Recursion Relations for Anomalous Dimensions of the 6d (2,0) Theory

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AdS/CFT

Worldvolume		**Gravity Dual**
• D3 branes	⟷	IIB string theory on $AdS_5 \times S^5$
• M2 branes	⟷	M-theory on $AdS_4 \times S^7$
• M5 branes	⟷	M-theory on $AdS_7 \times S^4$

M5-branes

- Abelian theory: 5 scalars, 8 fermions, self-dual 2-form (Howe,Sierra,Townsend)
- Non-abelian theory strongly coupled, so what can we say about it?
- OSp(8|4) symmetry
- When $N \longrightarrow \infty$, described by 11d supergravity in $AdS_7 \times S^4$
- Central charge c is $O(N^3)$ (Henningson,Skenderis)
- Goal: Go beyond the supergravity approximation

Stress Tensor Correlators

- The stress tensor belongs to a ½ BPS multiplet whose lowest component is a dimension 4 scalar T_{IJ} in the symmetric traceless representation (**14**) of the R-symmetry group SO(5)

- In the large-N limit, 4-point correlators of stress tensor multiplets can be computed using Witten diagrams for 11d supergravity in $AdS_7 \times S^4$.

- Strategy: Use superconformal and crossing symmetry to deduce 1/N corrections to 4-point correlators, which correspond to higher derivative corrections to 11d supergravity arising from M-theory.

4-Point Function

- Superconformal symmetry fixes the 4-point function in terms of a pre-potential

$$\lambda^4 (g_{13}g_{24})^{-2} \langle T_1 T_2 T_3 T_4 \rangle = \mathcal{D}(\mathcal{S}F(z,\bar{z})) + \mathcal{S}_1^2 F(z,z) + \mathcal{S}_2^2 F(\bar{z},\bar{z})$$

where $\lambda = z - \bar{z}$, $\mathcal{D} = -(\partial_z - \partial_{\bar{z}} + \lambda \partial_z \partial_{\bar{z}})\lambda$, $T_i = T_{IJ} Y_i^I Y_i^J$, $g_{ij} = Y_i \cdot Y_j / x_{ij}^4$

$$y\bar{y} = \frac{Y_1 \cdot Y_2 Y_3 \cdot Y_4}{Y_1 \cdot Y_3 Y_2 \cdot Y_4}, \qquad (1-y)(1-\bar{y}) = \frac{Y_1 \cdot Y_4 Y_2 \cdot Y_3}{Y_1 \cdot Y_3 Y_2 \cdot Y_4},$$

$$\mathcal{S}_1 = (z-y)(z-\bar{y}),\ \mathcal{S}_2 = (\bar{z}-y)(\bar{z}-\bar{y}),\ \mathcal{S} = \mathcal{S}_1 \mathcal{S}_2$$

Arutyunov,Sokatchev/Heslop

- Crossing symmetry: $F(u,v) = F(v,u), \quad F(u/v, 1/v) = v^2 F(u,v)$

CPW Expansion

- Decompose 4-point function as follows:

$$F(z,\bar{z}) = \frac{A}{u^2} + \frac{g(z) - g(\bar{z})}{u\,\lambda} + \lambda G(z,\bar{z})$$

- These functions can be written as a sum over operators appearing in TT OPE

- A, g, G encode identity, protected, and unprotected operators, respectively:

$$A \sim 1,\;\; g \sim T,\;\; G \sim T\partial^l \,\Box^n\, T$$

where unprotected ops have scaling dimension $\Delta = 2n + l + 8 + \mathcal{O}(1/c)$

- In more detail,

$$\lambda^2 G(z,\bar{z}) = \sum_{n,l\geq 0} A_{n,l}\, G^{\mathrm{S}}_{\Delta,l}(z,\bar{z})$$

where superconformal blocks can be written in terms of hypergeometrics
Dolan,Osborne/Heslop/Beem,Lemos,Rastelli,van Rees

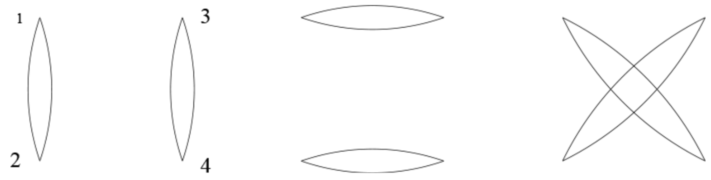
- Expand OPE data in 1/c: $A_{n,l} = A^{(0)}_{n,l} + \frac{1}{c}A^{(1)}_{n,l} + \ldots, \qquad \Delta = 2n + l + 8 + \frac{1}{c}\gamma_{n,l} + \ldots$

- Crossing: $$\sum_{n,l\geq 0}\left[A^{(1)}_{n,l}\, G^{\mathrm{S}}_{\Delta,l}(z,\bar{z}) + \frac{1}{2}A^{(0)}_{n,l}\,\gamma_{n,l}\,\partial_n G^{\mathrm{S}}_{\Delta,l}(z,\bar{z})\right] + (u \leftrightarrow v) = 0$$

Supergravity Prediction

- Free disconnected contribution:

$$F(u,v)_{\text{free-disc}} = 1 + \frac{1}{u^2} + \frac{1}{v^2}$$



- Decomposing into A, g, G and performing CPW expansion of G gives

$$A^{(0)}_{n,l} = \frac{(l+2)\,(n+3)!\,(n+4)!\,(l+2n+9)\,(l+2n+10)\,(l+n+5)!\,(l+n+6)!}{72\,(2n+5)!\,(2l+2n+9)!}$$

- Dynamical contribution: (Arutyunov,Sokatchev)

$$F^{\text{sugra}} = -\frac{1}{c}\frac{\lambda^2}{uv}\bar{D}_{3337}$$

where the D functions arise from AdS integrals:

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(x_1,x_2,x_3,x_4) = \int \frac{\mathrm{d}^d w \; \mathrm{d}w_0}{w_0^{d+1}} \prod_{i=1}^{4} K_{\Delta_i}(w,x_i)\,,\; K_\Delta(w,x) = \left(\frac{w_0}{w_0^2+(\vec{w}-x)^2}\right)^{\Delta}$$

- Performing CPW decomposition gives anomalous dimensions which scale like n^5

$$\gamma_{n,l}^{\text{sugra}} = -\frac{6}{c}\left(1+\frac{(n-2)(n+1)}{2(2n+l+4)(l+3)}\right)\frac{(n-1)_6}{(l+1)(l+2)(2n+l+5)(2n+l+6)}$$

- The CPW coefficients satisfy (Heslop,Lipstein)

$$A_{n,l}^{(1)} = \frac{1}{2}\partial_n\left(A_{n,l}^{(0)}\gamma_{n,l}\right)$$

Corrections to Supergravity

- Heemskerk, Penedones, Polchinski, Sully considered 4-point functions in a generic 2d or 4d CFT with a large-N expansion and solved the crossing equations to leading order in 1/c by truncating the CPW expansion in spin.

- They showed that the solutions are in 1 to 1 correspondence with local quartic interactions for a massive scalar field in AdS, which can be thought of as a toy model for the low-energy effective theory of the gravitational dual.

- The number of derivatives in the bulk interaction is related to the large-twist behaviour of the anomalous dimensions.

Examples

spin	interactions	anomalous dim.
0	ϕ^4	n^{const}
2	$\phi^2 \left(\nabla_\mu \nabla_\nu \phi\right)^2$ $\phi^2 \left(\nabla_\mu \nabla_\nu \nabla_\rho \phi\right)^2$	$n^{const+4}$, $n^{const+6}$
4	$\phi \nabla_{\mu_1\mu_2\nu_1\nu_2} \phi \nabla_{\mu_1\mu_2} \phi \nabla_{\nu_1\nu_2} \phi$ $\nabla_{\rho_1} \phi \nabla_{\mu_1\mu_2\nu_1\nu_2\rho_1} \phi \nabla_{\mu_1\mu_2} \phi \nabla_{\nu_1\nu_2} \phi$ $\nabla_{\rho_1\rho_2} \phi \nabla_{\mu_1\mu_2\nu_1\nu_2\rho_1\rho_2} \phi \nabla_{\mu_1\mu_2} \phi \nabla_{\nu_1\nu_2} \phi$	$n^{const+8}$, $n^{const+10}$, $n^{const+12}$

Spin-0

- Spin-0 solution: (Helsop,Lipstein)

$$F^{\text{spin-0}}(u,v) = C^{(0)}\lambda^2 uv\bar{D}_{5755}(u,v)$$

- Anomalous dimensions:

$$\gamma_{n,0}^{\text{spin}-0} = -\frac{C^{(0)}(n+1)_8(n+2)_6}{2240(2n+7)(2n+9)(2n+11)}$$

- Scales like n^{11} in the large-n limit.

Effective Action

- At large twist, $\gamma^{\text{spin}-0}/\gamma^{\text{sugra}} \sim n^6$

- This suggests that term in the bulk effective action corresponding to the spin-0 solution has six more derivatives than the supergravity Lagrangian, and is therefore of the form $(\text{Riemann})^4$.

- This is the M-theoretic analogue of $(\alpha')^3$ corrections in string theory and was previously deduced in flat space by uplifting string amplitudes (Green,Vanhove)

- Similarly, we obtained solutions up to 20 derivatives (truncated spin 4) by guessing crossing symmetric functions and checking their CPW expansions.

Recursion Relations

- Recall crossing eq:

$$\sum_{n,l\geq 0}\left[A^{(1)}_{n,l}\,G^{\mathrm{S}}_{\Delta,l}(z,\bar{z})+\frac{1}{2}A^{(0)}_{n,l}\,\gamma_{n,l}\,\partial_n G^{\mathrm{S}}_{\Delta,l}(z,\bar{z})\right]+(u\leftrightarrow v)=0$$

- Conformal blocks have schematic structure

$$G^{\mathrm{S}}_{\Delta,l}(z,\bar{z})\sim\sum u^n h_\alpha(z)h_\beta(\bar{z})$$

where $h_\beta(z)={}_2F_1\left(\beta/2,\beta/2-1,\beta,z\right)$

- $\partial_n G^{\mathrm{B}}_{\Delta,l}(z,\bar{z})$ gives a term with $\log(u)=\log(z\bar{z})$ so isolate $\gamma_{n,l}$ by taking

$$z\to 0 \text{ and } \bar{z}\to 1.$$

- In order for crossing equations to be consistent, the $\log(z)$ coming from $\partial_n G^{\mathrm{B}}_{\Delta,l}(z,\bar{z})$ must be accompanied by a $\log(1-\bar{z})$. Such terms arise from

$$h_\beta(\bar{z}) = \log(1-\bar{z})\,(1-\bar{z})\,\tilde{h}_\beta(1-\bar{z}) + \text{holomorphic at } \bar{z}=1$$

where

$$\tilde{h}_\beta(z) = \frac{\Gamma(\beta)}{\Gamma(\beta/2)\Gamma(\beta/2-1)}\,{}_2F_1\left(\beta/2+1, \beta/2, 2, z\right)$$

- Collecting terms proportional to $\log(z)\log(1-\bar{z})$ then gives a refined crossing eq:

$$\sum_{n,l\geq 0} A^{(0)}_{n,l}\,\gamma_{n,l}\left(\partial_n G^{\mathrm{S}}_{\Delta,l}(z,\bar{z})\right)\Big|_{\log z \log(1-\bar{z})} = -\sum_{n,l\geq 0} A^{(0)}_{n,l}\,\gamma_{n,l}\left(\partial_n G^{\mathrm{S}}_{\Delta,l}(1-z,1-\bar{z})\right)\Big|_{\log z \log(1-\bar{z})}$$

- To get numerical recursion relations, multiply by

$$\frac{h_{-2q}(z)}{z^q\,(1-z)} \times \frac{h_{-2p}(1-\bar{z})}{(1-\bar{z})^p\,\bar{z}}$$

and perform contour integrals around $(z,\bar{z}) = (0,1)$

- Use orthogonality of hypergeometrics

$$\delta_{m,m'} = \oint \frac{dz}{2\pi i}\,\frac{z^{m-m'-1}}{1-z}\,h_{2m+4}(z)\,h_{-2m'-2}(z)$$

and define

$$\mathcal{I}_{m,m'} = \oint \frac{dz}{2\pi i}\,\frac{(1-z)^{m-3}}{z^{m'-1}}\,\tilde{h}_{2m}(z)\,h_{-2m'}(z)$$

- Master equation:

$$
\begin{aligned}
0 = \sum_{l=0}^{L} \sum_{n=0}^{\infty} A_{n,l}^{(0)} \gamma_{n,l} \Big[& P_{n,l} \left(\delta_{q,n} \mathcal{I}_{n+l+6,p+2} - \delta_{q,n+l+3} \mathcal{I}_{n+3,p+2} \right) \\
& + Q_{n,l} \left(\delta_{q,n+2} \mathcal{I}_{n+l+6,p+2} - \delta_{q,n+l+3} \mathcal{I}_{n+5,p+2} \right) \\
& + R_{n,l} \left(\delta_{q,n+l+2} \mathcal{I}_{n+4,p+2} - \delta_{q,n+1} \mathcal{I}_{n+l+5,p+2} \right) \\
& + S_{n,l} \left(\delta_{q,n+l+4} \mathcal{I}_{n+4,p+2} - \delta_{q,n+1} \mathcal{I}_{n+l+7,p+2} \right) - (q \leftrightarrow p) \Big]
\end{aligned}
$$

where

$$
P_{n,l} = \frac{l+1}{(n+3)(n+l+5)}, \quad Q_{n,l} = \frac{(l+3)(n+5)(2n+l+8)}{4(2n+7)(2n+9)(n+l+5)(2n+l+10)},
$$

$$
R_{n,l} = \frac{l+3}{(n+3)(n+l+5)}, \quad S_{n,l} = \frac{(l+1)(n+l+7)(2n+l+8)}{4(n+3)(2n+l+10)(2n+2l+11)(2n+2l+13)}
$$

- Recursion relations follow from choosing (p,q) appropriately and solutions are labelled by spin truncation L.

Solutions

- Let's first consider L=0. Choosing q=0 gives the following recursion relation in terms of p, which is readily solved on a computer to give

$$\gamma_{n,0}^{\mathrm{spin}-0} = \gamma_{0,0} \frac{11\,(n+1)_8\,(n+2)_6}{2304000\,(2n+7)\,(2n+9)\,(2n+11)}$$

  where $\gamma_{0,0}$ is an unfixed parameter.

- For spin-L truncation, the solution will depend on (L+2)(L+8)/4 free parameters, in agreement with holographic arguments based on counting bulk vertices

Conclusions

- Found recursion relations for anomalous dimensions of double-trace operators in CPW expansion of 4-point stress tensor correlators in M5-brane theory.

- Solutions encode the low-energy effective action for M-theory on $AdS_7 \times S^4$, at least up to four-point interactions with unfixed coefficients.

- **Next**: Fix coefficients in M-theory effective action using chiral algebra conjecture Beem,Rastelli,van Rees/Chester,Perlmutter

- Explore loop expansion using methods developed for N=4 SYM by Aprile,Drummond,Heslop,Paul/Alday,Bissi