IR structure of two-loop four-gluon amplitude in $\mathcal{N} = 2$ SQCD

ongoing work with Gregor Kälin and Gustav Mogull

Alexander Ochirov
ETH Zürich

Amplitudes in the LHC era,
GGI, Florence, October 26, 2018

Invitation

Two-loop amplitudes $\mathcal{A} = \int \mathcal{I}$ beyond $2 \to 2$

- 5-gluon all-plus

  $\mathcal{I}$: Badger, Frellesvig, Zhang (2013)
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- n-gluon all-plus

  $\mathcal{I}$: Badger, Mogull, Peraro (2016)
  $\mathcal{A}$: Dunbar, Jehu, Perkins (2016)

- 5-gluon all helicities ($\mathcal{I}$ implicit, numerical $\int$)

  $\mathcal{A}$: Badger, Bronnum-Hansen, Hartanto, Peraro (2017)
  $\mathcal{A}$: Abreu, Febres Cordero, Ita, Page, Zeng (2017)

- 5-point general massless QCD

  talks by Peraro and Zeng

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- control complexity of $\mathcal{I} \approx$ complexity of $\mathcal{A}$
- understand provenance of IR divergences through $\mathcal{O}(\epsilon^{-2L})$

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This talk:

- inspect 4-pt integrand in $\mathcal{N} = 2$ SQCD (full color and N_f)
- analytic IR+UV structure at $\mathcal{O}(\epsilon^{-4})$, $\mathcal{O}(\epsilon^{-3})$ and $\mathcal{O}(\epsilon^{-2})$

Outline

1. IR factorization review
2. $\mathcal{N} = 2$ integrand
3. Analytic IR structure

4. Summary & outlook

IR factorization review

IR factorization *

Kunszt, Signer, Trocsanyi (1994)
Catani, Seymour (1996)
Catani (1998)

$$\widetilde{\mathcal{M}}_n^{(1)} = \mathbf{I}^{(1)}(\epsilon)\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$
$$\widetilde{\mathcal{M}}_n^{(2)} = \mathbf{I}^{(1)}(\epsilon)\widetilde{\mathcal{M}}_n^{(1)} + \mathbf{I}^{(2)}(\epsilon)\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$

$$\mathbf{I}^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \sum_{i=1}^{n} \left(\frac{1}{\epsilon^2} - \frac{\gamma_i^{(1)}}{\mathbf{T}_i^2}\frac{1}{\epsilon} \right) \sum_{j\neq i}^{n} \left(\frac{-s_{ij}}{\mu^2} \right)^{-\epsilon} \mathbf{T}_i \cdot \mathbf{T}_j$$

$$\mathbf{I}^{(2)}(\epsilon) = -\frac{1}{2}\mathbf{I}^{(1)}(\epsilon)\left(\mathbf{I}^{(1)}(\epsilon) + \frac{2\beta_0}{\epsilon} \right) + \left(\frac{\beta_0}{\epsilon} + 2K_{\mathrm{R.S.}} \right) \mathbf{I}^{(1)}(2\epsilon) + \mathcal{O}(\epsilon^{-1})$$

* More modern view — soft exponentiation

$$\widetilde{\mathcal{M}}_n(p_i, \mu, \alpha_s(\mu)) = \mathcal{P}\exp\left\{ -\int_0^{\mu} \frac{d\lambda}{\lambda} \mathbf{\Gamma}\left(\frac{p_i}{\lambda}, \alpha_s(\lambda) \right) \right\} \mathcal{H}_n(p_i, \mu, \alpha_s(\mu))$$

IR factorization *

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NB! $\mathcal{O}(\epsilon^{-2})$ and $\mathcal{O}(\epsilon^{-1})$ depend on dimreg scheme, e.g.

$$K_{\mathrm{FDH}} = \left(\frac{32-4\epsilon}{9} - \frac{\pi^2}{6}\right)C_A - \frac{5}{9}C_f n_f - \frac{6-2\epsilon}{27}C_s n_s$$

Bern, De Freitas, Dixon (2002)

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$$\widetilde{\mathcal{M}}_n(p_i,\mu,\alpha_s(\mu)) = \mathcal{P}\exp\left\{-\int_0^{\mu}\frac{d\lambda}{\lambda}\mathbf{\Gamma}\left(\frac{p_i}{\lambda},\alpha_s(\lambda)\right)\right\}\mathcal{H}_n(p_i,\mu,\alpha_s(\mu))$$

Undoing renormalization

$$\widetilde{\mathcal{M}}_n^{(1)} = S_\epsilon^{-1}\mathcal{M}_n^{(1)} - \frac{(n-2)\beta_0}{2\epsilon}\mathcal{M}_n^{(0)}$$

$$\widetilde{\mathcal{M}}_n^{(2)} = S_\epsilon^{-2}\mathcal{M}_n^{(2)} - \frac{n\beta_0}{2\epsilon}S_\epsilon^{-1}\mathcal{M}_n^{(1)} + \frac{(n-2)}{2}\left[\frac{n\beta_0^2}{4\epsilon^2} - \frac{\beta_1}{\epsilon}\right]\mathcal{M}_n^{(0)}$$

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Rearrange Catani into

$$\mathcal{M}_n^{(1)} = \mathbf{I}_\alpha^{(1)}(\epsilon)\mathcal{M}_n^{(0)} + \frac{1}{\epsilon}\left[\frac{n-2}{2}\beta_0 + \sum_{i=1}^{n}\gamma_i^{(1)}\right]\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$

$$\mathcal{M}_n^{(2)} = \mathbf{I}_\alpha^{(1)}(\epsilon)\mathcal{M}_n^{(1)} - \frac{1}{2}\mathbf{I}_\alpha^{(1)}(\epsilon)\mathbf{I}_\alpha^{(1)}(\epsilon)\mathcal{M}_n^{(0)} + \left[\frac{\beta_0}{\epsilon} + 2K_{\mathrm{R.S.}}\right]\mathbf{I}_\alpha^{(1)}(2\epsilon)\mathcal{M}_n^{(0)}$$

$$+ \frac{1}{2\epsilon^2}\left[\frac{n}{2}\beta_0 + \sum_{i=1}^{n}\gamma_i^{(1)}\right]\left[\frac{n-2}{2}\beta_0 + \sum_{i=1}^{n}\gamma_i^{(1)}\right]\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^{-1})$$

$$\mathbf{I}_\alpha^{(1)}(\epsilon) = \frac{e^{\epsilon\gamma_E}}{\epsilon^2\Gamma(1-\epsilon)}\sum_{i=1}^{n}\sum_{j\neq i}^{n}(-s_{ij})^{-\epsilon}\,\mathbf{T}_i\cdot\mathbf{T}_j$$

NB! Mixed IR with UV

Assume IR factorization for $\mathcal{N} = 4$ SYM

- Specialize to ext. gluons $\gamma_i^{(1)} = -\beta_0/2$
- Consider difference w.r.t. $\mathcal{N} = 4$

$$\mathcal{M}_n^{(1)} - \mathcal{M}_n^{(1)[\mathcal{N}=4]} = -\frac{\beta_0}{\epsilon}\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$

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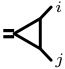
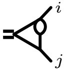
$$\mathcal{M}_n^{(2)} - \mathcal{M}_n^{(2)[\mathcal{N}=4]} = \left(\sum_{i\neq j} s_{ij}$$



$$\mathbf{T}_i \cdot \mathbf{T}_j\right) \left[\mathcal{M}_n^{(1)} - \mathcal{M}_n^{(1)[\mathcal{N}=4]}\right]$$

$$+ \beta_0 \left(\sum_{i\neq j} s_{ij}$$



$$\mathbf{T}_i \cdot \mathbf{T}_j\right) \mathcal{M}_n^{(0)}$$

$$+ \frac{n}{2}\frac{N_c}{\epsilon^2}\left(\beta_0 - 2[K - K_{\mathcal{N}=4}]\right)\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^{-1})$$

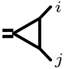
IR factorization for $\mathcal{N} = 2$ SQCD

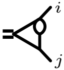
Specialize to $\mathcal{N} = 2$:

$$\beta_0 = 2N_c - N_f, \qquad K_{\mathcal{N}=2} = K_{\mathcal{N}=4} + \frac{\beta_0}{2} + \mathcal{O}(\epsilon)$$

Singularity structure encoded by triangles through $\mathcal{O}(\epsilon^{-2})$:

$$\mathcal{M}_n^{(1)} - \mathcal{M}_n^{(1)[\mathcal{N}=4]} = -\frac{\beta_0}{\epsilon}\mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^0)$$

$$\mathcal{M}_n^{(2)} - \mathcal{M}_n^{(2)[\mathcal{N}=4]} = \left(\sum_{i\neq j} s_{ij}$$  $$\mathbf{T}_i \cdot \mathbf{T}_j\right) \left[\mathcal{M}_n^{(1)} - \mathcal{M}_n^{(1)[\mathcal{N}=4]}\right]$$

$$+ \beta_0 \left(\sum_{i\neq j} s_{ij}$$  $$\mathbf{T}_i \cdot \mathbf{T}_j\right) \mathcal{M}_n^{(0)} + \mathcal{O}(\epsilon^{-1})$$

$\mathcal{N} = 2$ integrand

$\mathcal{N} = 2$ supersymmetric QCD

hel. \ th.	$\mathcal{N}=4$ SYM	$\mathcal{N}=2$ SQCD				QCD		
$+1$	1	1		$\overbrace{\quad\quad}^{N_f}$		1	$\overbrace{\quad\quad}^{N_f}$	
$+1/2$	4	2		1	1		1	1
0	6	1	1	2	2			
$-1/2$	4		2	1	1		1	1
-1	1		1			1		
rep.	G	G		N_c	$\overline{N}_c$	G	N_c	$\overline{N}_c$

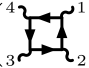
$$\alpha_s(\mu) = \frac{\mu_0^{2\epsilon}}{\mu^{2\epsilon}} \frac{\alpha_s(\mu_0)}{1 + \frac{\beta_0 \alpha_s(\mu_0)}{4\pi\epsilon}\left[1 - \frac{\mu_0^{2\epsilon}}{\mu^{2\epsilon}}\right]}, \qquad \beta_0 = 2N_c - N_f$$

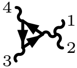
Seiberg (1988)

One-loop $\mathcal{N}=2$ integrand

Johansson, AO (2014)

Start with BCJ numerators $n_i^{[\mathcal{N}=2,\text{pure}]} = n_i^{[\mathcal{N}=4]} - 2n_i^{[\mathcal{N}=2,\text{fund}]}$,
where $n_i^{[\mathcal{N}=2,\text{fund}]}$ are

$n\Big($  $\Big) = \dfrac{\kappa_{13}}{u^2}\,\mathrm{tr}_- + \dfrac{\kappa_{24}}{u^2}\,\mathrm{tr}_+ + \mu^2\left(\dfrac{\kappa_{12}+\kappa_{34}}{s} + \dfrac{\kappa_{23}+\kappa_{14}}{t} + \dfrac{\kappa_{13}+\kappa_{24}}{u}\right)$

$n\Big($  $\Big) = \left(\dfrac{\kappa_{13}}{u^2} + \dfrac{\kappa_{23}}{t^2}\right)\mathrm{tr}_- + \left(\dfrac{\kappa_{24}}{u^2} + \dfrac{\kappa_{14}}{t^2}\right)\mathrm{tr}_+ + \dfrac{s}{t^2}\,(\kappa_{23}+\kappa_{14})\,(\ell+p_4)^2$

$n\Big($  $\Big) = \left(\dfrac{\kappa_{23}+\kappa_{14}}{t} - \dfrac{\kappa_{13}+\kappa_{24}}{u}\right)\big(s-\ell^2-(\ell-p_{12})^2\big)$

$$\mathrm{tr}_\pm = \mathrm{tr}_\pm(1(\ell-p_1)(\ell-p_{12})3)$$

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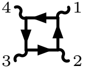
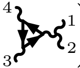
$n\Bigg($ 

$\Bigg) = \dfrac{\kappa_{13}}{u^2}\,\text{tr}_- + \dfrac{\kappa_{24}}{u^2}\,\text{tr}_+ + \mu^2\left(\dfrac{\kappa_{12}+\kappa_{34}}{s} + \dfrac{\kappa_{23}+\kappa_{14}}{t} + \dfrac{\kappa_{13}+\kappa_{24}}{u}\right)$

$n\Bigg($ 

$\Bigg) = \left(\dfrac{\kappa_{13}}{u^2} + \dfrac{\kappa_{23}}{t^2}\right)\text{tr}_- + \left(\dfrac{\kappa_{24}}{u^2} + \dfrac{\kappa_{14}}{t^2}\right)\text{tr}_+ + \dfrac{s}{t^2}\,(\kappa_{23}+\kappa_{14})\,(\ell+p_4)^2$

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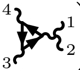
Integrand-reduce

$n\Bigg($ 

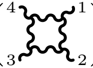
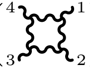
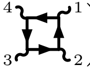
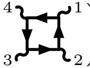
$\Bigg) = \left(\dfrac{\kappa_{13}}{u^2} + \dfrac{\kappa_{23}}{t^2}\right)\text{tr}_+(13\ell 4) + \left(\dfrac{\kappa_{24}}{u^2} + \dfrac{\kappa_{14}}{t^2}\right)\text{tr}_-(13\ell 4)$

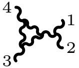
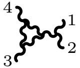
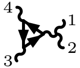
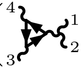
$n\Bigg($ 

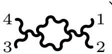
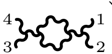
$\Bigg) = s\left(\dfrac{\kappa_{13}+\kappa_{24}}{u} - \dfrac{\kappa_{23}+\kappa_{14}}{t}\right)$

Pro: triangle integrates to zero; Con: color-kinematics broken

One-loop $\mathcal{N}=2$ integrand

$\mathcal{M}_4^{(1)} = \frac{i}{2}\sum_{\text{perms}} I\Big[\frac{1}{8}c\Big($  $\Big)n\Big($  $\Big) + \frac{N_f}{4}c\Big($  $\Big)n\Big($  $\Big)$

$+\frac{1}{4}c\Big($  $\Big)n\Big($  $\Big) + \frac{N_f}{2}c\Big($  $\Big)n\Big($  $\Big)$

$+\frac{1}{16}c\Big($  $\Big)n\Big($  $\Big) + \frac{N_f}{8}c\Big($  $\Big)n\Big($  $\Big)\Big]$

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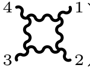
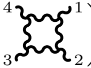
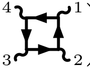
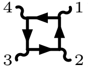
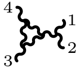
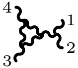
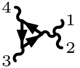
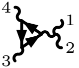
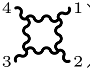
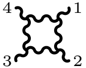
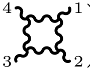
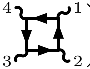
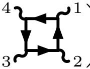
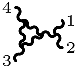
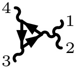
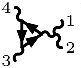
$\mathcal{M}_4^{(1)} = \frac{i}{2}\sum_{\text{perms}} I\Big[\frac{1}{8}c\Big($  $\Big) n \Big($  $\Big) + \frac{N_f}{4} c\Big($  $\Big) n\Big($  $\Big)$

$+ \frac{1}{4} c\Big($  $\Big) n\Big($  $\Big) + \frac{N_f}{2} c\Big($  $\Big) n\Big($  $\Big)$

$+ \frac{1}{16} c\Big($  $\Big) n\Big($  $\Big) + \frac{N_f}{8} c\Big($  $\Big) n\Big($  $\Big)\Big]$

$= \frac{i}{2}\sum_{\text{perms}} I\Big[\frac{1}{8}c\Big($  $\Big) n^{[\mathcal{N}=4]}\Big($  $\Big) \quad \Rightarrow \mathcal{M}_4^{(1)[\mathcal{N}=4]}$

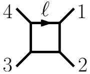
$- \frac{1}{4}\Big(c\Big($  $\Big) - N_f c\Big($  $\Big)\Big) n\Big($  $\Big) \quad \Rightarrow \mathcal{O}(\epsilon^0)$

$- \frac{1}{2}\Big(c\Big($  $\Big) - N_f c\Big($  $\Big)\Big) n\Big($  $\Big) \quad \Rightarrow 0$

$- \frac{1}{8}\Big(c\Big($  $\Big) - N_f c\Big($  $\Big)\Big) n\Big($  $\Big)\Big]$

$$= \mathcal{M}_4^{(1)[\mathcal{N}=4]} - \frac{\beta_0}{\epsilon}\mathcal{M}_4^{(0)} + \mathcal{O}(\epsilon^0)$$

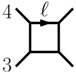
Finite boxes

$I\Big($ 

$\Big)\left[\,\mathrm{tr}_+[1(\ell-p_1)(\ell-p_{12})3)]\right] = -\frac{r_\Gamma}{2}\left[\,\log^2(\chi)+\pi^2\right]+O(\epsilon)$

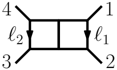
e.g. Badger, Mogull, Peraro (2016)

Finite boxes

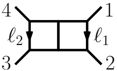
$I\Big($ 

$$\Big)\left[\,\mathrm{tr}_+[1(\ell-p_1)(\ell-p_{12})3)]\right] = -\frac{r_\Gamma}{2}\left[\log^2(\chi)+\pi^2\right]+O(\epsilon)$$

e.g. Badger, Mogull, Peraro (2016)

$I\Big($ 

$$\Big)\left[\,\mathrm{tr}_+(1\bar{\ell}_1 23\bar{\ell}_2 4)\right] = -2H_{-1,-1,0,0}(\chi) + \frac{\pi^2}{3}\mathrm{Li}_2(-\chi) - \left(\frac{\pi^2}{2}\log(1+\chi) - \frac{\pi^2}{3}\log\chi + 2\zeta(3)\right)\log(1+\chi) + 6\chi\zeta(3)$$

$I\Big($ 

$$\Big)\left[\,\mathrm{tr}_+(1\bar{\ell}_1 24\bar{\ell}_2 3)\right] = -2H_{0,-1,0,0}(\chi) + \pi^2\mathrm{Li}_2(-\chi) + \frac{\pi^2}{6}\log^2\chi + 4\zeta(3)\log\chi + \frac{\pi^4}{10} + 6(1+\chi)\zeta(3)$$

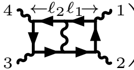
Caron-Huot, Larsen (2012)

where $\chi = t/s$

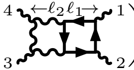
Two-loop $\mathcal{N} = 2$ integrand

Johansson, Kälin, Mogull (2017)

Start with BCJ numerators

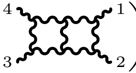
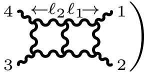
$n\Big($  $\Big)$ $= -\mu_{12}(\kappa_{12} + \kappa_{34}) + \frac{1}{u^2}\left(\kappa_{13}\,\mathrm{tr}_-(1\bar{\ell}_1 24\bar{\ell}_2 3) + \kappa_{24}\,\mathrm{tr}_+(1\bar{\ell}_1 24\bar{\ell}_2 3)\right)$

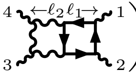
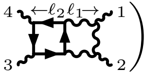
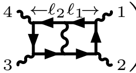
$$+ \frac{1}{t^2}\left(\kappa_{14}\,\mathrm{tr}_-(1\bar{\ell}_1 23\bar{\ell}_2 4) + \kappa_{23}\,\mathrm{tr}_+(1\bar{\ell}_1 23\bar{\ell}_2 4)\right)$$

$n\Big($  $\Big)$ $= \mu_{13}(\kappa_{12} + \kappa_{34}) - \frac{1}{u^2}\left(\kappa_{13}\,\mathrm{tr}_-(1\bar{\ell}_1 24\bar{\ell}_3 3) + \kappa_{24}\,\mathrm{tr}_+(1\bar{\ell}_1 24\bar{\ell}_3 3)\right)$

$$- \frac{1}{t^2}\left(\kappa_{14}\,\mathrm{tr}_-(1\bar{\ell}_1 23\bar{\ell}_3 4) + \kappa_{23}\,\mathrm{tr}_+(1\bar{\ell}_1 23\bar{\ell}_3 4)\right) \qquad \text{etc.}$$

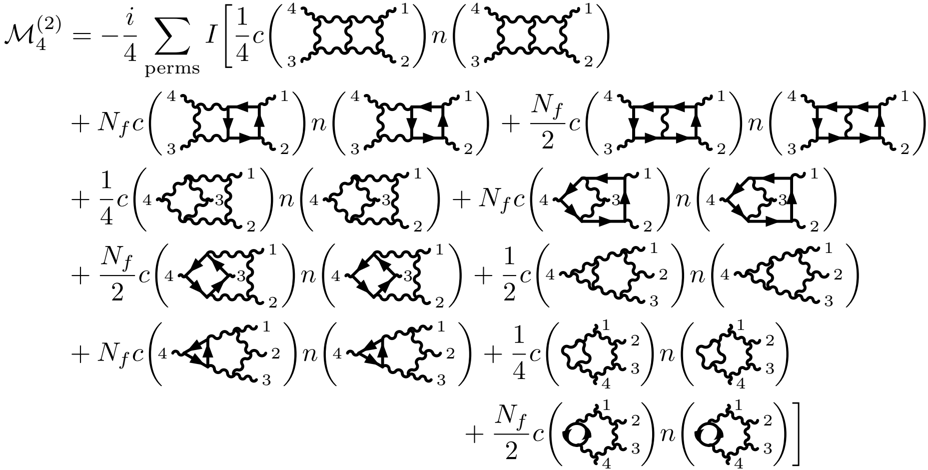
Relation to $\mathcal{N} = 4$ SYM:

$n^{[\mathcal{N}=4]}\Big($  $\Big) = n\Big($  $\Big)$

$+\, 2n\Big($  $\Big) + 2n\Big($  $\Big) + 2n\Big($  $\Big)$

Two-loop $\mathcal{N} = 2$ integrand

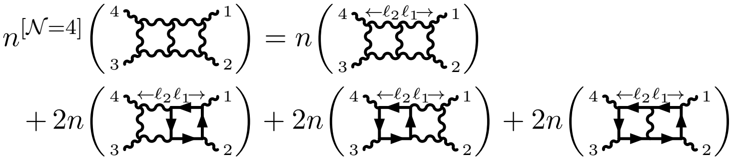
Integrand assembled with color factors



Analytic IR structure

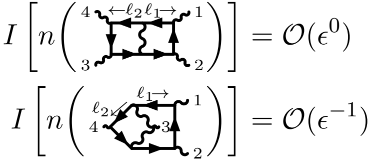
Nice features of $\mathcal{N}=2$ integrand

Two-loop numerators related to $\mathcal{N}=4$ SYM:

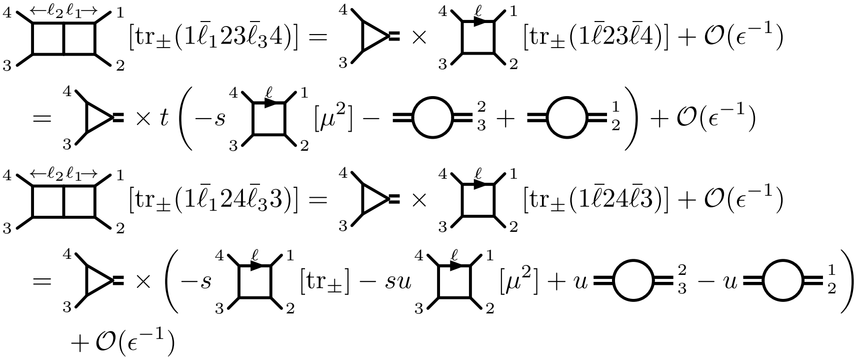


Johansson, Kälin, Mogull (2017)

- obtained from 6d cuts of $\mathcal{N}=(1,0)$ SYM with N_f hypers
- matter loops IR-regulated by numerators
- $\mathcal{O}(\epsilon^{-4})$ entirely inside $\mathcal{M}_4^{(2)[\mathcal{N}=4]}$

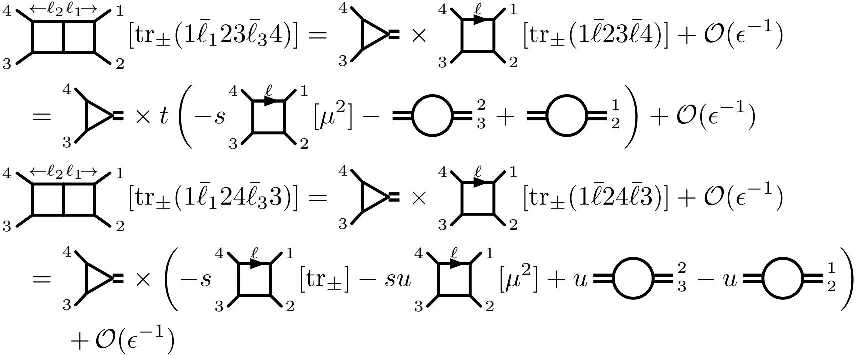


IR factorization of singular double boxes

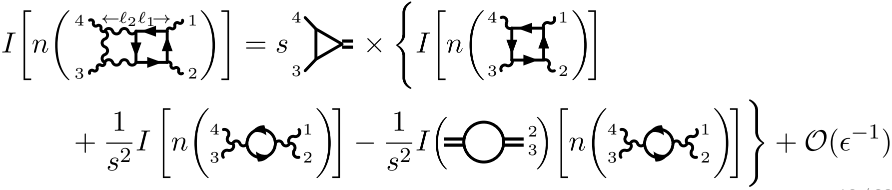


where recall 1-loop $\quad \mathrm{tr}_\pm = \mathrm{tr}_\pm(1(\ell - p_1)(\ell - p_{12})3)$

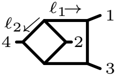
IR factorization of singular double boxes

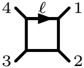


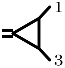
where recall 1-loop $\quad \mathrm{tr}_\pm = \mathrm{tr}_\pm(1(\ell - p_1)(\ell - p_{12})3)$



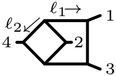
IR factorization of singular cross-boxes

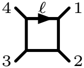


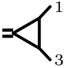
$[\mathrm{tr}_\pm(2\ell_3\ell_2 4)] =$


$[\mathrm{tr}_\mp(1\ell(\ell+p_4)3)] \times$


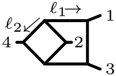
$+ \mathcal{O}(\epsilon^{-1})$

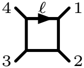


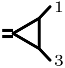
$[\mathrm{tr}_\pm(1\bar{\ell}_2 4 3 \bar{\ell}_3 2)] =$


$[\mathrm{tr}_\pm(1\bar{\ell}43(\bar{\ell}-p_1)2)] \times$


$+ \mathcal{O}(\epsilon^{-1})$

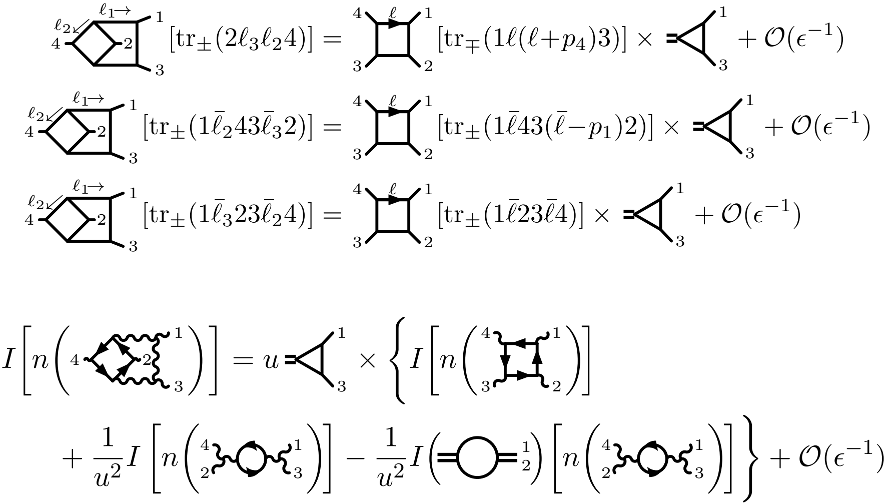


$[\mathrm{tr}_\pm(1\bar{\ell}_3 2 3 \bar{\ell}_2 4)] =$


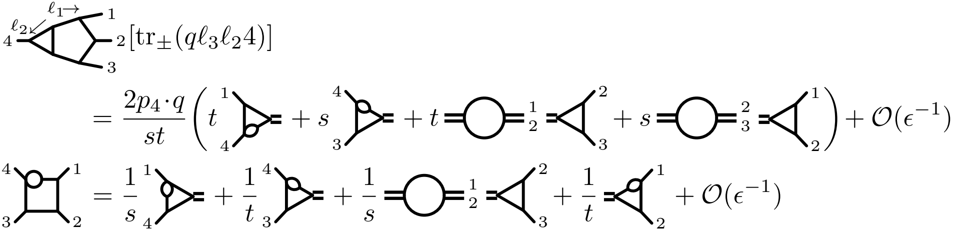
$[\mathrm{tr}_\pm(1\bar{\ell}23\bar{\ell}4)] \times$


$+ \mathcal{O}(\epsilon^{-1})$

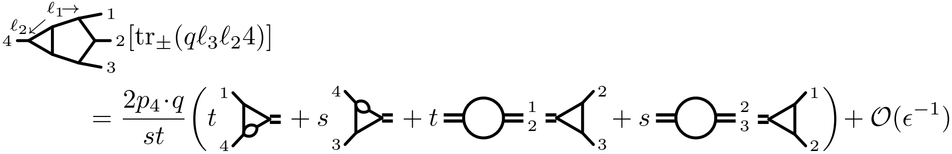
IR factorization of singular cross-boxes

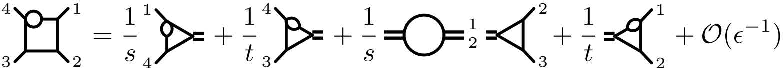


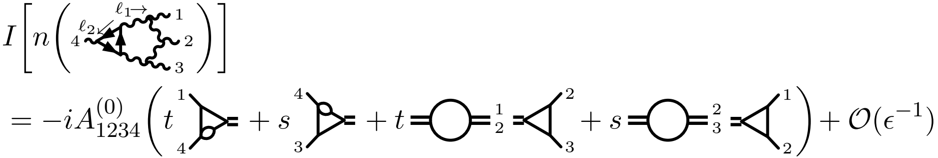
IR factorization of pentagon-triangles and box-bubbles

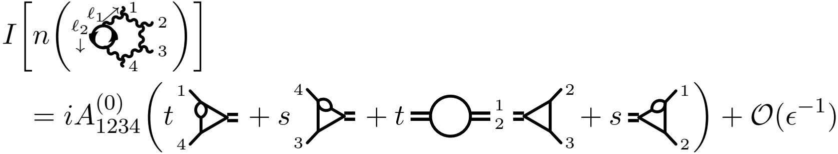


IR factorization of pentagon-triangles and box-bubbles



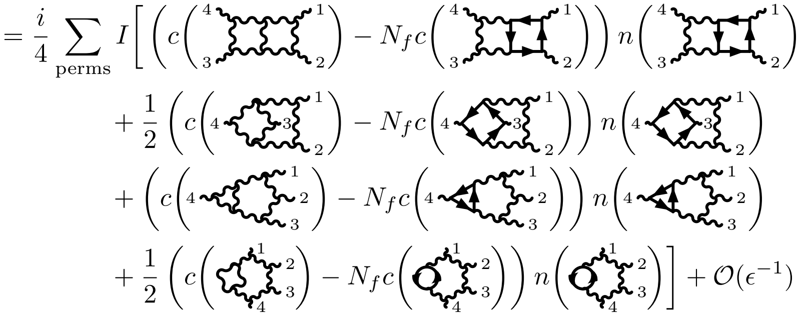






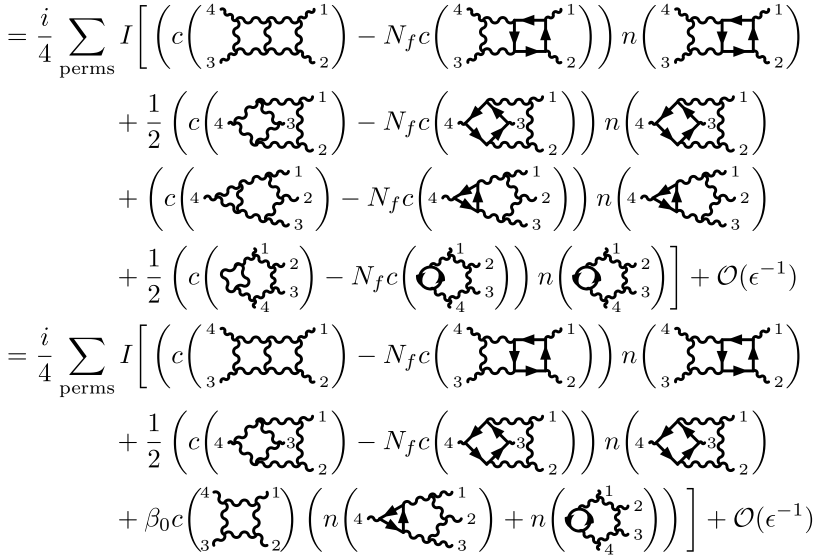
Extract difference from $\mathcal{N} = 4$

$$\mathcal{M}_4^{(2)} - \mathcal{M}_4^{(2)[\mathcal{N}=4]}$$



Extract difference from $\mathcal{N} = 4$

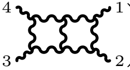
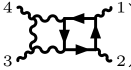
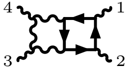
$$\mathcal{M}_4^{(2)} - \mathcal{M}_4^{(2)[\mathcal{N}=4]}$$

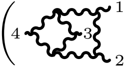
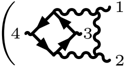
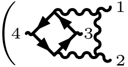


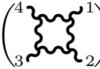
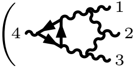
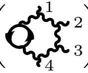
Match difference from $\mathcal{N} = 4$

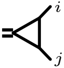
After analytic rearrangements match reorganized Catani

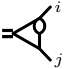
$\mathcal{M}_4^{(2)} - \mathcal{M}_4^{(2)[\mathcal{N}=4]}$

$= \frac{i}{4} \sum_{\text{perms}} I \Big[\Big(c \Big($  $\Big) - N_f c \Big($  $\Big) \Big) n \Big($  $\Big)$

$+ \frac{1}{2} \Big(c \Big($  $\Big) - N_f c \Big($  $\Big) \Big) n \Big($  $\Big)$

$+ \beta_0 c \Big($  $\Big) \Big(n \Big($  $\Big) + n \Big($  $\Big) \Big) \Big] + \mathcal{O}(\epsilon^{-1})$

$= \Big(\sum_{i \neq j} s_{ij}$  $\mathbf{T}_i \cdot \mathbf{T}_j \Big) \big[\mathcal{M}_4^{(1)} - \mathcal{M}_4^{(1)[\mathcal{N}=4]} \big]$

$+ \beta_0 \Big(\sum_{i \neq j} s_{ij}$  $\mathbf{T}_i \cdot \mathbf{T}_j \Big) \mathcal{M}_4^{(0)} + \mathcal{O}(\epsilon^{-1})$

Summary & outlook

- Discussed QCD-like IR structure beyond $\mathcal{N} = 4$
- Explicit 4-gluon integrand from 6d unitarity cuts

  Johansson, Kälin, Mogull (2017)
- Matter loops IR regulated

  e.g. box from Badger, Mogull, Peraro (2016)

  double boxes from Caron-Huot, Larsen (2012)
- Divergences of unregulated loops extracted analytically

  similar to Anastasiou, Sterman (talk 2018)
- External matter in progress

STAY TUNED!

Thank you!

Backup slides

General conventions

$$\widetilde{\mathcal{M}}_n = (4\pi\alpha_s)^{\frac{n-2}{2}} \left[\widetilde{\mathcal{M}}_n^{(0)} + \frac{\alpha_s}{4\pi}\widetilde{\mathcal{M}}_n^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \widetilde{\mathcal{M}}_n^{(2)} + \dots\right]$$

$$\mathcal{M}_n = (4\pi\alpha_0)^{\frac{n-2}{2}} \left[\mathcal{M}_n^{(0)} + \frac{\alpha_0}{4\pi}S_\epsilon\mathcal{M}_n^{(1)} + \left(\frac{\alpha_0}{4\pi}\right)^2 S_\epsilon^{-2}\mathcal{M}_n^{(2)} + \dots\right]$$

$$\alpha_0\mu_0^{2\epsilon}S_\epsilon = \alpha_s\mu_R^{2\epsilon}\left[1 - \frac{\alpha_s}{4\pi}\frac{\beta_0}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2\left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{\epsilon}\right) + \mathcal{O}(\alpha_s^3)\right]$$

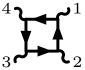
$$S_\epsilon = (4\pi)^\epsilon e^{-\epsilon\gamma_E}$$

$$I \sim \left(e^{\epsilon\gamma_E}\int\frac{\mathrm{d}^D\ell}{i\pi^{D/2}}\right)^L$$

$$r_\Gamma = e^{\epsilon\gamma_E}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} = 1 - \frac{1}{2}\zeta_2\epsilon^2 - \frac{7}{3}\zeta_3\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$\mathcal{N} = 0$ conventions

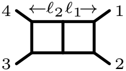
$$\mathcal{M}_4^{(0)} = -\frac{i}{4}\sum_{\text{perms}} c\Bigg($$



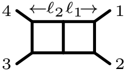
$$\Bigg)\frac{1}{st}(\kappa_{12} + \kappa_{13} + \kappa_{14} + \kappa_{23} + \kappa_{24} + \kappa_{34})$$

$$\kappa_{ij} = \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\delta^{(4)}(Q)\langle ij\rangle^2\eta_i^3\eta_j^3\eta_i^4\eta_j^3$$

All singularities of double box

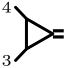


$[\mathrm{tr}_\pm(1\bar{\ell}_1 23(\bar{\ell}_1+\bar{\ell}_2)4)] =$



$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)] + \mathcal{O}(\epsilon^0)$

$=$



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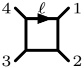
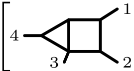
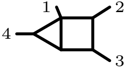


$[\mathrm{tr}_\pm(1\bar{\ell}23\bar{\ell}4)]$

$+ \frac{1}{s}\Bigg[$

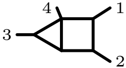


$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)] -$

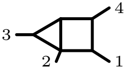


$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)]$

$+$



$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)] -$



$[\mathrm{tr}_\pm(1\bar{\ell}_1 23\bar{\ell}_1 4)]\Bigg]$

$+ \mathcal{O}(\epsilon^0)$