Announcement:

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Sketching a Proof of the Maldacena Conjecture at Small Radius

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Outline

- At small AdS radius, AdS-CFT duality is perturbative on both sides:

$$\lambda_{tHooft} = g_{YM}^2 N = R^4$$

- Worldsheet action in pure spinor formalism can be written as

$$S_{AdS_5 \times S^5} = \int d^2\tau [Q\Psi + R^2 B]$$

where B is antisymmetric in worldsheet derivatives

- B is independent of worldsheet metric $\Rightarrow$ action is topological
- Claim 1: Topological action at zero radius describes free super-Yang-Mills. Regions of worldsheet near AdS boundary are propagators of Feynman diagram (ala ´t Hooft) and regions of worldsheet near AdS horizon are holes.
- Claim 2: Interaction term $R^2 \int d^2\tau B$ generates cubic super-Yang-Mills vertex in Feynman diagram.

Related approaches to small radius

- Closed-open duality:

Gopakumar-Vafa (1999), Ooguri-Vafa (2002), Gaiotto-Rastelli (2005)

- Topological strings:

Polyakov (2002), Berkovits-Vafa (2008), Berkovits (2009)

- String bits:

Berenstein-Maldacena-Nastase (2002), H. Verlinde (2003), Gopakumar (2004), Alday-David-Gava-Narain (2006), Bargheer-Caetano-Fleury-Komatsu-Vieira (2018)

- Twistor string:

Witten (2004), Maldacena (private communication)

Half-BPS vertex operators

- To study superstring at small radius, useful to start with half-BPS vertex operators which are independent of radius
- Parameterize $AdS_5 \times S^5$ with $g(\theta, X, Y) \in \frac{PSU(2,2|4)}{SO(4,2)\times SO(6)} \times \frac{SO(4,2)}{SO(4,1)} \times \frac{SO(6)}{SO(5)}$

$$g(\theta, X, Y) = F(\theta)G(X)H(Y)$$

$$F = \exp(q^R_J \theta^J_R + q^J_R \theta^R_J), \quad X^{RS} = G^R_{\tilde{R}} \sigma_6^{\tilde{R}\tilde{S}} G^S_{\tilde{S}}, \quad Y^{JK} = H^J_{\tilde{J}} \sigma_6^{\tilde{J}\tilde{K}} H^K_{\tilde{K}}$$

$R = 1$ to 4 and $J = 1$ to 4 are SO(4,2) and SO(6) spinor indices.
$\tilde{R} = 1$ to 4 and $\tilde{J} = 1$ to 4 are SO(4,1) and SO(5) spinor indices.

- In Type IIB pure spinor formalism, unintegrated vertex operators are

$$V = \lambda^\alpha_L \lambda^\beta_R A_{\alpha\beta}(x, \theta_L, \theta_R)$$

λ^α_L and λ^β_R are pure spinors satisfying $\lambda_L \gamma^m \lambda_L = \lambda_R \gamma^m \lambda_R = 0$
for $m = 0$ to 9

- Physical vertex operators satisfy QV=0

$$Q = \int dz \lambda_L^\alpha \nabla_{L\alpha} + \int d\bar{z} \lambda_R^\alpha \nabla_{R\alpha} = \int dz \eta_{\alpha\beta} \lambda_L^\alpha J_1^\beta + \int d\bar{z} \eta_{\alpha\beta} \lambda_R^\alpha \bar{J}_3^\beta$$

$$J = g^{-1}\partial_z g, \quad \bar{J} = g^{-1}\partial_{\bar{z}} g, \quad \eta_{\alpha\beta} = (\gamma^{01234})_{\alpha\beta}$$

- Under BRST transformation,

$$Qg = g[(\lambda_L + i\lambda_R)^{\tilde{R}}_{\tilde{J}} q^J_R + (\lambda_L - i\lambda_R)^{\tilde{J}}_{\tilde{R}} q^R_J]$$

- Under local SO(4,1) x SO(5) gauge transformations,

$(\lambda_L)^{\tilde{J}}_{\tilde{R}}$ and $(\lambda_R)^{\tilde{J}}_{\tilde{R}}$ transform as SO(4,1) x SO(5) spinors

- Convenient to define gauge-invariant

$$(\tilde{\lambda}_L)^J_R = G^J_{\tilde{J}} (H^{-1})^{\tilde{R}}_R (\lambda_L)^{\tilde{J}}_{\tilde{R}}, \quad (\tilde{\lambda}_L)^R_J = (G^{-1})^{\tilde{J}}_J H^R_{\tilde{R}} (\lambda_L)^{\tilde{R}}_{\tilde{J}}$$
$$(\tilde{\lambda}_R)^J_R = G^J_{\tilde{J}} (H^{-1})^{\tilde{R}}_R (\lambda_R)^{\tilde{J}}_{\tilde{R}}, \quad (\tilde{\lambda}_R)^R_J = (G^{-1})^{\tilde{J}}_J H^R_{\tilde{R}} (\lambda_R)^{\tilde{R}}_{\tilde{J}}$$

$$QF(\theta) = F(\theta)[(\tilde{\lambda}_L + i\tilde{\lambda}_R)^R_J q^J_R + (\tilde{\lambda}_L - i\tilde{\lambda}_R)^J_R q^R_J]$$

- Consider state dual to $Tr[(y_0^{JK}\Phi_{JK}(x))^n]$

  $y_0 \cdot y_0 = x_0 \cdot x_0 = 0$ where $x_0^{RS} = [\epsilon^{AB}, x^m \sigma_m^{A\dot{A}}, \epsilon^{\dot{A}\dot{B}}(x^m x_m)]$ for $A, \dot{A} = 1, 2$

  State carries $J = \Delta = n$ and is annihilated by 24 susy's with $J - \Delta \geq 0$.

- Corresponding half-BPS vertex operator in -1 picture is

$$V_{-1} = (\lambda_L \lambda_R)\ P\ (\frac{Y \cdot y_0}{X \cdot x_0})^n$$

where $(\lambda_L \lambda_R) \equiv \eta_{\alpha\beta}\lambda_L^\alpha \lambda_R^\beta$ and P is the picture-lowering operator

$$P = \prod_{a=1}^{8} \theta_+^a \delta(Q(\theta_+^a)) = \prod_{a=1}^{8} \theta_+^a \delta(\tilde{\lambda}_+^a)$$

and θ_+^a are the 8 θ's which carry $J - \Delta = 1$

$$\theta_+^a = [(x_0)^{RS}(y_0)_{JK}\theta_S^K, \quad (x_0)_{RS}(y_0)^{JK}\theta_K^S]$$

- To obtain vertex operator $V = \lambda_L^\alpha \lambda_R^\alpha A_{\alpha\beta}$ in zero picture, define

$$V = C\ V_{-1} \text{ where } C = \prod_{a=1}^{8} Q(\xi_a) = \prod_{a=1}^{8} Q(\Theta(\tilde{w}_a^+))$$

- Similar construction in flat background produces Type IIB supergravity vertex operator

Type IIB supergravity vertex operator in flat backround

- D=10 Type IIB supergravity described by superfield $\Phi(x^+, \theta_L, \theta_R)$ with momentum k_+ satisfying the constraints

$$D_-^a \Phi = \frac{\partial}{\partial \bar\theta_L^{\dot a}} \Phi = \frac{\partial}{\partial \bar\theta_R^{\dot a}} \Phi = 0, \quad (D_+)^4_{abcd} \Phi = \epsilon_{abcdefgh} (D_-)^4_{efgh} \bar\Phi$$

$$D_\pm^a = \frac{\partial}{\partial \theta_\pm^a} \pm i\theta_\pm^a \partial_+ \text{ where } \theta_\pm^a = \theta_L^a \pm \theta_R^a \text{ implies } \Phi = e^{ik_+(x^+ + i\theta_L^a \theta_R^a)} f(\theta_-^a)$$

- BRST-invariant vertex operator is $V = V_0 + V_1 + V_2 + V_3 + V_4$

  where $V_n = (k_+)^{-n} (\bar\lambda_L \sigma_{j_1 \ldots j_{2n}} \bar\lambda_R) (\nabla_+)^n_{j_1 \ldots j_{2n}} \Phi$

- Vertex operator for lowest component of Φ can be written as

$$V = (\bar\lambda_L \bar\lambda_R)\ C\ P\ e^{ik_+(x^+ + i\theta_L^a \theta_R^a)}, \quad C = \prod_{i=1}^8 Q(\xi_a^+), \quad P = \prod_{i=1}^8 \theta_+^a \delta(\lambda_+^a)$$

To evaluate, use $\xi\delta(\lambda) = \xi e^{-\phi} = \frac{1}{\lambda}$. Poles in λ are absent since QV=0.

- After adding n-1 picture-raising operators C and n-1 picture-lowering operators P, can write the half-BPS vertex operator as

$$V = (\lambda_L \lambda_R) \;\; C \;\; P \frac{Y \cdot y_0}{X \cdot x_0} \;\; C \;\; P \frac{Y \cdot y_0}{X \cdot x_0} \;\; \ldots \;\; C \;\; P \frac{Y \cdot y_0}{X \cdot x_0}$$

- Using fact that the general non-BPS state can be described at small coupling by a spin chain of super-Yang-Mills fields, it is natural to conjecture that the general vertex operator at small radius as

$$V = (\lambda_L \lambda_R) \;\; : \; C \;\; E_1 \;\; C \;\; E_2 \;\; \ldots \;\; C \;\; E_n \;\; :$$

  where E_i is obtained from $P \frac{Y \cdot y_0}{X \cdot x_0}$ by performing the appopriate PSU(2,2|4) transformation that takes $y_0^{JK} \Phi_{JK}(x)$ to the desired super-Yang-Mills field

- Normal ordering in non-BPS vertex operators should be defined to be invariant under cyclic permutation of $E_1 \; E_2 \; \ldots \; E_n$

Free super-Yang-Mills

- $\tilde{\lambda}^a_+ = (G(X)H^{-1}(Y)\lambda)^a_+$ is proportional to $\sqrt{z}$ where z is the distance to the AdS boundary
- Since E_i is proportional to $\delta(\tilde{\lambda}^a_+)$ and C is proportional to $\Theta(\tilde{w}^+_a)$,

$$V = (\lambda_L \lambda_R) : C(\sigma_1)E_1(\sigma_2)C(\sigma_3)E_2(\sigma_4)...C(\sigma_{2n-1})E_n(\sigma_{2n}) :$$

implies that the worldsheet needs to be near the AdS boundary at $\sigma = \sigma_{2m}$ and near the AdS horizon at $\sigma = \sigma_{2m+1}$

Claim:

- Regions of the worldsheet near the AdS boundary describe Feynman propagators and connect two E's on different spin chains
- Regions of the worldsheet near the AdS horizon describe holes in the Feynman diagram

Topological string

- At arbitrary radius R, worldsheet action in $AdS_5 \times S^5$ background is

$$S = R^2 \int d^2\tau (\frac{1}{2} J_2 \bar{J}_2 - \frac{3}{4} J_1 \bar{J}_3 - \frac{1}{4} J_3 \bar{J}_1 + \text{ghost} \quad \text{terms})$$

$$= R^2 \int d^2\tau (Q\Psi + B) \quad \text{where}$$

$$Q\Psi = -\frac{\partial x^m \bar{\partial} x_m}{z^2} + ..., \qquad B = J_1 \wedge J_3 + \frac{(\lambda_L \gamma_{mn} \lambda_R)}{(\lambda_L \lambda_R)} dx^m \wedge dx^n + ...$$

$$\Psi = (\lambda_L \lambda_R)^{-1} [\lambda_L \gamma_m J_3) \bar{J}_2^m - (\lambda_R \gamma_m \bar{J}_1) J_2^m + ...]$$

- Coefficient Λ of BRST-trivial term $Q\Psi$ can be made large so that

$$S = \Lambda \int d^2\tau (-\frac{\partial x^m \bar{\partial} x_m}{z^2} + ...) + R^2 \int d^2\tau B$$

implies that when z is finite (away from horizon), x^m is frozen.

- At R=0, string is "topological" with regions of constant value of x^m near the AdS boundary (propagators) which are separated by discontinuities where worldsheet is near the AdS horizon (holes)

- Contribution from a region near boundary with r scalar fields is

$$\mathcal{A} = \int d^4x \int d^{11}\lambda \int d^{16}\theta E_1 E_2 \ldots E_r \text{ where } E_r = \frac{Y\cdot y_{0r}}{X\cdot x_{0r}} \prod_{a=1}^{8} \theta_+^a \delta(\lambda_+^a)$$

- Integration over θ zero modes implies r=2 and one finds as desired

$$\mathcal{A} \sim \frac{y_{01}\cdot y_{02}}{x_{01}\cdot x_{02}} = \frac{y_{01}\cdot y_{02}}{(x_1 - x_2)^2}$$

- Non-planar diagrams at genus g contribute
  $g_S^{2g-2} = (g_{YM})^{4g-4} \sim N^{2-2g}$ as expected from the 't Hooft expansion

Cubic super-Yang-Mills vertex

- Cubic super-Yang-Mills action (m,n=0 to 9)

$$S = Tr \int d^4x \; [\; (\partial_{[m} A_{n]} + D_{mn}) D^{mn} + \psi^\alpha \gamma^m_{\alpha\beta} \partial_m \psi^\beta$$

$$+ R^2 (\psi^\alpha \gamma^m_{\alpha\beta} [A_m, \psi^\beta] + D_{mn}[A^m, A^n]) \;]$$

- Claim: $R^2 \int d^2\tau B = R^2 \int d^2\tau [J_1 \wedge J_3 + \frac{(\lambda_L \gamma_{mn} \lambda_R)}{(\lambda_L \lambda_R)} dx^m \wedge dx^n + ...]$

  reproduces cubic vertex of super-Yang-Mills action
- Antisymmetric terms in B become commutators

$$\int d^2\tau f (\partial_{\tau_1} g \partial_{\tau_2} h - \partial_{\tau_2} g \partial_{\tau_1} h) \;\; \to \;\; Tr(f[g,h])$$

$$\int d^2\tau J_1 \wedge J_3 = \int d^2\tau (\theta \gamma^m)_\alpha dx_m \wedge d\theta^\alpha + ... \to Tr \int d^4x (\psi \gamma^m)_\alpha [A_m, \psi^\alpha]$$

$$\int d^2\tau \frac{(\lambda_L \gamma^{mn} \lambda_R)}{(\lambda_L \lambda_R)} dx_m \wedge dx_n + ... \to Tr \int d^4x D^{mn} [A_m, A_n]$$

- Argument:

$$\int d\tau \int d\sigma [f(\partial_\tau g \partial_\sigma h - \partial_\sigma g \partial_\tau h)]$$

$$= \Delta\tau \Delta\sigma f(\tau,\sigma)[\frac{g(\tau+\Delta\tau,\sigma)-g(\tau,\sigma)}{\Delta\tau}\frac{h(\tau,\sigma+\Delta\sigma)-h(\tau,\sigma)}{\Delta\sigma}$$

$$-\frac{g(\tau,\sigma+\Delta\sigma)-g(\tau,\sigma)}{\Delta\sigma}\frac{h(\tau+\Delta\tau,\sigma)-h(\tau,\sigma)}{\Delta\tau}]$$

$$= \langle f(\sigma_1)(g(\sigma_2)-g(\sigma_1))(h(\sigma_3)-h(\sigma_1)) - f(\sigma_1)(g(\sigma_3)-g(\sigma_1))(h(\sigma_2)-h(\sigma_1))\rangle$$

$$= \langle f(\sigma_1)g(\sigma_2)h(\sigma_3) - f(\sigma_1)h(\sigma_2)g(\sigma_3)\rangle$$

where $\sigma_1 \leq \sigma_2 \leq \sigma_3$ are cyclically ordered.

- To find contribution of terms in B, compute closed-open disk amplitude with one closed vertex operator B and 3 open string super-Yang-Mills vertex operators

$$V_{open} = \int d\tau [A_m(x)\partial x^m + \psi^\alpha(x) d_\alpha + F_{mn}(x)(w\gamma^{mn}\lambda) + ...]$$

- Term $\int d^2\tau (\theta\gamma^m)_\alpha dx_m \wedge d\theta^\alpha \to (\theta\gamma^m)_\alpha [x_m, \theta^\alpha]$
generates cubic vertex $Tr \int d^4x (\psi\gamma^m)_\alpha [A_m, \psi^\alpha]$

- Term $\int d^2\tau \frac{(\lambda_L\gamma^{mn}\lambda_R)}{(\lambda_L\lambda_R)} dx_m \wedge dx_n \to \frac{(\lambda_L\gamma^{mn}\lambda_R)}{(\lambda_L\lambda_R)}[x_m, x_n]$
generates cubic vertex $Tr \int d^4x F^{mn}[A_m, A_m]$

- Conjecture that other terms in B do not contribute to vertex
- Reasonable conjecture since both $\int d^2\tau B$ and the cubic onshell super-Yang-Mills vertex are PSU(2,2|4) invariant.

Conclusions and Open Problems

Arguments were sketched here that super-YM is described by a topological action for the $AdS_5 \times S^5$ superstring where

1) Propagators are described by regions of worldsheet near the AdS boundary;

2) Holes are described by regions of worldsheet near the AdS horizon;

3) Cubic super-Yang-Mills vertex is described by insertions of B field.

To turn sketchy arguments into proof, need to better understand

1) Scattering amplitude prescription for topological string;
2) Treatment of $(\lambda_L \lambda_R)^{-1}$ factors in vertex operators and action;
3) Relation of topological string prescription and usual string prescription involving integration over worldsheet moduli.

Unintegrated vertex operator for radius modulus is $(\lambda_L \lambda_R)$ and integrated vertex operator for radius modulus is $\int d^2\tau B$

Conjecture:

Perhaps usual string prescription with integrated vertex operators is related to topological prescription with unintegrated vertex operators by pulling down a factor of $\int d^2\tau B$ for each integrated vertex operator

⇒ usual prescription = $(\frac{\partial}{\partial R^2})^n$ topological prescription