Non-geometric Calabi-Yau backgrounds and heterotic/type II duality

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GGI String Workshop, April 2019

★ *Non-geometric Calabi-Yau Backgrounds and K3 automorphisms*,

- Chris Hull, D.I., Alessandra Sarti, arXiv:1710.00853, JHEP 1711 (2017) 084

★ *Heterotic/type II duality and non-geometric compactifications*

- Yoan Gautier, Chris Hull and D.I., to appear

Introduction

- What are the generic (SUSY) string compactifications?

  ➥ One may expect that most are not of geometrical nature
- Non-geometric compactifications have few massless moduli
- Interesting underlying mathematics
- Only sporadic classes known ➥ T-folds,...

Many view-points on non-geometry

- Worldsheet : asymmetric 2d CFTs
- Quotient of geometric solutions with stringy symmetries
- Generalized geometry
- 4d supergravity
- String dualities
- ...

★ Motivations

- Genuine non-geometric string backgrounds apart from free-fields ?
- How to construct *mirror-folds*?
- General $\mathcal{N} = 2$ vacua in 4d and string dualities

Scope of this presentation

- Supersymmetric vacua from non-geometric Calabi-Yau automorphisms
- Mathematical framework: Mirrored K3 automorphisms
- String backgrounds: Asymmetric $K3 \times T^2$ Gepner models
- New type of heterotic/type II duality
- Moduli spaces and quantum corrections

Non-geometric Calabi-Yau backgrounds

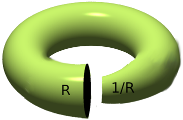
Generalized Scherk-Schwarz reductions *(Dabholkar, Hull '02)*

- String theory on compact manifolds: moduli space of vacua

$$\boxed{\mathcal{M} = O(\Gamma)\backslash G/H} \qquad O(\Gamma) \subset G \text{ isometry group of a charge lattice } \Gamma$$

- $O(\Gamma)$ contains "stringy" symmetries as T-dualities
- Those symmetries can appear in transition functions ➥ T-folds, U-folds,...
- Fibration over S^1 with (non-geometric) monodromy twist:

$$\phi(x^\mu, y) = e^{\frac{Ny}{2\pi R}}\phi(x^\mu) \ , \ M = e^N \in O(\Gamma)$$



- M of finite order ➥ critical points with Minkowski vacuum
- Critical point corresponds to fixed points of M ➥ orbifold CFTs

A simple toroidal model

T^2 compactification

$$\boxed{\mathrm{d}s^2 = \frac{T_2}{U_2}|\mathrm{d}x_1 + U\,\mathrm{d}x_2|^2,\ T_1 = B_{12}}$$

- Moduli space:

$$\underbrace{\frac{SL(2,\mathbb{R})}{SL(2,\mathbb{Z})\times U(1)}}_{\text{complex structure } U} \times \underbrace{\frac{SL(2,\mathbb{R})}{SL(2,\mathbb{Z})\times U(1)}}_{\text{Kähler } T}$$

$$\xleftrightarrow{\textit{T-dual}}$$

Order 4 automorphism

- $\sigma_4 : \begin{cases} x^1 & \mapsto -x^2 \\ x^2 & \mapsto x^1 \end{cases}$
- Induced $O(2,2;\mathbb{Z})$ action: $U \mapsto -1/U$
- Fixed point $U = i \leftrightarrow$ square torus
- Orbifold by $\langle \sigma_4 \rangle$ breaks all SUSY

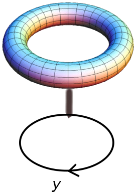
Supersymmetric T-fold reduction *(Hellerman, Walcher '06)*

- Fibration $T^2 \hookrightarrow \mathcal{M}_3 \to S^1$ with $O(2,2;\mathbb{Z})$ monodromy

$$\boxed{(x^i_{\mathrm{L}}, x^i_{\mathrm{R}}; y) \sim (-x^i_{\mathrm{L}}, x^i_{\mathrm{R}}; y + 2\pi R)}$$

➥ Monodromy twist $\begin{cases} U & \mapsto -1/U \\ T & \mapsto -1/T \end{cases}$

- Half-SUSY vacua with spacetime SUSY from right-movers



$\mathcal{N} = 2$ vacua from type IIA on $K3 \times T^2$ *(Hull,D.I., Sarti '17)*

- Type IIA superstrings on $K3 \hookrightarrow \mathcal{M}_6 \to T^2$ fibrations with monodromy twists

Low-energy limit of type IIA on $K3 \times T^2$

- $\mathcal{N} = 4$ SUGRA in four dimensions
- Field content: SUGRA multiplet $(g_{\mu\nu}, \psi^i_\mu, A^{1,\dots,6}_\mu, \chi^i, \tau)$
  22 vector multiplets $(A^a_\mu, \lambda^a_i, \mathcal{M})$
- Scalars $\mathcal{M}$, τ take value in the coset $\frac{O(6,22)}{O(6)\times O(22)} \times \frac{SL(2)}{O(2)}$

- Moduli space of K3 compactifications $O(\Gamma_{4,20})\backslash O(4,20)/O(4) \times O(20)$

  ➥ Consider monodromies $\boxed{\mathcal{M} \in O(\Gamma_{4,20}) \subset O(4,20)}$

- Goal: $\mathcal{N} = 4 \to \mathcal{N} = 2$ *spontaneous SUSY breaking*

Gauged supergravity analysis

(Ried-Edwards, Spanjaard '08, Horst, Louis, Smyth '12)

- $K3 \times T^2$ with monodromy twists $M_i = e^{N_i} \in O(\Gamma_{4,20})$ along T^2

  ➥ structure constants $\boxed{t_{iI}{}^J = N_{iI}{}^J}$ of $\mathcal{N} = 4$ gauged supergravity
- Potential and SUSY breaking mass terms computed from t_{MNP}

Vacua with spontaneous SUSY breaking $\mathcal{N} = 4 \ \to \mathcal{N} = 2$

- Gravitini transform in $(\mathbf{2},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2},\mathbf{1})$ of $\{SU(2) \times SU(2) \cong SO(4)\} \times SO(20) \subset O(4,20)$
- Minkowski vacua from *elliptic* monodromies in $\{SO(4) \times SO(20)\} \cap O(\Gamma_{4,20}) \subset O(4,20)$
- Half-SUSY vacua from monodromies in $\{SU(2) \times SO(20)\} \cap O(\Gamma_{4,20}) \subset O(\Gamma_{20}) \subset O(4,20)$

- Such solutions, if any, are necessarily non-geometric (as $K3$ diffeos in $O(3,19) \subset O(4,20)$) ➥ mirror-folds?
- Their construction relies on recent works on mirror symmetry of $K3$ surfaces

Non-linear sigma models on K3 and mirrored automorphisms

K3 surfaces: elementary facts

K3-surfaces

- K3 surface X: Kähler 2-fold with a nowhere vanishing holomorphic 2-form Ω
- Hodge diamond: $$\begin{array}{ccccc} & & h^{0,0} & & \\ & h^{1,0} & & h^{0,1} & \\ h^{2,0} & & h^{1,1} & & h^{0,2} \\ & h^{2,1} & & h^{1,2} & \\ & & h^{2,2} & & \end{array} \quad = \quad \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array}$$
- Inner product: $(\alpha, \beta) \in H^2(X, \mathbb{Z}) \times H^2(X, \mathbb{Z}) \mapsto \langle \alpha, \beta \rangle = \int \alpha \wedge \beta \in \mathbb{Z}$
- $H^2(X, \mathbb{Z})$ isomorphic to unique even, unimodular lattice of signature $(3, 19)$: $$\Gamma_{3,19} \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \;, \qquad U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
- Lattice of total cohomology $H^\star(X, \mathbb{Z})$: $\boxed{\Gamma_{4,20} \cong E_8 \oplus E_8 \oplus U \oplus U \oplus U \oplus U}$

Moduli space of Ricci-flat metrics on K3

- Ricci-flat metric on X $\leftrightarrow$ space-like oriented 3-plane
  $\Sigma = (\Omega, J) \subset \mathbb{R}^{3,19} \cong H^2(X, \mathbb{R})$, modulo large diffeos
- $\boxed{\mathcal{M}_{\text{KE}} \cong O(\Gamma_{3,19}) \backslash O(3,19) \Big/ O(3) \times O(19) \; \times \mathbb{R}_+}$

String theory compactifications on K3

Non-linear sigma-models on K3 surfaces

- $\int_\Sigma \mathrm{d}^2 z \left\{ g_{i\bar{\jmath}}\left(\partial_z \phi^i \partial_{\bar{z}} \phi^{\bar{\jmath}} + \partial_{\bar{z}} \phi^i \partial_z \phi^{\bar{\jmath}}\right) + b_{i\bar{\jmath}}\left(\partial_z \phi^i \partial_{\bar{z}} \phi^{\bar{\jmath}} - \partial_{\bar{z}} \phi^i \partial_z \phi^{\bar{\jmath}}\right) \right\}$
- g Ricci-flat and $\mathrm{d}b = 0$ ➥ CFT
- $\int_{\phi(\Sigma)} b$ ➥ 22 real parameters

Moduli space of NLSMs

- Choice of metric & B-field ↔ choice of space-like oriented 4-plane $\Pi \subset \mathbb{R}^{4,20}$
- $\boxed{\mathcal{M}_\sigma \cong O(\Gamma_{4,20}) \backslash \, O(4,20) \, / \, O(4) \times O(20)}$ *(Seiberg, Aspinwall-Morrison)*
- $O(\Gamma_{4,20})$ contains non-geometric symmetries as mirror symmetry

- K3 surfaces hyper-Kähler

➥ what does mirror symmetry mean?

➥ how to define mirror-folds?

Lattice-polarized mirror symmetry

- Picard lattice $S(X) = H^2(X,\mathbb{Z}) \cap H^{1,1}(X) \subset \Gamma_{3,19}$
  ➥ rank $\rho(X) \geqslant 1$ for an algebraic surface, signature $(1, \rho - 1)$

Polarized K3 surfaces

- Lattice M of signature $(1, r-1)$ with primitive embedding in $S(X)$
  ➥ M-polarized surface (X, M)
- Moduli space of complex structures compatible with polarization:
  $\mathcal{M}_M \cong {}_{O(M^\perp)}\backslash\, O(2, 20-r) \,/_{O(2)\times O(20-r)}$

Lattice-polarized mirror symmetry *(Dolgachev, Nikulin)*

M-polarized surface (X, M) and $\tilde{M}$-polarized surface $(\tilde{X}, \tilde{M})$ LP-mirror if

$$\boxed{\Gamma^{3,19} \cap M^\perp = U \oplus \tilde{M}}$$

Greene-Plesser mirror symmetry

- Is lattice-polarized mirror symmetry related to "physicist's" mirror symmetry?

Example of Greene-Plesser construction *(Greene, Plesser '90)*

- Hypersurface $w^2 + x^3 + y^8 + z^{24} = 0 \subset \mathbb{P}_{[12,8,3,1]}$
- Greene-Plesser mirror surface: quotient of the same hypersurface by the group G of supersymmetry-preserving automorphisms
- Here $G \simeq \mathbb{Z}_2$ generated by $g : \left\{ \begin{array}{ccc} w & \mapsto & -w \\ y & \mapsto & -y \end{array} \right.$
- More general case (non-Fermat): Berglund-Hübsch *(Berglund-Hübsch '91)*

- The key point, to compare both notions, is the choice of lattice polarization

Automorphisms of K3 surfaces *(Nikulin)*

- Non-symplectic order p automorphism σ_p: $\sigma_p^\star(\Omega) = e^{\frac{2i\pi}{p}}\Omega$
- Invariant sublattice of $\Gamma_{3,19}$: $S(\sigma_p) \subseteq S(X)$
- Orthogonal complement $T(\sigma_p) = S(\sigma_p)^\perp \cap \Gamma_{3,19}$

Previous example

- Hypersurface $w^2 + x^3 + y^8 + z^{24} = 0 \subset \mathbb{P}_{[12,8,3,1]}$
- Order 3 automorphism $\sigma_3 : x \mapsto e^{2i\pi/3}x$
- Sub-lattices $S(\sigma_3) \cong E_6 \oplus U$ and $T(\sigma_3) \cong E_8 \oplus A_2 \oplus U \oplus U$

Greene-Plesser mirror surface

- Orbifold $\tilde{w}^2 + \tilde{x}^3 + \tilde{y}^8 + \tilde{z}^{24} = 0 \subset \mathbb{P}_{[12,8,3,1]}\ /\mathbb{Z}_2$
- Order 3 automorphism $\tilde{\sigma}_3 : \tilde{x} \mapsto e^{2i\pi/3}\tilde{x}$
- Sub-lattices $S(\tilde{\sigma}_3) \cong E_8 \oplus A_2 \oplus U$ and $T(\sigma_3) \cong E_6 \oplus U \oplus U$

➥ Lattice-polarized mirror symmetry relates the first surface polarized by $S(\sigma_3)$ to the second surface polarized by $S(\tilde{\sigma}_3)$

The general story

Non-symplectic automorphisms and mirror symmetry

- *p-cyclic* K3 surface X: $\boxed{W = w^p + f(x,y,z)} \circlearrowleft \sigma_p : w \mapsto e^{\frac{2i\pi}{p}} w$
- Berglund-Hübsch mirror $\tilde{X}$: $\boxed{\tilde{W} = \tilde{w}^p + \tilde{f}(\tilde{x},\tilde{y},\tilde{z})/G} \circlearrowleft \tilde{\sigma}_p : \tilde{w} \mapsto e^{\frac{2i\pi}{p}} \tilde{w}$
- Theorem (Artebani et al., Comparin et al., Bott et al.): *The $S(\sigma_p)$-polarized surface X and the $S(\tilde{\sigma}_p)$-polarized surface $\tilde{X}$ are lattice-polarized mirrors.*

Corollary: lattice decomposition *(Hull, DI, Sarti)*

- $T(\tilde{\sigma}_p)$ is the orthogonal complement of $T(\sigma_p)$ in $\Gamma_{4,20}$:

$$T(\tilde{\sigma}_p) \cong T(\sigma_p)^{\perp} \cap \Gamma_{4,20} \,.$$

- Orthogonal decomposition over $\mathbb{R}$ (and over $\mathbb{Q}$):

$$\boxed{\Gamma_{4,20} \otimes \mathbb{R} \;\cong\; \Big(T(\sigma_p) \oplus T(\tilde{\sigma}_p) \Big) \otimes \mathbb{R}}$$

Mirrored K3 automorphisms

(Hull, DI, Sarti)

Lattice definition

- Let X be a p-cyclic K3 surface, and $\tilde{X}$ its LP/BH mirror,
- One can extend the diagonal action of $(\sigma_p, \tilde{\sigma}_p)$ on $T(\sigma_p) \oplus T(\tilde{\sigma}_p)$ to an action on the whole lattice $\Gamma_{4,20}$.
- This defines a lattice isometry in $O(\Gamma_{4,20})$ associated with the action of a NLSM automorphism $\hat{\sigma}_p$, that we name *mirrored automorphism*.

Intrinsic definition

- Denoting by μ the BH/LP mirror involution, $\boxed{\hat{\sigma}_p := \mu \circ \tilde{\sigma}_p \circ \mu \circ \sigma_p}$
- "Gluing" of a Calabi-Yau symmetry and of a symmetry of the mirror CY

Reduction with monodromy twists

- $T(\sigma_p)$ and $T(\tilde{\sigma}_p)$ of signatures $(2, r)$ and $(2, 20 - r)$.
- Action of $\hat{\sigma}_p$ ➥ diagonal space-like $O(2) \times O(2) \subset O(4, 20)$ of order p
- Leads to $\mathcal{N} = 2$ Minkowski vacua ➥ orbifold theories at the fixed points?

Asymmetric Landau-Ginzburg/Gepner orbifolds

Gepner models/LG orbifolds for K3 surfaces

Landau-Ginzburg models

- $\mathcal{N} = (2,2)$ QFTs in 2d, chiral multiplets Z_ℓ and superpotential $W(Z_\ell)$
- Quasi-homogeneous polynomial with an isolated critical point:

$$\boxed{W(\lambda^{w_\ell} Z_\ell) = \lambda^d W(Z_\ell)}$$

- Flows to a $(2,2)$ SCFT in the IR

LG orbifold model for K3 surfaces

- Quantum non-linear sigma-model on a K3 surface in small-volume limit
- LG model $\boxed{W = Z_1^{p_1} + Z_2^{p_2} + Z_3^{p_3} + Z_4^{p_4}}$, $K = \mathrm{lcm}(p_1, \dots, p_4)$
- GSO projection: diagonal $\mathbb{Z}_K$ orbifold $j : Z_\ell \mapsto e^{2i\pi/p_\ell} Z_\ell$

  ➥ fields in twisted sectors $\gamma = 0, \dots, K-1$
- IR fixed point: $\mathcal{N} = (4,4)$ SCFT with $c = \bar{c} = 6$ ➥ Gepner model

K3 Gepner/Landau-Ginzurg orbifolds

Symmetries of Gepner models

- $W = Z_1^{p_1} + \cdots + Z_4^{p_4}$ ➥ discrete symmetry group $(\mathbb{Z}_{p_1} \times \cdots \times \mathbb{Z}_{p_4})/\langle j \rangle$
- $Z_\ell \mapsto e^{\frac{2i\pi r_\ell}{p_\ell}} Z_\ell$ with $\sum_\ell \frac{r_\ell}{p_\ell} \in \mathbb{Z}$ ➥ SUSY-preserving symmetries
- Quantum symmetry of LG orbifold: $\boxed{\sigma_K^{\Omega} \, : \, \phi_\gamma \mapsto e^{2i\pi\gamma/K} \phi_\gamma}$

Orbifolds of Gepner models

- Supersymmetric orbifold of a K3 Gepner model

  ➥ other point in K3 NLSM moduli space
- Quotient by $\langle \sigma_{p_\ell} \rangle$, with $\boxed{\sigma_{p_\ell} \, : \, Z_\ell \mapsto e^{2i\pi/p_\ell} Z_\ell}$ for given ℓ

  ➥ breaks all space-time SUSY

★Latter case: space-time SUSY can be partially restored using discrete torsion

Asymmetric K3 Gepner models

A simple class of asymmetric $K3$ Gepner models *(DI '15)*

- $\boxed{\sigma_{p_1} : Z_1 \mapsto e^{2i\pi/p_1} Z_1}$ orbifold ➥ field $(Z_1^{n_1} \cdots)$ has charge $Q_{p_1} \equiv \frac{n_1}{p_1}$ mod 1

  ➥ twisted sectors $r = 0, \ldots, p_1 - 1$

★ Project w.r.t. shifted $\mathbb{Z}_{p_1}$ orbifold charge: $\hat{Q}_{p_1} = Q_{p_1} + \frac{\gamma}{p_1}$

★ (diagonal $\mathbb{Z}_{\mathrm{K}}$ orbifold charge shifted by $-\frac{r}{p_1}$)

$\Big\}$ discrete torsion

- Interpretation: order p subgroup of the quantum symmetry group

  $\boxed{\sigma^{\mathfrak{Q}}_{p_1} := (\sigma^{\mathfrak{Q}}_{K})^{K/p_1}}$ ➥ γ-tw. sector field has charge $Q^{\mathfrak{Q}}_{p_1} \equiv \frac{\gamma}{p_1} \mod 1$
- Space-time supercharges from left-movers only

Related works

- Asymmetric models from simple currents *(Schellekens & Yankielowicz 90)*
- LG orbifolds *(Intriligator & Vafa 90)*

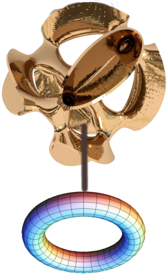
K3 fibrations with non-geometric monodromies

Asymmetric $K3 \times T^2$ Gepner models in type IIA

(DI, Thiéry '14)

- $K3$ Gepner model ($W = Z_1^{p_1} + Z_2^{p_2} + Z_3^{p_3} + Z_4^{p_4}$) times $\mathbb{R}^2$ (x, y) in type IIA/B
- Freely-acting $\mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2}$ quotient with discrete torsion as above

$$\begin{cases} Z_1 & \mapsto e^{2i\pi/p_1} Z_1 \\ x & \mapsto x + 2\pi R_1 \end{cases} \qquad \begin{cases} Z_2 & \mapsto e^{2i\pi/p_2} Z_2 \\ y & \mapsto y + 2\pi R_2 \end{cases}$$



Main features

Supersymmetry breaking

- All space-time supercharges from left-movers ➥ non-geometric
- No massless Ramond-Ramond states
- Spontaneous breaking $\mathcal{N} = 4 \; \to \; \mathcal{N} = 2$ in four dimensions

Moduli space

- U and T moduli of the T^2 and axio-dilaton S always massless
- For about 50% of the models: *all* K3 moduli become massive

Low-energy 4d theory

- $\mathcal{N} = 2$ vacua of $\mathcal{N} = 4$ gauged SUGRA
- Axio-dilaton and torus moduli in vector multiplets ➥ $\mathcal{N} = 2$ STU SUGRA
- Surviving K3 moduli (if any): hypermultiplets

Mirrored K3 automorphisms vs. asymmetric Gepner models

Mirror symmetry and quantum symmetry of Gepner models

- In the Gepner model construction we have used:
  1. order p_1 symmetry group of the superpotential $Z_1 \mapsto e^{2i\pi/p_1} Z_1$
  2. order p_1 subgroup of the quantum sym. group generated by $\sigma^{\Omega}_{p_1} := (\sigma^{\Omega})^{K/p_1}$
- These symmetries are exchanged by mirror symmetry $(\bar{Q}_R \mapsto -\bar{Q}_R)$

Non-geometric orbifolds from mirrored automorphisms

- $K3$ orbifold with discrete torsion ➥ projection $Q_{p_1} + Q^{\Omega}_{p_1} \in \mathbb{Z}$
- Corresponds to the diagonal action of $(\sigma_{p_1}, \tilde{\sigma}_{p_1})$!
- Therefore, a $K3$ bundle over T^2 with mirrored automorphisms twists gives at the fixed points an asymmetric $K3 \times T^2$ Gepner model

New 4d heterotic/type II dualities

Heterotic/type II dualities in 4d

Six-dimensional duality *(Hull, Townsend '94)*

- Type IIA on K3 ↔ Heterotic on T^4, $\phi_{\text{IIA}} = -\phi_{\text{IIB}}$
- $O(\Gamma_{4,20})\backslash\, O(4,20) \,/\, O(4)\times O(20)$ as heterotic Narain moduli space
- Non-Abelian heterotic gauge groups ↔ non-perturbative IIA vacua

Four-dimensional $\mathcal{N}=4$ duality

- Type IIA/IIB on $K3\times T^2$ ↔ Heterotic on T^6
- Moduli space $O(\Gamma_{6,22})\backslash\, O(6,22) \,/\, O(4)\times O(20)$

Four-dimensional $\mathcal{N}=2$ dualities

- Type II $\mathcal{N}=2$ compactification on CY_3 manifold with $K3$ fibration
- Large base volume limit: apply the 6d duality fiberwise

  ➥ *adiabatic argument* *(Vafa, Witten 95)*

★Analoguous $\mathcal{N}=4$ models: type IIA duals of heterotic CHL *(Schwarz,Sen ' 95)*

Example :FHSV construction

Enriques CY 3-fold

- There exists a unique non-symplectic involution σ_2 of $K3$ surfaces without fixed points ➥ Enriques involution
- Quotient of $K3 \times T^2$ by $(\sigma_2, \mathcal{I}_{T^2})$ ➥ freely acting orbifold
- Calabi-Yau 3-fold with $SU(2) \times \mathbb{Z}_2$ holonomy

Heterotic dual

(Ferrara, Harvey, Strominger, Vafa 95)

- Dual: Freely-acting orbifold of heterotic on $T^4 \times T^2$
- Heterotic modular invariance? ➥ winding shift along T^4 required
- Type IIA interpretation: discrete Wilson line for RR forms ➥ non-perturbative consistency condition!

★ *General story* : heterotic on $K3 \times T^2$ ↔ IIA on K3-fibered CY$_3$

New $\mathcal{N} = 2$ dualities from non-geometric backgrounds

The type IIA story

- $K3 \hookrightarrow \mathcal{M}_6 \rightarrow T^2$ fibration with mirrored automorphisms twists
- Free action on T^2 (translation)
- Monodromies $\hat{\sigma}_p \in O(\Gamma_{4,20})$ of $K3$ fiber
- $\mathcal{N} = 2$ SUSY vacua, without BPS D-branes
- Dilaton sits in a *vector* multiplet

The heterotic story

(Gautier, Hull, DI '19)

- $\hat{\sigma}_p \in O(\Gamma_{4,20}) \leftrightarrow$ order p isometry of the$(4, 20)$ Narain lattice
- Action on the T^4 left-movers (SUSY side): rotation of angles $(2\pi/p, -2\pi/p)$, $p \in \{2, \ldots, 13\}$, $p \neq 11$
- Action on the 24 right-moving compact bosons: rotation leaving no sub-lattice invariant

  ➥ unlike ordinary orbifolds, twist, not shift, in the gauge sector
- Dual of IIA Gepner points have no enhanced gauge symmetry from T^4

Heterotic perturbative consistency

- Asymmetric orbifolds of heterotic on $T^4 \times T^2$ ➥ level matching?
- Modular invariance of the partition function requires a winding shift along T^2:

  Shift vector $\boxed{\delta = \frac{1}{p}(1,0,1,0) \in \mathbb{R}^{2,2} \mod \Gamma_{2,2}}$
- Invisible in large T^2 limit

  ➥ compatible with "adiabatic argument" of Vafa and Witten

Type IIA interpretation

- Fundamental heterotic wrapped on $S^1 \subset T^2$

  $\updownarrow$

  Type IIA NS5-brane wrapped on $S^1 \subset T^2$ and $K3$ fiber *(Sen '95)*
- Consistency condition found in heterotic becomes non-perturbative: wrapped NS5-branes charged under the mirrored automorphisms
- Is there a *generalized* non-perturbative concept of modular invariance?

Hypermultiplet moduli space (single monodromy)

- Hypermuliplets in type IIA frame: surviving K3 moduli (if any)
- *Exact* hypermultiplets moduli space determined from the heterotic description
- Mirrored automorphism of order 2 : $\hat{\sigma}_2 = -\mathbb{I}_{24}$ hence no restriction

  ➥ as usual, choice of space-like 4-plane $\Pi_L(\Gamma_{4,20})$ into $\mathbb{R}^{4,20}$

$$\mathcal{M} \cong O(\Gamma_{4,20})\backslash O(4,20)/O(4)\times O(20)$$

Mirrored automorphism of order $p > 2$

- There exists a basis of $\Pi_L(\Gamma_{4,20})\otimes\mathbb{C}$ with $\hat{\sigma}_p = (e^{2i\pi/p}\mathbb{I}_2, e^{-2i\pi/p}\mathbb{I}_2)$
- Eigenspace for $e^{2i\pi/p}$ of dimension $24/\phi(p)$ (Euler's totient)
- Freedom of choosing space-like complex plane into $\mathbb{C}^{24/\phi(p)}$ ➥ moduli space

$$\mathcal{T} \cong SU(2,\tfrac{24}{\phi(p)}-2)/S\big[U(2)\times U(\tfrac{24}{\phi(p)}-2)\big]$$

- Duality group: $\hat{\Gamma}_p = \big\{\gamma\in O(\Gamma_{4,20})\big|\gamma\otimes\hat{\sigma}_p^* = \hat{\sigma}_p^*\otimes\gamma\big\}$

Vector multiplet moduli space: type IIA

- Classical moduli space: $\mathcal{T} \cong \left(\frac{SL(2;\mathbb{R})}{U(1)}\right)_S \times \left(\frac{SL(2;\mathbb{R})}{U(1)}\right)_T \times \left(\frac{SL(2;\mathbb{R})}{U(1)}\right)_U$
- Dilaton T in vector multiplet ➥ prepotential does receive quantum corrections

$$\boxed{F(S,T,U) = STU + h_{\rm II}^{1-loop}(S,U) + \mathcal{O}\left(e^{-T}\right)}$$

- Perturbative dualities should preserve the shift vector $\delta_{\rm II} = \frac{1}{p}(1,0,0,0)$

$$G_{\rm II} = \{\gamma \in O(\Gamma_{2,2}) | G_{\rm II}\delta = \delta_{\rm II} \mod \Gamma_{2,2}\}$$

- One finds $\Gamma_1(p)_S \times \Gamma_1(p)_U \subset G_{\rm II} \subset SL(2;\mathbb{Z}) \times SL(2;\mathbb{Z})$

★ Congruence subgroup: $\Gamma_1(p) = \left\{ g = \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix} \mod p \right\}$

Modular propreties of $h_{\rm II}^{1-loop}(S,U)$

1. No enhanced gauge symmetry ➥ modular form of weight $(-2,-2)$ *(Antoniadis et al., de Wit et al. '95)*
2. Should vanish at the cusps (decompactification limits)

No negative weights modular form for congruence subgroups: $\boxed{h_{\rm II}^{1-loop}(S,U) = 0}$

Vector multiplet moduli space: heterotic

- $\boxed{F(S,T,U) = STU + h_{\text{HET}}^{1-loop}(T,U) + \mathcal{O}\left(e^{-S}\right)}$
- Perturbative dualities should preserve the shift vector $\delta_{\text{HET}} = \frac{1}{p}(1,0,1,0)$

$$G_{\text{HET}} = \{\gamma \in O(\Gamma_{2,2}) | G_{\text{HET}}\delta = \delta_{\text{HET}} \mod \Gamma_{2,2}\} \cong SL(2;\mathbb{Z})_{\textsf{diag}} \times \Gamma(p) \ltimes \mathbb{Z}_2^{T\leftrightarrow -1/U}$$

- Congruence subgroup: $\Gamma(p)_T \times \Gamma(p)_U \subset G_{\text{HET}}$ with $\Gamma(p) = \{g = \mathbb{I} \mod p\}$

Modular propreties of $h_{\text{II}}^{1-loop}(S,U)$

- Enhanced $SU(2)$ gauge symmetry for $T = U \mod G_{\text{HET}}$ ➥ singularities

$$h_{\text{HET}}^{1-loop}(T,U) \sim -\frac{1}{16\pi^2}(T-U)^2 \log(T-U)^2$$

- $h^{\text{HET}}(T,U)$ not a modular form but $\partial_U^3 h^{\text{HET}}(T,U)$ and $\partial_T^3 h^{\text{HET}}(T,U)$ are
- Determined from $\Gamma(p) \times \Gamma(p)$ covariance, singularities & vanishing at cusps

★So far, exact form of $\partial^3 h_{\text{HET}}^{1-loop}(T,U)$ for $p = 2$ with the expected singularities

Conclusions

- ❑ Non-geometric compactifications of superstring theory are likely the most generic ones yet poorly understood
- ❑ Large class of non-geometric compactifications based on Calabi-Yau rather than toroidal geometries ➥ first construction of "mirrorfolds"
- ❑ Analysis from 4 viewpoints:
  1. Worldsheet CFT
  2. Algebraic geometry
  3. Gauged SUGRA
  4. Heterotic/type II duality
- ❑ New classes of symmetries of CY sigma-models: mirrored CY automorphisms
- ❑ Heterotic/type II duality: new $\mathcal{N} = 2$ string dualities in 4d
- ❑ Some open questions:
  1. Relation with the Mathieu moonshine?
  2. Insights on NS5-brane winding shifts in the type IIA frame
  3. CY$_3$-based constructions ➥ $\mathcal{N} = 1$ type II vacua without RR fluxes!
  4. How to get non-Abelian gauge groups in type II?

- First glimpse of a new continent inside the string landscape – or unicorn?