A PROPOSAL FOR THE CFT DUAL OF ADS3 AT THE STRING SCALE

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- REVIEW OF THE PERTURBATIVE ADS3 STRING SPECTRUM
- ADS3 AT THE STRING SCALE $\alpha' = L^2$
- THE FATE OF THE VACUUM
- A PHASE TRANSITION AT $\alpha' = L^2$?
- THE FULL NON-PERTURBATIVE SPECTRUM AT $\alpha' = L^2$?
- AN EXACT CFT DUAL?
- WHAT ABOUT SHORT STRINGS, SPECTRAL DENSITY, INTERACTIONS ETC.?
- A TENTATIVE ANSWER INSTEAD OF A CONCLUSION

PERTURBATIVE SPECTRUM OF ADS3 STRINGS (WZW, N=1)

WORLD-SHEET SIGMA MODEL

$$SL(2,R)_{k+2} \times SL(2,R)_{-2} \times \mathcal{N}$$

BOSONIC AFFINE ALGEBRA c=3+6/k

3 FREE FERMIONS c=3/2

c=15-9/2-6/k

THE SIGMA MODEL IS EXACT TO ALL ORDERS IN α'

IT IS VALID EVEN WHEN $k = L^2/\alpha'$ IS SMALL

SINCE $L_P/L \sim g_S^2$, THIS IS WHEN THE ADS3 RADIUS IS OF ORDER OF THE STRING SCALE, BUT STILL MUCH LARGER THAN THE PLANCK SCALE, FOR WEAKLY-COUPLED STRINGS

PERTURBATIVE SPECTRUM OF ADS3 STRINGS (WZW, N=1)

UNFLOWED REPRESENTATIONS ARE AFFINE ALGEBRA DESCENDANTS OF AFFINE PRIMARIES

NEW REPRESENTATIONS OBTAINED BY DEFINING:

$$J^3_n = \tilde{J}^3_n + \frac{k}{2} w \delta_{n,0}, \quad J^{\pm}_n = \tilde{J}^{\pm}_{n \mp w}, \quad L_n = \tilde{L}_n - w \tilde{J}^3_n - \frac{k}{4} w^2 \delta_{n,0}$$

THE (STANDARD) TILDED REPRESENTATION DEFINES A FLOWED REPRESENTATION OF THE (NON-TILDED) ALGEBRA

THE AFFINE PRIMARIES CAN BELONG TO EITHER LOWEST-WEIGHT OR PRINCIPAL CONTINUOUS REPRESENTATIONS OF SL(2,R)

THE PERTURBATIVE STRING SPECTRUM IS MADE OF THE FOLLOWING REPRESENTATIONS

1) LOWEST-WEIGHT REPRESENTATIONS OF THE BOSONIC AFFINE ALGEBRA:

DISCRETE REPRESENTATION $\mathcal{D}_j$

$\mathcal{C}^{\alpha}_{j=1/2+is}$ CONTINUOUS REPRESENTATION

2) REPRESENTATIONS OF THE FERMIONIC AFFINE AFFINE ALGEBRA:

FOCK SPACE OF THE THREE FREE FERMIONS, WITH EITHER NS OR R BOUNDARY CONDITIONS

REPEAT FOR LEFT MOVERS

3) ALL THE REPRESENTATIONS OBTAINED BY LEFT-RIGHT SYMMETRIC SPECTRAL FLOW (w=INTEGER) OF THE ABOVE

THE PHYSICAL STATE CONDITION IN THE NS SECTOR IS

$$L_0 - 1/2 = 0 \to \tilde{L}_0 - wJ_0 + kw^2/4 - 1/2 = 0$$

IN THE R SECTOR IT IS

$$L_0 = 0 \to \tilde{L}_0 - wJ_0 + kw^2/4 = 0$$

THE TILDED VARIABLES ARE VIRASORO GENERATORS OBTAINED BY THE STANDARD SUGAWARA CONSTRUCTION OUT OF TILDED CURRENTS.

THE REPRESENTATIONS OF THE TILDED CURRENTS ARE STANDARD LOWEST-WEIGHT REPRESENTATIONS OF THE AFFINE ALGEBRA. SO:

$$\tilde{L}_0 = -\frac{j(j-1)}{k} + N + h$$

THE SPACE-TIME ENERGY AND SPIN OF STRING STATES ARE

$$E = J_0 + \bar{J}_0^3, \qquad s = J_0 - \bar{J}_0^3$$

$$\tilde{L}_0 = -\frac{j(j-1)}{k} + N + h$$

CONFORMAL WEIGH IN $\mathcal{N}$

LEVEL IN CURRENT ALGEBRA

$$N = \frac{1}{k}\left[\sum_{m=1}^{\infty} j^{+}_{-m} j^{-}_{m} + j^{-}_{-m} j^{+}_{m} - 2 j^{3}_{-m} j^{3}_{m} + \sum_{r>0} r(\psi^{+}_{-r}\psi^{-}_{r} + \psi^{-}_{-r}\psi^{+}_{r} - 2\psi^{3}_{r}\psi^{3}_{r})\right]$$

THE GSO PROJECTION DOES NOT ELIMINATE ANY CONFORMAL WEIGHT IN THE R SECTOR. IN THE NS SECTOR IT DOES BECAUSE IT SAYS

$$N + N_{\mathcal{N}} + (w+1)/2 \in \mathbb{Z}$$

THE SL(2,C) INVARIANT VACUUM OF THE TARGET SPACE BELONGS TO THE j=1 REPRESENTATION

MALDACENA AND OOGURI SHOWED THAT A MODULAR-INVARIANT, UNITARY STRING THEORY IS OBTAINED BY PERFORMING A SPECTRAL FLOW,
WITH ARBITRARY INTEGER w,
OF ALL THE PRINCIPAL CONTINUOUS REPRESENTATIONS BUT ONLY THE DISCRETE REPRESENTATIONS OBEYING

$$1/2 < j < (k+1)/2$$

WHEN $k<1$ THE SPACE-TIME VACUUM DOES NOT BELONG TO THE PHYSICAL SPECTRUM.
THIS IS ALSO TRUE OF BTZ STATES.
THESE FEATURES AND THE BEHAVIOR OF THE COUPLING TO DILATONS IN THE EFFECTIVE THEORY OF LONG STRINGS LED GIVEON, KUTASOV, RABINOVICI AND SEVER TO CONJECTURE IN hep-th/0503121 THAT STRING ON ADS3 UNDERGO A PHASE TRANSITION AT $k=1$

k=1 CORRESPONDS PHYSICALLY TO AN ADS3 RADIUS

$$\alpha' = L^2$$

GKRS PROPOSE THAT AT k<1 THE SPECTRUM OF STRING THEORY IS DOMINATED AT HIGH ENERGY BY WEAKLY INTERACTING LONG STRINGS INSTEAD OF BTZ BLACK HOLES. IN SUCH A REGIME, THE DENSITY OF STATES OBEYS CARDY'S FORMULA BUT WITH AN EFFECTIVE CENTRAL CHARGE

$$c_{eff} = c\left[1 - \frac{(k-1)^2}{k^2}\right] \leq c$$

UNITARITY OF WORLD-SHEET CFT CONSTRAINS $k \geq \frac{4}{7}$

k=1 APPEARS TO BE A PHASE TRANSITION POINT
SEVERAL INTERESTING FACTS POINT TOWARD IT:

- THE CONTINUOUS SPECTRUM OF LONG STRINGS BECOMES GAPLESS AND THE EFFECTIVE WORLD-SHEET CFT OF THE REDUCES TO THAT OF A FREE BOSON.

- THE EFFECTIVE COUPLING CONSTANT OF THE LONG-STRING WORLD-SHEET CFT SWITCHES FROM DIVERGING AT THE BOUNDARY OF ADS3 TO VANISHING. AT k=1 IT IS CONSTANT THROUGHOUT ADS3

- THE SPACE-TIME VACUUM IS NO LONGER NORMALIZABLE BUT ONLY PLANE-WAVE NORMALIZABLE. IT BECOMES PART OF THE CONTINUUM.

- BTZ STATES ALSO BECOME ONLY PLANE-WAVE NORMALIZABLE AND PART OF A CONTINUUM.

- THE CENTRAL CHARGE OF THE (PUTATIVE) DUAL CFT BECOMES EQUAL TO THE EFFECTIVE CENTRAL CHARGE COMPUTED BY ASSUMING THAT THE SPECTRUM IS DOMINATED AT HIGH ENERGY BY A GAS OF FREE LONG STRINGS INSTEAD OF BTZ BLACK-HOLE MICROSTATES.

ALL THESE FACTS MAKE IT PLAUSIBLE THAT THE NONPERTURBATIVE SPECTRUM OF STRING THEORY AT $k=1$ (THAT IS AT THE STRING RADIUS) IS MADE ONLY OF THE (PERTURBATIVE) STATES OF MALDACENA AND OOGURI.

THE EXACT KNOWLEDGE OF THE STRING SPECTRUM MAKES IT POSSIBLE TO CONJECTURE A CFT DUAL OF SUPERSTRINGS AT $k=1$ THAT DESCRIBES THE COMPLETE THEORY RATHER THAN JUST THE BPS SECTOR

THE SPECTRUM OF LONG STRINGS IN ADS3

$$(E+S)/2 = \frac{1}{w}\left(s^2 + N + N_{\mathcal{N}} + h_0 + \frac{a}{2}\right) + \frac{1}{4}\left(w - \frac{1}{w}\right)$$

$$(E-S)/2 = \frac{1}{w}\left(s^2 + \bar{N} + \bar{N}_{\mathcal{N}} + \bar{h}_0 + \frac{b}{2}\right) + \frac{1}{4}\left(w - \frac{1}{w}\right)$$

SUBJECT TO THE CONSTRAINT

$$N + N_{\mathcal{N}} + h_0 + \frac{a}{2} - \bar{N} - \bar{N}_{\mathcal{N}} - \bar{h}_0 - \frac{b}{2} \in w\left(\mathbb{Z} + \frac{a}{2} - \frac{b}{2}\right)$$

a,b=0 IN NS SECTOR, 1 IN R SECTOR

THIS IS THE SPECTRUM OF ONE-PARTICLE STATES. FOR SMALL STRING COUPLING CONSTANT g, THE SPECTRUM OF THE THEORY IS OBTAINED BY TENSORING ONE-PARTICLE STATES AND EITHER SYMMETRIZING OR ANTISYMMETRIZING ACCORDING TO THE SPACE-TIME STATISTICS OF THE PARTICLES

THE MULTIPLICITY OF ONE-PARTICLE STATES OBEYING THE PHYSICAL-STATE AND AUXILIARY CONDITIONS CAN BE FOUND BY THE METHOD USED BY MALDACENA AND OOGURI IN THE BOSONIC CASE

MALDACENA AND OOGURI SHOW THAT PHYSICAL STATES ARE IN ONE-TO-ONE CORRESPONDENCE WITH THE FOCK STATES OF ONE FREE BOSON

THE SUPERSYMMETRIC EXTENSION OF THEIR RESULT IS THAT PHYSICAL SUPERSTRING STATES IN ADS3 MAP ONE-TO-ONE TO STATES IN THE FOCK SPACE OF ONE FREE BOSON PLUS ONE FREE FERMION

A PROPOSAL FOR A DUAL CFT

RELATED TO WORK BY ARGURIO, GIVEON, SHOMER, GABERDIEL, GOPAKUMAR

ASSUME THAT WE KNOW INDEED THE COMPLETE, NONPERTURBATIVE SPECTRUM AT k=1.
WE CAN THEN CONJECTURE AN EXACT DUAL THAT REPRODUCES THE FULL SPECTRUM

WE PROPOSE THAT STRING THEORY AT k=1 ON

$$AdS_3 \times \mathcal{N}$$

IS DUAL TO THE PERMUTATION ORBIFOLD

$$(\mathbb{R} \times \mathcal{N})^N / S_N$$

THE STRING COUPLING CONSTANT AND THE NUMBER OF COPIES OF THE SEED THEORY ARE RELATED BY

$$g_S^2 = 1/N$$

TO BE PRECISE, WE WILL SHOW THAT THE PERMUTATION ORBIFOLD CFT EXACTLY MATCHES THE LONG-STRING SPECTRUM WHEN THE INTERNAL CFT IS

$$\mathcal{N} = S^1 \times SU(2)_2 \times SU(2)_2 = S^1 \times S^3 \times S^3$$

1 FREE BOSON + (1+3+3) FREE FERMIONS: c=9/2

THE TWISTED SECTORS OF THE ORBIFOLD ARE LABELED BY THE CONJUGACY CLASSES OF THE PERMUTATION GROUP OF N ELEMENTS. THESE ARE ASSOCIATED TO PARTITIONS OF N INTO POSITIVE INTEGERS

$$[g] = (1)^{M_1}(2)^{M_2}....(N)^{M_N}, \qquad \sum_{i=1}^{N} nM_n = N$$

THE HILBERT SPACE OF THE TWISTED SECTOR [g] IS THE INVARIANT SUBSPACE OF THE COMMUTANT OF [g]

$$H_g^{C_g} = \otimes_{n=1}^{N} S^{M_n} H_{(n)}^{Z_n}$$

FIRST CHECK OF THE CORRESPONDENCE

WE MAP A STATE CONTAINING

$$(M_1, M_2, .., M_N)$$

LONG STRINGS WITH WINDING NUMBERS w=1,2,..N

INTO THE HILBERT SPACE OF THE TWISTED SECTOR [g]

THE (ANTI)SYMMETRIZATION IN THE ORBIFOLD AGREES PRECISELY WITH THE (ANTI)SYMMETRIZATION OF MULTI-STRING STATES DICTATED BY SPIN-STATISTICS

A MORE DETAILED CHECK

THE SPECTRUM OF SINGLE-PARTICLE STATES IN THE LONG-STRING SECTOR AT WINDING NUMBER w=n MATCHES WITH

$$H_{(n)}^{Z_n}$$

TWISTED SECTOR (n): MAPS FROM WORLDSHEET

$$\mathbb{R} \times S^1, \qquad (\sigma, t) \sim (\sigma + \pi, t)$$

TO TARGET SPACE $\quad (\mathbb{R} \times \mathcal{N})^N / S_N$

$$X_j(\sigma + \pi) = X_{j+1}(\sigma), \;\; \psi_j(\sigma + \pi) = -\psi_{j+1}(\sigma), \qquad j + n \equiv j$$

FOR n ODD FERMIONS ARE NS, FOR n EVEN THEY ARE R

THE MAP IS FROM A CIRCLE n TIMES LONGER THAN THE WORLDSHEET CIRCLE SO THE CONFORMAL WEIGHTS ARE

$$h_n = \frac{h}{n} + \frac{c}{24}\left(n - \frac{1}{n}\right)$$

CASIMIR ENERGY

NOTICE THAT FOR n EVEN THE MINIMUM WEIGHT IS h=d/16, THE ENERGY OF THE R VACUUM OF d FERMIONS.

IN OUR CASE d=8 SO h=1/2

THE CONFORMAL WEIGHTS IN OUR ORBIFOLD ARE

$$h_n = \frac{1}{n}\left(\frac{1}{2}p^2 + N + N_{\mathcal{N}} + h_0\right) + \frac{1}{4}\left(n - \frac{1}{n}\right)$$
$$\bar{h}_n = \frac{1}{n}\left(\frac{1}{2}p^2 + \bar{N} + \bar{N}_{\mathcal{N}} + \bar{h}_0\right) + \frac{1}{4}\left(n - \frac{1}{n}\right)$$

THEY ARE EVIDENTLY VERY SIMILAR TO THE SPACE-TIME CONFORMAL WEIGHT IN ADS3 IN FACT THEY ARE IDENTICAL ONCE THE PROJECTION OVER INVARIANT STATE OF THE ORBIFOLD IS TAKEN INTO PROPER ACCOUNT

IT IS STRAIGHTFORWARD TO SHOW THAT THE PROJECTION OVER $\mathbb{Z}_n$ INVARIANT STATES IS

$$N + N_{\mathcal{N}} + h_0 - \bar{N} - \bar{N}_{\mathcal{N}} - \bar{h}_0 \in n(\mathbb{Z} + F/2 + \bar{F}/2)$$

WITH THE IDENTIFICATIONS

$$p = \sqrt{2}s, \qquad n = w$$

THE SPECTRUM AND MULTIPLICITIES OF STATES OF THE ORBIFOLD MATCHES EXACTLY THAT OF THE LONG STRINGS IF THE ORBIFOLD CONSTRAINT

$$N + N_{\mathcal{N}} + h_0 - \bar{N} - \bar{N}_{\mathcal{N}} - \bar{h}_0 \in n(\mathbb{Z} + F/2 + \bar{F}/2)$$

IS EQUIVALENT TO THE ADS3 STRING CONSTRAINTS

$$N + N_{\mathcal{N}} + (w+1)/2 \in \mathbb{Z}$$

$$N + N_{\mathcal{N}} + h_0 + \frac{a}{2} - \bar{N} - \bar{N}_{\mathcal{N}} - \bar{h}_0 - \frac{b}{2} \in w\left(\mathbb{Z} + \frac{a}{2} - \frac{b}{2}\right)$$

TO PROVE THIS WE MUST DISTINGUISH TWO CASES:
n ODD AND n EVEN

n ODD

THE FERMIONS OF THE SYMMETRIC ORBIFOLD ARE IN THE NS SECTOR SO THEIR GROUND STATE IS UNIQUE AND HAS ZERO ENERGY

EVEN NUMBER OF ORBIFOLD FERMIONS MAPS ONE-TO-ONE TO FULL NS SECTOR OF THE SUPERSTRING

$$d^I_{-1/2-m} d^J_{-1/2-n} |0\rangle_O \Leftrightarrow \psi^I_{-1/2-m} \psi^J_{-1/2-n} |0\rangle_S$$

THANKS TO THE GSO PROJECTION OF THE SUPERSTRING

$$(-1)^F = (-1)^{n+1} = 1 = (-1)^{\bar{F}}$$

ODD NUMBER OF ORBIFOLD FERMIONS MAPS ONE-TO-ONE TO R SECTOR OF THE SUPERSTRING

n ODD

$$d^I_{-1/2-m} d^J_{+1/2-n} d^k_{-1/2} |0\rangle_O \Leftrightarrow \psi^I_{-m} \psi^J_{-n} |K\rangle_S, \quad |K\rangle_S = 8_s$$

BY SO(8) TRIALITY

$$d^I_{-1/2-m} d^J_{+1/2-n} d^K_{-p-1/2} |0\rangle_O \Leftrightarrow \psi^I_{-m} \psi^J_{-n} \psi^K_{-p} |0\rangle_S, \quad |0\rangle_S = 8_c$$

BY SO(8) TRIALITY

WITH THESE IDENTIFICATIONS, THE ORBIFOLD PROJECTION OVER Z_n INVARIANT STATES AND THE LEFT-RIGHT LEVEL MATCHING CONDITION OF THE SUPERSTRING COINCIDE

n EVEN

ORBIFOLD FERMIONS ARE IN THE R SECTOR
WHEN THE SUPERSTRING FERMIONS ARE IN THE R SECTOR
IDENTIFY

$$|\alpha\rangle_S = |\alpha\rangle_O, \qquad d^I_{-n} = \psi^I_{-n}$$

THE ORBIFOLD IS NOT GSO PROJECTED SO AT EACH LEVEL IT HAS TWICE THE NUMBER OF STATES OF THE NS SECTOR OF THE SUPERSTRING

THE EXTRA STATES MATCH THE NS-SECTOR SUPERSYMMETRIC PARTNERS OF THE R-SECTOR SUPERSTRING STATES

AGAIN, WITH THESE IDENTIFICATIONS, THE ORBIFOLD PROJECTION OVER Z_n INVARIANT STATES AND THE LEFT-RIGHT LEVEL MATCHING CONDITION OF THE SUPERSTRING COINCIDE

SHORT STRINGS AND INTERACTIONS

- CAN WE NEGLECT SHORT STRING?
- ALWAYS? OR ONLY FOR SOME DYNAMICAL PROCESS?
- DO WE EVEN GET THE RANGE OF p RIGHT?
- DO WE GET THE RIGHT SPECTRAL MEASURE FOR THE CONTINUOUS SPECTRUM?
- CAN WE MATCH THREE- AND HIGHER POINT CORRELATION FUNCTIONS BETWEEN ORBIFOLD AND ADS3 SUPERSTRING?

CAN WE NEGLECT SHORT STRINGS?

IN SOME OBSERVABLES, YES, BECAUSE THEY HAVE A DISCRETE SPECTRUM AND THE CONTINUUM BEGINS AT ZERO ENERGY

EXAMPLE: CANONICAL PARTITION FUNCTION

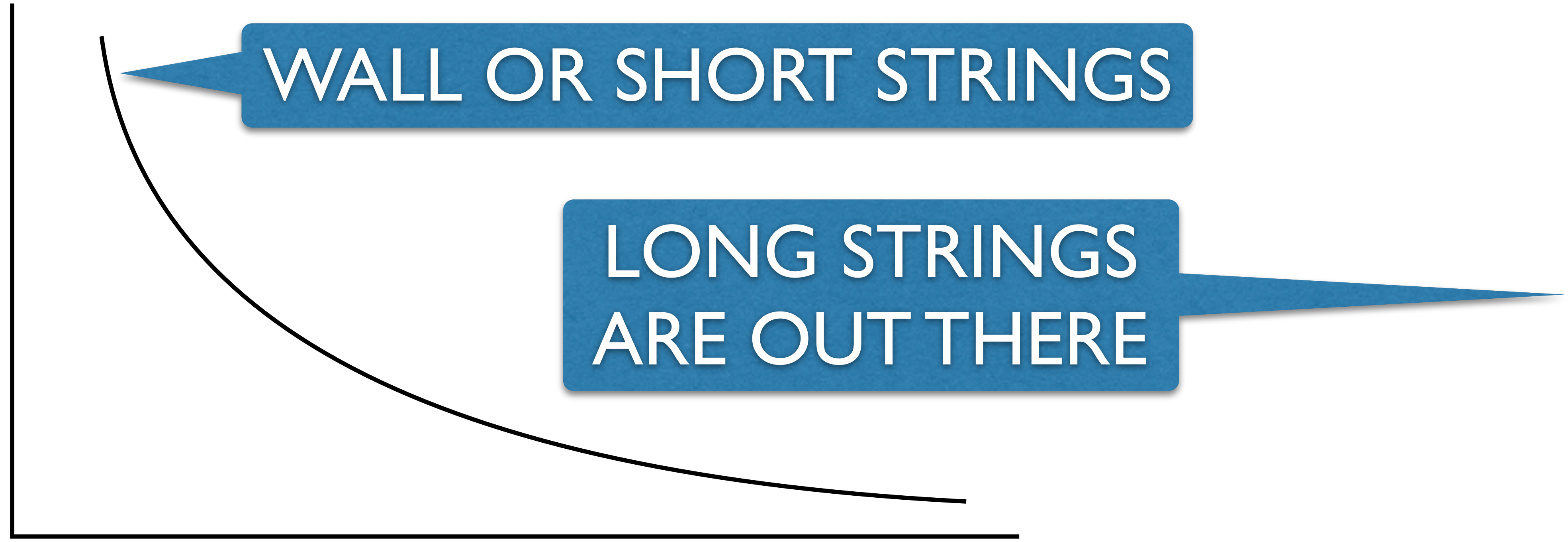
$$\exp(-\beta F) = \lim_{L\to\infty} L \int dE \rho(E) \exp(-\beta E) + \sum_j D_j \exp(-\beta E_j)$$

THIS TERM IS NEGLIGIBLE AT ALL TEMPERATURES BECAUSE L DIVERGES AND THE SPECTRUM IS GAPLESS

BETTER EXAMPLE: LONG STRINGS NEAR THE BOUNDARY

THEY TAKE INFINITE CFT TIME TO REACH THE BOUNDARY SO THEY BEHAVE AS PARTICLES IN A WALL POTENTIAL

$$\lim_{\rho\to+\infty} V(\rho) = 0, \qquad \lim_{\rho\to-\infty} V(\rho) = +\infty$$



FINITE-TIME LONG-STRING DYNAMICS IS INSENSITIVE TO THE EXISTENCE OF A WALL OR A SHORT STRING (CONFINED INSIDE ADS3). BOTH ARE AT DISTANCE

$$L \to +\infty$$

IT TAKES A TIME O(2L/v) v=RADIAL SPEED OF LONG STRING, TO DETECT THEM.

DYNAMICS FAR FROM THE WALL DEPENDS ONLY ON THE (UNIVERSAL) DIVERGENT PART OF THE SPECTRAL DENSITY

$$d\mu(p) = [L + \delta(p)]dp/2\pi$$

PRIMARY OPERATORS IN THE UNTWISTED STATES OF THE ORBIFOLD HAVE THE FORM

$$O = \exp(ipX)\hat{O}, \quad X \in \mathbb{R}, \quad \hat{O} \in S^1 \times SL(2,R)_{-2} \times SL(2,R)_{-2}$$

CORRELATORS OF SUCH OPERATORS VANISH UNLESS THE SUM OF MOMENTA IS ZERO

3-POINT CORRELATORS OF THE CORRESPONDING VERTICES IN STRING THEORY CAN BE COMPUTED

$$\langle V_{p_1} V_{p_2} V_{p_3} \rangle \propto (k-1)^{i\sum_j p_j}$$

THE SELECTION RULE IS RECOVERED IN THE DISTRIBUTIONAL SENSE, BUT ONLY IF THE VERTICES ARE NOT RENORMALIZED AS

$$V_p^R = (k-1)^{-ip} V_p$$

PROPOSAL AND CONCLUSIONS

- THE ORBIFOLD CFT DESCRIBES EXACTLY THE DYNAMICS OF LONG STRINGS NEAR THE BOUNDARY OF ADS3
- IT IS SUFFICIENT TO DETERMINE THE THERMODYNAMICS AND FINITE-TIME DYNAMICS OF LONG STRINGS, WHICH ARE BOTH INSENSITIVE TO SHORT STRINGS , TO THE RANGE IN p AND TO THE FINITE PART OF THE SPECTRAL DENSITY.
- TO GO BEYOND THIS APPROXIMATION WE NEED TO UNDERSTAND INTERACTIONS. THE ORBIFOLD SEEMS TO MATCH THE SUPERSTRINGS IN A SINGULAR LIMIT AT k=1.
- COMPUTING INTERACTIONS IS NECESSARY TO DISTINGUISH BETWEEN VARIOUS POSSIBILITIES FOR THE CONTINUOUS-SPECTRUM PART OF THE ORBIFOLD CFT:

$$\mathbb{R}, \quad \mathbb{R}/Z_2, \quad \text{Runkel-Watts } c=1 \text{ CFT}, \ldots.$$