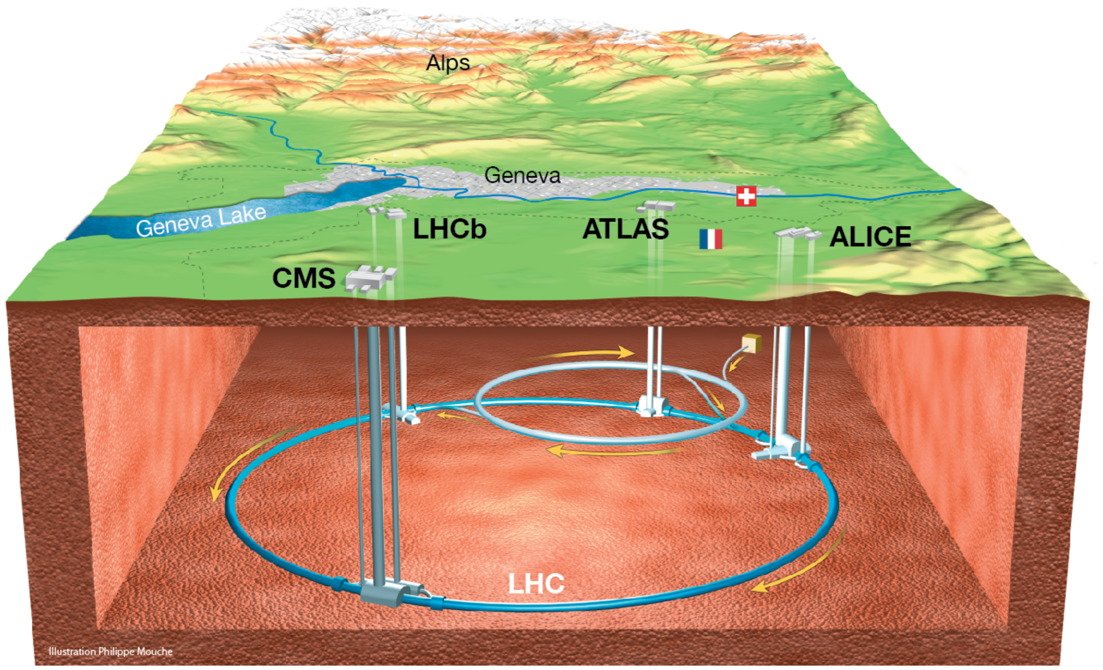
BERNHARD MISTLBERGER

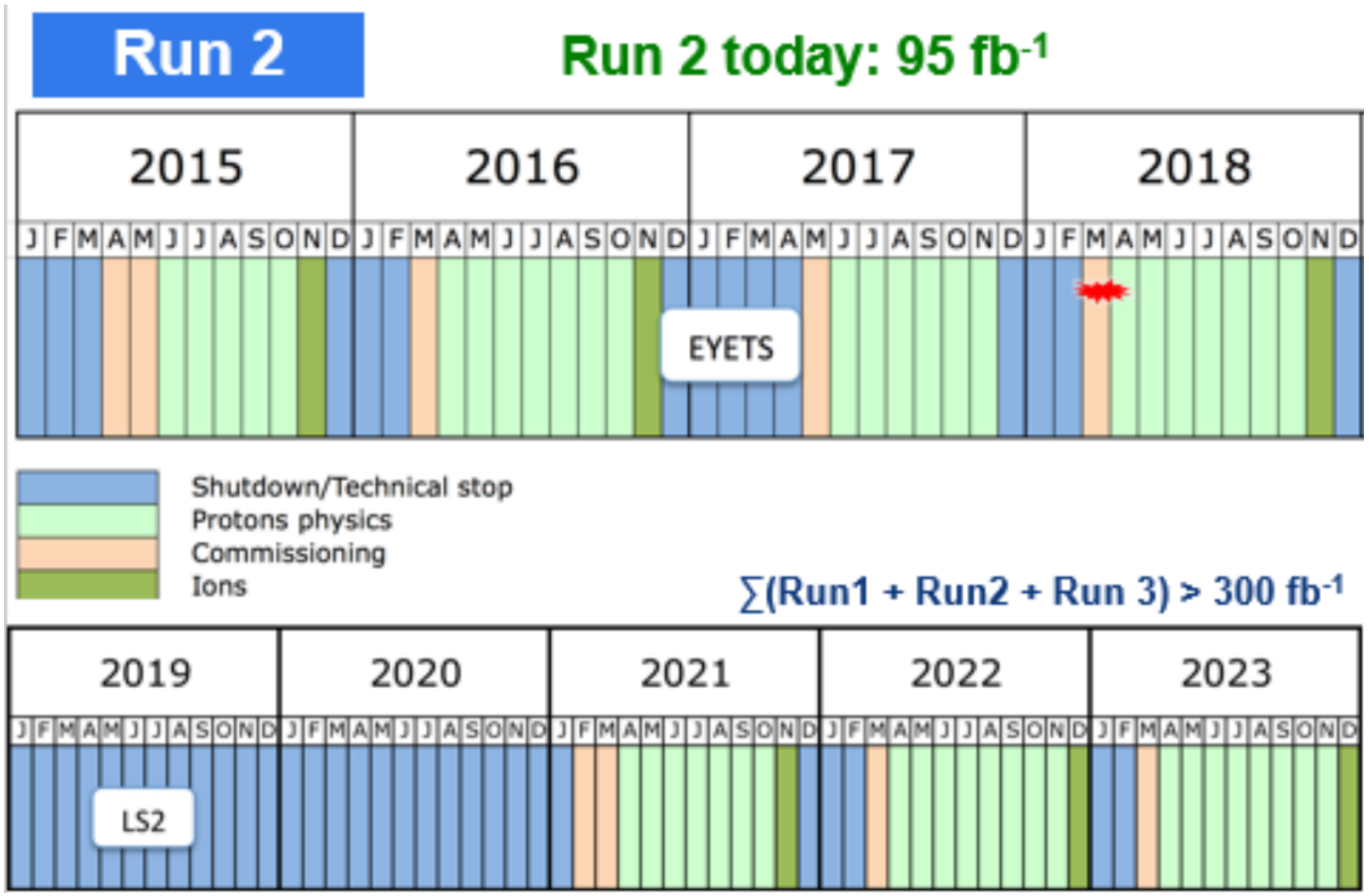
PRECISION PREDICTIONS AT N3LO FOR THE HIGGS BOSON RAPIDITY DISTRIBUTION

with Falko Dulat and Andrea Pelloni

THE LHC - AN INCREDIBLE SUCCESS



- Running since 2009
- Experimental performance excellent and exceeding expectations!



- **We are still at the beginning of LHC physics!**
- **300 fb^-1 until end of 2023**
- **3000 fb^-1 in HL - LHC**

THE LHC – AN INCREDIBLE SUCCESS

- 4th of July 2012: The beginning of the precision physics age of Higgs boson phenomenology
  - The SM of Particle Physics is now a complete / self-consistent theory!
  - The vast amount of data allows us to study the Higgs physics in detail.

✳ **Gain insight in the mechanism of electro-weak symmetry breaking**

✳ **Investigate the generation of fundamental masses**

✳ **Determine couplings / interactions with established matter**

✳ **Explore the limitations of the Standard Model of particle physics.**

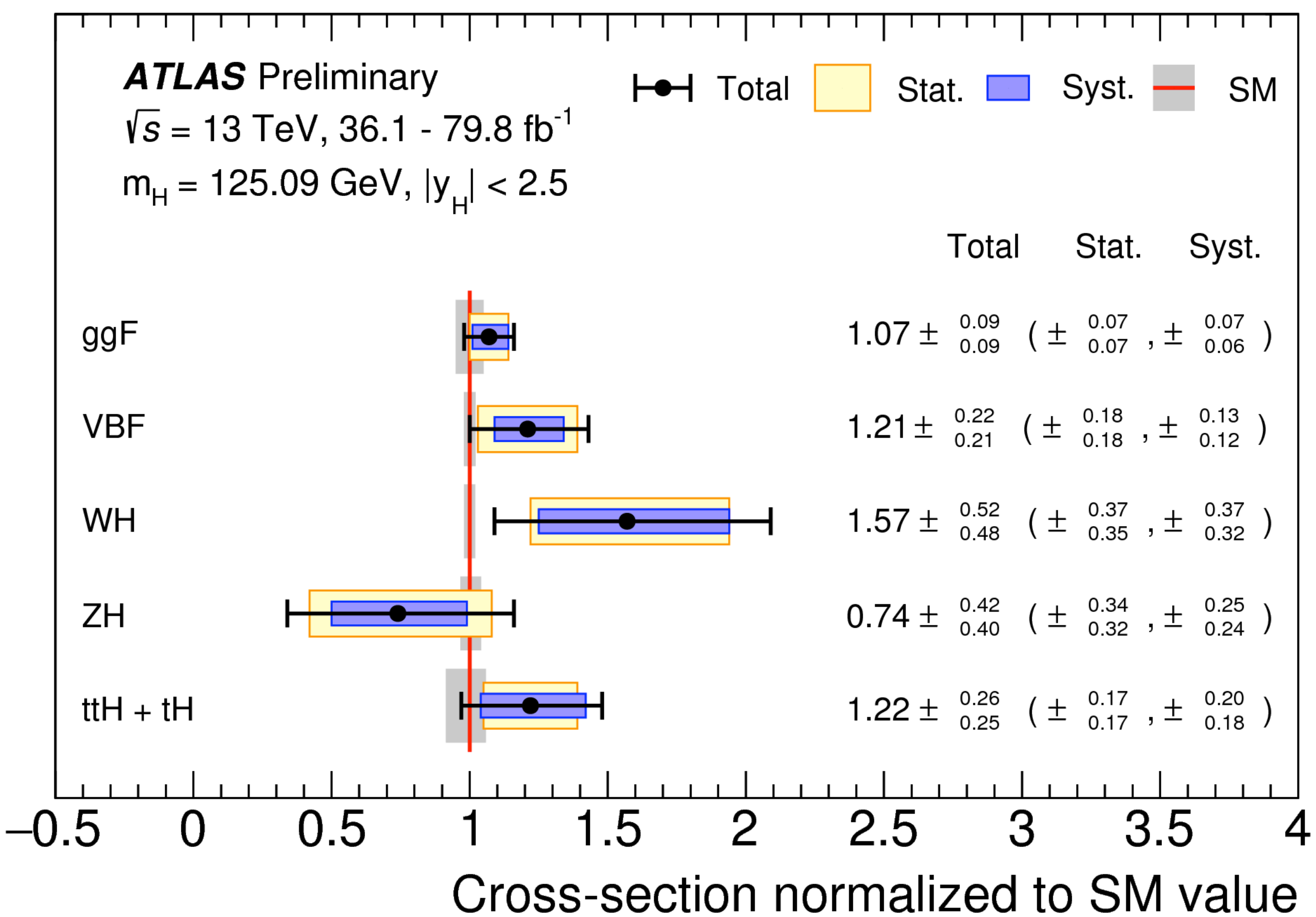
THE LHC - AN INCREDIBLE SUCCESS

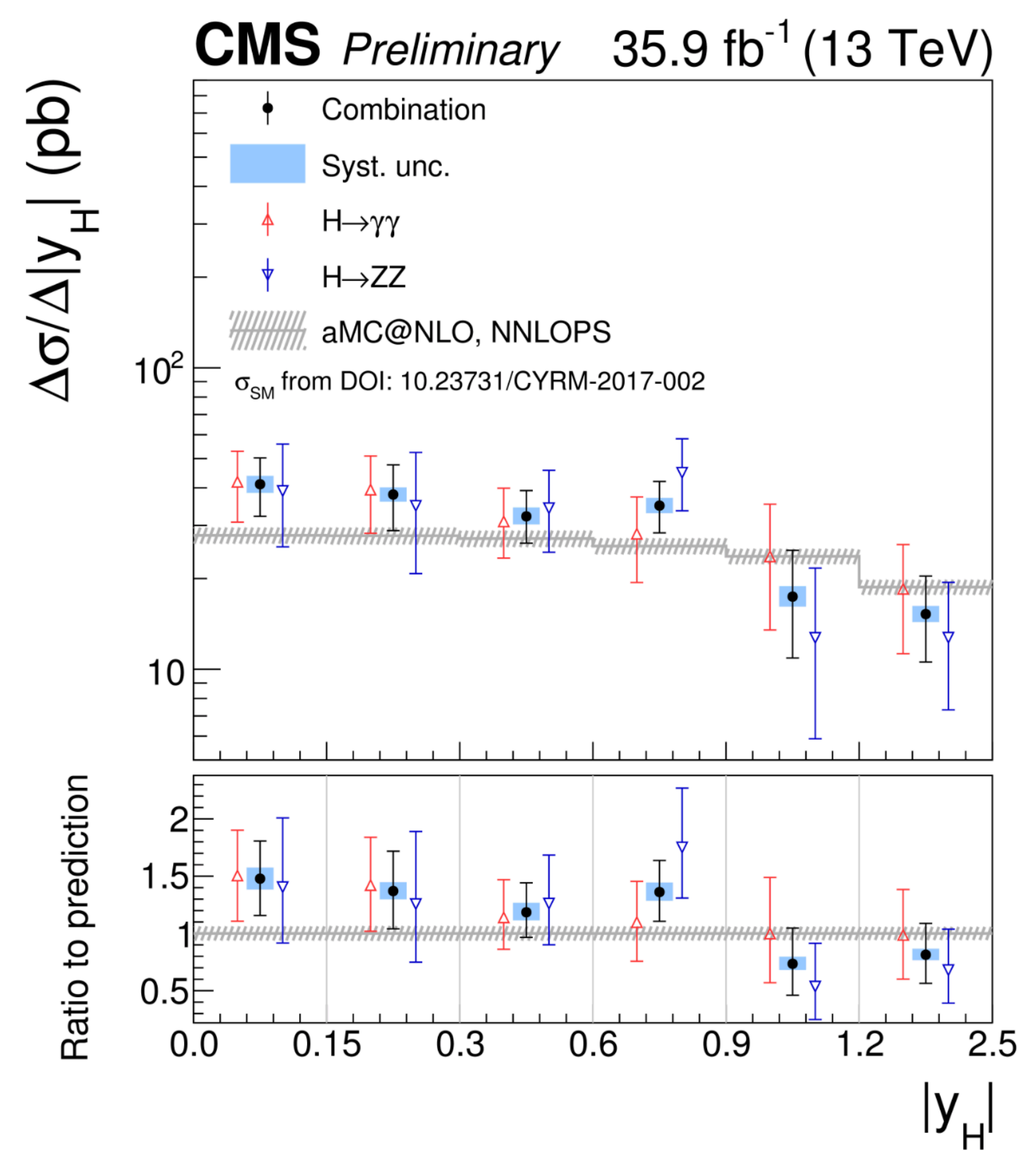
- 4th of July 2012: The beginning of the precision physics age of Higgs boson phenomenology
  - ✳ **Gain insight in the mechanism of electro-weak symmetry breaking**
  - ✳ **Investigate the generation of fundamental masses**
  - ✳ **Determine couplings / interactions with established matter**
  - ✳ **Explore the limitations of the Standard Model of particle physics.**

The Method: Predict & Compare.

Precision is key!

CURRENT STATUS





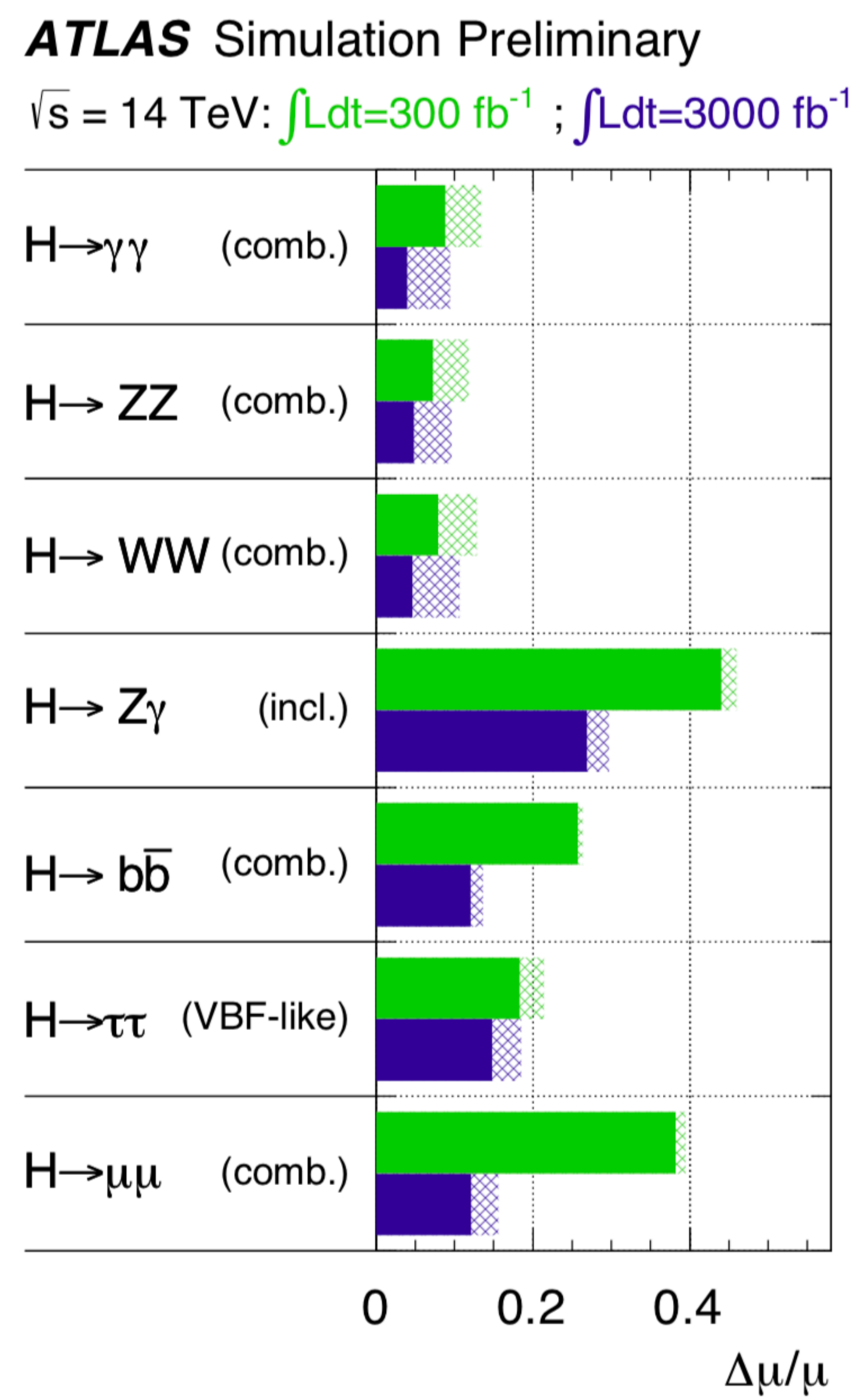
Physics at 10 % level

THE FUTURE – 3000 FB^-1

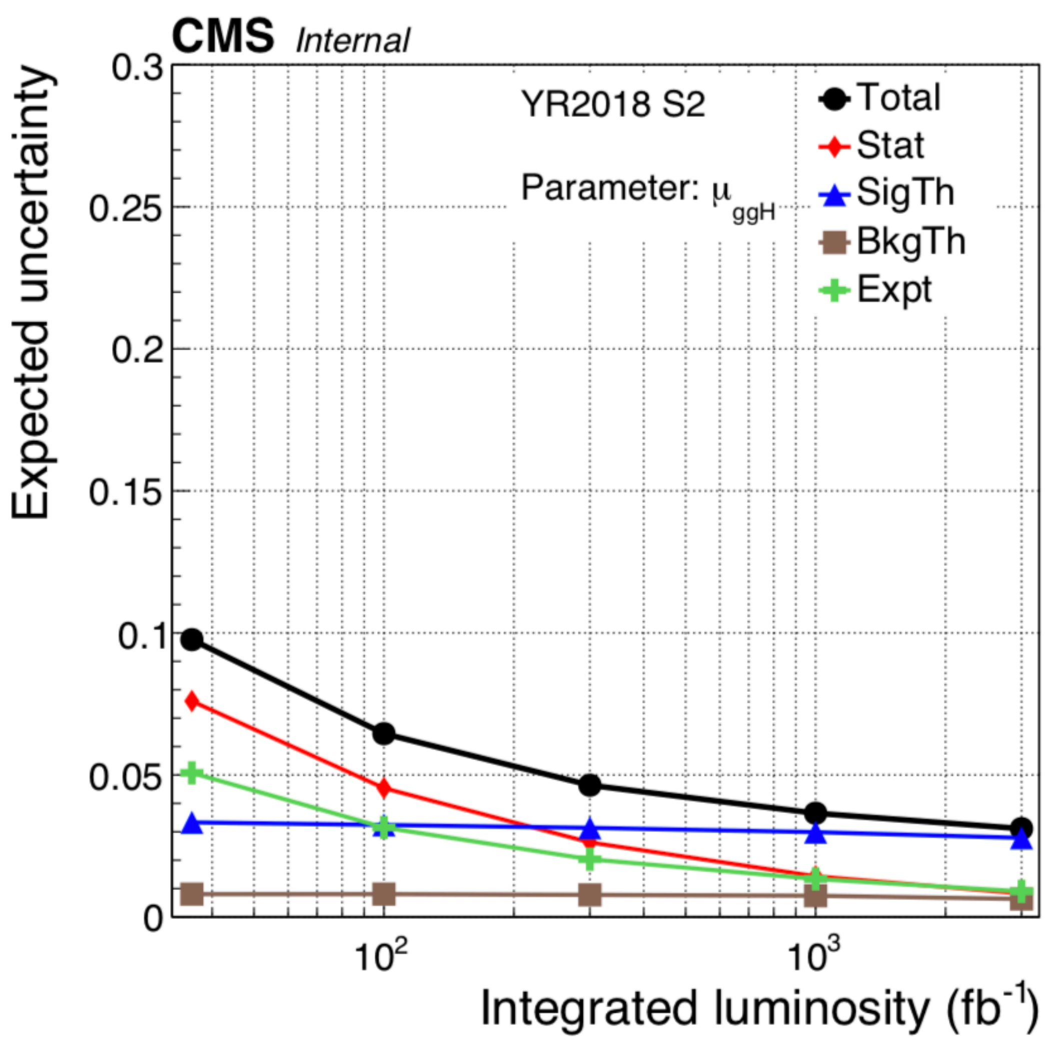
Inclusive signal strength

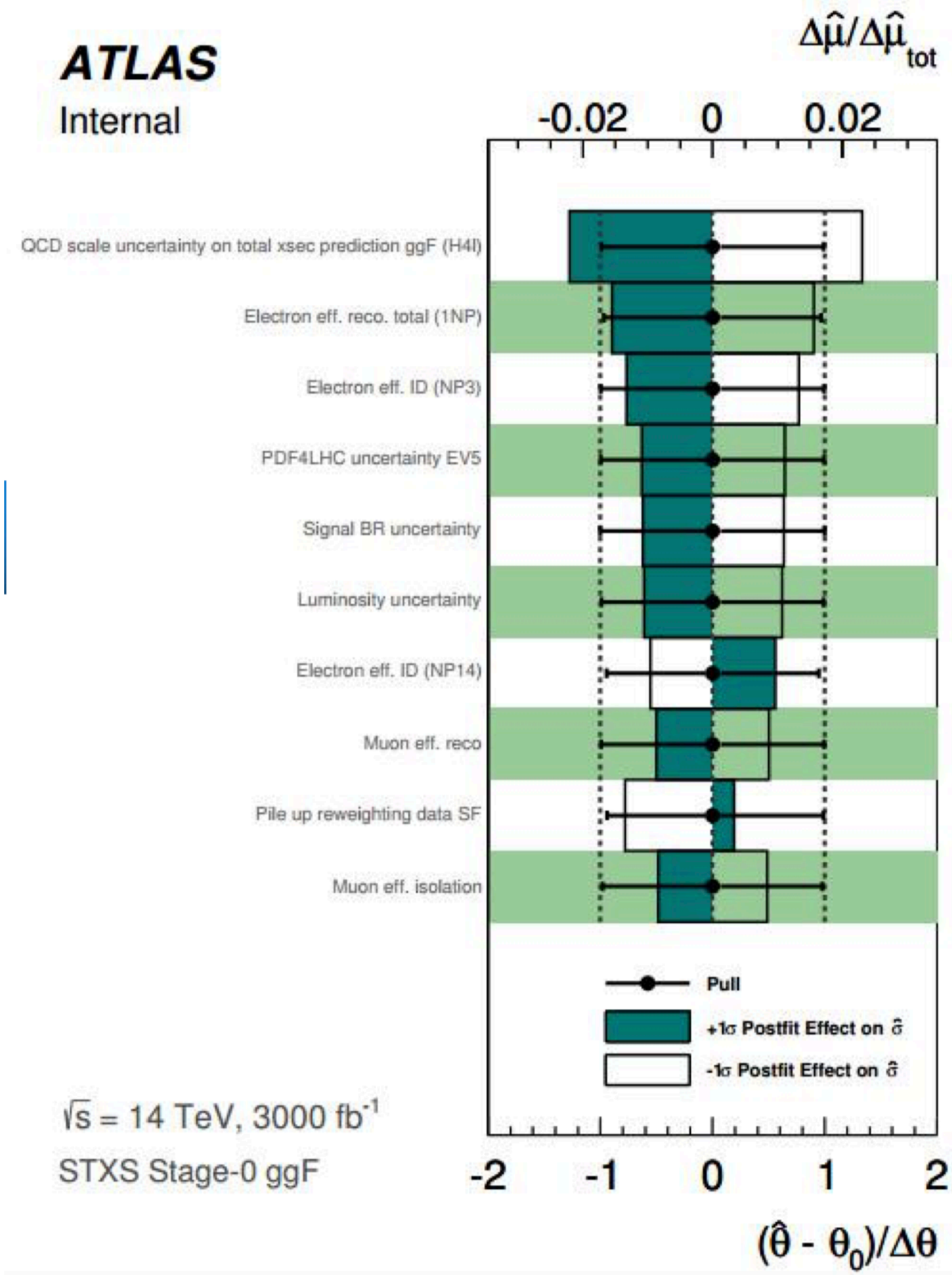
Relative uncertainty	Total	Stat	Exp.
S1	3.5%	0.6%	1.6%
S2	2.4%	0.6%	1.3%

- Luminosity at 1 %
- Couplings better then 5%
- Differential Cross Sections get precise



THE FUTURE – 3000 FB^-1

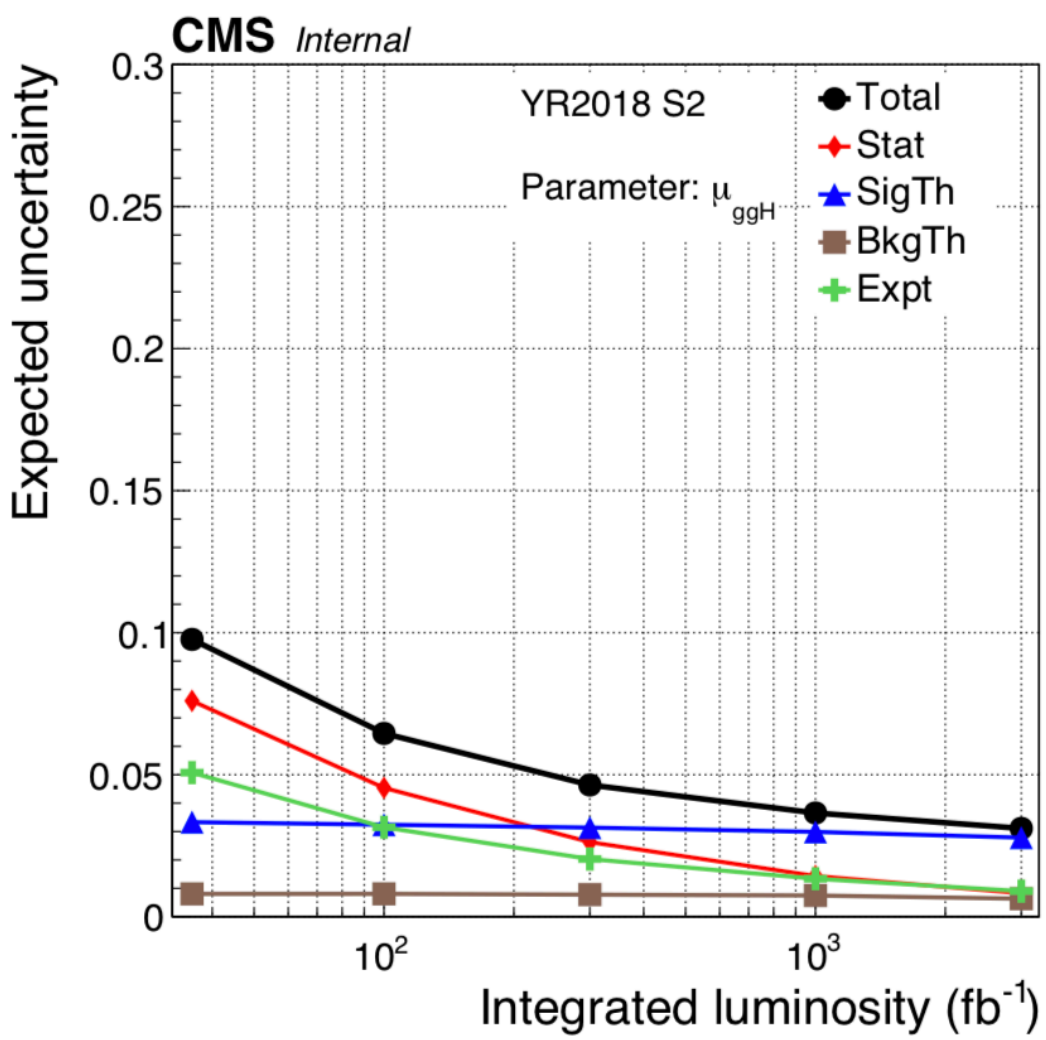


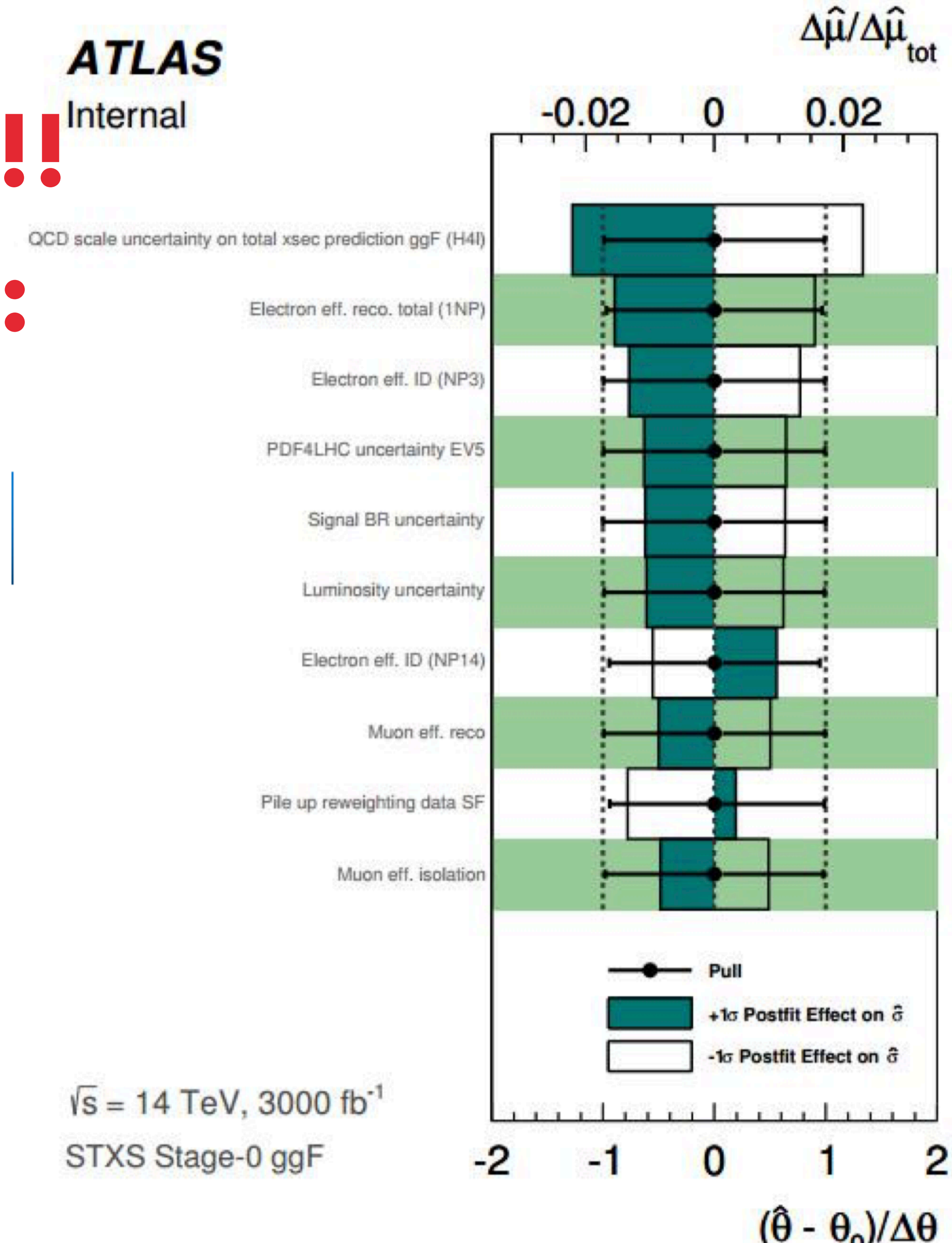


THE FUTURE – 3000 FB^-1

Theory uncertainties!!!

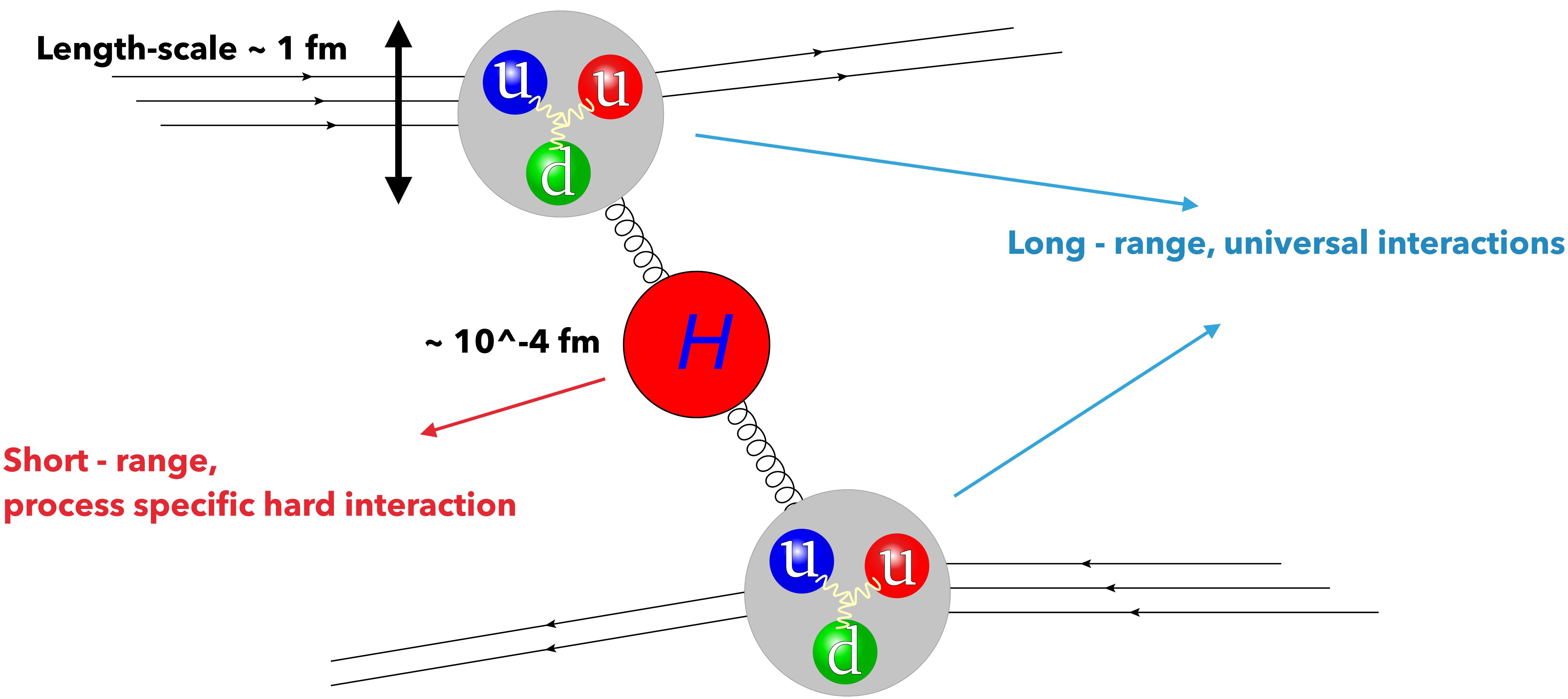
OPTIMISTIC Scenario:



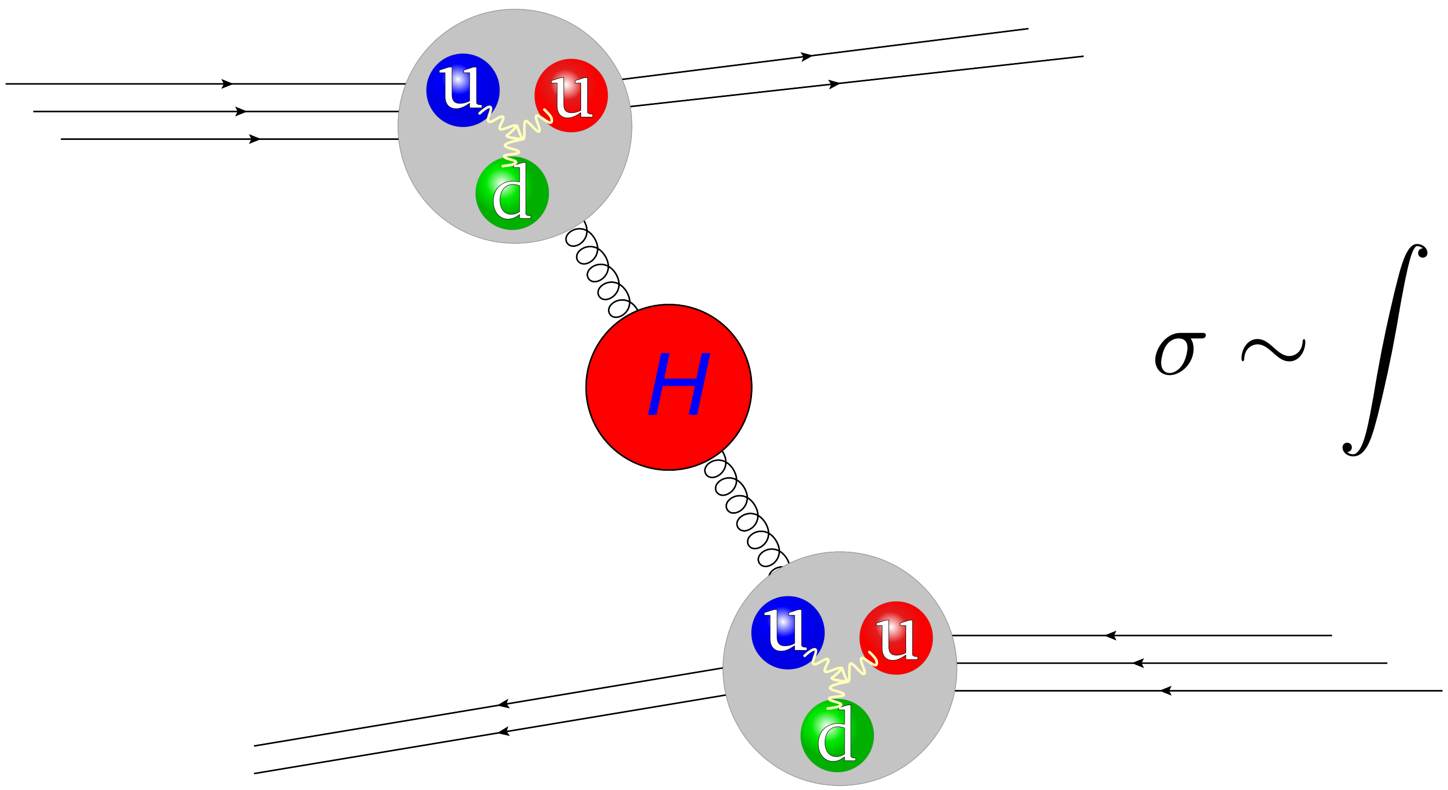


THE WAY TO PRECISION HIGGS BOSON PREDICTIONS

- Interaction of proton constituents within one proton negligible compared to collision energy: Free Quarks!
- Description of proton scattering in terms of interaction of fundamental particles.



THE WAY TO PRECISION HIGGS BOSON PREDICTIONS



$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

- Probabilities for the outcome of scattering events: cross sections.
- Partonic cross sections $\hat{\sigma}$: From first principle QFT.
- Intrinsic limitation: $\mathcal{O}(\frac{\Lambda}{Q}) \sim 1\%$ = level of target precision.

THE WAY TO PRECISION HIGGS BOSON PREDICTIONS

$$\sigma \sim \int dxdy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

- Perturbative approach to computing partonic cross sections.
- QCD perturbation theory is dominant $\alpha_S = 0.118$
- Naively:

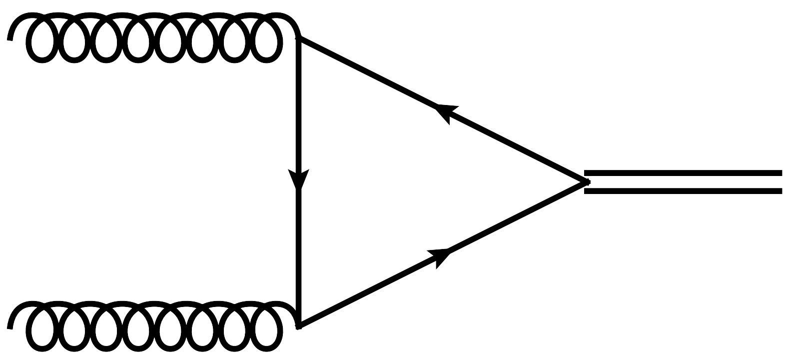
LO **NLO** **NNLO** **N3LO**

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(0)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \ldots$$

10% 1% 0.1%

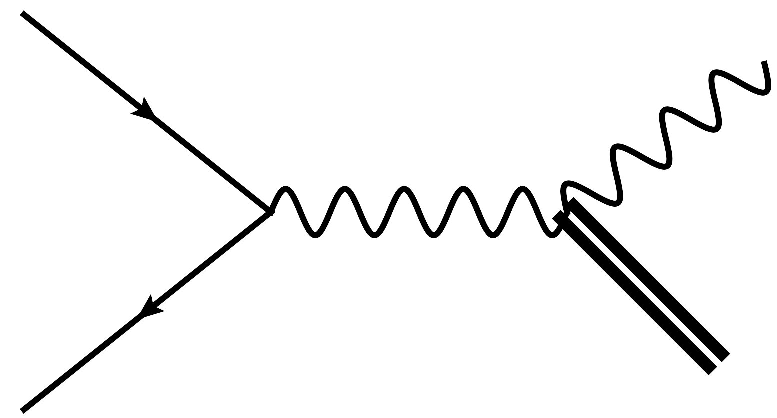
- How well does it actually work?

4 WAYS TO PRODUCE A HIGGS



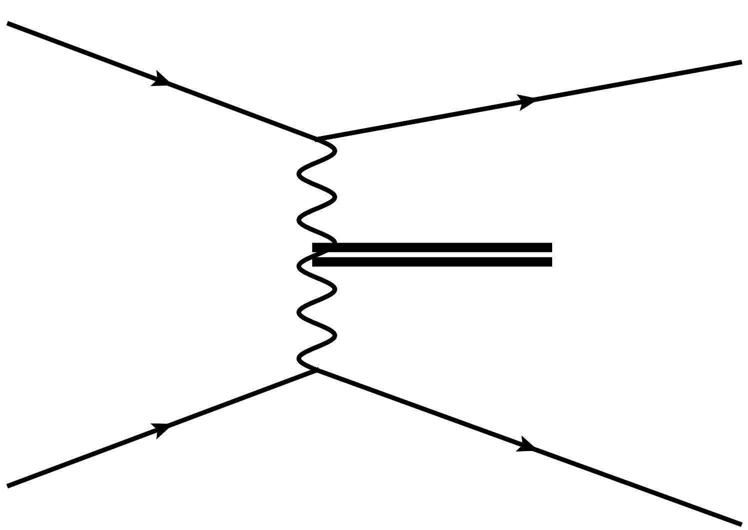
ggF

~88.2%



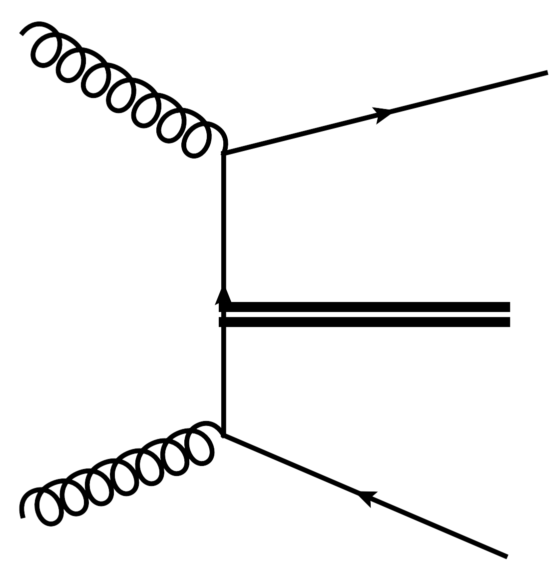
Associated

~4.1%



VBF

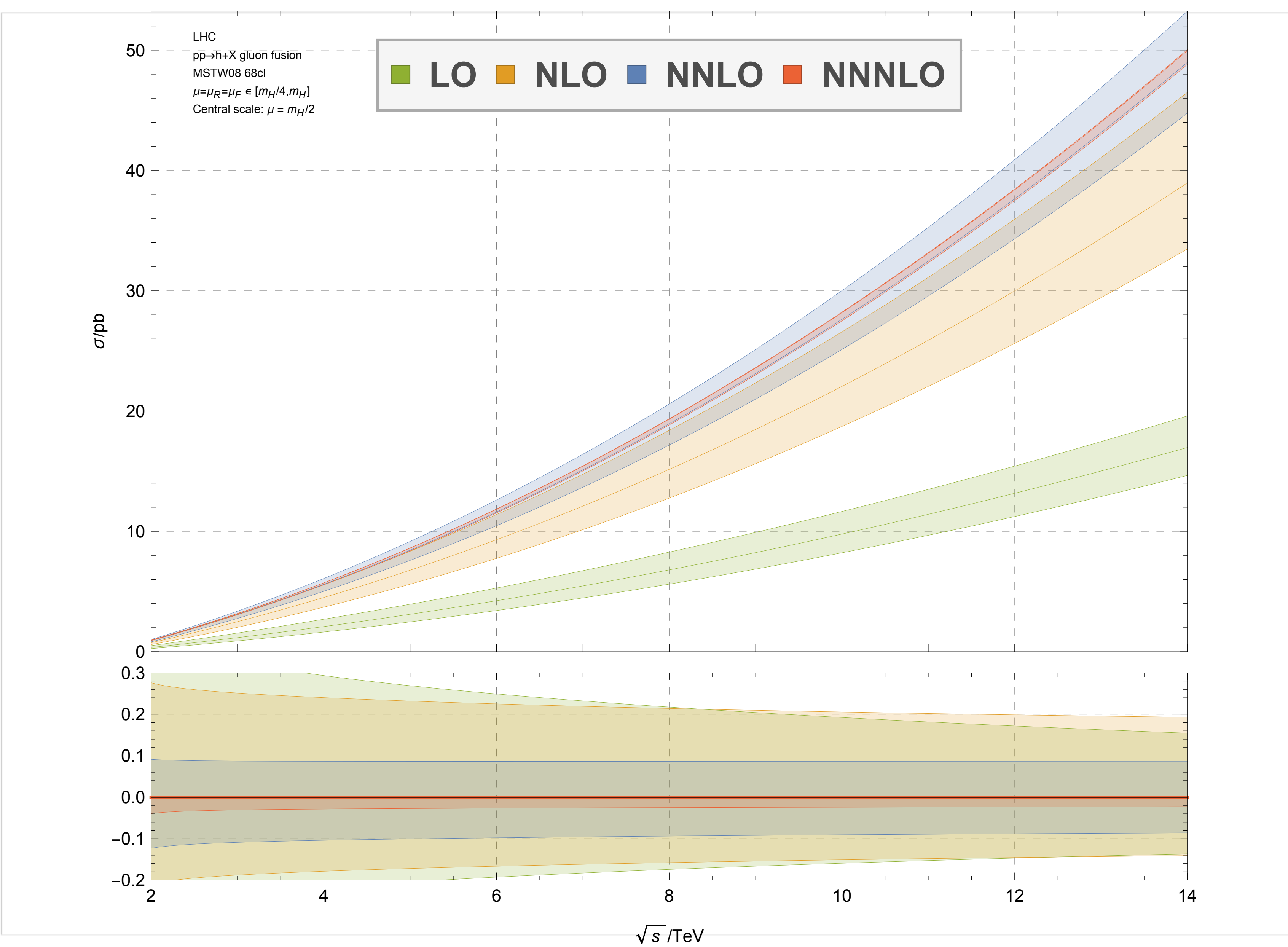
~6.8%



ttH

~0.9%

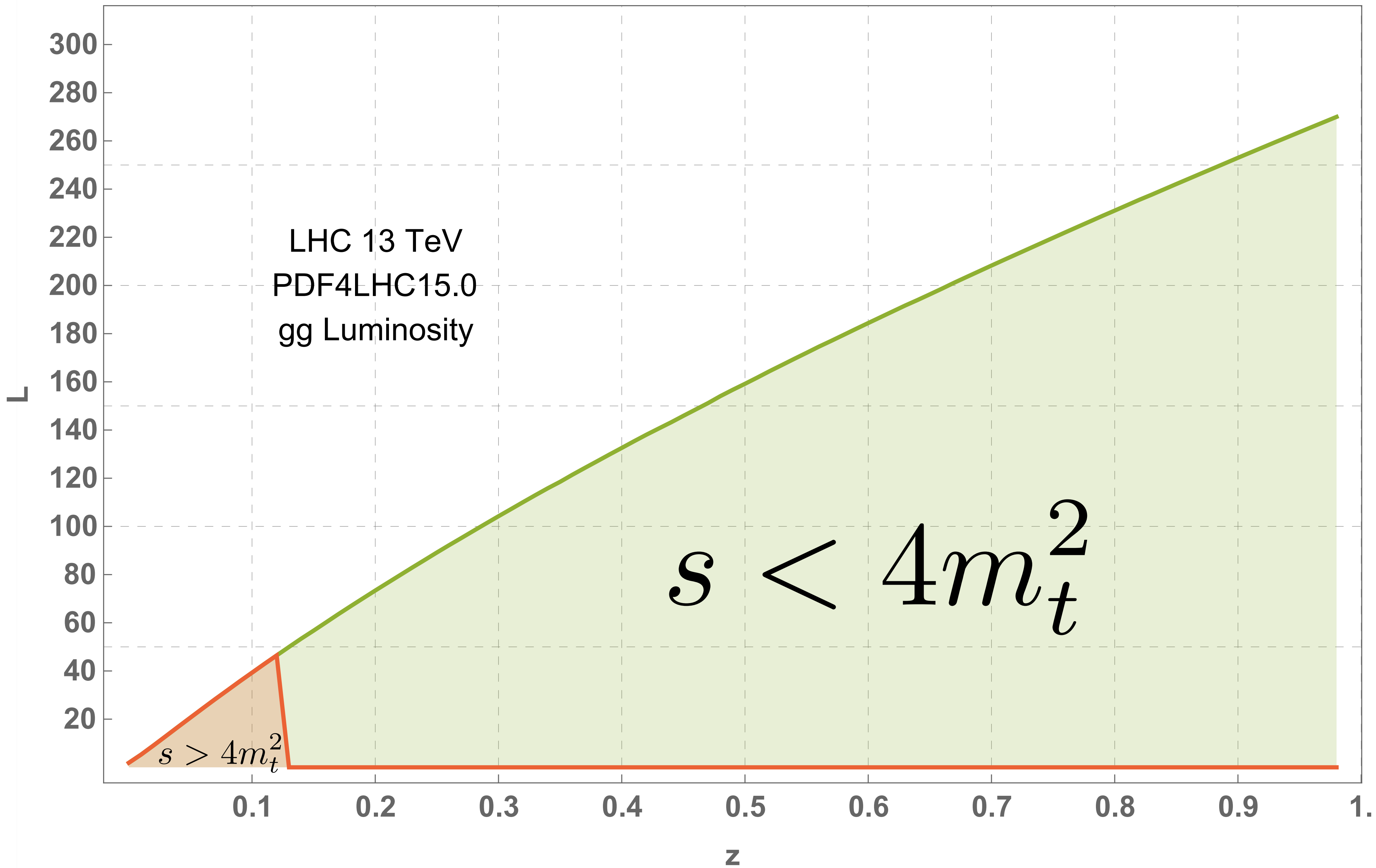
PERTURBATIVE CORRECTIONS TO HIGGS BOSON PRODUCTION



- N3LO corrections stabilise perturbative expansion.
- Significant reduction in residual perturbative uncertainty estimates.
- High orders are required!

INGREDIENT NR1: GLUONS.

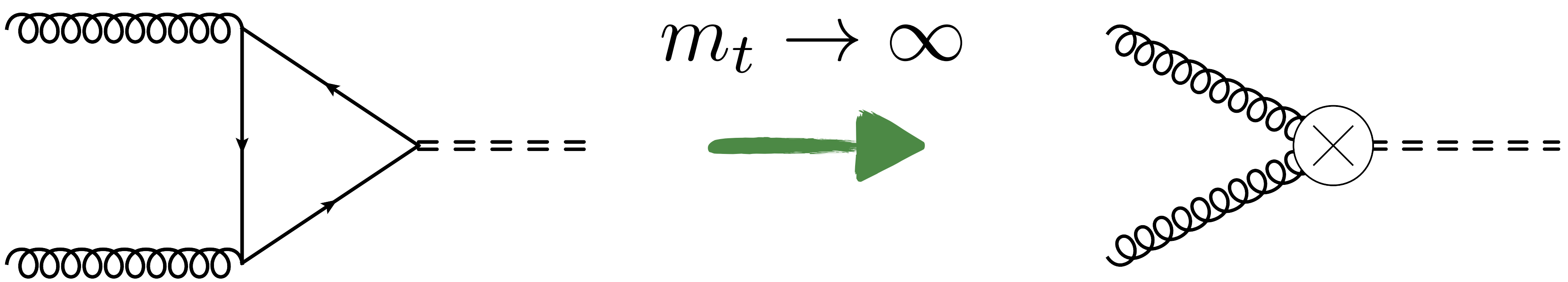
Probability to get two gluons out of the proton as a function of the partonic centre of mass energy:



COMPUTING N3LO CROSS SECTIONS IS CHALLENGING

Simplifications:

- Work in an EFT



- Removes one loop!

- Excellent approximation: Captures dominant QCD effects.

$$\delta_t^{\mathrm{LO}} \sim 7\%$$
$$\delta_t^{\mathrm{NLO}} \sim 0.7\%$$

- Supplement with mass corrections, EWK corrections etc.

THIS TALK: THE RAPIDITY DISTRIBUTION OF THE HIGGS

- Towards realistic LHC physics

$$Y = \frac{1}{2}\log\left(\frac{2P_1 p_h}{2P_2 p_h}\right)$$

- **THE** Differential Observable of inclusive Higgs

$$\frac{d\sigma_{P\,P\to H+X}}{dY} = \hat{\sigma}_0 \sum_{i,j}\int_0^1 dx_1 dx_2 dy_1 dy_2 f_i(y_1) f_j(y_2)$$

$$\times\delta(\tau - x_1 x_2 y_1 y_2)\delta\left(Y - \frac{1}{2}\log\left(\frac{x_1 y_1}{x_2 y_2}\right)\right)\eta_{ij}(x_1, x_2).$$

Our strategy:

- 1) Obtain analytic results.
- 2) Find systematically improvable, high quality approximation.
- 3) Ensure compatibility with the inclusive cross section.

ANALYTIC RESULTS

Motivation

- Numerical computations are complex! NNLO:

$2 \to 1$
Inclusive cross sections - analytic formulae: ~ seconds

$2 \to 1$
Differential cross sections for Higgs boson final states: 10 - 100 CPU hours

$2 \to 2$
Differential cross sections for Higgs + J boson final states: 100000+ CPU hours

- Extension to N3LO poses a considerable challenge.

ANALYTIC RESULTS HIGGS DIFFERENTIAL CROSS SECTIONS

- **Focus on the degrees of freedom of the Higgs boson:**

$$p_h = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \sqrt{p_T^2 + m_h^2}\cosh Y \\ p_T \cos\phi \\ p_T \sin\phi \\ \sqrt{p_T^2 + m_h^2}\sinh Y \end{pmatrix}$$

  - Trivial dependence on the azimuthal angle ϕ
  - Together with the Bjorken / PDF variables we have a 4 dimensional problem

$$\{x_1, x_2, p_T, Y\}$$

  - Framework would allow to even compute pT!

HIGGS – DIFFERENTIAL CROSS SECTIONS

Inspired by [Anastasiou,Dixon,Melnikov,Petriello]

- **How to compute a partonic Higgs - differential cross section:**

Inclusive:

$$\int d\Phi_{h+X} \sim \int d^d p_h \prod_i^n d^d p_i$$

Higgs - differential:

$$\int d\Phi_n \sim \int \cancel{d^d p_h} \prod_i^n d^d p_i$$

- **Partonic Higgs - differential cross section:**

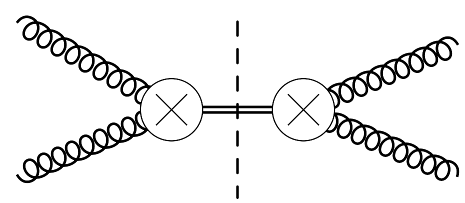
$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \left|\mathcal{M}_{ij\to H+X}\right|^2$$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

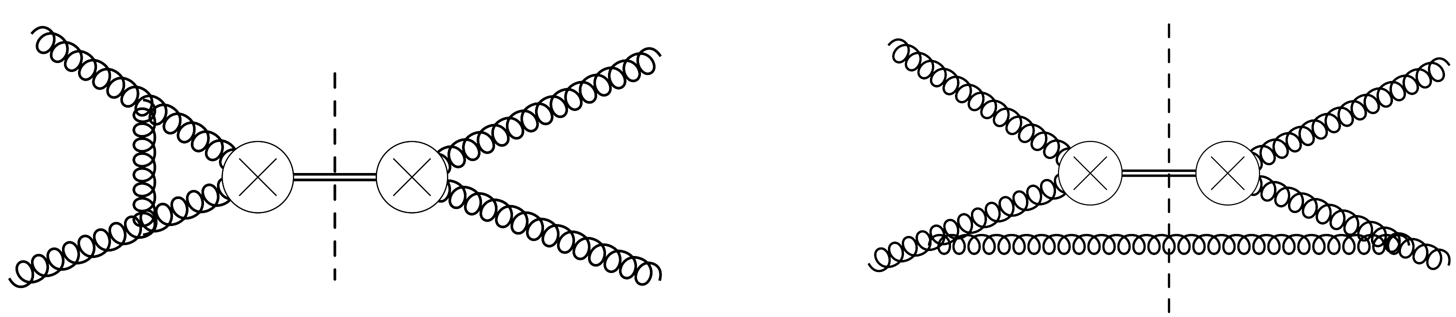
$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \left| \mathcal{M}_{ij \to H+X} \right|^2$$

- Compute all required matrix elements of different final states X to a given order in perturbation theory.

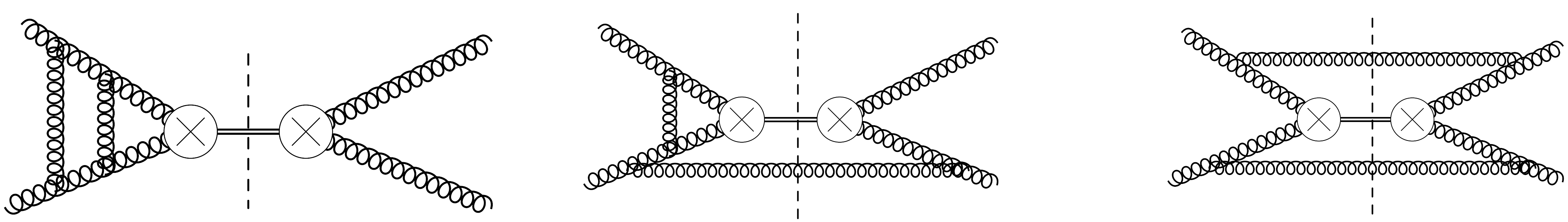
LO:



NLO:

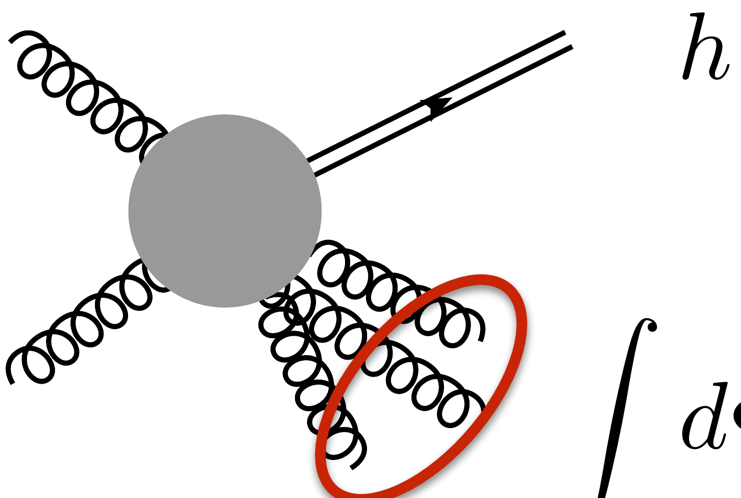


NNLO:



PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \left|\mathcal{M}_{ij\to H+X}\right|^2$$



$$\int d\Phi_n$$

- Perform integration over parton phase space analytically
- Rely on tools to perform analytic computation learned from inclusive N3LO
- Make singularities of final state parton integrations manifest using dimensional regularisation. $d = 4 - 2\epsilon$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

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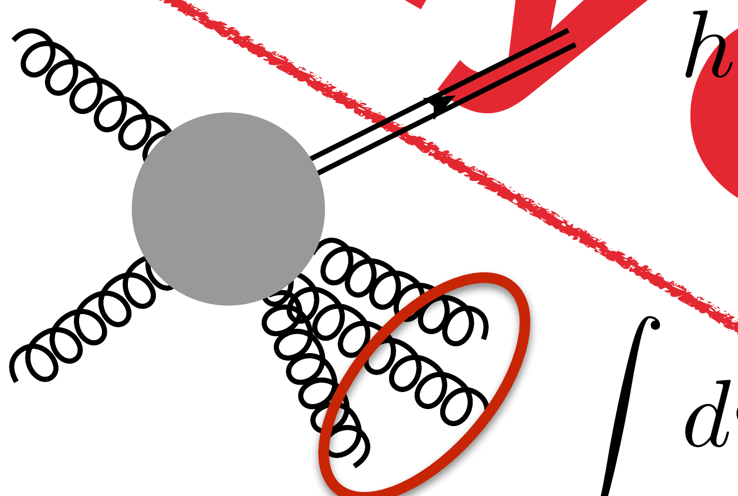


$$\int d\Phi_n$$

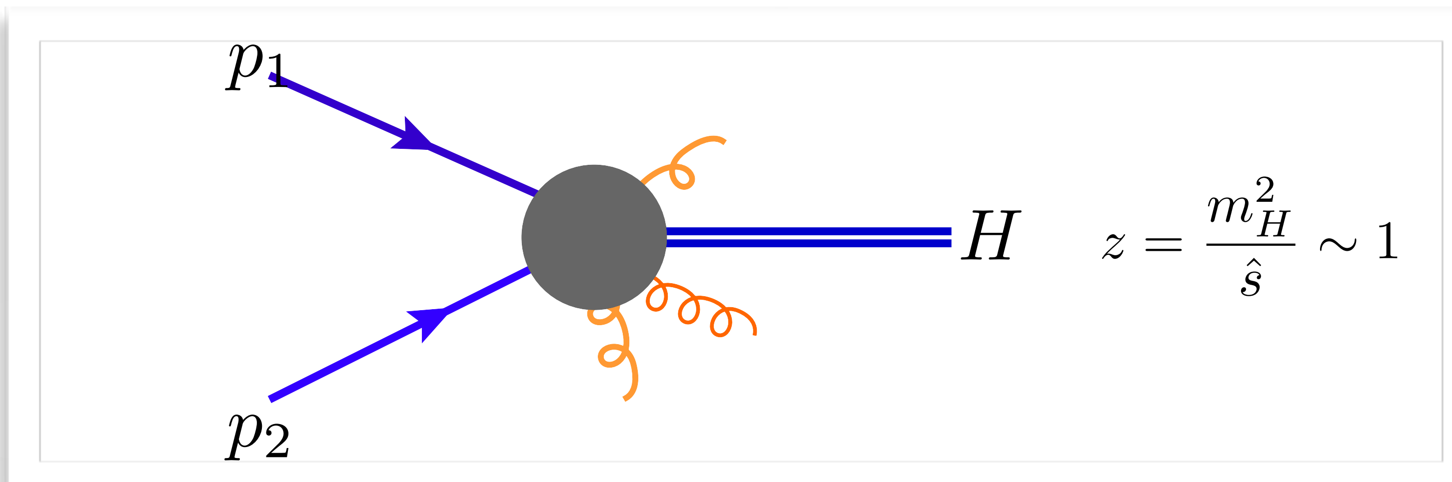
Very Complicated!

- Perform integration over parton phase space analytically
- Rely on tools to perform analytic computation learned from inclusive N3LO
- Make singularities of final state parton integrations manifest using dimensional regularisation. $d = 4 - 2\epsilon$

//EXPAND

Simplifications:

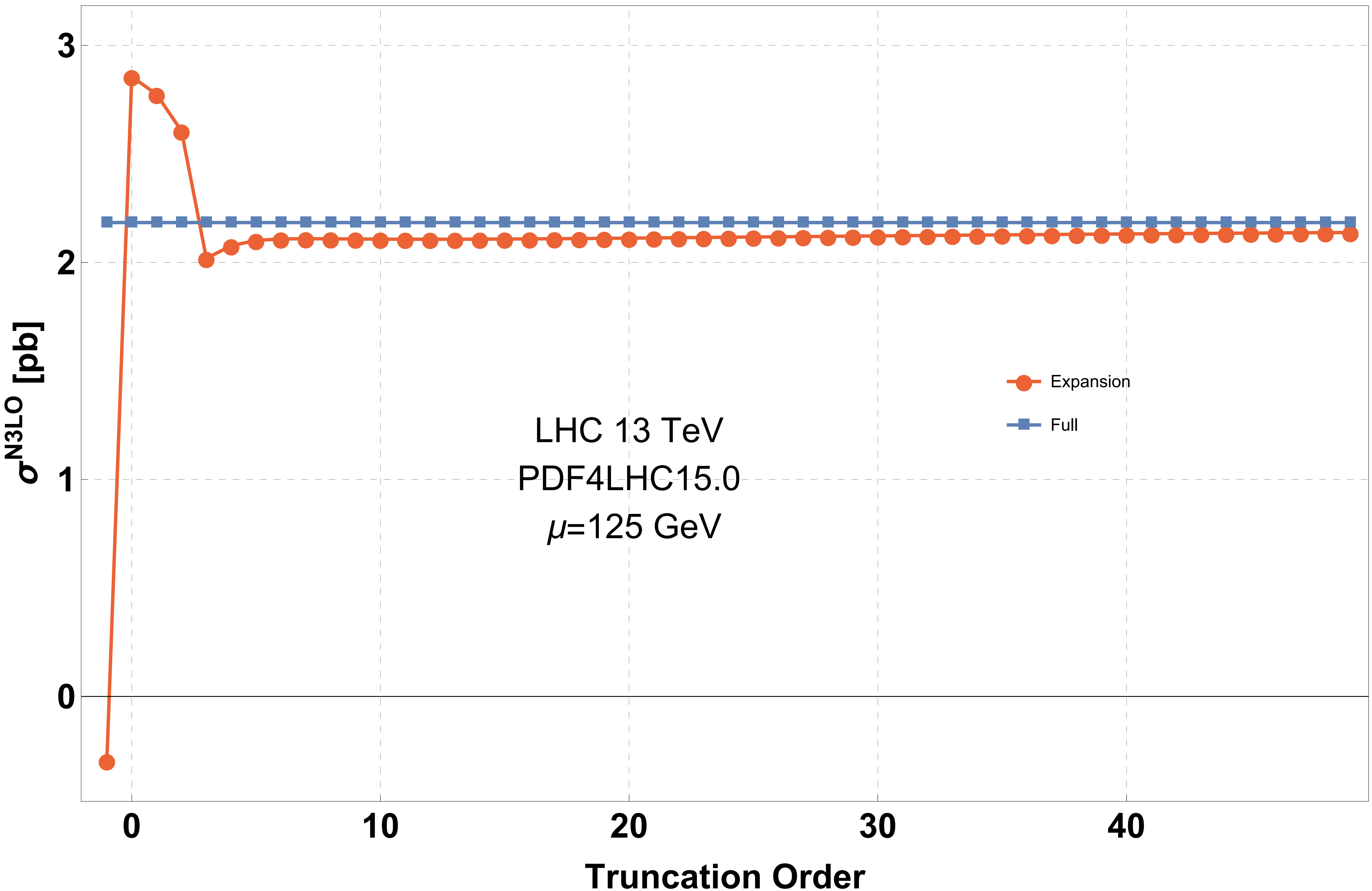
- Perform expansion around kinematic limit: **Production Threshold**



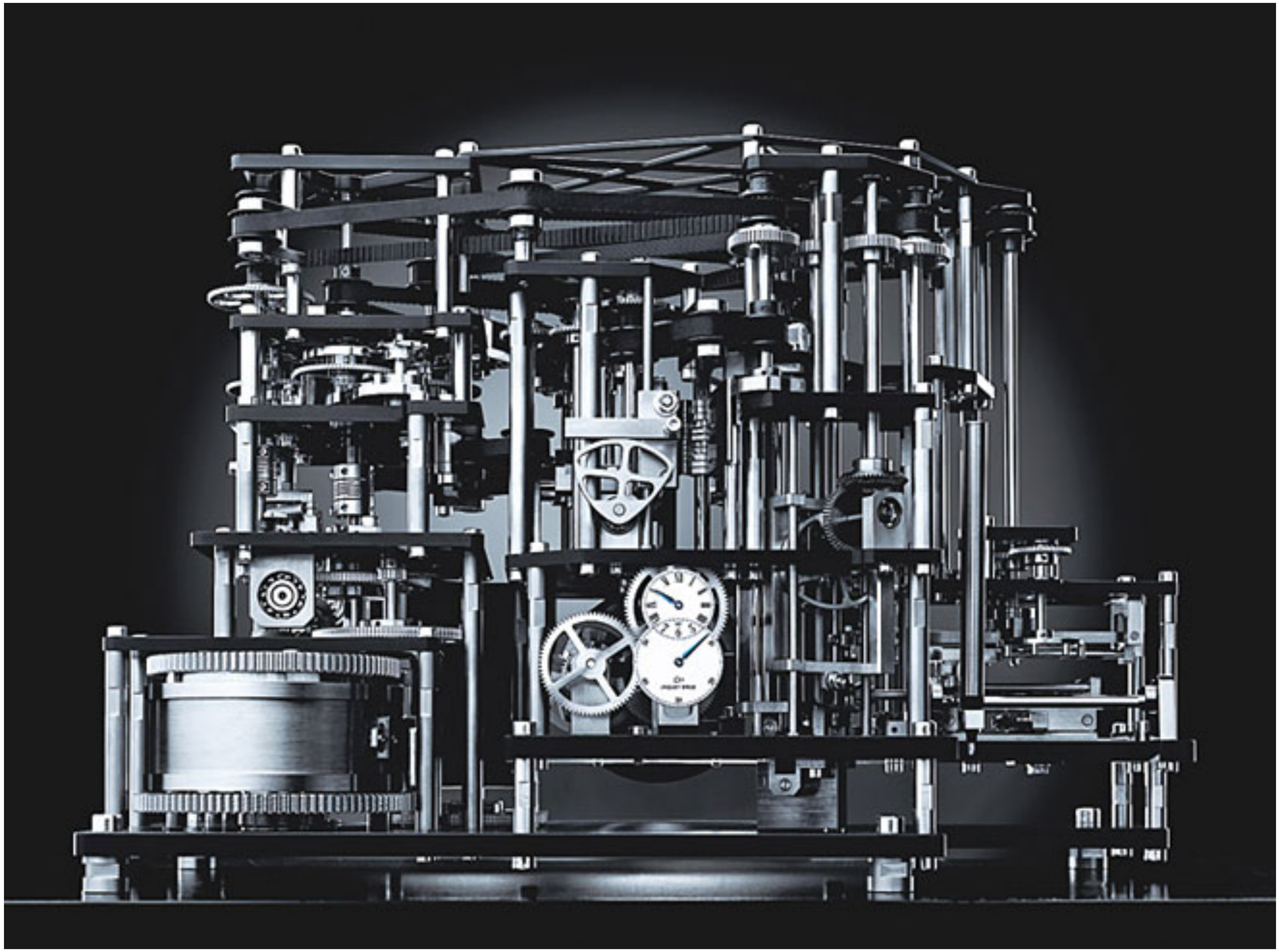
$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

- Expand to sufficiently high order to ensure stable results.
- Remarkably successful for inclusive N3LO.

EXPANDED VS. EXACT



SO WE USE OUR CROSS SECTION MACHINE AND COMPUTE!



THRESHOLD EXPANSION

- Obtain in total 6 terms in the threshold expansion.
- Integrate out the transverse momentum degree of freedom.
- Perform analytic renormalisation and infra-red subtraction.

→ **Finite N3LO coefficient function** $\eta_{ij}(x_1, x_2)$.

- Perform expansion such that each term corresponds **exactly** to one term in the expansion of the inclusive cross section!

$$\bar{x}_1 \to \delta\bar{x}_1 \frac{1-\bar{x}_2}{1-\delta\bar{x}_2}, \qquad \bar{x}_2 \to \delta\bar{x}_2.$$

EXPLOITING POLE CANCELLATION

- Partonic coefficient functions take the form

$$\eta^{(3)}_{ij,\,\mathrm{bare}}(\bar{x}_1,\bar{x}_2) = \eta^{(3)}_{ij,\,\mathrm{virt.}}\delta(\bar{x}_1)\delta(\bar{x}_2) + \sum_{n,m=1}^{3} \bar{x}_1^{-1-m\epsilon}\bar{x}_2^{-1-n\epsilon}\eta^{(3,m,n)}_{ij,\,\mathrm{bare}}(\bar{x}_1,\bar{x}_2).$$

- Different generalised exponents n,m -> different momentum modes of loop particles.

- Remaining functions holomorphic around 0.
  All logs from pre-factors.

EXPLOITING POLE CANCELLATION

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- Expand in Distributions

$$\int_0^1 d\bar{x}_i\phi(\bar{x}_i)\bar{x}_i^{-1+a\epsilon} = \int_0^1 d\bar{x}_i\phi(\bar{x}_i)\frac{\delta(\bar{x}_i)}{a\epsilon} + \int_0^1 d\bar{x}_i\phi(\bar{x}_i)\sum_{n=0}^{\infty}\frac{(a\epsilon)^n}{n!}\left[\frac{L_i^n}{\bar{x}_i}\right]_+ ,$$

Explicit Pole

Test Function:
- Luminosity, Observable

Plus - Distribution

EXPLOITING POLE CANCELLATION

- Partonic coefficient functions take the form

$$\eta^{(3)}_{ij,\,\mathrm{bare}}(\bar{x}_1,\bar{x}_2) = \eta^{(3)}_{ij,\,\mathrm{virt.}}\delta(\bar{x}_1)\delta(\bar{x}_2) + \sum_{n,m=1}^{3} \bar{x}_1^{-1-m\epsilon}\bar{x}_2^{-1-n\epsilon}\eta^{(3,m,n)}_{ij,\,\mathrm{bare}}(\bar{x}_1,\bar{x}_2).$$

- m=1 or n=1 known exactly! Genuine two loop contributions.
- Something curious: $\lim_{\bar{x}_2\to 0}\eta^{(3,m,n)}_{ij,\,\mathrm{bare}}(\bar{x}_1,\bar{x}_2)$

Only m $\leq$ n non-zero.

EXPLOITING POLE CANCELLATION

- Partonic coefficient functions take the form

$$\eta^{(3)}_{ij,\,\mathrm{bare}}(\bar{x}_1,\bar{x}_2) = \eta^{(3)}_{ij,\,\mathrm{virt.}}\delta(\bar{x}_1)\delta(\bar{x}_2) + \sum_{n,m=1}^{3} \bar{x}_1^{-1-m\epsilon}\bar{x}_2^{-1-n\epsilon}\eta^{(3,m,n)}_{ij,\,\mathrm{bare}}(\bar{x}_1,\bar{x}_2).$$

- 5 powers of log(1-x)

- Pole cancellation:

$$\eta^{(3)}_{ij}(x_1,x_2) = \lim_{\epsilon\to 0}\left[\eta^{(3)}_{ij,\,\mathrm{bare}}(x_1,x_2) + CT^{(3)}_{ij}(x_1,x_2)\right]$$

DGLAP+renormalisation

EXPLOITING POLE CANCELLATION

- Knowing that poles will cancel allows to derive relations between different singular pieces.
- We can extract a good portion of coefficients of logarithmically enhanced terms from those equations.
- Here is what we **cannot** extract!

$$\eta^{(3)}_{ij,\,\mathrm{missing}}(x_1,x_2) = \Big[\delta(\bar{x}_1)\log(\bar{x}_2)\eta^{(3)}_{ij,(1,9)}(0,x_2) + \delta(\bar{x}_1)\eta^{(3)}_{ij,(1,8)}(0,x_2) + \left[\frac{1}{\bar{x}_1}\right]_+ \eta^{(3)}_{ij,(2,8)}(0,x_2) + \log(\bar{x}_2)\eta^{(3)}_{ij,(8,9)}(x_1,x_2)\Big] + \Big[(x_1 \leftrightarrow x_2)\Big] + \eta^{(3)}_{ij,(8,8)}(x_1,x_2) + \log(\bar{x}_1)\log(\bar{x}_2)\eta^{(3)}_{ij,(9,9)}(x_1,x_2).$$

RELATION TO INCLUSIVE CROSS SECTION

- Inclusive partonic coefficient function known exactly at N3LO
- Relation to our rapidity PCF:

$$\eta_{ij}^{(3),\text{inc.}}(z) = \int_0^1 \frac{\bar{z}d\bar{x}}{(1-\bar{z}\bar{x})}\eta_{ij}^{(3)}\left(\frac{(1-\bar{x})\bar{z}}{1-\bar{x}\bar{z}}, \bar{x}\bar{z}\right) .$$

- Fantastic Check!
- Can we use it?

RELATION TO INCLUSIVE CROSS SECTION

- Modify our approximated result:

$$\eta_{ij}^{(3),\text{matched}}(x_1, x_2) = \eta_{ij}^{(3),\text{app.}}(x_1, x_2) + \frac{x_1 + x_2}{2(1 - x_1 x_2)} \underbrace{\left[\eta_{ij}^{(3),\text{inc}}(x_1 x_2) - \eta_{ij}^{(3),\text{inc, app.}}(x_1 x_2)\right]}_{\mathcal{O}(\bar{z}^5)} .$$

- Ensures that we reproduce the exact N3LO inclusive cross section when integrating over the rapidity!
- Exact for every value in z!

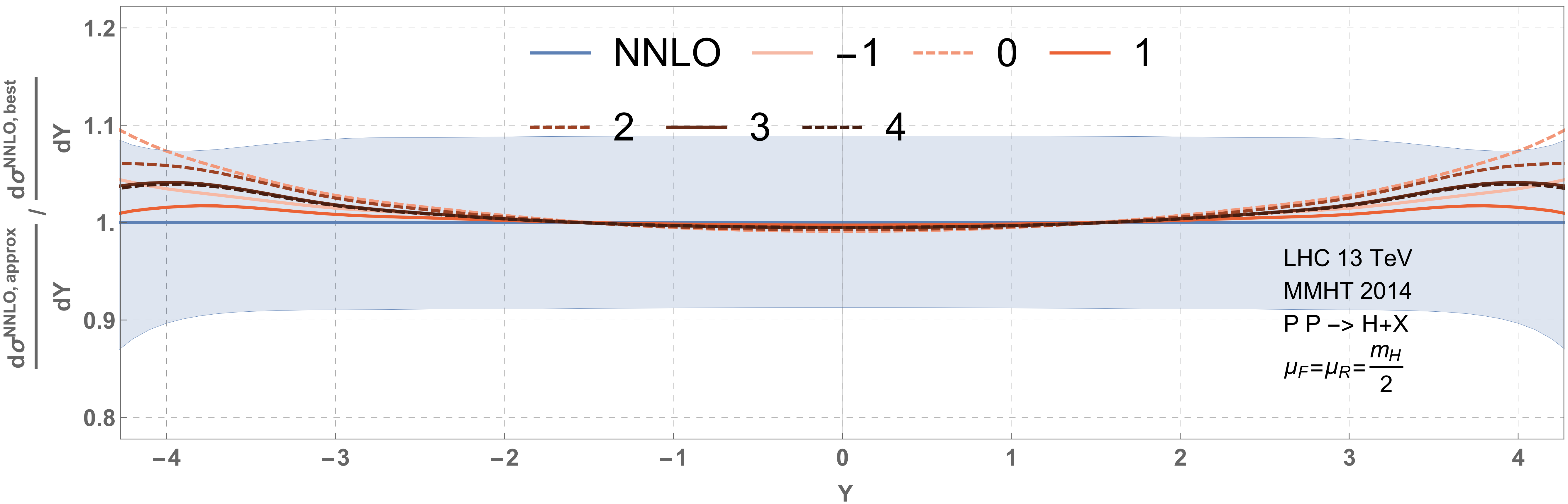
OUR PARTONIC COEFFICIENT FUNCTION – SUMMARY

$$\eta^{(3)}(x_1, x_2)$$

Ingredients:

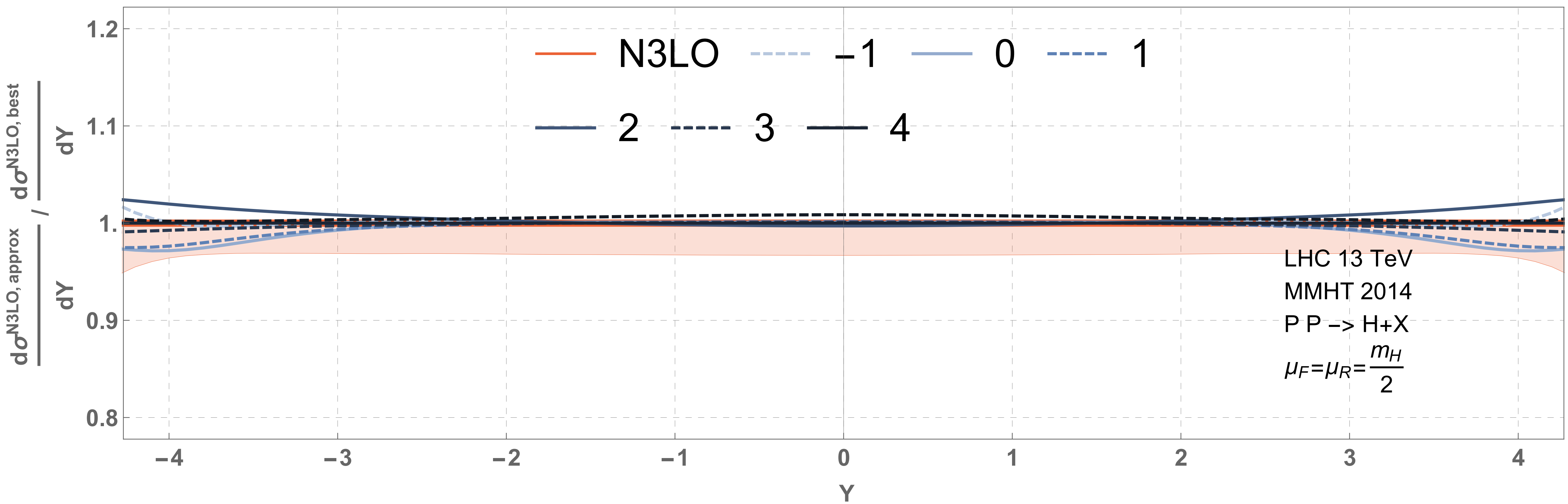
- ★ Integrates to the exact N3LO cross section.
- ★ Contains leading logarithmic contributions exactly.
- ★ Complemented by **six** terms in the expansion around the partonic threshold.

VALIDATION AT NNLO



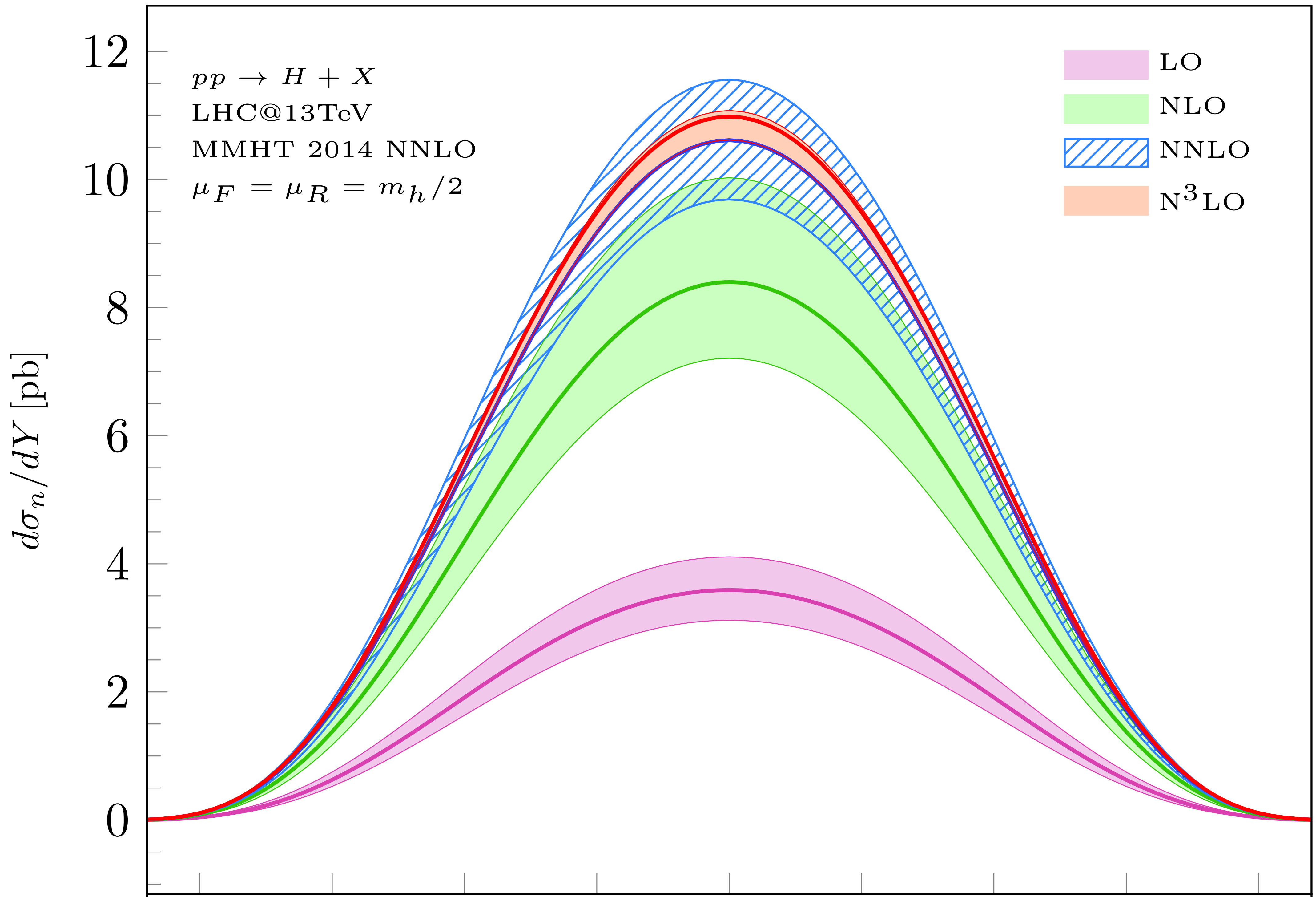
- Our approximation performs nicely!
- Especially for central rapidities |Y| < 3
  Larger Rapidities ~ More energetic final states = further from threshold
- After first couple of orders: Systematic improvement by including more terms in threshold expansion.
- To cover the remaining difference to exact NNLO other ingredients than threshold expansion are necessary.

VALIDATION AT N3LO

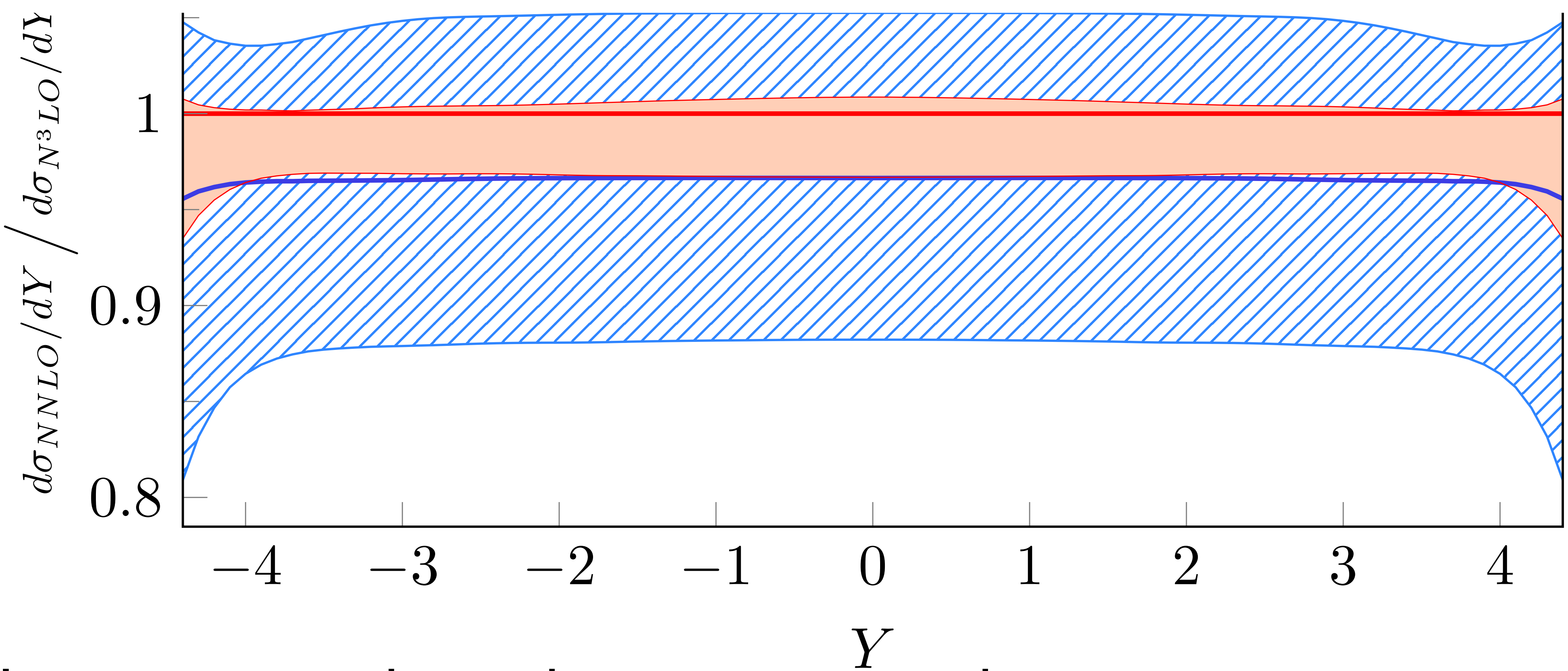


- Similar picture as at NNLO.
- Central rapidities very stable under adding more threshold terms.
- Larger rapidities: expansion varies more.
- High confidence in central rapidity region.

HIGGS BOSON RAPIDITY



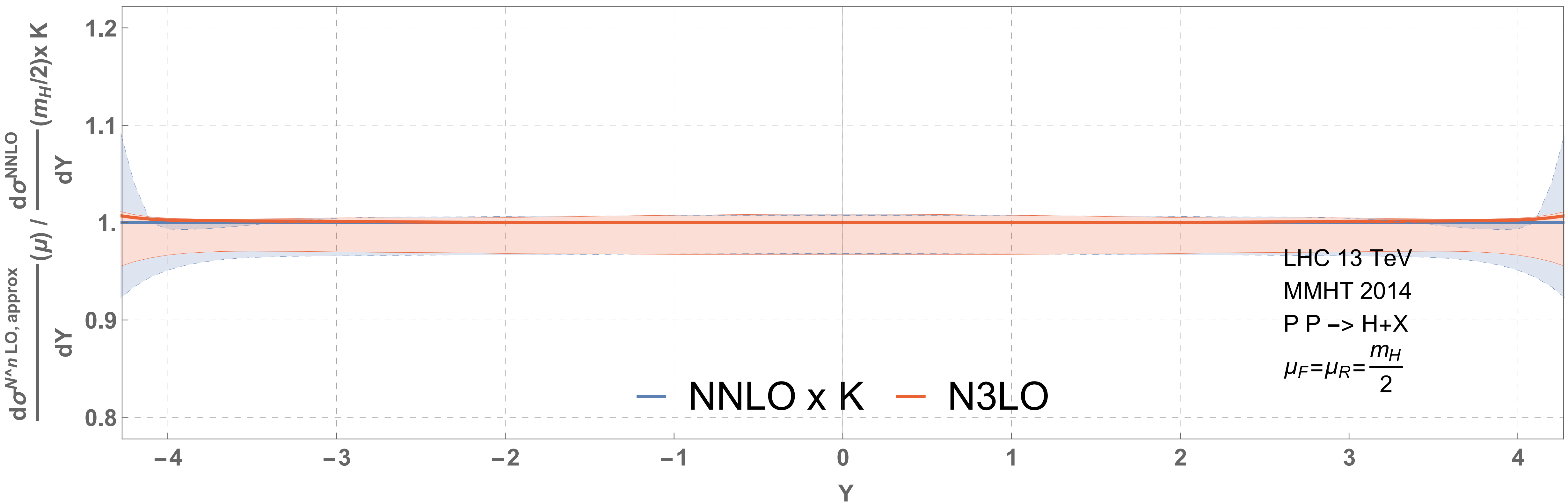
HIGGS BOSON RAPIDITY - RATIO



- Flat correction throughout entire rapidity range.
- Significant reduction in scale uncertainty.
- Excellent agreement with earlier approximation of

[Cieri,Chen,Gehrmann,Glover,Huss]

HIGGS BOSON RAPIDITY – RATIO



- Very compatible with rescaling of NNLO distribution
- Good news for current experimental usage!
  Re-weighted Parton-Shower MC.

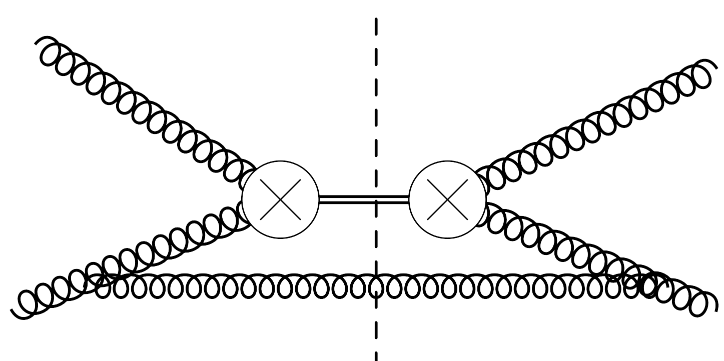
FIDUCIAL PREDICTIONS

- Real LHC observables are constrained by fiducial selection of Higgs boson decay products.

→ **Fully Differential Cross Sections**

- High orders: Challenging to compute due to presence of complex infra-red singularities!

$$\int d^4p_h d^4p_g$$



$$|M|^2$$

$$\sim \int \frac{dE_g}{E_g} \frac{d\cos\phi_{1g}}{1-\cos\phi_{1g}} F(E_g, \cos(\phi_{1g}))$$

soft collinear

SUBTRACTION

- Procedure to regulate infra-red and collinear singularities in all generality (at NLO):

$$\int d\Phi |M|^2 J(\Phi_B, \Phi_g) \rightarrow$$

Finite numerically

$$\int d\Phi \left(|M|^2 J(\Phi_B, \Phi_g) - \left| M^{(0)} \right|^2 J(\Phi_B, 0) \right)$$

Integrate in dim.-reg.

$$+ \int d\Phi_B J(\Phi_B, 0) \int d\Phi_g \left| M^{(0)} \right|^2$$

$\left| M^{(0)} \right|^2$ Typically an approximation of the matrix element in singular limit.

PROJECTION TO BORN

[Manohar,Nason,Salam,Zanderighi]

$$\left|M^{(0)}\right|^2 = |M|^2$$

The best possible subtraction scheme!

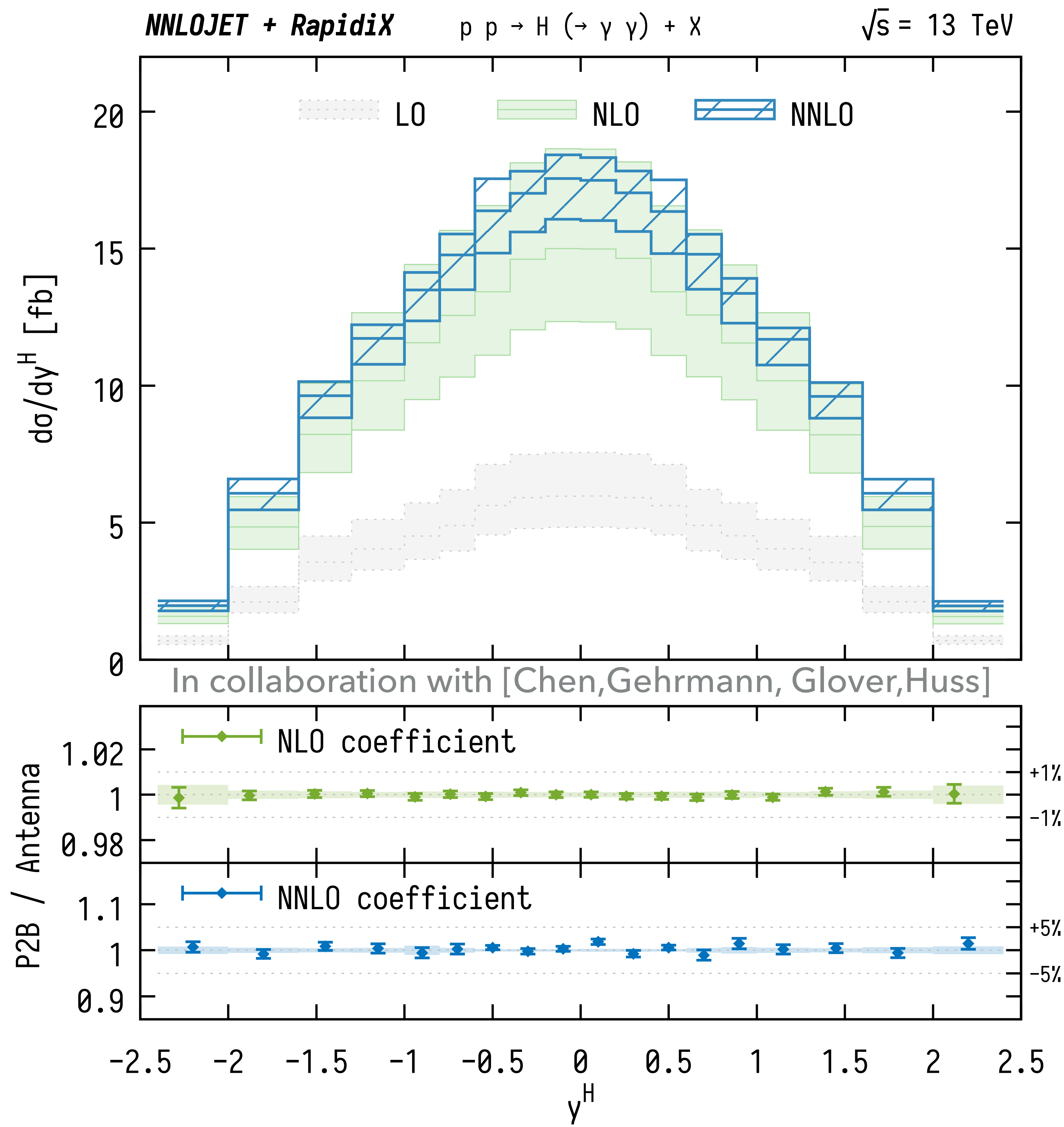
$$\int d\Phi \left(|M|^2 J(\Phi_B, \Phi_g) - \left|M^{(0)}\right|^2 J(\Phi_B, 0) \right)$$
$$\to \int d\Phi |M|^2 \left(J(\Phi_B, \Phi_g) - J(\Phi_B, 0) \right)$$

- Exact cancellation of singularities!
- Integrated counter term is the inclusive cross section, differential in all Born variables!
- Typically very hard to obtain! For the Higgs:

$$\Phi_B = \{z, Y\} \quad \rightarrow \quad \boxed{\frac{d\sigma}{dz\,dY}}$$

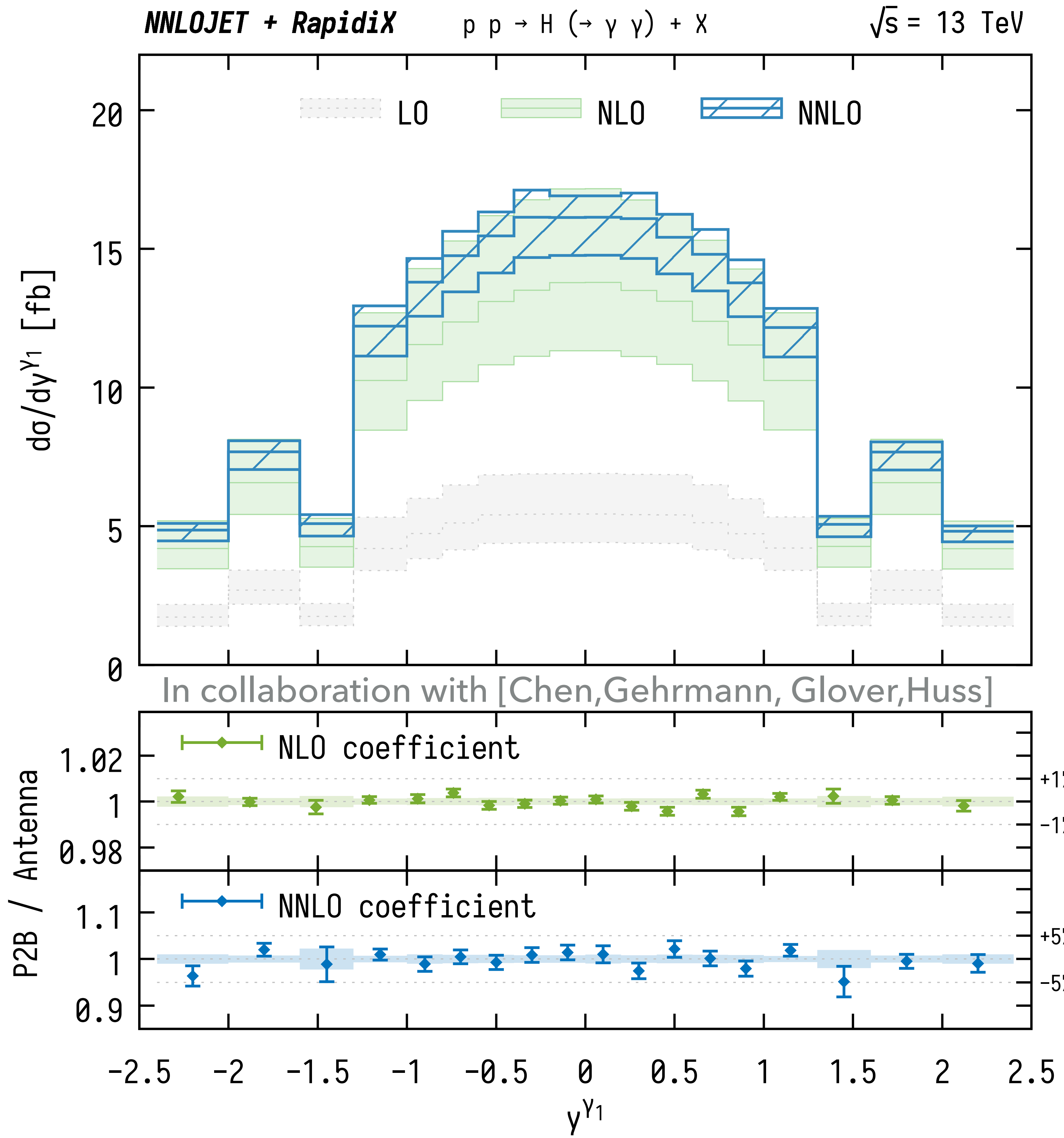
$$H \to \gamma\gamma$$

- Combination with H+J
- Validation at NNLO
- Fiducial Cross Sections for LHC Phenomenology!



$$H \to \gamma\gamma$$

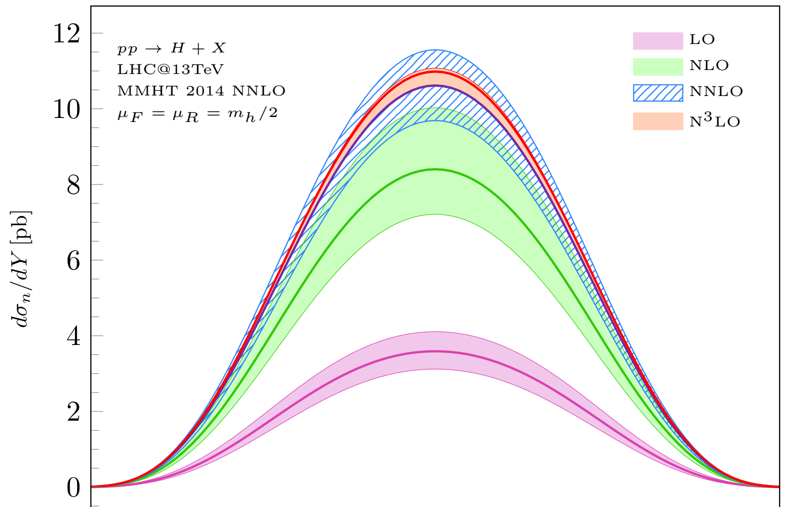
- Leading Photon Y
- Extension to N3LO in progress
- Combination with other uncertainties required!



In collaboration with [Chen,Gehrmann, Glover,Huss]

CONCLUSIONS

- We computed the Higgs boson rapidity distribution at N3LO.
- We observe a stabilisation of perturbative corrections and a significant reduction in the variation of the cross section as a function of the perturbative scale.
- N3LO corrections are uniform throughout the entire rapidity range.
- Our result is the cornerstone for future fully differential predictions of Higgs boson phenomenology.



Thank you!