String-brane interactions from large to small distances

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Outline

- Strings, high energy and curved spacetimes
- The eikonal operator: classical gravity effects and stringy corrections
- The eikonal operator from the worldsheet σ-model
- String absorption: closed-open transition and the infall of a probe into a singularity

String backgrounds

String theory from a worldsheet perspective is a perturbative series

sum over surfaces or genus espansion

around a given classical background

a (super)conformal σ-model with $c = 26$ (15)

A good understanding of string compactifications, i.e. vacua of the form

$$\mathbb{R}^{1,d-1} \times \mathcal{K}$$

Main examples: toroidal compactifications, orbifolds, compact Wess-Zumino-Witten models and their GKO cosets, Gepner models.
Many lessons: importance of winding states and twisted sectors, T-duality, non-geometric backgrounds...

String backgrounds

Our understanding of curved spacetimes in string theory is more limited.

Several classes of known examples:

- Plane waves (Horowitz and Steif, 1990)
- Lorentzian orbifolds (Horowitz and Steif, 1991; Liu, Moore and Seiberg, 2002)
- Non-compact WZW models and their cosets (Witten, 1991; Nappi and Witten, 1992 and 1993)

Change of strategy: analyze the high-energy dynamics of a string in the background of a collection of D-branes.

String-brane system at high energy

The string-brane system provides an excellent framework for the study of string dynamics in curved spacetimes

- Microscopic description at weak coupling in terms of open strings
- Geometric description at strong coupling, extremal p-branes
- Unitary S-matrix

The high-energy limit makes the dynamics both interesting and (to a certain extent...) tractable

- The large energy causes an enhancement of classical and quantum gravity effects
- The large energy reveals the full string dynamics: interactions of states of arbitrary mass and spin. Inelastic processes grows in variety and importance.

Extremal p-branes

Low energy effective action in the string frame

$$S = \frac{1}{(2\pi)^7 l_s^8} \int d^{10}x \sqrt{-g} \left[e^{-2\varphi} \left(R + 4(\nabla\varphi)^2 \right) - \frac{2}{(8-p)!} F_{p+2}^2 \right] .$$

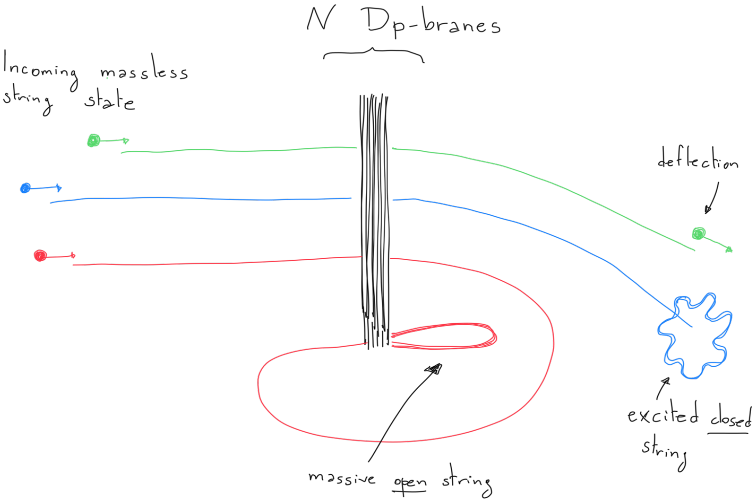
BPS solutions carrying R-R charges Horowitz and Strominger, 1991

$$ds^2 = \frac{1}{\sqrt{H(r)}} \eta_{\mu\nu} \, dx^\mu dx^\nu + \sqrt{H(r)} \, \delta_{ij} \, dx^i dx^j , \qquad e^\varphi = g H^{\frac{3-p}{4}} ,$$

$$\int_{S^{8-p}} {}^*F_{p+2} = N , \qquad H(r) = 1 + \left(\frac{R}{r} \right)^{7-p} , \qquad \frac{R^{7-p}}{l_s^{7-p}} = d_p \, g N ,$$

$$\lambda = gN , \qquad d_p = (4\pi)^{\frac{5-p}{2}} \Gamma \left(\frac{7-p}{2} \right) , \qquad \tau_p = \frac{R^{7-p}}{(2\pi)^7 d_p g^2 \alpha'^4} .$$

String-brane collisions



String-brane collisions

Relevant scales

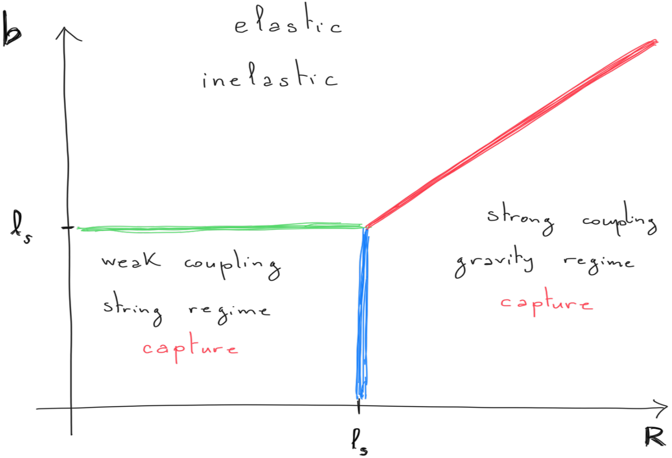
$$\alpha' s \gg 1 \ , \qquad \left(\frac{R}{l_s}\right)^{7-p} \sim \ g\,N \ , \qquad b_T^{8-p} \sim \alpha' E R^{7-p} \ , \qquad b_c \sim R$$

Various possible processes as the impact parameter is varied

- $b \gg b_T \gg R$ — elastic scattering
- $b_T \geq b \gg R$ — string tidal excitations
- $b < b_c$ $\begin{cases} R \ll l_s & \text{creation of open strings} \\ R \gg l_s & \text{infall into the singularity} \end{cases}$

Fixed effective background: extremal p-brane metric

String-brane collisions



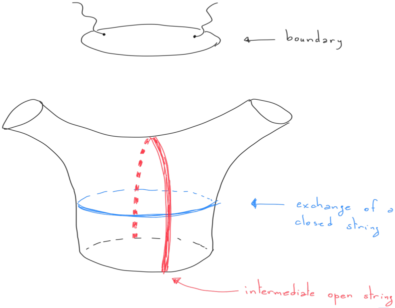
The eikonal operator

Regge limit of the disk amplitude (tree level)

$$\mathcal{A}_1(s,t) \sim \Gamma\left(-\frac{\alpha'}{4}t\right) e^{-i\pi\frac{\alpha' t}{4}} \, (\alpha' s)^{1+\frac{\alpha' t}{4}} \ ,$$

$$s = E^2 \ , \quad t = -(p_1+p_2)^2$$

Grows too fast with energy. Include higher-orders.



The eikonal operator

The result is the eikonal operator

$$S(s,b) = e^{2i\hat{\delta}(s,b)} \ ,$$

$$2\hat{\delta}(s,b) = \int_0^{2\pi} \frac{d\sigma}{2\pi} \frac{:\mathcal{A}_1\left(s, \mathbf{b} + \mathbf{X}(\sigma)\right):}{2E}$$

Amati, Ciafaloni e Veneziano (1987)
GD, Di Vecchia, Russo e Veneziano (2010).
In impact parameter space the tree-level amplitude is

$$\mathcal{A}_1(s,b) \sim s\sqrt{\pi}\,\frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\frac{R_p^{7-p}}{b^{6-p}} + \frac{i\pi\sqrt{s}}{\Gamma\left(\frac{7-p}{2}\right)}\sqrt{\frac{\pi\alpha' s}{\ln\alpha' s}}\left(\frac{R_p}{l_s(s)}\right)^{7-p} e^{-\frac{b^2}{l_s^2(s)}}$$

$l_s(s)$ is the effective string length $l_s(s) = l_s\sqrt{\ln\alpha' s}$

The eikonal operator

Two main effects

- deflection of the trajectory
- excitation of the internal degrees of freedom of the string: tidal forces

Scattering angle $\theta = -\frac{2}{E}\frac{\partial \delta(s,b)}{\partial b}$

The leading and next-to-leading terms

$$\Theta_p = \sqrt{\pi}\left[\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\left(\frac{R_p}{b}\right)^{7-p} + \frac{1}{2}\frac{\Gamma\left(\frac{15-2p}{2}\right)}{\Gamma\left(6-p\right)}\left(\frac{R_p}{b}\right)^{2(7-p)} + \dots\right]$$

are in perfect agreement with the classical deflection of a massless point-like probe in the p-brane background.

Tidal excitation at leading order

When $b \gg R \gg l_s \sqrt{\ln(\alpha' s)}$

$$2\,\hat{\delta}(s, \mathbf{b} + \hat{\mathbf{X}}) \sim \frac{1}{2E}\left[\mathcal{A}_1(s,b) + \frac{1}{2}\frac{\partial^2 \mathcal{A}_1(s,b)}{\partial b^i \partial b^j}\ \overline{\hat{X}^i \hat{X}^j} + \ldots\right]$$

where $\bar{Q} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\sigma : Q(\sigma) :$ and the string coordinates are

$$X^i = i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left(\frac{A_n^i}{n}\mathrm{e}^{in\sigma} + \frac{\bar{A}_n^i}{n}e^{-in\sigma}\right)\ , \quad [A_n^i, A_m^j] = n\delta^{ij}\delta_{n+m,0}$$

Matrix of the second derivatives of the eikonal phase

$$\frac{1}{4\sqrt{s}}\frac{\partial^2 \mathcal{A}_1(s,b)}{\partial b_i \partial b_j} = Q_\perp(s,b)\left[\delta_{ij} - \frac{b_i b_j}{b^2}\right] + Q_\parallel(s,b)\ \frac{b_i b_j}{b^2}$$

where

$$Q_\perp(s,b) = \frac{1}{4\sqrt{s}}\,\frac{1}{b}\frac{d\mathcal{A}_1(s,b)}{db}\ , \quad Q_\parallel(s,b) = \frac{1}{4\sqrt{s}}\,\frac{d^2\mathcal{A}_1(s,b)}{db^2}$$

Tidal excitation at leading order

At leading order in $\frac{R}{b}$

$$Q_{\perp}(s,b) = -\frac{\sqrt{\pi}}{2}\sqrt{s}\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\frac{R^{7-p}}{b^{8-p}}\,, \qquad Q_{\parallel}(s,b) = -(7-p)\; Q_{\perp}(s,b)$$

String corrections contribute a non-vanishing imaginary part to the eikonal phase

$$\left|\langle 0|e^{2i\hat{\delta}(s,\mathbf{b})}|0\rangle\right| \sim e^{-\frac{1}{2\sqrt{s}}\mathrm{Im}\mathcal{A}_1(s,b)}\,\left(2\pi\alpha'|Q_{\perp}|\right)^{\frac{8-p}{2}}\sqrt{7-p}\;e^{-\pi\alpha'(7-p)|Q_{\perp}|}$$

The absorption of the elastic channel due to string excitations becomes non negligible for $b \leq b_T$

$$b_T^{8-p} = \frac{\pi}{2}\alpha'\sqrt{\pi s}(7-p)\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}R_p^{7-p}$$

Tidal excitation and the pp-wave limit

Can we reproduce these results starting with the sigma-model for the extremal p-brane metric?

$$ds^2 = \alpha(r)\left(-dt^2 + \sum_{a=1}^{p}(dx^a)^2\right) + \beta(r)\left(dr^2 + r^2(d\theta^2 + \sin^2\theta d\Omega_{7-p}^2)\right)$$

where $\beta(r) = 1/\alpha(r) = \sqrt{H(r)}$.
We focus on a small neighborhood around a null geodesic expanding the sigma model action in Fermi coordinates. The leading term in energy corresponds to the Penrose limit of the brane background

$$\begin{aligned} ds^2 &= 2dud\hat{v} + \sum_{a=1}^{p} d\hat{x}_a^2 + \sum_{i=1}^{7-p} d\hat{y}_i^2 + d\hat{y}_0^2 + \mathcal{G}(u, \hat{x}^a, \hat{y}^i, \hat{y}^0)du^2 \quad , \\ \mathcal{G} &= \frac{\partial_u^2\sqrt{\alpha}}{\sqrt{\alpha}}\sum_{a=1}^{p}\hat{x}_a^2 + \frac{\partial_u^2(\sqrt{\beta}r\sin\bar{\theta})}{\sqrt{\beta}r\sin\bar{\theta}}\sum_{i=1}^{7-p}\hat{y}_i^2 + \frac{\partial_u^2\sqrt{\beta r^2 - b^2\alpha}}{\sqrt{\beta r^2 - b^2\alpha}}\hat{y}_0^2 \ . \end{aligned}$$

Blau, Figueroa-O'Farrill and Papadopoulos, 2002

The bosonic part of the string sigma model becomes

$$S \sim S_0 - \frac{1}{4\pi\alpha'} \int d\tau \int_0^{2\pi} d\sigma \; \eta^{\alpha\beta} \; \partial_\alpha U \partial_\beta U \; \mathcal{G}(U, X^a, Y^i, Y^0)$$

where S_0 is the string action in Minkowski space.
We work in the light-cone gauge $U(\sigma, \tau) = \alpha' E \tau$ and evaluate the transition amplitudes in the impulsive approximation. The integrals over u then decouple from the string coordinates and simply provide c-number coefficients to the quadratic action of the fluctuations

$$S - S_0 \sim \frac{E}{2} \int_0^{2\pi} \frac{d\sigma}{2\pi} \left(c_x \sum_{a=1}^{p} X_a^2(\sigma, 0) + c_y \sum_{i=1}^{7-p} Y_i^2(\sigma, 0) + c_0 Y_0^2(\sigma, 0) \right)$$

$$c_y = \int_{-\infty}^{+\infty} du \mathcal{G}_y(u) = 2 \int_0^{\infty} \frac{\partial_u^2(\sqrt{\beta} r \sin\bar{\theta})}{\sqrt{\beta} r \sin\bar{\theta}} \quad \Longrightarrow \quad \frac{E}{2} c_y = Q_\perp(s, b)$$

$$c_0 = \int_{-\infty}^{+\infty} du \mathcal{G}_0(u) = 2 \int_0^{\infty} \frac{\partial_u^2 \sqrt{\beta r^2 - b^2 \alpha}}{\sqrt{\beta r^2 - b^2 \alpha}} \quad \Longrightarrow \quad \frac{E}{2} c_0 = Q_\parallel(s, b)$$

High-energy limit of the string propagator

Working in Fermi coordinates it is possible to establish a precise dictionary to compare the amplitudes derived from the σ-model in the curved background and those obatined by the eikonal resummation of string amplitudes in flat space in the presence of D-branes.

It is also possible to include higher-order terms in the string coordinates, adapting to Fermi coordinates a recursive algorithm developed by Sunil Mukhi (1986) for the expansion in Riemann coordinates.

Using Mukhi's method one can for instance derive the full leading eikonal operator (i.e. including all the corrections in α') from the worldsheet σ-model.

Work in progress with Alessio Caddeo.

The eikonal operator

Existence and form of the eikonal operator deduced from the elastic amplitude. An inclusive sum over the intermediate states.

Natural interpretation: Hilbert space of the string quantized in a light-cone gauge aligned to the collision axis.

This can be proved in two different ways

- Light-cone derivation: Regge limit of the three-string vertex of Green and Schwarz
- Covariant derivation: Reggeon vertex operator

GD, Di Vecchia, Russo and Veneziano, 2013

The Reggeon vertex operator

Elegant description in terms of an effective string state: the Reggeon

$$\mathcal{A}(s,t) \sim \Pi_R^{D_p}\, C_{12R}\, \bar{C}_{12R}\ .$$

- Factorized form for the four (two) point amplitudes
- Process independent propagator (tadpole)
- Evaluation of three-point couplings

Reggeon tadpole

$$\Pi_R^{D_p} = \mathcal{A}_1(s,t) = \frac{\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})}\, R_p^{7-p}\, \Gamma\left(-\frac{\alpha' t}{4}\right) \mathrm{e}^{-i\pi\frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}}$$

Three-point coupling with the Reggeon

$$C_{S_1,S_2,R} = \left\langle V_{S_1}^{(-1)} V_{S_2}^{(0)} V_R^{(-1)} \right\rangle = \epsilon_{\mu_1\ldots\mu_r}\ \zeta_{\nu_1\ldots\nu_s}\ T_{S_1,S_2,R}^{\mu_1\ldots\mu_r;\nu_1\ldots\nu_s}$$

The Reggeon vertex operator

The Reggeon vertex is a superconformal primary of dimension one half in the high-energy limit Ademollo, Bellini, Ciafaloni (1989) Brower, Polchinski, Strassler, Tan (2006)

Reggeon vertex operator (picture (-1))

$$V_R^{(-1)} = \frac{\psi^+}{\sqrt{\alpha'}E}\left(\sqrt{\frac{2}{\alpha'}}\frac{i\partial X^+}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}} \mathrm{e}^{-iqX} \; .$$

Reggeon vertex operator (picture (0))

$$\begin{aligned} V_R^{(0)} &= \left[-\frac{2}{\alpha'}\frac{\partial X^+ \partial X^+}{\alpha' E^2} - iq\psi\frac{\psi^+\partial X^+}{\alpha' E^2} - \frac{\alpha' t}{4}\frac{\psi^+\partial\psi^+}{\alpha' E^2}\right] \\ & \left(\sqrt{\frac{2}{\alpha'}}\frac{i\partial X^+}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}-1} \mathrm{e}^{-iqX_L} \end{aligned}$$

Absorption of closed strings by D-branes

In the background of a Dp-brane a closed string can split turning itself into an open string.
Initial closed string at level N_c

$$p_c = (E, \vec{0}_p, \vec{p}_c) \ , \qquad \alpha' M^2 = 4(N_c - 1) \ .$$

Final open string at level n

$$p_o = (-m, \vec{0}_p, \vec{0}_{25-p}) \ , \qquad \alpha' m^2 = n - 1 \ .$$

The transition is possible if

$$\alpha' \vec{p}_c^{\,2} = n - 4N_c + 3 \geq 0 \ .$$

The Chan-Paton factor corresponds to the $U(1)$ in the decomposition $U(N) = U(1) \times SU(N)$.

The closed-open vertex

The closed-open light-cone vertex describes the transition from an arbitrary closed string to an arbitrary open string

Cremmer and Scherk (1972), Clavelli and Shapiro (1973), Green and Schwarz (1984), Shapiro and Thorn (1987), Green and Wai (1994).

Same ideas and techniques used to derive the vertex for three open strings

- Path integral (Mandelstam, 1973)
- Operator solution of the continuity conditions for the string coordinates and supersymmetry(Green and Schwarz,1983)
- Calculation of the three-point couplings of DDF operators (Ademollo, Del Giudice, Di Vecchia and Fubini, 1974)

Historically, showing that Mandelstam $\leftrightarrow$ ADDF was important to realize that

Dual models $\leftrightarrow$ Theory of interacting relativistic strings

The closed-open vertex

Chosen two light-cone directions $e^{\pm}$, the vertex is given by

$$|V_B\rangle = \beta \exp\left[\sum_{r,s=1}^{3}\sum_{k,l=1}^{\infty}\frac{1}{2}A_{-k}^{r,i}N_{kl}^{rs}A_{-l}^{s,i} + \sum_{r=1}^{3}\sum_{k=1}^{\infty}P^i N_k^r A_{-k}^{r,i}\right]\prod_{r=1}^{3}|0\rangle^{(r)} .$$

The index i runs along the $d-2$ directions orthogonal to $e^{\pm}$
The index $r = 1, 2 \rightarrow$ left, right parts of the closed string
The index $r = 3 \rightarrow$ open string

$$p^{(1)} = \frac{p_c}{2} , \qquad p^{(2)} = \frac{Dp_c}{2} , \qquad p^{(3)} = p_o ,$$

$$p_c = (E, \vec{0}_p, \vec{p}_t, p) , \qquad p_o = (-m, \vec{0}_p, \vec{0}_{24-p}, 0) .$$

$$A_k^{1,i} = A_k^i , \qquad A_k^{2,i} = (D\bar{A}^i)_k , \qquad A_k^{3,i} = a_k^i .$$

The closed-open vertex

The quantities α_r are given by

$$\alpha_r = 2\sqrt{2\alpha'}(e^+ p^{(r)}) \ , \qquad r = 1, 2, 3 \ , \qquad \alpha_1 + \alpha_2 + \alpha_3 = 0 \ .$$

We also have

$$P_i \equiv \sqrt{2\alpha'} \left[\alpha_r p_i^{(r+1)} - \alpha_{r+1} p_i^{(r)} \right] \ .$$

The Neumann matrices N in the vertex are

$$N_k^r = -\frac{1}{k\alpha_{r+1}} \begin{pmatrix} -k\frac{\alpha_{r+1}}{\alpha_r} \\ k \end{pmatrix} = \frac{1}{\alpha_r k!} \frac{\Gamma\left(-k\frac{\alpha_{r+1}}{\alpha_r}\right)}{\Gamma\left(-k\frac{\alpha_{r+1}}{\alpha_r} + 1 - k\right)} \ ,$$

$$N_{kl}^{rs} = -\frac{kl\alpha_1\alpha_2\alpha_3}{k\alpha_s + l\alpha_r} N_k^r N_l^s \ .$$

The closed-open vertex

Light-cone gauge lying in the branes worldvolume ($p \geq 1$)

$$e^{+} = \frac{1}{\sqrt{2}}(-1, 1, \ldots, 0, 0) \ , \qquad e^{-} = \frac{1}{\sqrt{2}}(1, 1, \ldots, 0, 0) \ .$$

We find

$$\alpha_1 = \alpha_2 = \sqrt{\alpha'} E \ , \qquad \alpha_3 = -2\sqrt{\alpha'} E \ ,$$

and

$$P_i = \alpha_3 \sqrt{\frac{\alpha'}{2}}\, p_{c,i} \ .$$

This is the form usually displayed in the literature.

The closed-open vertex

Light-cone gauge along the direction of collision

$$e^{+} = \frac{1}{\sqrt{2}}(-1, 0, \ldots, 0, 1) \ , \qquad e^{-} = \frac{1}{\sqrt{2}}(1, 0, \ldots, 0, 1) \ .$$

We find

$$\begin{aligned}
\alpha_1 &= \sqrt{\alpha'}\,(E + p) = \sqrt{n-1} + \sqrt{n-1-4\omega} \ , \\
\alpha_2 &= \sqrt{\alpha'}\,(E - p) = \sqrt{n-1} - \sqrt{n-1-4\omega} \ , \\
\alpha_3 &= -2\sqrt{\alpha'}E = -2\sqrt{n-1} \ ,
\end{aligned}$$

where

$$\omega = \frac{\alpha'}{4}(M^2 + \vec{p}_t^{\,2}) = N_c - 1 + \frac{\alpha' \vec{p}_t^{\,2}}{4} \ .$$

The closed-open vertex

Closed-open couplings and discontinuity from the vertex

$$B_{\psi\chi} = \langle\chi|V_{\vec{p}_t}|\psi\rangle \ , \qquad \mathrm{Im}A_{\psi\psi}(s,t) = \pi\alpha'\langle\psi_2|V^\dagger_{\vec{q}/2}V_{\vec{q}/2}|\psi_1\rangle \ .$$

At high energy (large level limit)

$$V_{\vec{p}_t} \sim \mathcal{V}_{\vec{p}_t} \ , \qquad n \gg 1 \ .$$

$$\mathcal{B}_{\psi\chi} = \langle\chi|\mathcal{V}_{\vec{p}_t}|\psi\rangle \ , \qquad \mathrm{Im}\mathcal{A}_{\psi\psi}(s,t) = \pi\alpha'\langle\psi|\mathcal{V}^\dagger_{\vec{0}}\mathcal{V}_{\vec{q}}|\psi\rangle \ .$$

In impact parameter space

$$\widetilde{\mathcal{V}}_{\vec{b}} = \int \frac{d^{24-p}q}{(2\pi)^{24-p}}\, e^{-i\vec{b}\vec{q}}\,\mathcal{V}_{\vec{q}} \ .$$

$$\mathcal{B}_{\psi\chi}(\vec{b}) = \langle\chi|\widetilde{\mathcal{V}}_{\vec{b}}|\psi\rangle \ , \qquad \mathrm{Im}\mathcal{A}_{\psi\psi}(s,\vec{b}) = \pi\alpha'\langle\psi|\mathcal{V}^\dagger_0\widetilde{\mathcal{V}}_{\vec{b}}|\psi\rangle \ .$$

The closed-open vertex

When n tends to infinity, the coefficients quadratic in the open modes behave like

$$N_{kl}^{33} \quad \sim \quad -\frac{\omega}{n}\frac{1}{k+l}\,\frac{k^{-\omega\frac{k}{n}}}{\Gamma\left(1-\omega\frac{k}{n}\right)}\,\frac{l^{-\omega\frac{l}{n}}}{\Gamma\left(1-\omega\frac{l}{n}\right)}\ .$$

If the ratios k/n and l/n tend to zero this is

$$N_{kl}^{33} \sim -\frac{\omega}{n}\frac{1}{k+l}\ .$$

If k and l are of order n, setting $k = nx$ and $l = ny$ we find

$$N_{kl}^{33} \quad \sim \quad -\, n^{-\omega(x+y)-2}\,\frac{x^{-\omega x}y^{-\omega y}}{\Gamma(1-\omega x)\Gamma(1-\omega y)}\frac{\omega}{x+y}\ .$$

The coefficients N_{kl}^{13} (coupling the open modes and the left modes of the closed string) are enhanced by an additional power of n when $k = l$.

Absorption of a tachyon

Let us start from the absorption of a tachyon with $\vec{p}_t = 0$

$$\mathcal{V}_{\vec{0}}|0\rangle \;\; = \;\; \beta\, \mathcal{P}_n\, e^{\frac{1}{2}\sum_{k,l} N^{33}_{kl} a^i_{-k} a^i_{-l}} |0\rangle \; .$$

Series representations for the open state

$$\mathcal{V}_{\vec{0}}|0\rangle = \beta \sum_{Q=0}^{\infty} \frac{1}{Q!2^Q} \mathcal{P}_n \prod_{\alpha=1}^{Q} \sum_{k_\alpha, l_\alpha} N^{33}_{k_\alpha l_\alpha} a^{i_\alpha}_{-k_\alpha} a^{i_\alpha}_{-l_\alpha} |0\rangle \; ,$$

and for the imaginary part of the elastic amplitude

$$\mathrm{Im}\mathcal{A}_{TT} = \pi\alpha'\beta^2 \sum_{Q=0}^{\infty} \frac{1}{(Q!)^2 2^{2Q}} \sum_{\{k_\alpha, l_\alpha\}} \prod_{\alpha=1}^{Q} \left(N^{33}_{k_\alpha l_\alpha}\right)^2$$

$$\langle 0| \prod_{\alpha=1}^{Q} a^{i_\alpha}_{k_\alpha} a^{i_\alpha}_{l_\alpha} \prod_{\alpha=1}^{Q} a^{i_\alpha}_{-k_\alpha} a^{i_\alpha}_{-l_\alpha} |0\rangle \; , \qquad \sum_{\alpha=1}^{Q} (k_\alpha + l_\alpha) = n \; .$$

Absorption of a tachyon

The first few terms give an accurate representation of the state

$$\begin{aligned} \mathrm{Im}\mathcal{A}_{TT} &\sim \pi\alpha'\beta^2 \sum_{\substack{k,l \\ k+l=n}} \frac{1}{4} \left(N^{33}_{kl}\right)^2 \langle 0| a^i_k a^i_l a^j_{-k} a^j_{-l} |0\rangle \\ &= \pi\alpha'\beta^2 \sum_{\substack{k,l \\ k+l=n}} 12 \left(N^{33}_{kl}\right)^2 kl \ , \end{aligned}$$

$$\mathrm{Im}\mathcal{A}_{TT} \sim 12\,\pi\alpha'\beta^2\, n \int_0^1 dx \int_0^1 dy \frac{x^{2x+1} y^{2y+1} \delta(x+y-1)}{\Gamma^2(1+x)\Gamma^2(1+y)} \ .$$

We find

$$\mathrm{Im}\mathcal{A}_{TT} \sim 0.929\,\pi\alpha'\beta^2\, n \ .$$

Absorption of a tachyon

Including terms with four oscillators we find

$$\mathrm{Im}\mathcal{A}_{TT} \sim 0.998\,\pi\alpha'\beta^2\,n \ .$$

Therefore only 0.2% of the full forward imaginary part is left to terms with six or more oscillators. This is strong evidence that the series converges rapidly and that

$$\langle 0|e^{Z_0^\dagger}\mathcal{P}_n e^{Z_0}|0\rangle = n \ .$$

Classical solutions

- Intuitive picture of the absorption process: the closed string hits the branes and it is turned into an open string that stretches along the direction of motion of the closed string.
- Since the open string cannot have a momentum zero-mode in a direction orthogonal to the branes, the momentum p of the closed string is turned into open string excitations (these can carry, at any given time, a net transverse momentum)
- The string performs a periodic motion converting, as it stretches and contracts, its kinetic energy into tensile energy and vice versa.

Classical solutions

- It is convenient to choose the same range $0 \leq \sigma \leq \pi$ for the worldsheet coordinate σ of the closed and the open string and work in a static gauge.
- We will assume that the closed-open transition happens at $\tau = 0$ and require continuity of the string coordinates and their first time derivatives at $\tau = 0$.
- We denote with X^a, $a = 1, ..., p$ the directions parallel to the branes, with $Z = X^{25}$ the direction of collision and with X^i, $i = p + 1, ..., 24$ the remaining coordinates orthogonal to the branes.
- The position of the branes in the transverse directions is $X^i = Z = 0$.
- Without loss of generality we can choose a frame where the only non vanishing components of the closed momentum are $p^0 = E$ and $p^Z = p$.

Classical solutions

Head-on collision at zero impact parameter of a closed string without excitations

$$X^0 = 2\alpha' E\tau \ , \qquad Z = 2\alpha' E\tau \ , \qquad \tau < 0 \ .$$

The open string solution at $\tau > 0$ is

$$X^0 = 2\alpha' E\tau \ , \qquad Z = \alpha' E \left(W(\tau + \sigma) - W(\tau - \sigma) \right) \ ,$$

where $W(\xi)$ is a triangular wave with period 2π

$$W(\xi) = \xi \ , \quad \xi \in [0, \pi] \ , \qquad W(\xi) = 2\pi - \xi \ , \quad \xi \in [\pi, 2\pi] \ ,$$

whose Fourier series is

$$W(\xi) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \cos(2k+1)\xi \ .$$

Classical solutions

The open string undergoes a periodic motion in the Z direction with period 2π. When $\tau \in [0, \pi/2]$

$$Z(\sigma,\tau) = \begin{cases} 2\alpha' E\sigma & 0 \leq \sigma \leq \tau \\ 2\alpha' E\tau & \tau \leq \sigma \leq \pi - \tau \\ 2\alpha' E(\pi - \sigma) & \pi - \tau \leq \sigma \leq \pi \end{cases} .$$

The sum $(\dot{Z})^2 + (Z')^2$ remains constant all along the open string and equal to $(\alpha' E)^2$.
At a generic positive $\tau < \pi/2$ the points of the string at $0 < \sigma < \tau$ and $\pi - \tau < \sigma < \pi$ have stretching energy (Z') and no kinetic energy $(\dot{Z})$.
The complementary points at $\tau < \sigma < \pi - \tau$ have kinetic but no stretching energy.

Classical solutions

When $\tau = \pi/2$ all the energy of the string is due to its stretching as it reaches the maximum extension in the Z direction, $Z_{\text{max}} = \pi\alpha' E$.
The average length of the string over a period of oscillation is

$$\langle L \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\tau \int_0^{\pi} d\sigma \sqrt{(\partial_\sigma Z)^2} = \pi\alpha' E \ .$$

The mean square distance of the points of the string from the position of the branes is

$$\langle \Delta Z^2 \rangle = \frac{1}{2\pi^2} \int_0^{2\pi} d\tau \int_0^{\pi} d\sigma \, Z^2(\tau, \sigma) = \frac{\pi^2}{6} \alpha'^2 E^2 \ .$$

The way a D-brane absorbs an arbitrary incident energy is by converting it into the length of its open string excitations (at tree level).

Classical solutions

T-dual picture of the solution: both the branes and the initial closed string are wrapped around a circle. The initial energy is entirely due to the stretching of the closed string

$$E = \frac{wR}{\alpha'} \ .$$

At $\tau = 0$ the closed string splits into an open string satisfying Neumann boundary conditions

$$Z = \alpha' E \left(W(\tau + \sigma) + W(\tau - \sigma) \right) \ .$$

The T-dual picture makes it intuitive why the total decay rate

$$\Gamma = \frac{\mathrm{Im}\mathcal{A}}{2E} \sim E \ ,$$

grows linearly with the energy: it simply reflects the constant splitting probability per unit length of the closed string.

Conclusions

The string-brane system provides a convenient framework to address several problems in quantum gravity and to study the structure and symmetries of string theory

- emergence of an effective geometry from the scattering data
- existence of a unitary S-matrix for high-energy collisions
- microscopic description of the infall of a particle into a singularity

Some work in progress

- study less symmetric backgrounds (e.g. intersecting branes)
- study the open string state created by the absorption of the closed string at weak coupling
- derive a general formula for the phase shift in an asymptotically flat spacetime
- derive the full structure of the eikonal operator at the next order in $\frac{R^{7-p}}{b^{7-p}}$