Towards the 1-loop effective action of type IIB orientifolds

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Overview

- Motivation
- Calculational setup
- Results

Motivation

(Perturbative) quantum corrections to effective action can be important

- if "zero effect" at tree level
  (e.g. *no-scale structure* of potential)
- certain minima with fixed moduli only apparent if quantum corrections are included
  (cf. *Large Volume Scenario*)
- for string phenomenology
  (e.g. embedding inflation in string theory)

$\mathcal{N} = 1,\ d = 4$ Supergravity

$$\frac{\mathcal{L}_{\rm bos}}{(-g)^{1/2}} = \frac{1}{2\kappa^2}R - K_{,\bar{I}J}D_\mu\bar{\Phi}^I D^\mu\Phi^J - \frac{1}{4}\mathrm{Re}(f_{ab}(\Phi))F^a_{\mu\nu}F^{b\mu\nu}$$
$$-\frac{1}{8}\mathrm{Im}(f_{ab}(\Phi))\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma} - V(\Phi,\bar{\Phi})$$

with $V(\Phi,\bar{\Phi}) = e^K(G^{\bar{I}J}D_{\bar{I}}\overline{W}D_JW - 3|W|^2) + \mathrm{Re}(f_{ab})\mathcal{D}^a\mathcal{D}^b$

$$D_JW = \partial_{\Phi^J}W + (\partial_{\Phi^J}K)W$$

- Kähler potential K
- Gauge kinetic function f_{ab}
- Superpotential W

Quantum Corrections

- Superpotential $W = W^{\mathrm{tree}} + W^{\mathrm{non-pert}}$

- Gauge kinetic function $f = f^{\mathrm{tree}} + f^{\mathrm{1-loop}} + f^{\mathrm{non-pert}}$

- Kähler potential $K = K^{\mathrm{tree}} + \sum_{n=1}^{\infty} K^{n-\mathrm{loop}} + K^{\mathrm{non-pert}}$

Goal and Method

- Goal: Calculate string 1-loop corrections to Kähler potential K of moduli fields in type I theory

  In applications this would give you direct access to corrections to the potential V

- Method: 1-loop scattering amplitudes in type I

Complications:

- Result from string amplitudes not in Einstein frame, but

$$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}[(1+\delta E)\tfrac{1}{2}R + (\mathrm{scalar})\ \mathrm{kinetic\ terms}]$$

Need Weylrescaling: $g^{(E)}_{\mu\nu} = \underbrace{(1+\delta E)}_{\equiv\,\Omega^{-2}} g_{\mu\nu}$

$\Longrightarrow$ kinetic terms multiplied by Ω^2

- String theory naturally calculates

$$\tilde{G}_{\bar{I}J}\partial_\mu\bar{\varphi}^I\partial^\mu\varphi^J = \Big(G^{(0)}_{\bar{I}J}(\varphi) + \tilde{G}^{(1)}_{\bar{I}J}(\varphi)\Big)\partial_\mu\bar{\varphi}^I\partial^\mu\varphi^J$$

with $\Phi^I = \Phi^I(\varphi)$

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with $\Phi^I = \Phi^I(\varphi)$ (e.g. $T = c + i\tau$)

RR field

$e^{-\Phi_{10}}\mathcal{V}$

- Thus, string theory gives you (after Weyl-rescaling):

$$G_{\bar{I}J}\partial_\mu\bar{\varphi}^I\partial^\mu\varphi^J = \Big(G^{(0)}_{\bar{I}J}(\varphi) + \underbrace{G^{(1)}_{\bar{I}J}(\varphi)}_{\tilde{G}^{(1)}_{\bar{I}J}(\varphi) - G^{(0)}_{\bar{I}J}(\varphi)\delta E(\varphi)}\Big)\partial_\mu\bar{\varphi}^I\partial^\mu\varphi^J$$

- Suppose $G^{(0)}_{\bar{I}J}(\varphi)\partial_\mu\bar{\varphi}^I\partial^\mu\varphi^J = \dfrac{\partial^2 K^{(0)}(\Phi^{(0)})}{\partial\bar{\Phi}^{(0)I}\partial\Phi^{(0)J}}\partial_\mu\bar{\Phi}^{(0)I}\partial^\mu\Phi^{(0)J}$ $(\star)$

then in general $G^{(1)}_{\bar{I}J}(\varphi)\partial_\mu\bar{\varphi}^I\partial^\mu\varphi^J \neq \dfrac{\partial^2 K^{(1)}(\Phi^{(0)})}{\partial\bar{\Phi}^{(0)I}\partial\Phi^{(0)J}}\partial_\mu\bar{\Phi}^{(0)I}\partial^\mu\Phi^{(0)J}$

- Solution: $\Phi^I = \Phi^I(\varphi) = \Phi^{(0)I}(\varphi) + \Phi^{(1)I}(\varphi)$

$\Longrightarrow$ Get additional contributions to the 1-loop metric from inserting $\Phi^{(0)I} = \Phi^I - \Phi^{(1)I}$ into $(\star)$ such that

$$\left(G^{(0)}_{\bar{I}J}(\varphi) + G^{(1)}_{\bar{I}J}(\varphi)\right)\partial_\mu \bar{\varphi}^I \partial^\mu \varphi^J = \frac{\partial^2\left(K^{(0)}(\Phi) + K^{(1)}(\Phi)\right)}{\partial \bar{\Phi}^I \partial \Phi^J} \partial_\mu \bar{\Phi}^I \partial^\mu \Phi^J$$

$\Longrightarrow$ read off $K^{(1)}$

- Upshot: Need 1-loop corrections to

(i) scalar metric $\tilde{G}^{(1)}_{\bar{I}J}(\varphi)$

(ii) Einstein-Hilbert term δE

(iii) definition of field variables $\Phi^{(1)I}(\varphi)$

Calculational setup

Some generalities of the amplitude calculations:

- Aim: read off scalar metric from scalar 2-pt fct.
- 2-pt fct. $= 0$ on-shell with momentum conservation
- Trick: use $p_1 + p_2 \neq 0 \Longleftrightarrow \lambda \equiv p_1 \cdot p_2 \neq 0$
  in intermediate steps

[Atick, Dixon, Sen; Minahan; Antoniadis, Bachas, Fabre, Partouche, Taylor; Antoniadis, Kirtsis, Rizos; cf. also Kiritsis, Kounnas, ...]

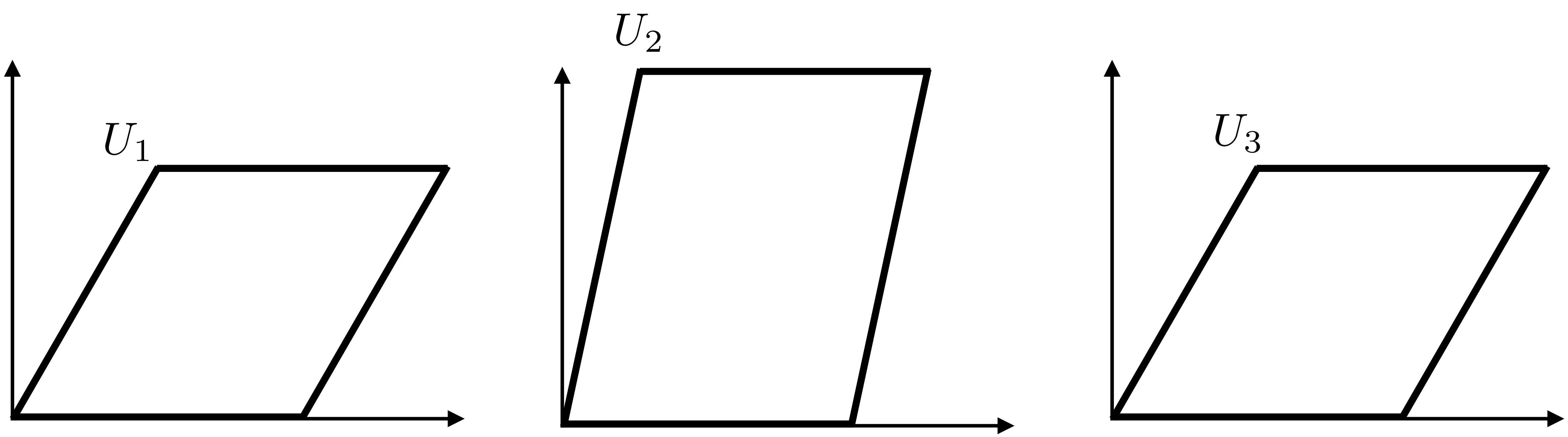
- $\langle \varphi_i \varphi_j \rangle = \lambda G_{ij} + \mathcal{O}(\lambda^2)$
- Similarly for gravitons: $\langle hh \rangle \sim \delta E p_2^\mu \epsilon_{1\mu\nu} \eta^{\nu\sigma} \epsilon_{2\sigma\rho} p_1^\rho$

$T^6/\mathbb{Z}_6'$

- $\Theta Z^1 = e^{2\pi i v_1} Z^1 \quad \Theta Z^2 = e^{2\pi i v_2} Z^2 \quad \Theta Z^3 = e^{2\pi i v_3} Z^3$

  $(v_1, v_2, v_3) = \left(\frac{1}{6}, -\frac{1}{2}, \frac{1}{3}\right)$

-



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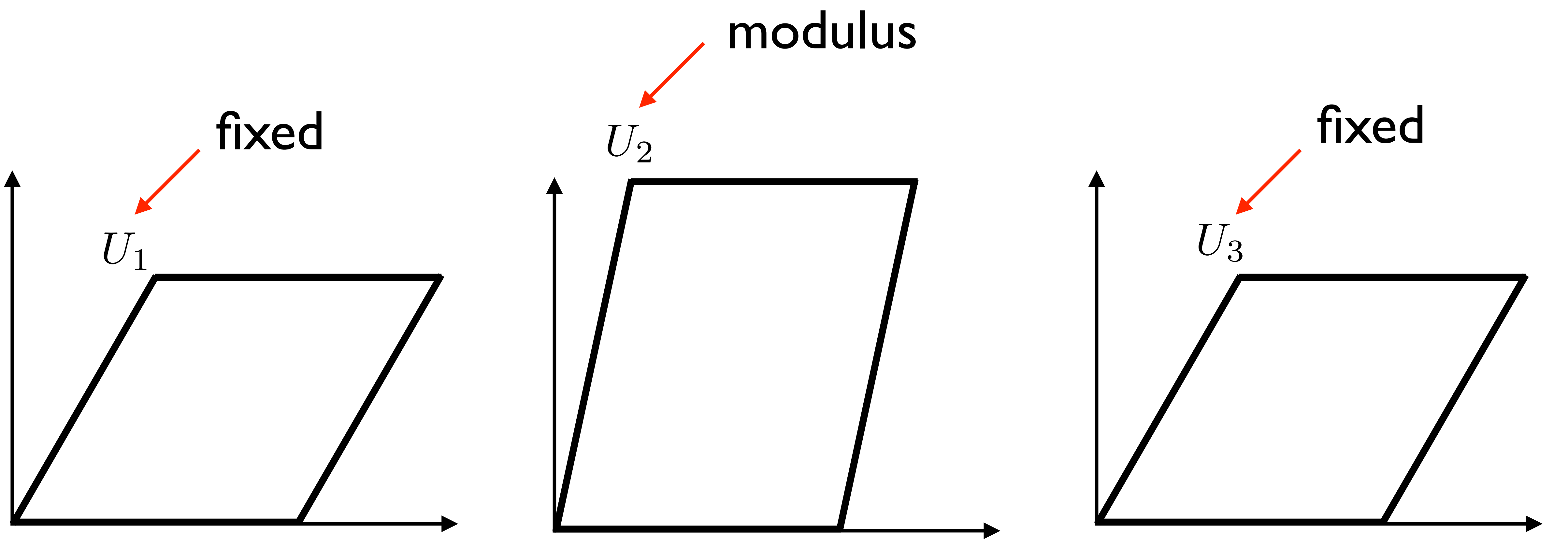
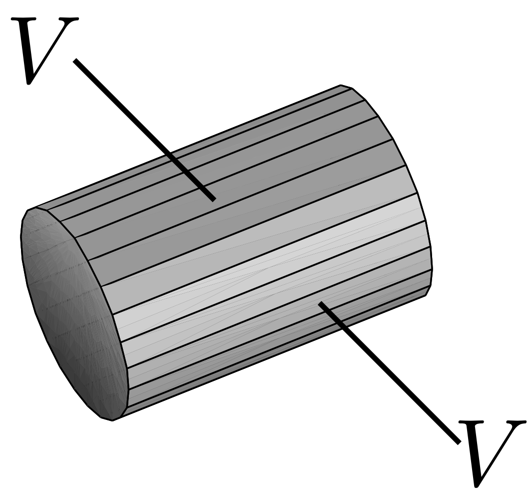
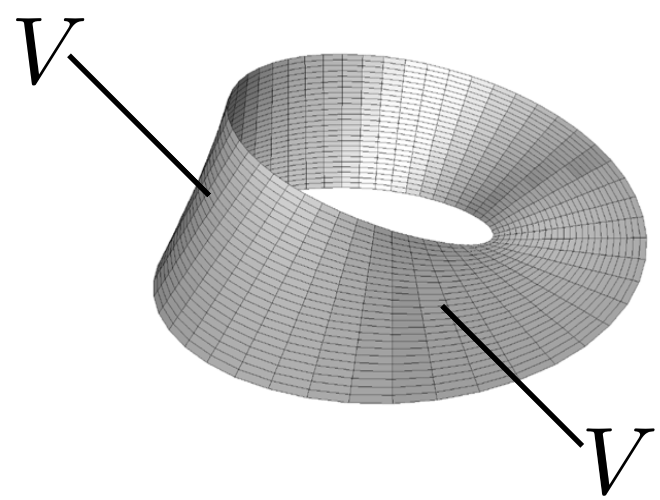
- 

- Resulting 4D effective action has $\mathcal{N} = 1$
- Model contains D9- and D5-branes (wrapped around 3rd torus)
- In addition to torus, at 1-loop need to calculate:

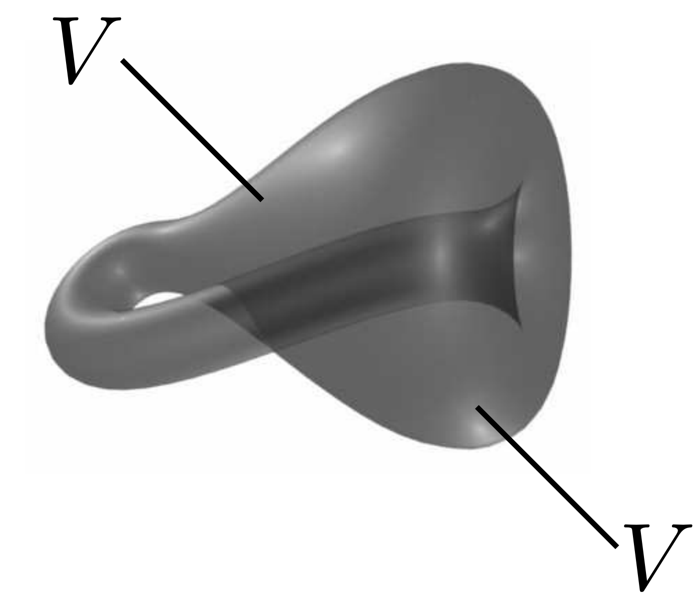
Annulus:



Möbius strip:



Klein bottle:



All of them have Euler number $\chi = 0$

- E.g. annulus:

projection operator
on orbifold invariant states

$$\mathcal{A} \sim \int_0^\infty \frac{dt}{t} \mathrm{Tr}_{open} \Big(\Big[\frac{1}{6} \sum_{k=0}^{5} \Theta^k \Big] q^{(p^2+m^2)/2} V V \Big)$$

$$= \frac{1}{6} \sum_{k=0}^{5} \int_0^\infty \frac{dt}{t} \mathrm{Tr}_{open} \Big(\Theta^k q^{(p^2+m^2)/2} V V \Big)$$

$e^{-\pi t}$

vertex operator

- $\mathcal{N} = 1$ -sector: no zero mode contributions

  $\mathcal{N} = 2$ -sector: zero mode contributions along 1 torus

Results: Scalar kinetic term

$10D$ dilaton

Concretely considered: $\tau = \mathrm{Im}(T_3)$ with $\tau^{(0)} \sim e^{-\Phi}\mathcal{V}_3$

volume of 3rd torus measured
with string frame metric

$4D$ dilaton

Note: $e^{2\Phi} = e^{2\Phi_4}\mathcal{V}$

overall volume measured
with string frame metric

Results: Scalar kinetic term

Prior results:

- No contribution from disk-level [Hashimoto, Klebanov; Lüst, Mayr, Richter, Stieberger]

2-point function of closed string vertex operators:

$$\sim 1 + \mathcal{O}(k^4)$$

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$$\sim 1 + \mathcal{O}(k^4)$$

What about projective plane?

Results: Scalar kinetic term

Prior results:

- No contribution from disk-level

[Hashimoto, Klebanov; Lüst, Mayr, Richter, Stieberger]

- Sphere, torus and $\mathcal{N}=2$ -sectors:

$$G^{(0)} = -\frac{1}{4(\tau^{(0)})^2}\left(1+\frac{\zeta(3)\chi}{\mathcal{V}}\right)$$

Euler number (48 for $\mathbb{Z}_6'$)

$$\tilde{G}^{(1)} \sim e^{2\Phi_4}\left(\frac{\chi}{(\tau^{(0)})^2} + a_1\frac{1}{(\tau^{(0)})^2\mathcal{V}_3}E_2(U_3) + a_2\frac{\mathcal{V}_2}{(\tau^{(0)})^2}E_2(-1/U_2)\right)$$

complex structure of 3rd torus

[Antoniadis, Ferrara, Minasian, Narain; Becker, Becker, M.H., Louis; Berg, M.H., Körs]

$$E_2(U) \equiv \sum_{(m,n)\neq(0,0)} \frac{(\mathrm{Im}(U))^2}{|m+nU|^4}$$

- New result: $\mathcal{N} = 1$ sectors [Berg, M.H., Kang, Sjörs]

- Usual lore: $\mathcal{N} = 1$ sectors less interesting, because they do not lead to moduli dependent results

- Moduli dependence in $\mathcal{N} = 1$ sectors via:
  - ★ normalization of vertex operators
  - ★ dilaton factor in Einstein frame

- Expect further moduli dependence in $\mathcal{N} = 1$ in presence of world volume fluxes or for branes at angles (cf. gauge couplings [Lüst, Stieberger; ...])

- For $\tau = \mathrm{Im}(T_3)$ in $\mathbb{Z}_6'$ from $\mathcal{A}, \mathcal{M}, \mathcal{K}$: [Berg, M.H., Kang, Sjörs]

$$\tilde{G}^{(1)}_{(\mathcal{N}=1)} = e^{2\Phi_4} \frac{5}{2^9 \pi^3} \overbrace{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}^{\approx 3.197 \cdot 10^{-4}} \frac{1}{(\tau^{(0)})^2}$$

2nd Clausen function $\mathrm{Cl}_2(\varphi) = \sum_{k=1}^{\infty} \frac{\sin(k\varphi)}{k^2}$

- This is a correction to the usual torus contribution

$$\sim e^{2\Phi_4} \frac{\chi}{(\tau^{(0)})^2}$$

Results: EH-term

- Prior results:
  - ★ Type II: [Antoniadis, Ferrara, Minasian, Narain]

IIB

$$S^{(II)} = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\Big[e^{-2\Phi_4} + \chi\Big(\zeta(3)\frac{e^{-2\Phi_4}}{\mathcal{V}} \pm \frac{\pi^2}{6}\Big)\Big]\frac{R}{2}$$

IIA

  - ★ Type I (on $K_3 \times T^2$): [Antoniadis, Bachas, Fabre, Partouche, Taylor]

$$S^{(I)} = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\Big[e^{-2\Phi_4} + a\underbrace{\frac{1}{\mathcal{V}_{T^2}}E_2(U)}_{\text{from } \mathcal{A},\mathcal{M},\mathcal{K}}\Big]\frac{R}{2} + \dots$$

★ Heterotic string: No 1-loop contribution, i.e.

$$S^{(het)} = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\Big[e^{-2\Phi_4}\Big(1+\frac{\chi\zeta(3)}{\mathcal{V}}\Big)\Big]\frac{R}{2}+\dots$$

[Antoniadis, Gava, Narain; Kiritsis, Kounnas]

This can be understood via

- World sheet calculation: integrand of torus & higher loop graviton 2-point function is total derivative

[Kiritsis, Kounnas, Petropoulos,Rizos]

- $10D$ R^4-terms

- Type II: [Gross, Witten; Green, Schwarz; Grisaru, van de Ven, Zanon]

$$\underbrace{(\zeta(3)e^{-2\Phi} + \frac{\pi^2}{6})t_8 t_8 R^4}_{\substack{\text{leads to correction to 4D} \\ \text{kinetic terms of scalars}}} - \underbrace{(\zeta(3)e^{-2\Phi} \overset{\text{IIB}}{\underset{\text{IIA}}{\pm}} \frac{\pi^2}{6})\epsilon_{10}\epsilon_{10}R^4}_{\substack{\text{leads to correction} \\ \text{to 4D EH-term}}}$$

- Heterotic: [Cai, Nunez; Gross, Sloan; Sakai, Tanii; Abe, Kubota, Sakai]

$$(\zeta(3)e^{-2\Phi} + \frac{\pi^2}{6})t_8 t_8 R^4 - \zeta(3)e^{-2\Phi}\epsilon_{10}\epsilon_{10}R^4$$

- How is this compatible with heterotic / type I duality?

[Tseytlin; Green, Rudra]

$$g_{\mu\nu}^{(I)} = e^{-\Phi_{het}} g_{\mu\nu}^{(het)} \quad , \quad \Phi_I = -\Phi_{het}$$

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- Possible answer: $S^{(het)}$ in 10D contains

[Green, Rudra]

$$\sqrt{g^{(het)}} e^{-\Phi_{het}/2} J_0 E_{3/2}(e^{-\Phi_{het}}) - \sqrt{g^{(het)}} \frac{\pi^2}{6} \mathcal{I}_2$$

★ $J_0 = t_8 t_8 R^4 - \frac{1}{8}\epsilon_{10}\epsilon_{10} R^4$

★ $\mathcal{I}_2 = -\frac{1}{8}\epsilon_{10}\epsilon_{10} R^4 + \ldots$

★ $E_{3/2} = \zeta(3) e^{-3/2\Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non - pert.$

S-duality invariant

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$$\rightarrow \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})$$

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Disk level!

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Disk level!

Disk level correction to 4D EH-term?

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★ $E_{3/2} = \zeta(3) e^{-3/2\Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non - pert.$

S-duality invariant

- New results for $\mathbb{Z}_6'$: [M.H., Kang]

from $\mathcal{N}=1$ -sectors of $\mathcal{A},\mathcal{M},\mathcal{K}$

$$\delta E = \frac{\chi}{(2\pi)^3}\left(2\zeta(3)+\frac{\pi^2}{3}e^{2\Phi_4}\right)+e^{2\Phi_4}\frac{5}{64\pi^2}\mathrm{Cl}_2\left(\frac{\pi}{3}\right)$$
$$-\,e^{2\Phi_4}\frac{5}{256\pi^2}\left[\frac{64\pi^2\alpha'}{\mathcal{V}_3}E_2(U_3)-\frac{12\pi^2\alpha'}{\mathcal{V}_2}E_2(U_2)-\frac{3\mathcal{V}_2}{4\pi^2\alpha'}E_2(-1/U_2)\right]$$

from $\mathcal{N}=2$-sectors of $\mathcal{A},\mathcal{M},\mathcal{K}$

- Follows closely a calculation by [Epple]

- Generalization to $\mathbb{Z}_3$ [M.H., Kang] and $\mathbb{Z}_6,\mathbb{Z}_7,\mathbb{Z}_{12}$ [with Tailin Li, unpublished]

Results: Field redefinitions

- Examples:
  - ★ Type I (on $K_3 \times T^2$): $\tau = \tau^{(0)} + A^2 + \dfrac{E_2(U)}{e^{-\Phi}\mathcal{V}}$

    [Antoniadis, Bachas, Fabre, Partouche, Taylor]

    open string scalars (cf. inflation in string theory)

  - ★ Type IIA on CY: $e^{-2\tilde{\Phi}_4} = e^{-2\Phi_4}\left(1 + a\dfrac{\chi}{\mathcal{V}} + \dots\right)$

    [Antoniadis, Minasian, Theisen, Vanhove]

    $$\tilde{\mathcal{V}} = \mathcal{V}\left(1 - 12\frac{\zeta(2)}{(2\pi)^3}\chi e^{2\Phi_4} + \dots\right)$$

Determine form of *1-loop* field redefinitions using:

- ★ Kählerness of metric
- ★ Shift symmetries
- ★ Ansatz for 1-loop correction of metric (dependence on volume moduli and dilaton)

- Tree level coordinates:

$$\bigstar \quad T_j^{(0)} = c_j^{(0)} + i\tau_j^{(0)} \;, \qquad j = 1,2,3$$

component of RR 2-form C_2 along j-th torus

$e^{-\Phi}\mathcal{V}_j$

volume of j-th torus (in string frame metric)

$$\bigstar \quad T_0^{(0)} = c_0^{(0)} + i\tau_0^{(0)}$$

dual to $C_{\mu\nu}$

$e^{-\Phi}\mathcal{V}$

overall volume

- Tree level Kähler potential:

$$K = -\sum_{I=0}^{3} \ln\left(T_I^{(0)} - \bar{T}_I^{(0)}\right) \quad \Rightarrow \quad G_{I\bar{J}} \sim \frac{\delta_{IJ}}{(\tau_I^{(0)})^2}$$

- In principle one can calculate 1-loop corrections to metric using well-known vertex operators for $\mathcal{V}_j$, Φ_4 and RR-fields

- Express these (after Weyl-rescaling to Einstein frame) in terms of $c_I^{(0)}$, $\tau_I^{(0)}$ via

$$e^{2\Phi_4} = \frac{1}{\sqrt{\prod_{I=0}^{3} \tau_I^{(0)}}} \quad , \quad \mathcal{V}_j = \tau_j^{(0)} \sqrt{\frac{\tau_0^{(0)}}{\prod_{i=1}^{3} \tau_i^{(0)}}}$$

- Result:

$$\mathcal{L} = G_{c_I^{(0)} c_J^{(0)}}(\tau^{(0)}) \partial_\mu c_I^{(0)} \partial^\mu c_J^{(0)} + G_{\tau_I^{(0)} \tau_J^{(0)}}(\tau^{(0)}) \partial_\mu \tau_I^{(0)} \partial^\mu \tau_J^{(0)}$$

G independent of $c^{(0)}$ due to perturbative shift symmetry

$$\mathcal{L} = G_{c_I^{(0)} c_J^{(0)}}(\tau^{(0)}) \partial_\mu c_I^{(0)} \partial^\mu c_J^{(0)} + G_{\tau_I^{(0)} \tau_J^{(0)}}(\tau^{(0)}) \partial_\mu \tau_I^{(0)} \partial^\mu \tau_J^{(0)} \quad (\star)$$

- Aim: Find variables

$$T_I = c_I^{(0)} + c_I^{(1)}(c^{(0)}, \tau^{(0)}) + i\Big(\tau_I^{(0)} + \tau_I^{(1)}(c^{(0)}, \tau^{(0)})\Big)$$

such that $(\star)$ becomes

$$\mathcal{L} = K_{I\bar{J}} \partial_\mu T_I \partial^\mu \bar{T}_J$$

- Shift symmetry of c, i.e. $K = K(\tau)$, implies:

$$T = c + i\tau$$

★ $$K_{T\bar{T}}\partial_\mu T\partial^\mu \bar{T} = \frac{1}{4}K_{\tau\tau}(\partial_\mu c\partial^\mu c + \partial_\mu \tau\partial^\mu \tau)$$

$$(i) \quad G_{c_i\tau_j} = 0$$

$$(ii) \quad G_{\tau_i\tau_j} = \frac{1}{4}\frac{\partial^2 K}{\partial\tau_i\partial\tau_j} = G_{c_ic_j}$$

$$(iii) \quad \partial_{\tau_k}G_{c_ic_j} = \partial_{\tau_i}G_{c_jc_k} = \partial_{\tau_j}G_{c_kc_i}$$

★ $$c_I^{(1)} = 0$$

- Use $c_I^{(0)} = c_I \quad , \quad \tau_I^{(0)} = \tau_I - \tau_I^{(1)}(\tau^{(0)})$ in

$$\mathcal{L} = \underbrace{G_{c_I^{(0)} c_J^{(0)}}(\tau^{(0)})}_{G^{(0)}_{c_I^{(0)} c_J^{(0)}} + G^{(1)}_{c_I^{(0)} c_J^{(0)}}} \partial_\mu c_I^{(0)} \partial^\mu c_J^{(0)} + \underbrace{G_{\tau_I^{(0)} \tau_J^{(0)}}(\tau^{(0)})}_{G^{(0)}_{\tau_I^{(0)} \tau_J^{(0)}} + G^{(1)}_{\tau_I^{(0)} \tau_J^{(0)}}} \partial_\mu \tau_I^{(0)} \partial^\mu \tau_J^{(0)}$$

to obtain

$$G_{c_I c_J} = \frac{\delta_{IJ}}{4\tau_I^2} + G^{(1)}_{c_I^{(0)} c_J^{(0)}} + \frac{\tau_I^{(1)}}{2\tau_I^3}\delta_{IJ}$$

$$G_{\tau_I \tau_J} = \frac{\delta_{IJ}}{4\tau_I^2} + G^{(1)}_{\tau_I^{(0)} \tau_J^{(0)}} + \frac{\tau_I^{(1)}}{2\tau_I^3}\delta_{IJ} - \frac{1}{4}\left[\frac{\partial_{\tau_J}\left(\tau_I^{(1)}\right)}{\tau_I^2} + \frac{\partial_{\tau_I}\left(\tau_J^{(1)}\right)}{\tau_J^2}\right]$$

- Now use $G_{\tau_I \tau_J}(\tau) = \frac{1}{4}\frac{\partial^2 K}{\partial \tau_I \partial \tau_J} = G_{c_I c_J}(\tau)$

  to obtain equations for $K^{(1)}$ and $\tau^{(1)}$

- $K^{(1)}$:

$$\boxed{\begin{aligned} &\frac{1}{4}\frac{\partial^2 K^{(1)}}{\partial \tau_I \partial \tau_J} = G^{(1)}_{c_I^{(0)} c_J^{(0)}} \quad \text{for} \quad I \neq J \\ &\frac{1}{4}\frac{\partial^2 K^{(1)}}{\partial \tau_I \partial \tau_I} = G^{(1)}_{c_I^{(0)} c_I^{(0)}} + \frac{\tau_I^{(1)}}{2\tau_I^3} \end{aligned}}$$

- $\tau^{(1)}$:

$$\boxed{\begin{aligned} &\frac{\partial_{\tau_I}\left(\tau_I^{(1)}\right)}{2\tau_I^2} = G^{(1)}_{\tau_I^{(0)} \tau_I^{(0)}} - G^{(1)}_{c_I^{(0)} c_I^{(0)}} \\ &\frac{\partial_{\tau_J}\left(\tau_I^{(1)}\right)}{2\tau_I^3} = \partial_{\tau_I}\left(G^{(1)}_{c_I^{(0)} c_J^{(0)}}\right) - \partial_{\tau_J}\left(G^{(1)}_{c_I^{(0)} c_I^{(0)}}\right), \quad \text{for} \quad I \neq J \end{aligned}}$$

- There are also some consistency conditions:

$$\partial_{\tau_k} G^{(1)}_{c_I^{(0)} c_J^{(0)}} = \partial_{\tau_I} G^{(1)}_{c_J^{(0)} c_K^{(0)}} = \partial_{\tau_J} G^{(1)}_{c_K^{(0)} c_I^{(0)}} , \quad \text{for} \quad I \neq J \neq K$$

$$\tau_I^2 \frac{\partial}{\partial \tau_J} \left[G^{(1)}_{c_I^{(0)} c_I^{(0)}} - G^{(1)}_{\tau_I^{(0)} \tau_I^{(0)}} \right] = \frac{\partial}{\partial \tau_I} \left[\tau_I^3 \left(\partial_{\tau_J} G^{(1)}_{c_I^{(0)} c_I^{(0)}} - \partial_{\tau_I} G^{(1)}_{c_I^{(0)} c_J^{(0)}} \right) \right] , \ \text{for } I \neq J$$

$$\tau_I \left[\partial_{\tau_I} G^{(1)}_{c_I^{(0)} c_J^{(0)}} - \partial_{\tau_J} G^{(1)}_{c_I^{(0)} c_I^{(0)}} \right] + (I \leftrightarrow J) = 2 \left[G^{(1)}_{\tau_I^{(0)} \tau_J^{(0)}} - G^{(1)}_{c_I^{(0)} c_J^{(0)}} \right] , \ \text{for } I \neq J$$

- There are also some consistency conditions:

$$\boxed{\begin{aligned}
&\partial_{\tau_k} G^{(1)}_{c_I^{(0)} c_J^{(0)}} = \partial_{\tau_I} G^{(1)}_{c_J^{(0)} c_K^{(0)}} = \partial_{\tau_J} G^{(1)}_{c_K^{(0)} c_I^{(0)}} \,, \quad \text{for} \quad I \neq J \neq K \\
&\tau_I^2 \frac{\partial}{\partial \tau_J} \left[G^{(1)}_{c_I^{(0)} c_I^{(0)}} - G^{(1)}_{\tau_I^{(0)} \tau_I^{(0)}} \right] = \frac{\partial}{\partial \tau_I} \left[\tau_I^3 \left(\partial_{\tau_J} G^{(1)}_{c_I^{(0)} c_I^{(0)}} - \partial_{\tau_I} G^{(1)}_{c_I^{(0)} c_J^{(0)}} \right) \right] \,, \ \text{for } I \neq J \\
&\tau_I \left[\partial_{\tau_I} G^{(1)}_{c_I^{(0)} c_J^{(0)}} - \partial_{\tau_J} G^{(1)}_{c_I^{(0)} c_I^{(0)}} \right] + (I \leftrightarrow J) = 2 \left[G^{(1)}_{\tau_I^{(0)} \tau_J^{(0)}} - G^{(1)}_{c_I^{(0)} c_J^{(0)}} \right] \,, \ \text{for } I \neq J
\end{aligned}}$$

To proceed further we make an ansatz for $G^{(1)}$

Ansatz for 1-loop correction of metric: (motivated by string amplitudes)

- $\mathcal{N}=1:$ $$G^{(1)}_{\tau_I^{(0)}\tau_J^{(0)}} = \alpha_{IJ}\frac{e^{2\Phi_4}}{\tau_I^{(0)}\tau_J^{(0)}}$$

- $\mathcal{N}=2:$ $$G^{(1)}_{\tau_I^{(0)}\tau_J^{(0)}} = \frac{e^{2\Phi_4}}{\tau_I^{(0)}\tau_J^{(0)}}\sum_{\mathcal{N}=2-\mathrm{sectors}}\alpha_{IJ}^{(l,n)}(U_l)\mathcal{V}_l^n$$

(similarly for $G^{(1)}_{c_I^{(0)}c_J^{(0)}}$)

$n=-1:$ Closed string winding state exchange

$n=+1:$ Closed string KK state exchange

- E.g. (for $\mathbb{Z}_6'$) [M.H., Kang]

$\mathcal{T}, \mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N}=1$

$$\delta\tau_3 = a_1 \sqrt{\frac{T_3^{(0)} - \bar{T}_3^{(0)}}{(T_0^{(0)} - \bar{T}_0^{(0)})(T_1^{(0)} - \bar{T}_1^{(0)})(T_2^{(0)} - \bar{T}_2^{(0)})}}$$

$$\left.\begin{aligned} &+a_2 \frac{(T_3^{(0)} - \bar{T}_3^{(0)})E_2(U_2)}{(T_0^{(0)} - \bar{T}_0^{(0)})(T_2^{(0)} - \bar{T}_2^{(0)})} \\ &+a_3 \frac{E_2(-1/U_2)}{(T_1^{(0)} - \bar{T}_1^{(0)})} + a_4 \frac{E_2(U_3)}{(T_0^{(0)} - \bar{T}_0^{(0)})} \end{aligned}\right\}$$

$\mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N}=2$

analog of correction by
[Antoniadis, Bachas, Fabre, Partouche, Taylor]

- E.g. (for $\mathbb{Z}_6'$) [M.H., Kang]

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$$\left.\begin{aligned}&+a_2\frac{(T_3^{(0)}-\bar{T}_3^{(0)})E_2(U_2)}{(T_0^{(0)}-\bar{T}_0^{(0)})(T_2^{(0)}-\bar{T}_2^{(0)})}\\ &+a_3\frac{E_2(-1/U_2)}{(T_1^{(0)}-\bar{T}_1^{(0)})}+a_4\frac{E_2(U_3)}{(T_0^{(0)}-\bar{T}_0^{(0)})}\end{aligned}\right\}$$

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$\mathcal{N}=2$

Coefficients determined by string theory, e.g. $a_2E_2(U_2) \sim \alpha_{03}^{(2,-1)}$

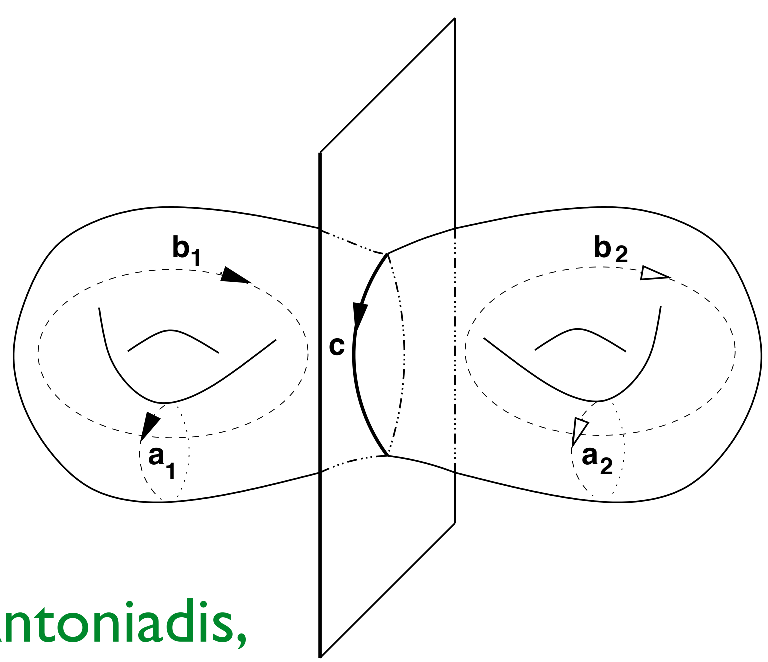
$$\alpha_{03} \sim G^{(1)}_{\Phi\Phi} - \mathcal{V}_3 G^{(1)}_{\Phi\mathcal{V}_3} - \mathcal{V}_1^2 G^{(1)}_{\mathcal{V}_1\mathcal{V}_1} - \mathcal{V}_2^2 G^{(1)}_{\mathcal{V}_2\mathcal{V}_2} + \mathcal{V}_3^2 G^{(1)}_{\mathcal{V}_3\mathcal{V}_3} - 2\mathcal{V}_1\mathcal{V}_2 G^{(1)}_{\mathcal{V}_1\mathcal{V}_2}$$

Check of field redefinition of T_3 from holomorphic gauge kinetic function of D5-branes (wrapped around 3rd torus)

- At disk level: $f_{D5} = T_3^{(0)}$ $\sim e^{-\Phi}$

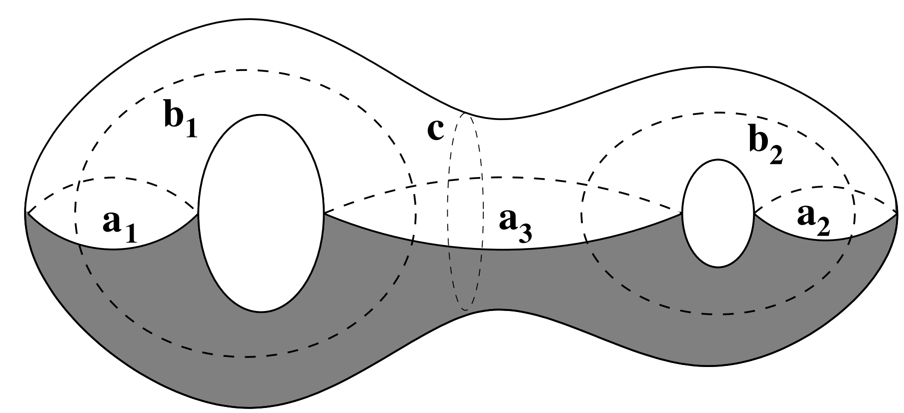
- 1-loop correction to $T_3^{(0)}$ appears at $\chi = 2 - 2h - b - c = -1$:

E.g.



[from: Antoniadis, Taylor]

,



[from: Antoniadis, Narain, Taylor]

+ 3 more diagrams

$$(b=1, c=2), (b=2, c=1), (h=1, c=1)$$

Kähler potential of dilaton and untwisted moduli for $T^6/\mathbb{Z}_6'$ (up to 1-loop):

[Berg, M.H., Körs; generalization to other $\mathbb{Z}_N$: M.H., Kang]

$$K = -\ln(T_0 - \bar{T}_0) - \ln[(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)] - \ln(U_2 - \bar{U}_2)$$

$$+c_1 \chi \zeta(3) \sqrt{\frac{(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)}{(T_0 - \bar{T}_0)^3}} \longleftarrow S^2$$

$$+c_2 \frac{E_2(U_2)}{(T_2 - \bar{T}_2)(T_0 - \bar{T}_0)} + c_3 \frac{E_2(-1/U_2)}{(T_1 - \bar{T}_1)(T_3 - \bar{T}_3)} + c_4 \frac{E_2(U_3)}{(T_3 - \bar{T}_3)(T_0 - \bar{T}_0)}$$

$$+c_5 \frac{1}{\sqrt{(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)(T_0 - \bar{T}_0)}}$$

$\mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N} = 2$

$\mathcal{T}, \mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N} = 1$

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$\mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N} = 2$

$\mathcal{T}, \mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N} = 1$

Note: $\boxed{K^{(1)} = 8\mathcal{V}_1^2 G^{(1)}_{\mathcal{V}_1\mathcal{V}_1}}$ (follows from relations between different metric components)

[M.H., Kang]

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$$+c_5 \frac{1}{\sqrt{(T_1 - \bar{T}_1)(T_2 - \bar{T}_2)(T_3 - \bar{T}_3)(T_0 - \bar{T}_0)}}$$

$\mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N} = 2$

$\mathcal{T}, \mathcal{A}, \mathcal{M}, \mathcal{K}$
$\mathcal{N} = 1$

Moreover e.g.

$$c_4 E_2(U_3) \sim G^{(1)|(3,-1)}_{\mathcal{V}_1 \mathcal{V}_2}$$

(follows from relations between different metric components)

Outlook

- Prove correctness of metric ansatz
- Work out coefficients in K and field redefinitions
- For non-vanishing result of $K^{(1)}$, certain off-diagonal metric components have to be non-vanishing, for example mixing 2 different Kähler moduli
- Additional field redefinitions from α'-corrections?

  [Antoniadis, Minasian, Theisen, Vanhove; Grimm, Savelli, Weissenbacher]
- Check correction to EH-term at disk level

  [Green, Rudra]

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Molte Grazie!