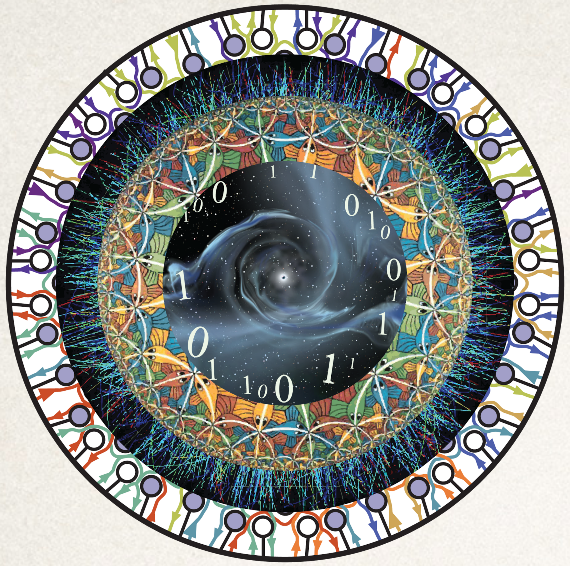


EFTs from the soft limits of scattering amplitudes

Jaroslav Trnka

Center for Quantum Mathematics and Physics (QMAP)

University of California, Davis

with Clifford Cheung, Karol Kampf, Jiri Novotny, Chia-Hsien Shen, Congkao Wen

Amplitudes in the LHC era, GGI, October 31, 2018

Motivation

- Tree-level amplitudes of massless particles in EFTs
- Normally not considered: bad powercounting, problems with loops
- Standard procedure: Lagrangian

  ↓

  Symmetry

  ↓

  Properties of amplitudes

Motivation

- In this talk: opposite approach
    - Start with generic Lagrangian with free couplings
      = free parameters in the amplitude
    - Impose kinematical constraints: fix all parameters
    - Find corresponding theory
    - Construct recursion relations to calculate amplitudes
- Classify interesting EFTs, perhaps find some new ones
- It is easier to impose kinematical constraints on amplitudes than to search in space of all symmetries

Typical example

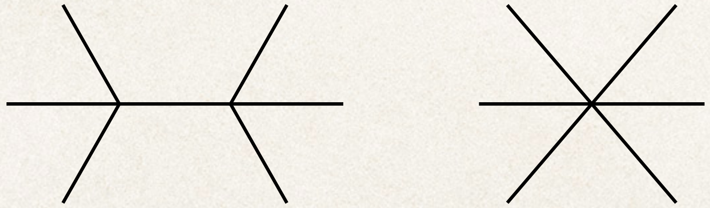
- Single scalar

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \ldots$$

Is there a symmetry which
fixes relates these couplings?

- 6pt amplitude

$A_6 =$



$$A_6 = \sum c_4^2 \frac{(\ldots)}{s_{123}} + c_6(\ldots)$$

Impose kinematical condition on A_6

EFT setup

Three point interactions

- Consider scalar field theory given by

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \mathcal{L}_{int}(\phi, \partial\phi, \dots)$$

- Simplest interaction is 3pt but there are no 3pt amplitudes except for $\mathcal{L}_{int} = \lambda\phi^3$

- Any derivatively coupled term can be written as

$\mathcal{L}_{int} = (\Box\phi)(\dots)$ and removed by EOM

Fundamental interaction

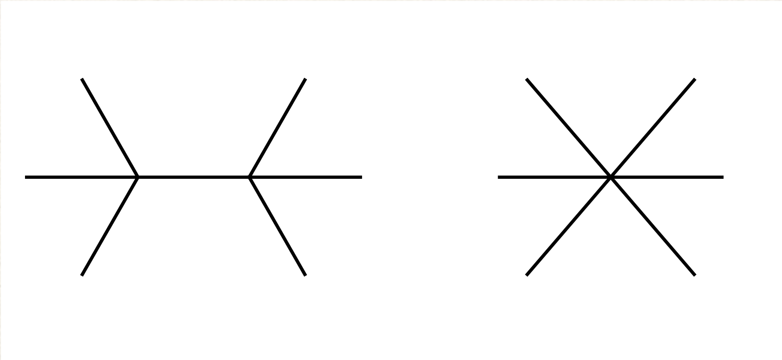
- Let us start with a 4pt interaction term

$$\mathcal{L}_{int} = \lambda_4(\partial^m \phi^4) \longrightarrow \text{many terms}$$

- Four point amplitude: special kinematics

- Six point amplitude: presence of contact terms

$$\frac{\partial^m \partial^m}{\partial^2} = \partial^{2m-2} \longrightarrow$$



Powercounting

$$\mathcal{L}_6 = \partial^{2m-2}\phi^6$$

- For $\mathcal{L}_{int} = \lambda_4 \phi^4$ no contact terms possible

EFT setup

- We consider the infinite tower of terms

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial^m \phi^4) + \lambda_6(\partial^{2m-4}\phi^6) + \ldots$$

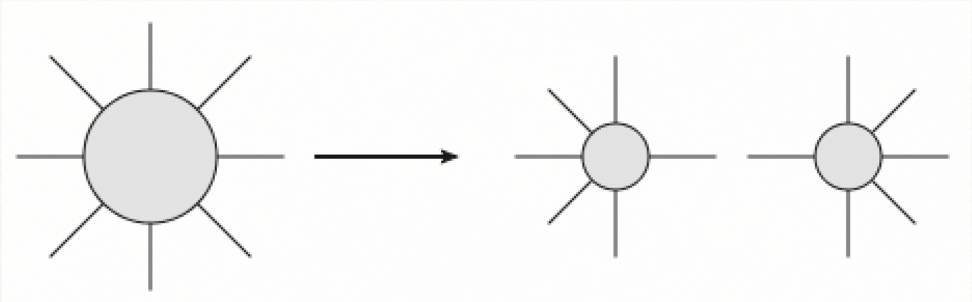
- Even if we start with the 4pt term we can do field redefinitions and generate infinite tower

- We get a generic amplitude $A_n(\lambda_4, \lambda_6, \ldots)$

- Find constraints which uniquely specifies all couplings

On-shell constructibility

- On the pole the amplitude must factorize

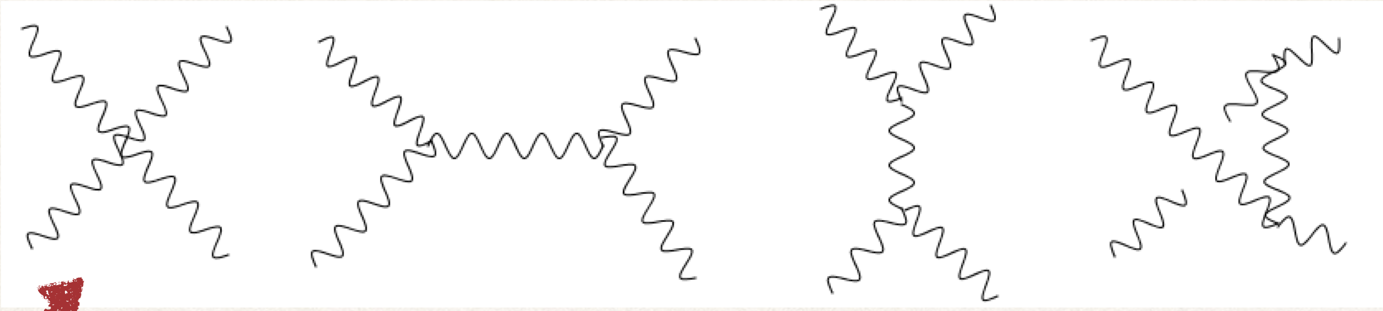


- Contact terms vanish on all poles: not detectable
- Therefore, EFT amplitudes are not specified only by factorization - unfixed kinematical terms

$$\frac{s_{12}s_{56}}{s_{123}} \sim \frac{(s_{12}+s_{123})s_{56}}{s_{123}} \quad \text{on the pole}$$

On-shell constructibility

- Naively, this problem arises also in YM theory
- In fact, the contact terms there is completely fixed



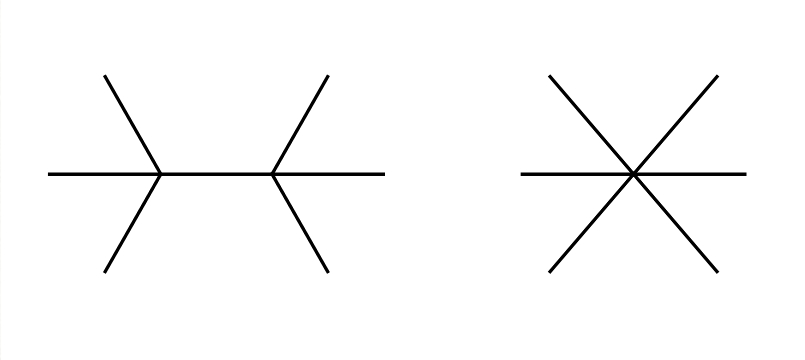
Contact term

Imposing gauge invariance fixes it

- In our case, contact terms are unfixed with free parameters, there is no gauge invariance

Extra constraints

- If we want to fix the amplitude completely we have to impose additional constraints!
- It must link the contact terms to factorization terms



None of them individually satisfy condition X

- Natural condition for EFTs at low energies

Soft limit $p \to 0$

Simplest case

Free theory

- Single scalar field ϕ
- Minimal derivative coupling

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4\phi^2(\partial\phi)^2 + c_6\phi^4(\partial\phi)^2 + \dots$$

- Looks like interesting interacting theory but it is not

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2$$

free theory with

$$\phi \to F(\phi)$$

all amplitudes are zero

$$\sum_{ij} s_{ij} = 0$$

Non-trivial example

- Multiple scalars $\phi = \phi^a T^a$

- Write the same Lagrangian: now it is not just free

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4\phi^2(\partial\phi)^2 + c_6\phi^4(\partial\phi)^2 + \dots$$

traces, more couplings

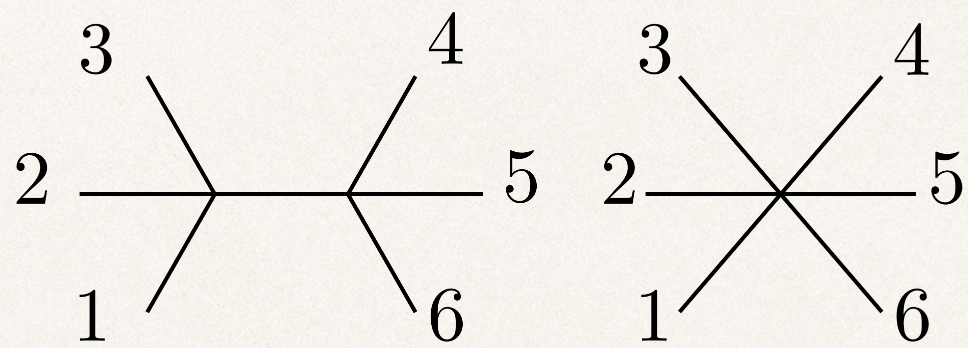
- We can do "color"- ordering (Kampf, Novotny, Trnka, 2013)

$$A_n = \sum_\sigma \text{Tr}(T^{a_1}T^{a_2}\dots T^{a_n})\, A(123\dots n)$$

Non-trivial example

- Example: six point amplitude

$$A_6 = \sum_{cycl}$$



important:
same power-counting

$$A_6 \sim c_4^2 \frac{p^2 \times p^2}{p^2} + c_6 p^2$$

- Impose: vanishing in soft limit

fixes $c_6 \sim c_4^2$

$A_6 \to 0$

for

$p \to 0$

Non-linear sigma model

(Weinberg 1966)

- Continue to higher points:

  fixes all coefficients and gives a unique theory (up to a gauge group)

  (Susskind, Frye 1970)

$$\mathcal{L} = \frac{F^2}{2}\langle(\partial_\mu U)(\partial^\mu U)\rangle \text{ where } \quad U = e^{\frac{i}{F}\phi^a T^a}$$

SU(N) non-linear sigma model

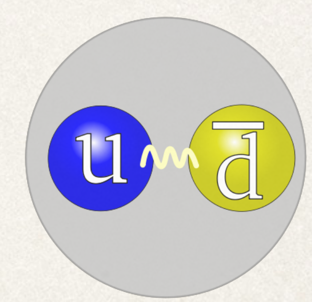
$$A_n \to 0$$

for

$$p \to 0$$

- Symmetry explanation: shift symmetry

$$\phi \to \phi + a$$



Low energy QCD

Uniqueness in minimality

- When renormalizing the SU(N) non-linear sigma model we need higher derivative terms

$$\mathcal{L}_{\chi PT} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$$(\partial_\mu U)(\partial^\mu U) \qquad (\partial_\mu \partial_\nu U)(\partial^\mu \partial^\nu U)$$

$$[(\partial_\mu U)(\partial^\nu U)]^2$$

etc

- They all have just a soft-limit vanishing

- Only the minimal coupling (NLSM) is uniquely fixed

Exceptional theories

(Cheung, Kampf, Novotny, JT 2014)

Single scalar

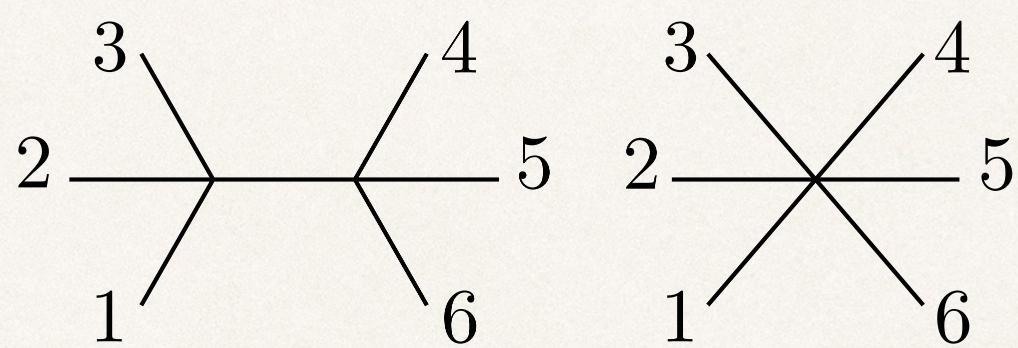
- The first non-trivial is the original example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \dots$$

- Calculate 6pt amplitude

$$(\partial\phi)^{2n} = [(\partial_\mu\phi)(\partial^\mu\phi)]^n$$

$$A_6 = \sum_\sigma$$



$$= 4\sum_\sigma c_4^2 \frac{(s_{12}s_{23} + s_{23}s_{13} + s_{12}s_{13})(s_{45}s_{56} + s_{45}s_{46} + s_{46}s_{56})}{s_{123}} + c_6\, s_{12}s_{34}s_{56}$$

trivial soft-limit vanishing $p_i \to 0 \quad \leftrightarrow$ Lagrangian trivially invariant $\phi \to \phi + a$

Single scalar

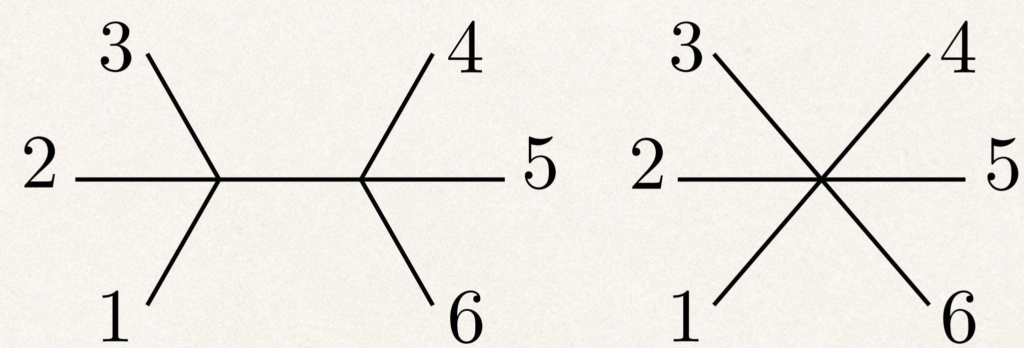
- The first non-trivial is the original example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \ldots$$

$$(\partial\phi)^{2n} = [(\partial_\mu\phi)(\partial^\mu\phi)]^n$$

- Calculate 6pt amplitude

$$A_6 = \sum_\sigma$$



$$= 4\sum_\sigma c_4^2 \frac{(s_{12}s_{23} + s_{23}s_{13} + s_{12}s_{13})(s_{45}s_{56} + s_{45}s_{46} + s_{46}s_{56})}{s_{123}} + c_6\, s_{12}s_{34}s_{56}$$

Impose quadratic vanishing

$$A_6 \to \mathcal{O}(t^2)$$

$$p_i \to tp_i$$
$$t \to 0$$

Single scalar

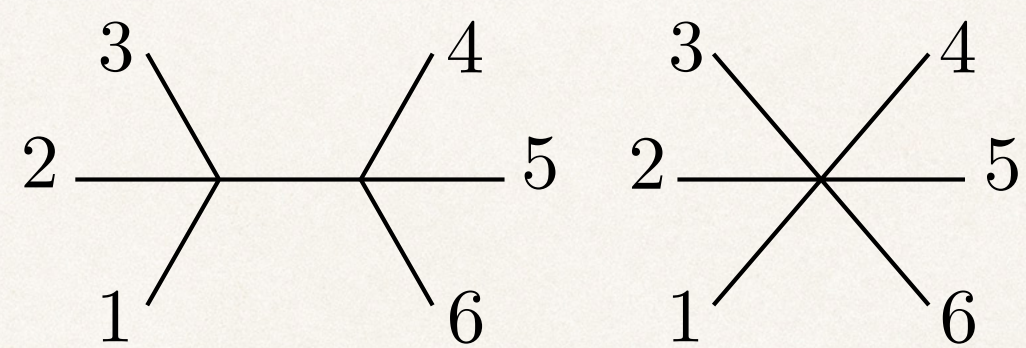
- The first non-trivial is the original example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \ldots$$

$$(\partial\phi)^{2n} = [(\partial_\mu\phi)(\partial^\mu\phi)]^n$$

- Calculate 6pt amplitude

$$A_6 = \sum_\sigma$$



$$= 4\sum_\sigma c_4^2\,\frac{(s_{12}s_{23}+s_{23}s_{13}+s_{12}s_{13})(s_{45}s_{56}+s_{45}s_{46}+s_{46}s_{56})}{s_{123}} + c_6\, s_{12}s_{34}s_{56}$$

There is a single solution and it fixes: $c_6 = 4c_4^2$

Single scalar

- The first non-trivial is the original example

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + c_6(\partial\phi)^6 + c_8(\partial\phi)^8 + \dots$$

$$(\partial\phi)^{2n} = [(\partial_\mu\phi)(\partial^\mu\phi)]^n$$

- Apply to higher point amplitudes

$$\boxed{A_n = \mathcal{O}(t^2)} \quad \text{for} \quad \begin{array}{c} p_i \to tp_i \\ t \to 0 \end{array}$$

- Cancelations between diagrams required, a unique solutions exists and relates $c_{2n} \sim c_4^{\#}$

Single scalar

- The Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + c_4(\partial\phi)^4 + 4c_4^2(\partial\phi)^6 + 20c_4^3(\partial\phi)^8 + \dots$$

$$(\partial\phi)^{2n} = [(\partial_\mu\phi)(\partial^\mu\phi)]^n$$

- Apply to higher point amplitudes

$$\boxed{A_n = \mathcal{O}(t^2)} \qquad \text{for} \quad \begin{array}{c} p_i \to tp_i \\ t \to 0 \end{array}$$

- Cancelations between diagrams required, a unique solutions exists and relates $c_{2n} \sim c_4^{\#}$

Single scalar

- The Lagrangian becomes

$$\mathcal{L} = -\frac{1}{g}\sqrt{1 - g(\partial\phi)^2} \quad \text{where} \quad g = 8c_4$$

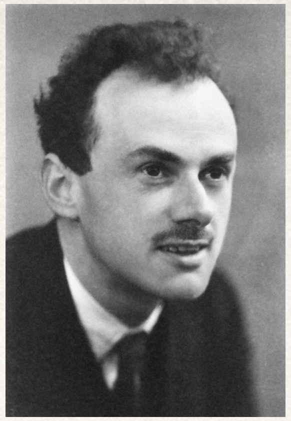
- Apply to higher point amplitudes

$$\boxed{A_n = \mathcal{O}(t^2)} \qquad \text{for} \qquad \begin{matrix} p_i \to tp_i \\ t \to 0 \end{matrix}$$

- Cancelations between diagrams required, a unique solutions exists and relates $c_{2n} \sim c_4^{\#}$

Result: DBI action

(Dirac, Born, Infeld 1934)



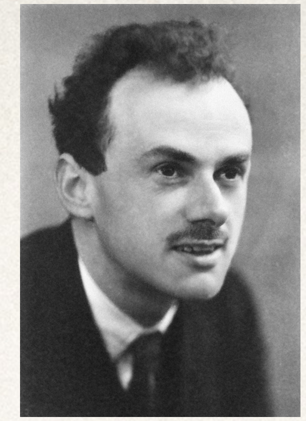
- The Lagrangian becomes

$$\mathcal{L} = -\frac{1}{g}\sqrt{1 - g(\partial\phi)^2} \quad \text{where} \quad g = 8c_4$$

- It describes the fluctuation of D-dimensional brane in (D+1) dimensions

- What is the symmetry principle behind this?

Result: DBI action



(Dirac, Born, Infeld 1934)

- Symmetry of the action: (D+1) Lorentz symmetry

$$\phi \to \phi + (b \cdot x) + (b \cdot \phi \partial \phi)$$

- It can be shown that this implies the soft limit behavior

- But we can also derive the action based on the soft limit

$$2\mathcal{L}'(X)/g = 2X\mathcal{L}'(X) - \mathcal{L}(X) \quad \to \quad \mathcal{L}(X) \sim \sqrt{1-gX}$$

where $X = (\partial\phi)^2$

Galileon

- Let us consider the next Lagrangian

$$\mathcal{L}_2 = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial^6\phi^4) + \lambda_6(\partial^{10}\phi^6) + \dots$$

- Calculate amplitudes: impose again $A_n = \mathcal{O}(t^2)$

Fully specifies a family of solutions

Galilean symmetry $\phi \to \phi + a + (b \cdot x)$

Galileons

Relevant for cosmological models

- There are (d-2) Lagrangians:

$$\mathcal{L}_n = \phi \det[\partial^{\mu_j}\partial_{\nu_k}\phi]^n_{j,k=1} \qquad n \leq d$$

Special Galileon

- Not enough for us: not minimal, not unique
- We impose even stronger condition

$$\boxed{A_n = \mathcal{O}(t^3)} \qquad \text{for} \qquad \begin{array}{c} p_i \to t p_i \\ t \to 0 \end{array}$$

- And there exists an unique solution, linear combination of Galileon Lagrangians: we called it special Galileon
- No symmetry explanation at that time

Special Galileon

- Not enough for us: not minimal, not unique
- We impose even stronger condition

$$\boxed{A_n = \mathcal{O}(t^3)} \qquad \text{for} \qquad \begin{array}{c} p_i \to t p_i \\ t \to 0 \end{array}$$

- And there exists an unique solution, linear combination of Galileon Lagrangians: we called it special Galileon

Effective Field Theories from Soft Limits
Clifford Cheung, Karol Kampf, Jiri Novotny, Jaroslav Trnka
(Submitted on 12 Dec 2014)

A Hidden Symmetry of the Galileon
Kurt Hinterbichler, Austin Joyce
(Submitted on 29 Jan 2015)

$$\phi \to s_{\mu\nu} x^\mu x^\nu + \frac{\lambda_4}{12} s^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

Classification

- Use soft-limit as classification tool

  (Cheung, Kampf, Novotny, Shen, JT 2016) (Elvang, Hadjiantonis, Jones, Paranjape 2018)

- No more interesting theories with 4pt vertices

  no theory with non-trivial $\mathcal{O}(t^4)$ behavior

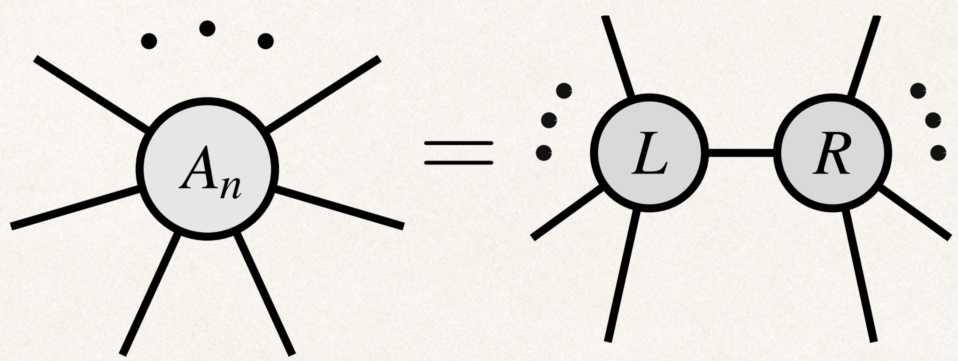
- Starting with 5pt vertices: WZW model but nothing more at higher points

- There are also analogues of DBI and Galileon for multiple scalars but nothing more

Recursion relations

On-shell reconstruction

- Tree-level factorization



on poles

$$P^2 = 0$$

- If the amplitude is fully fixed by factorizations we can reconstruct it using BCFW or other recursion relations

$$p_1 \to p_1 + zq$$
$$p_2 \to p_2 - zq$$
$$q^2 = (p_1 \cdot q) = (p_2 \cdot q) = 0$$

shifted amplitude

$$A_n(z)$$

is also an on-shell amplitude an factorizes properly

On-shell reconstruction

- Cauchy formula

$$\oint \frac{dz}{z} A_n(z) = 0$$

pole at z=0

$$A_n = A_n(z=0)$$

poles at other points

$$P^2(z) = 0 \;\rightarrow\; z = z^*$$

residue is the product of amplitudes

$$\mathop{\mathrm{Res}}_{z=z^*} A_n(z) \rightarrow \frac{A_L(z^*) A_R(z^*)}{P^2}$$

Express A_n using lower point amplitudes evaluated at shifted kinematics

On-shell reconstruction

- Cauchy formula

$$\oint \frac{dz}{z} A_n(z) = 0$$

Importantly, this can not have any pole at infinity

$$A_n(z \to \infty) = 0$$

This is violated for EFTs because of higher derivatives

$$A_n(z \to \infty) \sim z^{\#}$$

- We can use other shifts but it does not help if the amplitude is not fixed by factorizations

Soft limit recursion

(Cheung, Kampf, Novotny, Shen, JT 2015)

- Amplitudes fixed by factorizations + soft limit behavior

$$A_n = \mathcal{O}(t^\sigma)$$

- We can use soft limit behavior in the recursion

Shift

$$p_j \to p_j(1 - za_j)$$

Constraint

$$\sum_j a_j p_j = 0$$

- Shifted amplitude has zero

at $z = \frac{1}{a_j}$

$$A_n = \mathcal{O}((1 - za_j)^\sigma)$$

Modified Cauchy formula

$$\oint \frac{dz}{z} \frac{A_n(z)}{\prod_j (1 - za_j)^\sigma} = 0$$

Vector EFTs

(Cheung, Kampf, Novotny, Shen, Wen, JT 2018)

Setup for spin-1

- Single massless vector field A_μ (photon)

- Gauge invariance: Lagrangian depends on $F_{\mu\nu}$ only

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

trivial shift symmetry: soft-limit vanishing

- Leading derivative order: $\mathcal{L} = \mathcal{L}(F)$

- No cubic terms: no 3-photon interactions

Setup for spin-1

- General Lagrangian

$$\mathcal{L} = -\frac{1}{4}\langle FF\rangle + g_4^{(1)}\langle FFFF\rangle + g_4^{(2)}\langle FF\rangle^2 + g_6^{(1)}\langle FF\rangle^3$$
$$+g_6^{(2)}\langle FFFF\rangle\langle FF\rangle + g_6^{(3)}\langle FFFFFF\rangle + \dots$$

where the traces are defined as

$$\langle FF\rangle = F_{\mu\nu}F^{\mu\nu} \qquad \langle FFFF\rangle = F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} \quad \text{etc}$$

- Trivial soft limit vanishing, impose $A_n = \mathcal{O}(t^2)$

Setup for spin-1

- General Lagrangian

$$\mathcal{L} = -\frac{1}{4}\langle FF\rangle + g_4^{(1)}\langle FFFF\rangle + g_4^{(2)}\langle FF\rangle^2 + g_6^{(1)}\langle FF\rangle^3 + g_6^{(2)}\langle FFFF\rangle\langle FF\rangle + g_6^{(3)}\langle FFFFFF\rangle + \dots$$

where the traces are defined as

$$\langle FF\rangle = F_{\mu\nu}F^{\mu\nu} \qquad \langle FFFF\rangle = F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}$$ etc

- Trivial soft limit vanishing, impose $A_n = \mathcal{O}(t^2)$

No solution

Born-Infeld theory

- We know there is a special theory of this kind

Born-Infeld (BI) theory

$$\mathcal{L} = \sqrt{(-1)^{D-1}\det\left(\eta_{\mu\nu} + F_{\mu\nu}\right)}$$

U(1) gauge field on the brane

- Unfortunately, no known symmetry of this theory which would point to some amplitudes property

- This theory also shows up in the CHY formula, along with NLSM, DBI and special Galileon so it should be “unique”

Going to D=4

- Let us go to D=4: helicity amplitudes

  $(+,-)$ two polarizations e.g. $A_6(1^-2^-3^-4^+5^+6^+)$

- Use spinor helicity variables

$$p^\mu = \sigma^\mu_{a\dot{a}}\lambda_a\widetilde{\lambda}_{\dot{a}}$$

  only 3 degrees of freedom in momentum

- Little group scaling

$$\lambda_i \to t\lambda_i \qquad \widetilde{\lambda}_i \to \frac{1}{t}\widetilde{\lambda}_i \qquad p \to p$$

- Amplitudes of spin-1 particles transform as

$$A_n(j^-) \to t^2 A_n(j^-) \qquad A_n(j^+) \to \frac{1}{t^2} A_n(j^+)$$

Chiral soft limit

- Having spinor helicity variables we have two options how to approach the soft limit

$$p^\mu = \sigma^\mu_{a\dot{a}} \lambda_a \widetilde{\lambda}_{\dot{a}} \quad \begin{array}{l} \nearrow \; \lambda \to 0 \\ \searrow \; \widetilde{\lambda} \to 0 \end{array}$$

- In D=4 we can re-organize the Lagrangian using

$$f = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad g = -\frac{1}{4} F_{\mu\nu} \widetilde{F}^{\mu\nu}$$

thanks to Cayley-Hamilton relation

$$\langle F^n \rangle = -2f \langle F^{n-2} \rangle + g^2 \langle F^{n-4} \rangle$$

Chiral soft limit

- Rewrite Lagrangian

$$\mathcal{L} = f + a_1 f^2 + a_2 g^2 + b_1 f^3 + b_2 f g^2 + \ldots$$

- Calculate 4pt amplitudes

$$A_4(1^-2^-3^+4^+) = \frac{1}{2}(a_1 + a_2)\langle 12\rangle^2[34]^2 \qquad \text{etc}$$

then higher point amplitudes for all helicity configurations

- Impose the constraint:

$$A_n(1^-2^- \ldots j^-(j+1)^+ \ldots n^+) \to 0$$

multi-chiral soft limit

$$\widetilde{\lambda}_k \to 0$$

for all negative helicity photons

Unique solution

- This fixes all coefficients in the Lagrangian

$$\mathcal{L} = -\sqrt{1 - 2f - g^2}$$

indeed we got
BI action

- Note that the only non-zero amplitudes are helicity conserving $A_n(1^- 2^- \dots (n/2)^- (n/2+1)^+ \dots n^+)$

- Cancelation between all diagram: similar to DBI

- We also found recursion relations

Unique solution

- Note: there is no known symmetry of BI action and explanation of the soft limit behavior directly
- It can be proven using susy: breaking N=2 to N=1
- There should be some manifestation of this soft limit behavior in D dimensions
- We have alternative construction using dimensional decomposition to DBI action in lower dimension

Beyond photons

- Fermionic theories were inspected using supersymmetry

  (Elvang, Hadjiantonis, Jones, Paranjape 2018)
- We looked at higher derivative theories "vector Galileons" — they should not exist but we found some?!

- Main challenge: non-abelian Born-Infeld
  - It should exist but there is no known Lagrangian despite considerable effort, ideal problem for us to attack
  - Important role in string theory, also perhaps in cosmology
  - If exists, there is no "color"-ordering

Conclusion

Conclusion

- **On-shell amplitudes** as unique objects
  - EFTs: not fixed by factorizations
  - Special theories with non-trivial soft limit behavior
  - Recursion relations: reconstruction
- Search for new symmetries or even new theories using simple properties of tree-level amplitudes

Thank you for your attention