GGI, 18.04.2019

Transplanckian collisions of particles, strings and branes: results & challenges

Gabriele Veneziano

COLLÈGE DE FRANCE 1530

CERN

Outline

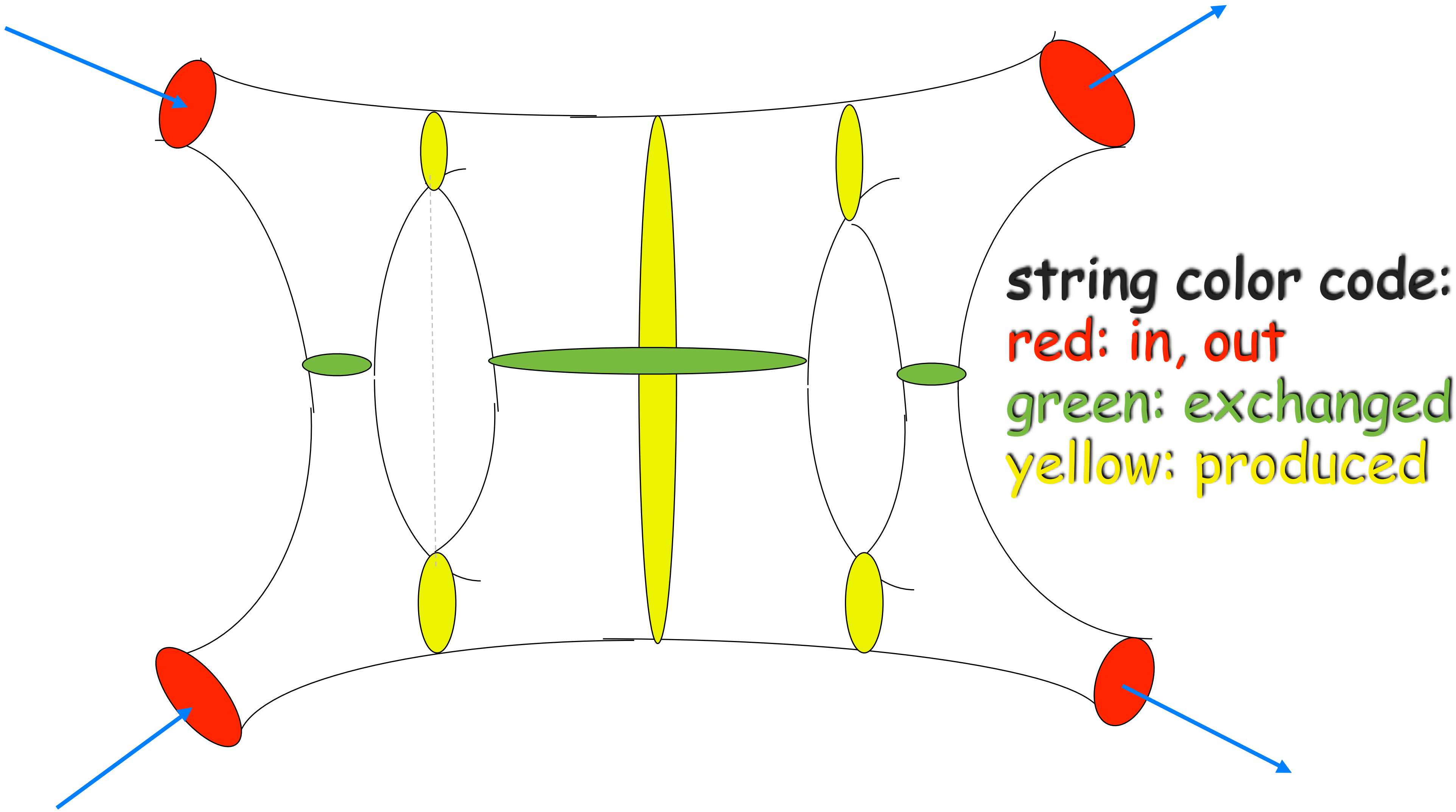
I: String (p.particle) collisions (1987->)

II: String-brane collisions (2010->)

III: Gravitational bremsstrahlung from ultra-relativistic collisions (2014->)

I: String (p.particle) collisions

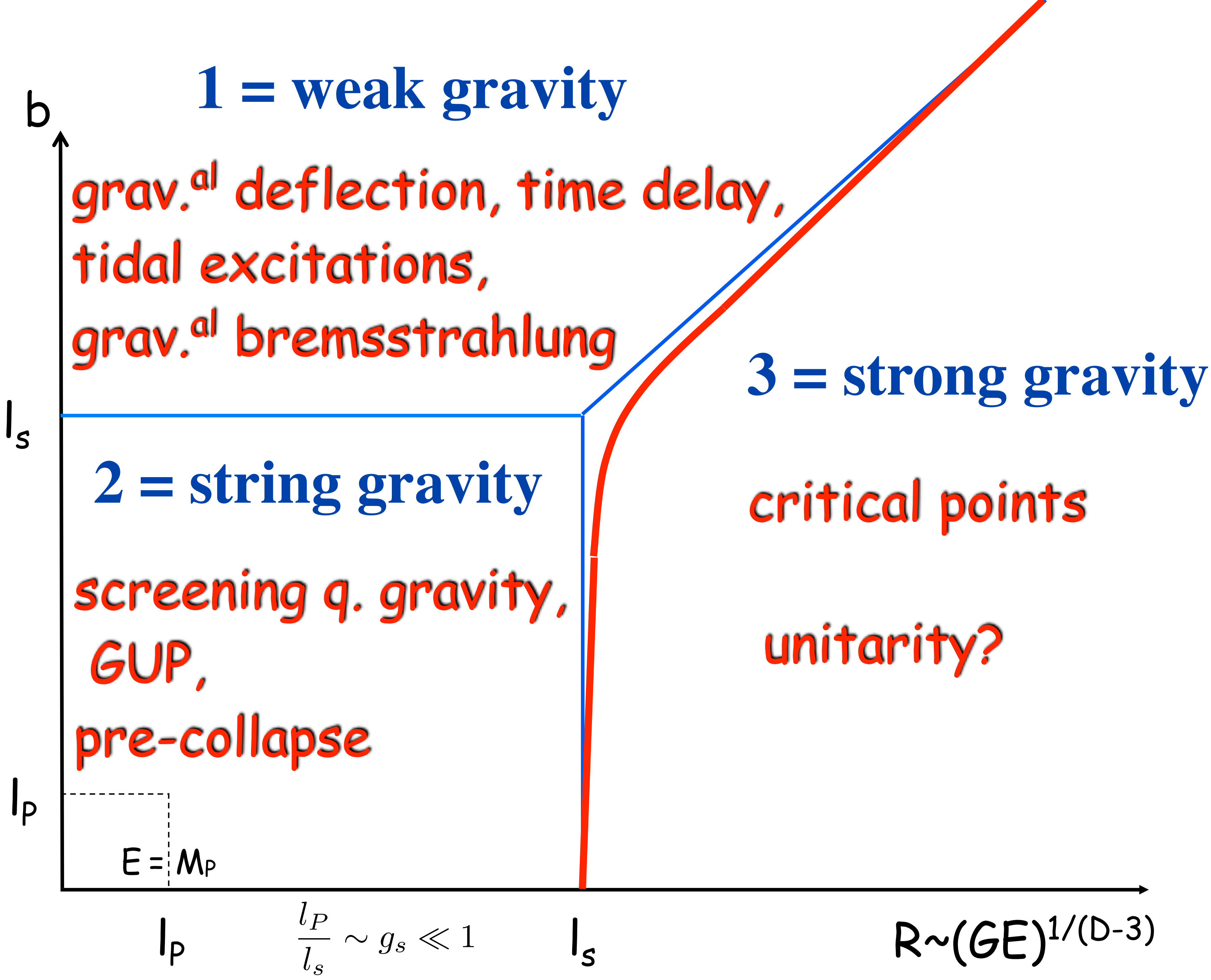
Transplanckian (closed)string-string collisions
(a two-loop contribution)



Parameter-space for string-string collisions @ $s \gg M_P^2$

$$b \sim \frac{2J}{\sqrt{s}} \quad ; \quad R_D \sim \left(G\sqrt{s}\right)^{\frac{1}{D-3}} \quad ; \quad l_s \sim \sqrt{\alpha' \hbar} \quad ; \quad G\hbar = l_P^{D-2} \sim g_s^2 l_s^{D-2}$$

- 3 relevant length scales (neglecting l_P @ $g_s \ll 1$)
- Playing w/s and g_s we can make R_D/l_s arbitrary
- Many different regimes emerge. Roughly:



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grav.^{al} deflection, time delay,
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critical points
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pre-collapse
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
$R \sim (GE)^{1/(D-3)}$

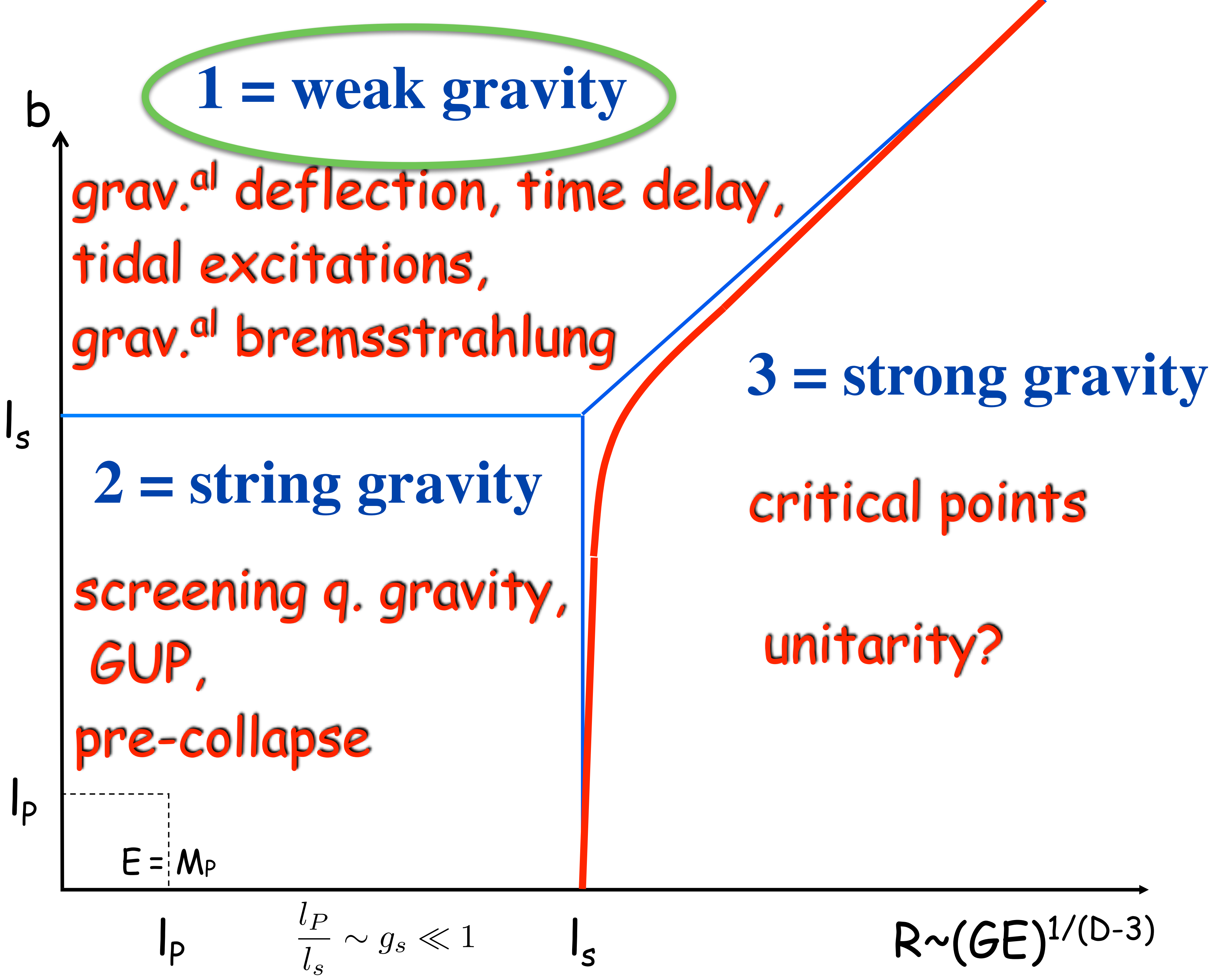
A semiclassical S-matrix @ high energy

(D is the number of large uncompactified dimensions, out of 10)

General arguments and explicit calculations suggest the following form for the TPE string-string elastic S-matrix:

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \ldots\right)$$

NB: Since leading term is real, for Im A_{cl} subleading terms may be more than just corrections. They give absorption ($|S_{el}| < 1$). This gives rise to subregions.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Results in region 1 (weak gravity)

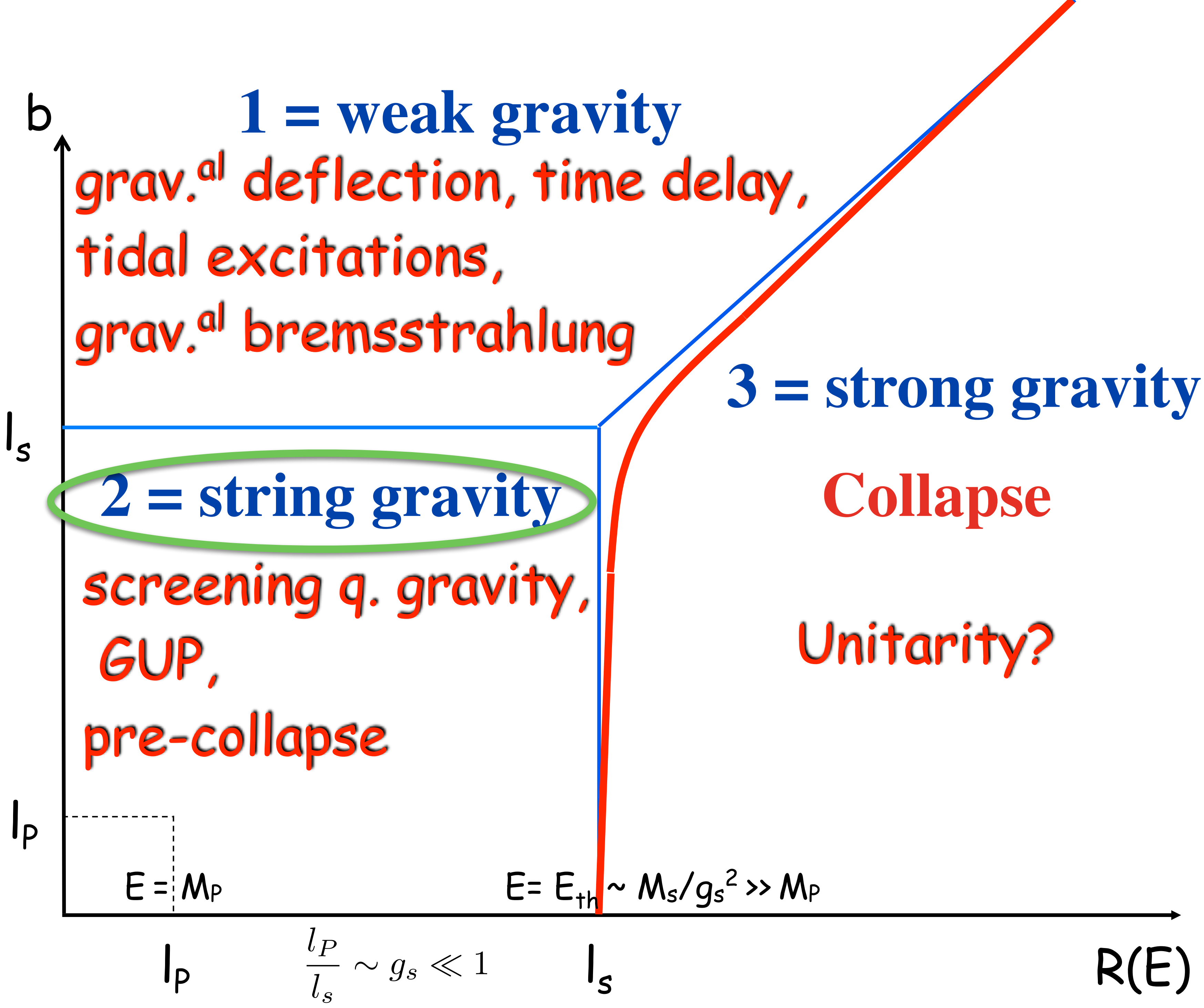
- Restoring (elastic) unitarity via eikonal resummation (trees violate p.w.u.)

- Gravitational deflection & time delay:an emerging Aichelburg-Sexl (AS) metric

- t-channel "fractionation" and hard scattering (large Q) from large-distance (b) physics

- Tidal excitation of colliding strings, inelastic unitarity, comparison with string in AS metric (not yet done beyond leading term in R/b => Challenge # 1)

- Gravitational bremsstrahlung (=> Part III)



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E= $E_{th} \sim M_s/g_s^2 >> M_P$
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l_s
R(E)

Results in region 2 (string gravity)

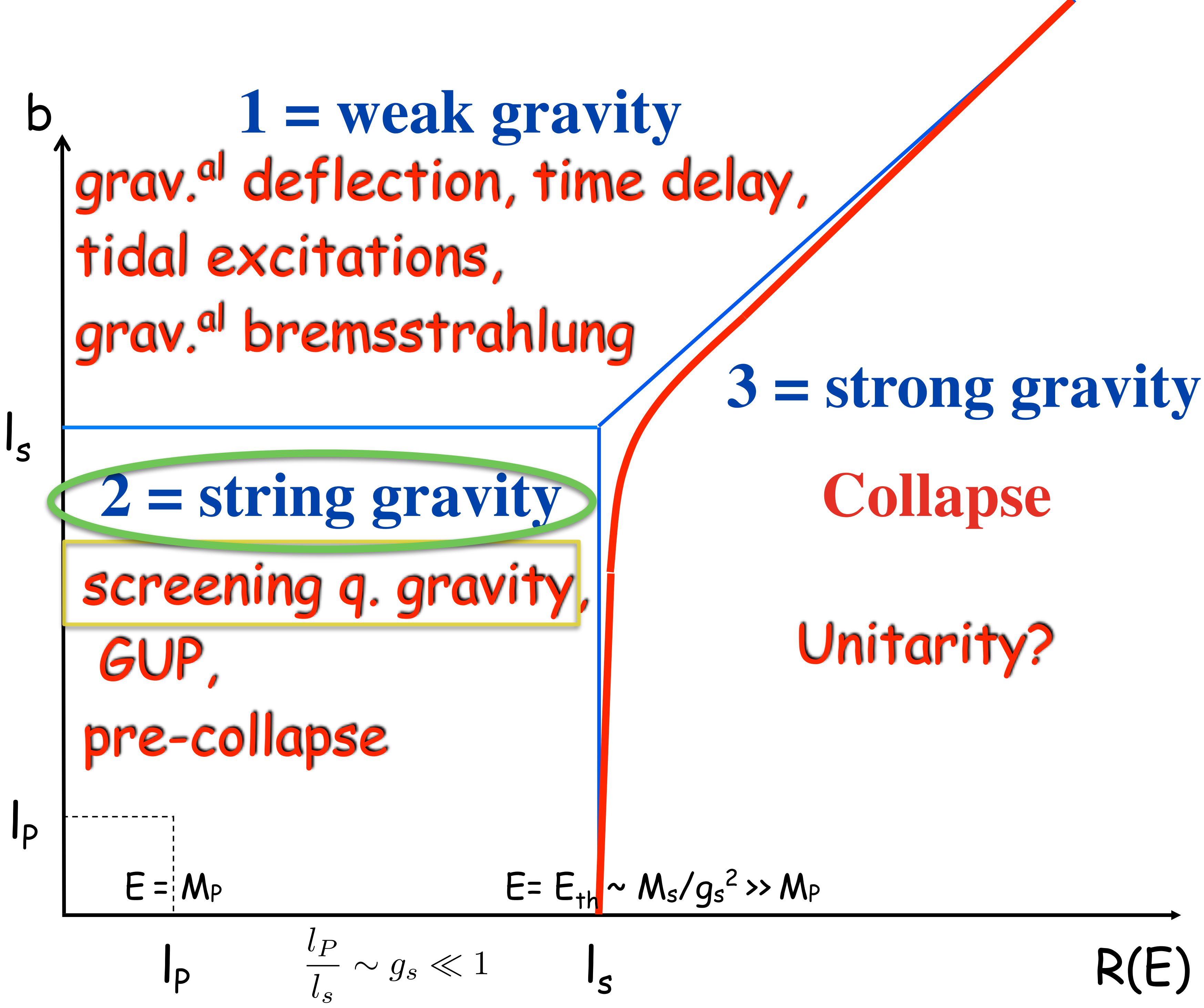
- String softening of quantum gravity @ small b: solving a causality problem via Regge-behavior

- Maximal class. deflection and comparison/ agreement w/ Gross-Mende-Ooguri

- Generalized uncertainty principle (GUP)

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq l_s$$

- s-channel "fractionation", antiscaling, and precocious black-hole-like behavior



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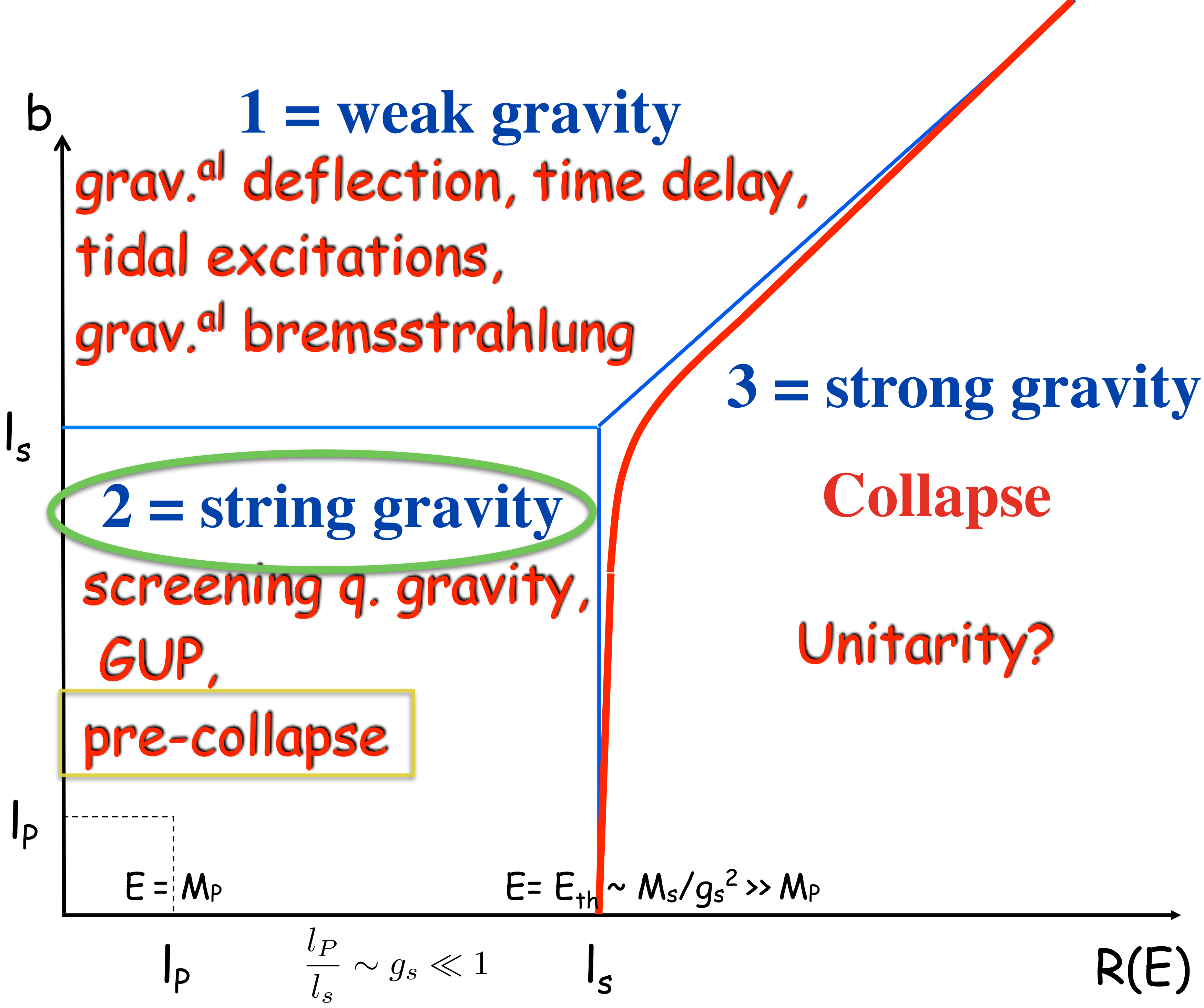
R(E)

String-string scattering @ b,R < l_s

$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + \cancel{O(R^2/b^2)} + O(l_s^2/b^2) + \cancel{O(l_P^2/b^2)} + \dots)\right)$$

"Classical corrections" screened, corrected, leading eikonal can be trusted even for b << R.

Solves a potential "causality problem", pointed out by Camanho et al (1407.5597), see part II.



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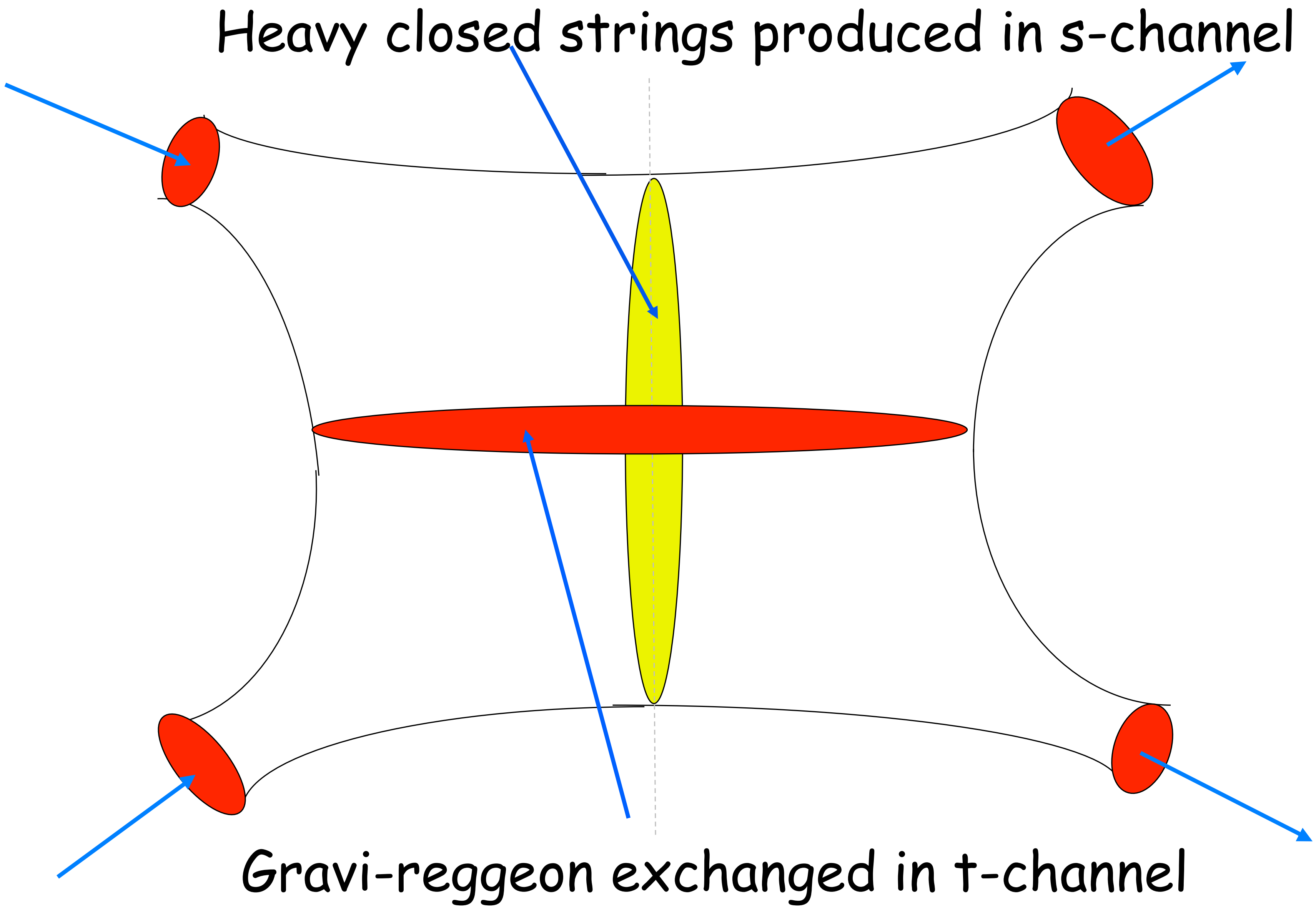
R(E)

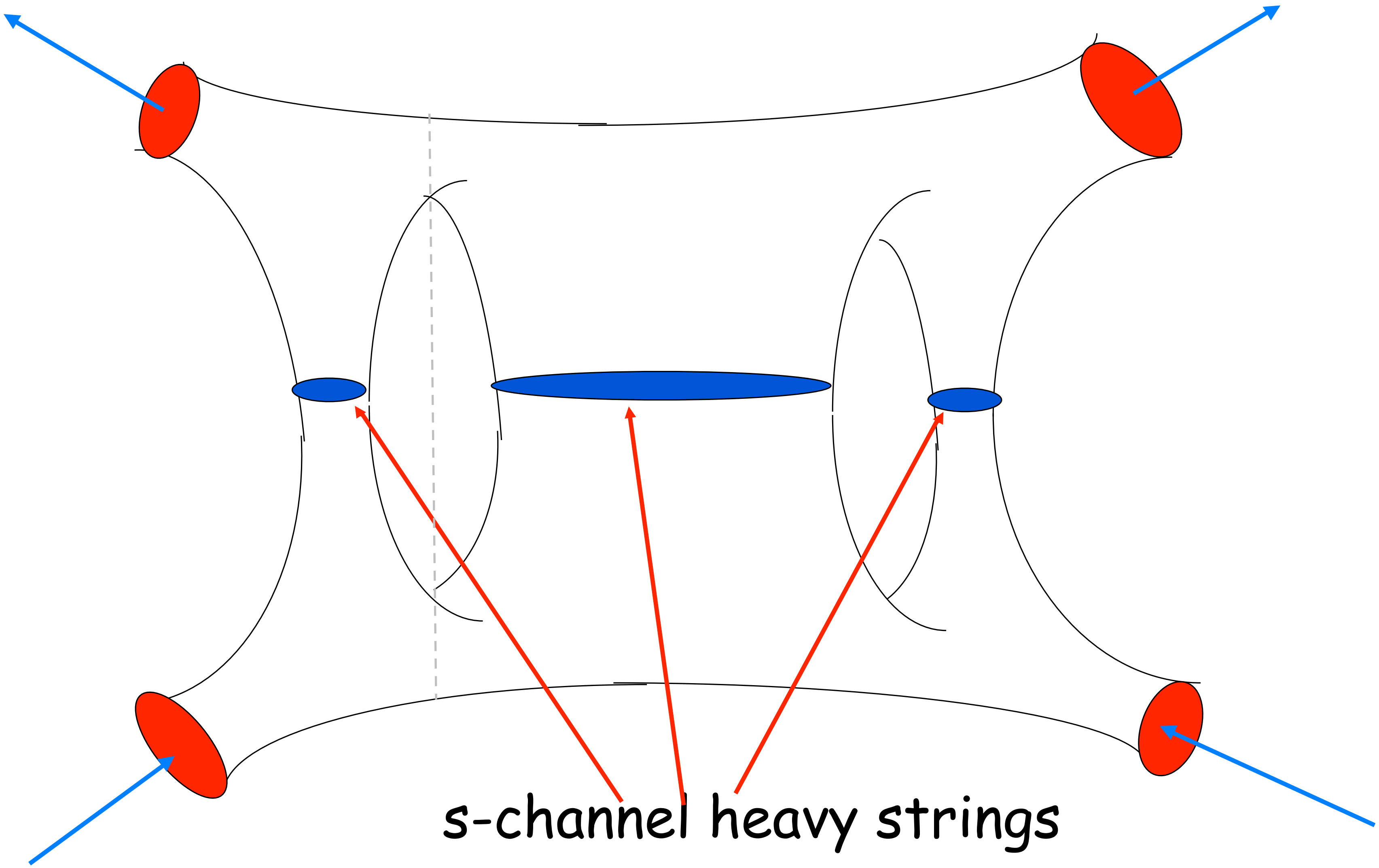
Because of (DHS) duality, even single gravi-reggeon exchange gives a complex scattering amplitude. Its imaginary part, due to formation of closed-strings in the s-channel, is exponentially small at $b \gg l_s$ (neglected previously, but important now).

It is also smooth for $b \to 0$.

Im A is due to closed strings in s-channel (DHS duality)



Turning the previous diagram by 90^o



$$\mathrm{Im} A_{cl}(E,b) \sim \frac{G\ s}{\hbar}\ (l_s\sqrt{Y})^{4-d} \exp\left(-\frac{b^2}{l_s^2 Y}\right) \quad ; \quad Y = \log(\alpha' s)$$

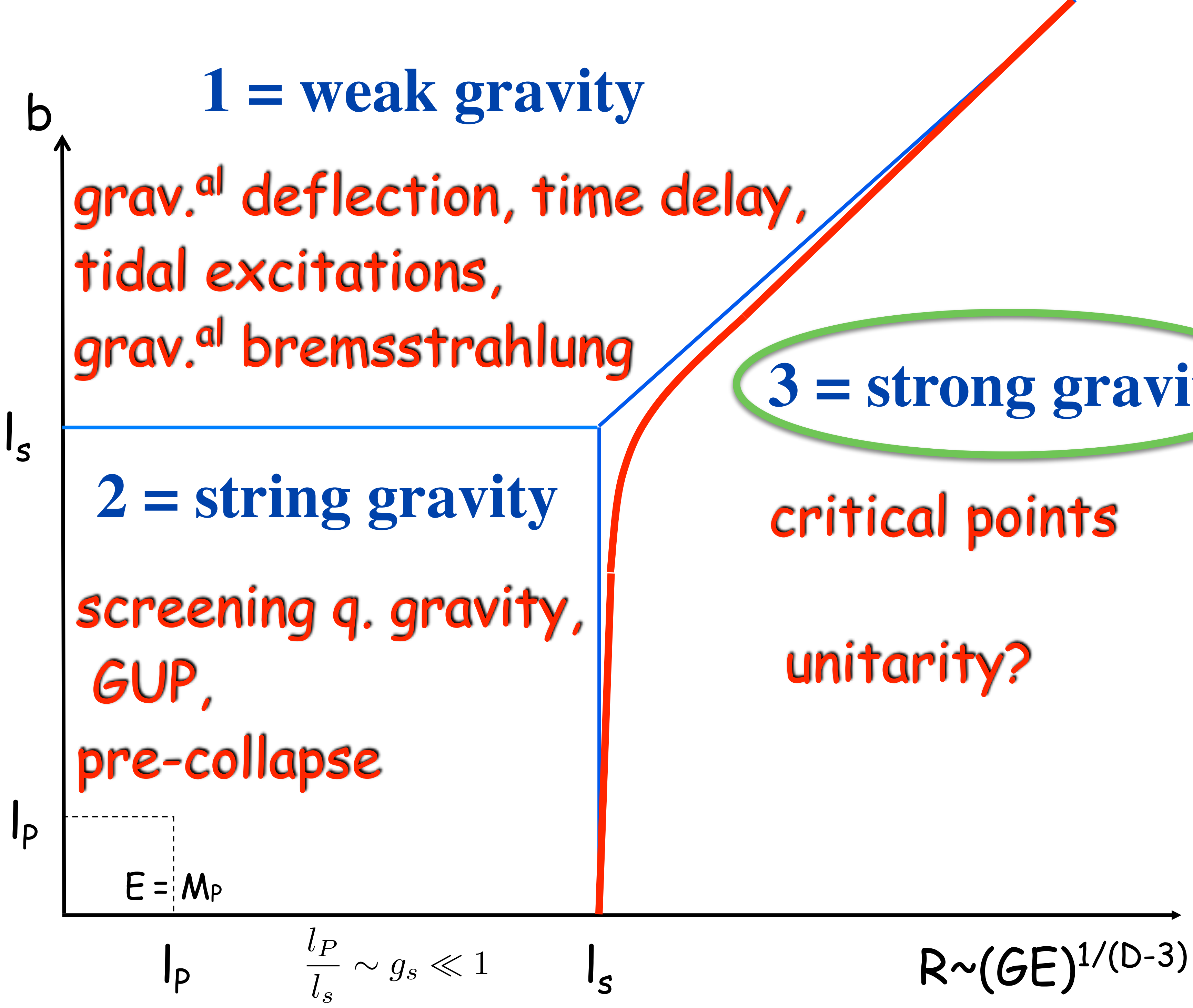
For $b < l_s Y^{1/2}$ more and more strings are produced. Their average number grows like $Gs \sim E^2$ (Cf. # of exchanged strings) so that, above $M_s/g \sim M_P$, the average energy of each final string starts decreasing as E is increased

$$\langle E_{final}\rangle \sim \frac{M_s^2}{g^2\sqrt{s}} \to M_s \text{ at } \sqrt{s} = E_{th} \qquad \mathrm{Im} A_{cl}(E,b) \sim \langle n\rangle \to g^{-2} \sim S_{BH}$$

Similar to what we expect in BH physics!

This is the s-channel analog of the "fractionation" we have seen earlier in the t-channel.

Region 3 (strong gravity)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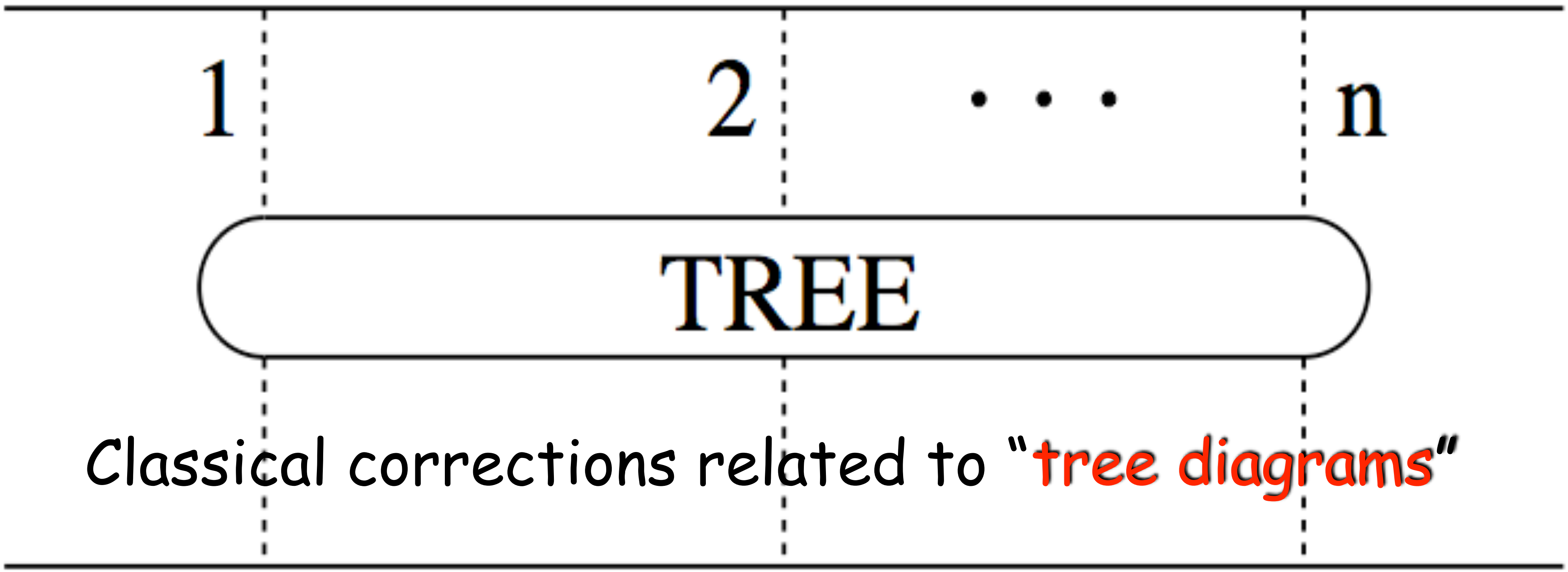
$$S(E,b) \sim exp\left(i\frac{A}{\hbar}\right) \sim exp\left(-i\frac{Gs}{\hbar}(logb^2 + O(R^2/b^2) + O(\cancel{l_s^2/b^2}) + O(\cancel{l_P^2/b^2}) + \dots)\right)$$



Classical corrections related to "tree diagrams"

Power counting for connected trees:

$$A_{cl}(E,b) \sim G^{2n-1}s^n \sim Gs\ R^{2(n-1)} \to Gs\ (R/b)^{2(n-1)}$$

Summing tree diagrams => solving a classical field theory.

Q: Which is the effective field theory for TP-scattering?

Results (ACV07->)

- D=4, point-particle limit. D>4 easier?
  - Identifying (semi) classical contributions as trees
  - An effective 2D field theory to resum trees
  - Emergence of critical surfaces (for existence of R.R. solutions) in good agreement with collapse criteria based on constructing a CTS.
  - Unitarity beyond cr. surf.? Challenge # 2!

II: String-brane collisions

Another basic process in which a pure initial state evolves into a complicated (yet presumably still pure) state.

An easier problem since the string acts as a probe of a geometry determined by the heavy brane system.

Once more: we are not assuming a metric: calculations performed in flat spacetime (D-branes introduced via boundary-state formalism)

(At very high E gravity dominates. Yet we can neglect closed-string loops by working below an E_{max} that goes to ∞ with N)

G. D'Appollonio, P. Di Vecchia, R. Russo & G.V.

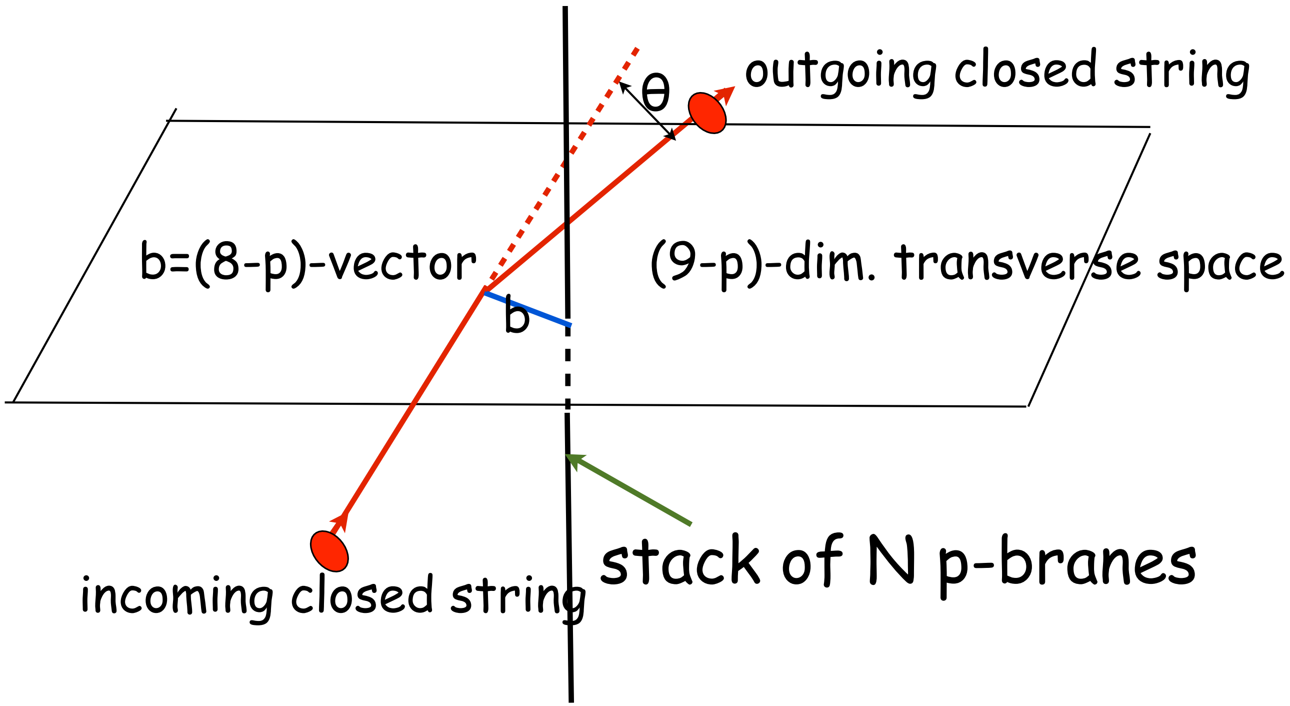
(1008.4773, 1310.1254, 1310.4478, 1502.01254, 1510.03837)

W. Black and C. Monni, 1107.4321

M. Bianchi and P. Teresi, 1108.1071

HE scattering on heavy string/target GV, 1212.0626

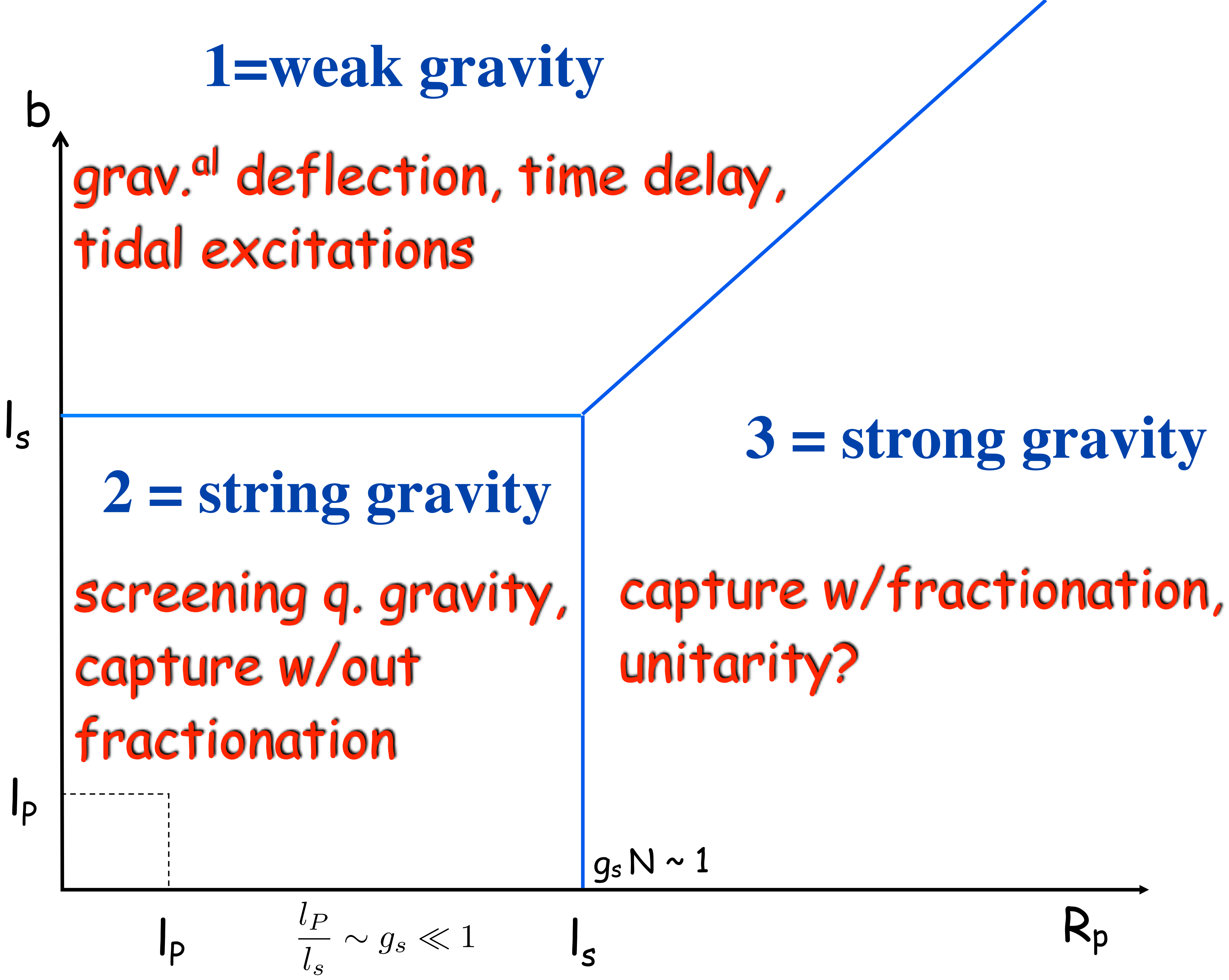
R. Akhoury, R. Saotome and G. Sterman, 1308.5204 +...



Parameter-space @ high-energy

- HE string-brane scattering ($N >> 1$, $g_s << 1$):
- 3 relevant length scales (neglecting again l_P)
- Playing w/ N and g_s we can make R_p/l_s arbitrary

$$b \sim \frac{J}{E} \quad ; \quad R_p \sim (g_s N)^{\frac{1}{7-p}} l_s \quad ; \quad l_s \sim \sqrt{\alpha' \hbar}$$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l_s
R_p

The semiclassical S-matrix @ high energy

In analogy with the string-string collisions case, the HE string-brane S-matrix takes the form

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{E\ b}{\hbar} c_p \left(\frac{R_p}{b}\right)^{7-p} \left(1 + O\left((\frac{R_p}{b})^{7-p}\right) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \ldots\right)$$

and here too there are subregions.

Results on string-brane collisions

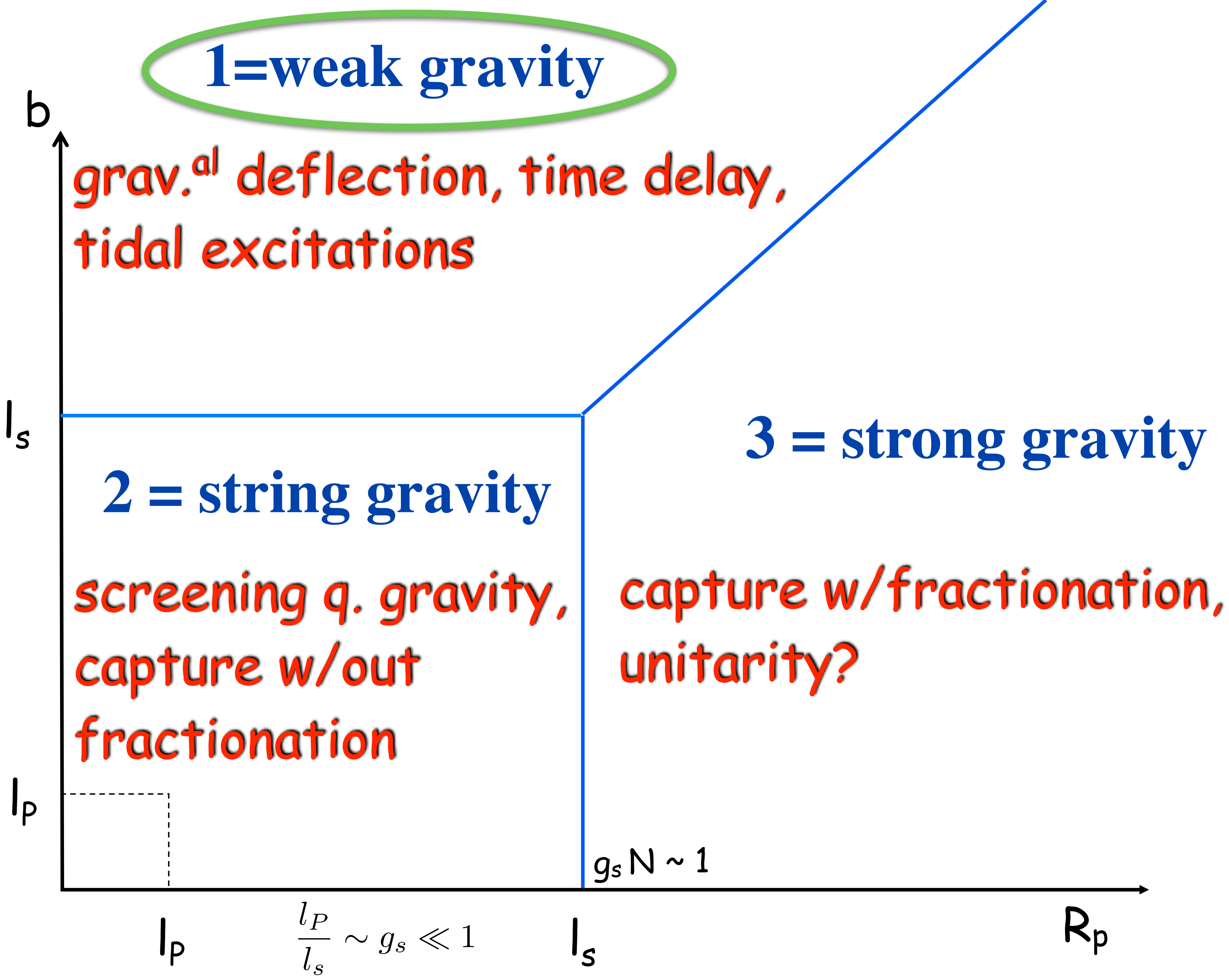
- Deflection angle, time delay, agreement with curved space-time calculations

- Unitarity preserving tidal excitation

- Short-distance corrections & resolution of potential causality problems

- Absorption via closed-open transition

- Dissipation into many open strings, thermalization? Unitarity?



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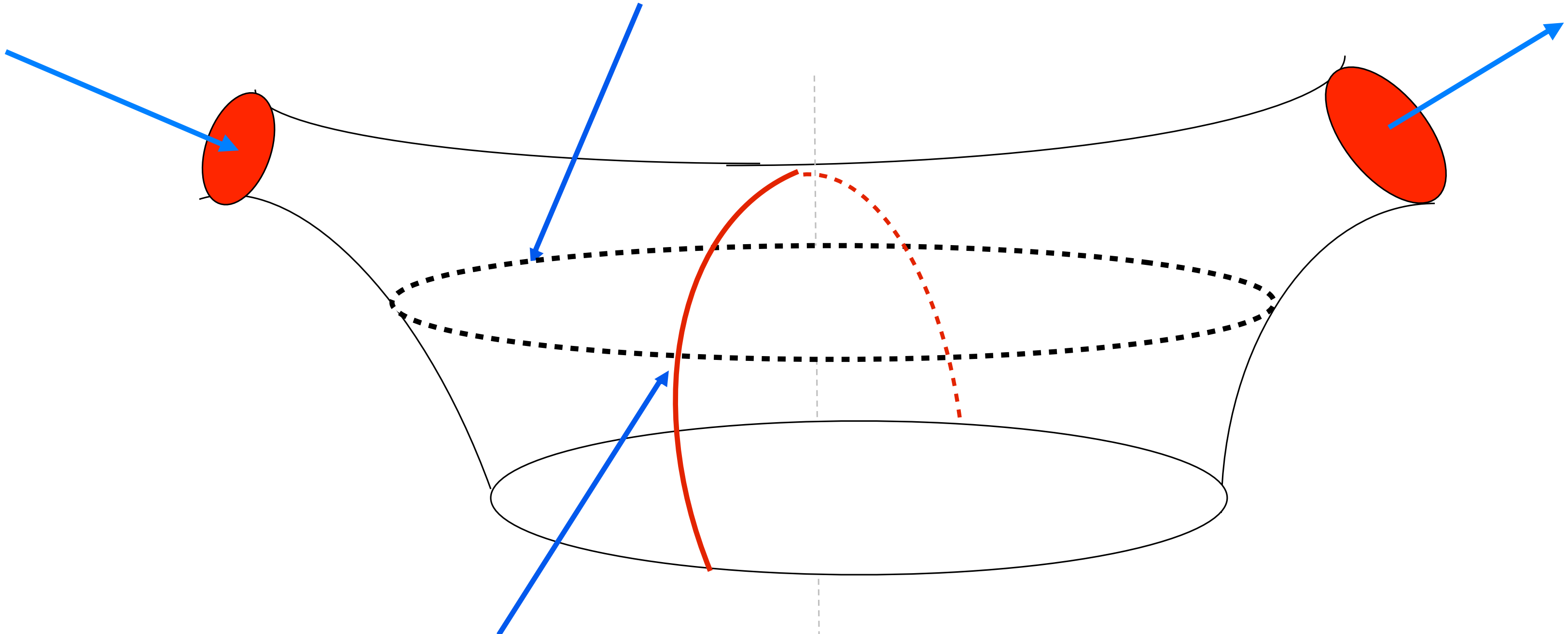
String-brane scattering at tree-level

gravi-reggeon (closed string) exchanged in t-channel



heavy open strings produced in s-channel

String-brane scattering @ large b

• An effective brane geometry emerges through the deflection formulae satisfied at saddle point in b. Calculation of leading and next to leading eikonal gives

$$\Theta_p = \sqrt{\pi}\left[\frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\left(\frac{R_p}{b}\right)^{7-p} + \frac{1}{2}\frac{\Gamma\left(\frac{15-2p}{2}\right)}{\Gamma\left(6-p\right)}\left(\frac{R_p}{b}\right)^{2(7-p)} + O\left(\left(\frac{R_p}{b}\right)^{3(7-p)}\right)\right]$$

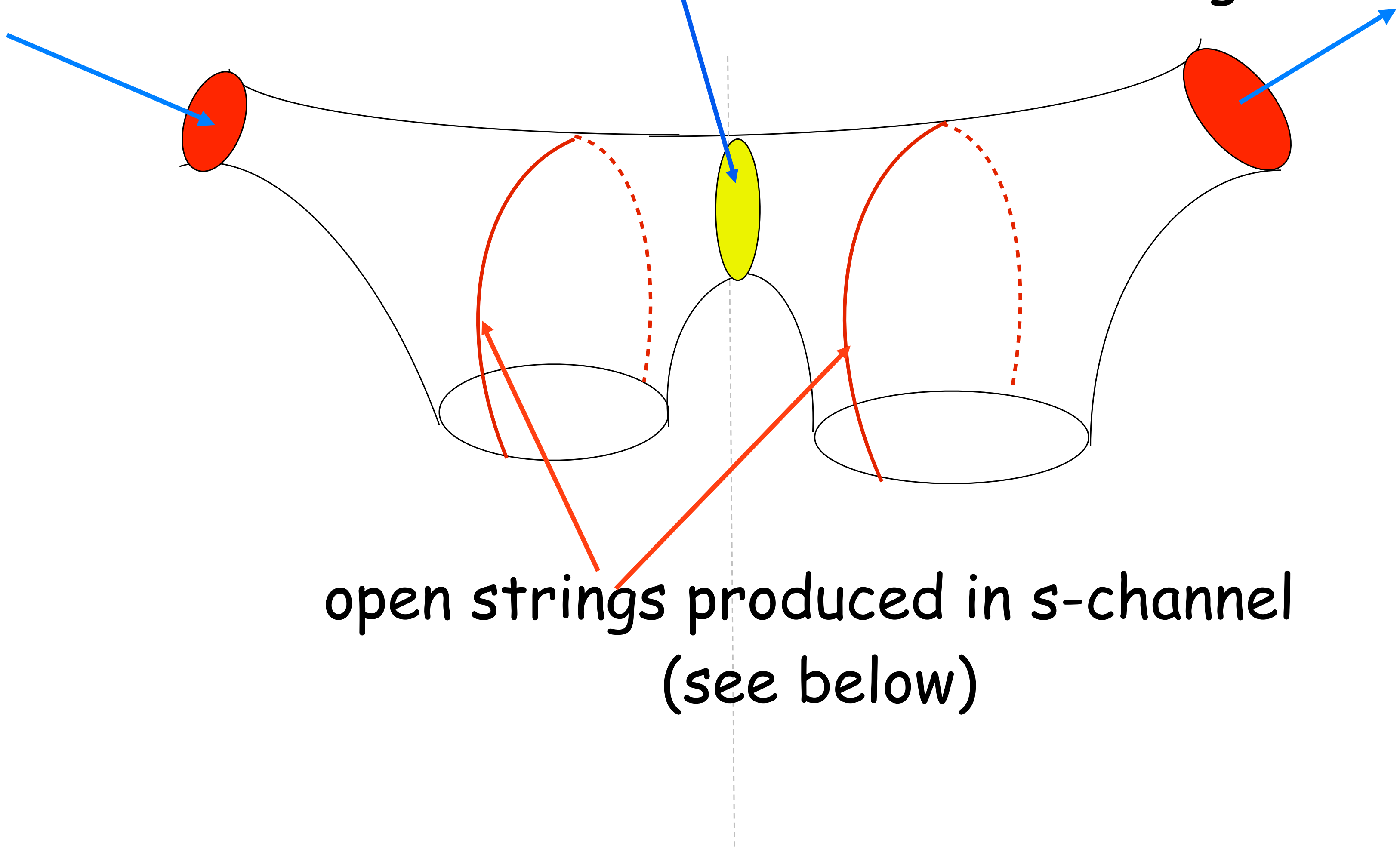
Agrees to that order w/ exact classical formula ($\rho_* = R_p/r_{tp}$):

$$\Theta_p = 2\int_0^{\rho_*} d\rho \frac{\hat{b}}{\sqrt{1+\rho^{7-p}-\hat{b}^2\rho^2}} - \pi \qquad \hat{b} \equiv \frac{b}{R_p}$$

that can be computed in the D-brane-induced metric

Annulus (1-loop) level scattering

Tidal excitation of initial string



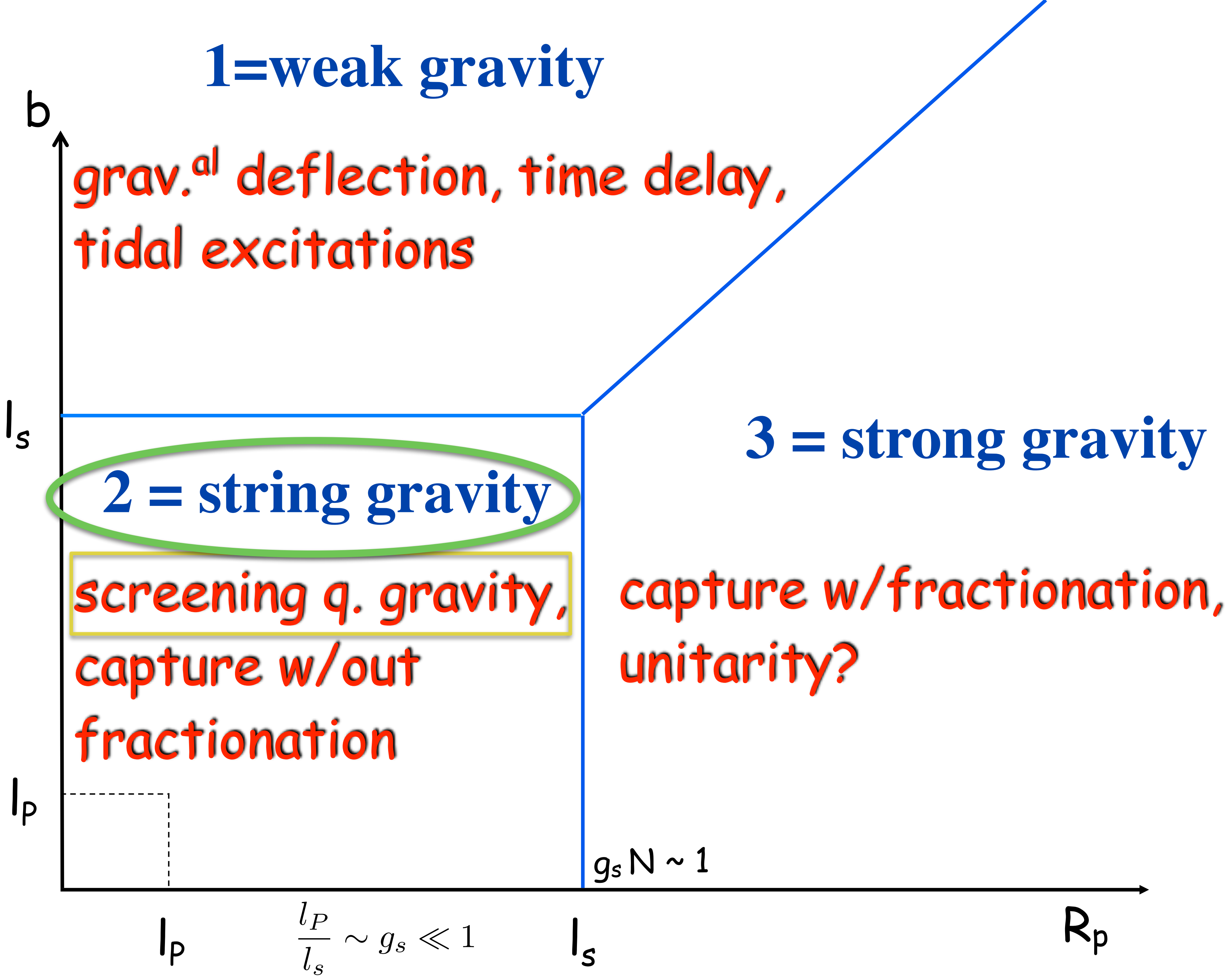
open strings produced in s-channel
(see below)

• Tidal effects can be computed. They come out in complete agreement with what one would obtain (to leading order in R_p/b and l_s/b) by quantizing the string in the D-brane metric.

• Tidal effects become relevant below a critical $b=b_t$

$$b_t^{8-p} \sim (\alpha' E)\ R_p^{7-p}$$

• In DDRV 1310.1254 (see also 1310.4478) we have studied in detail the actual microscopic structure of the excited states that insure (inelastic) unitarity in this regime (not yet done beyond leading term in R_p/b, Cf. Challenge # 1)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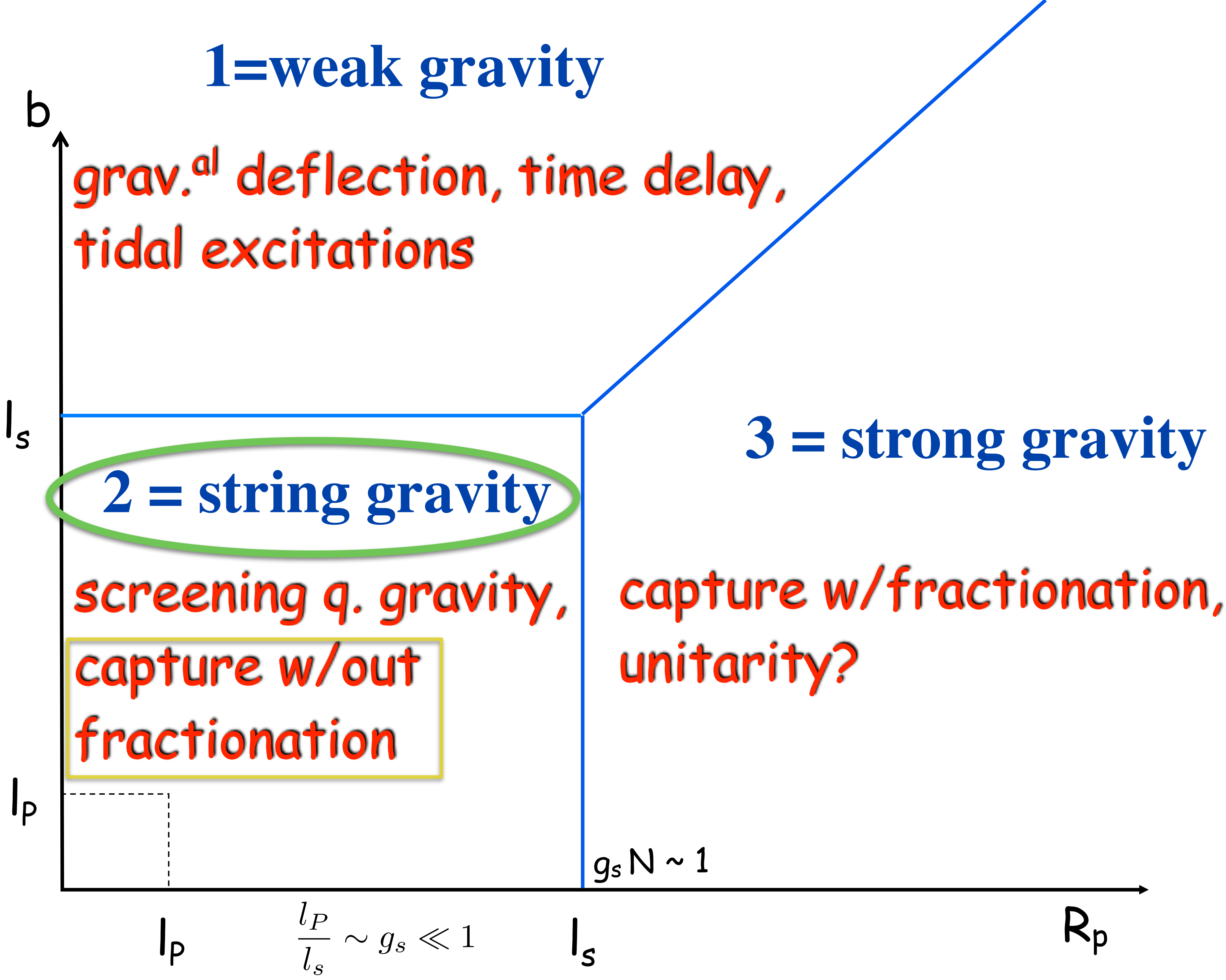
RP

Causality violation (resp. restoration) in Quantum Field (resp. String) Theory

Camanho, Edelstein, Maldacena & Zhiboedov, 1407.5597
D'Appollonio, Di Vecchia, Russo & GV, 1502.01254

Phase shift is finite at b=0 and has a smooth expansion in $b^2/$ (l_s^2 logs). Its derivative wrt E gives a well-behaved time delay even for b -> 0.

Regge behavior saves string theory from causality problems.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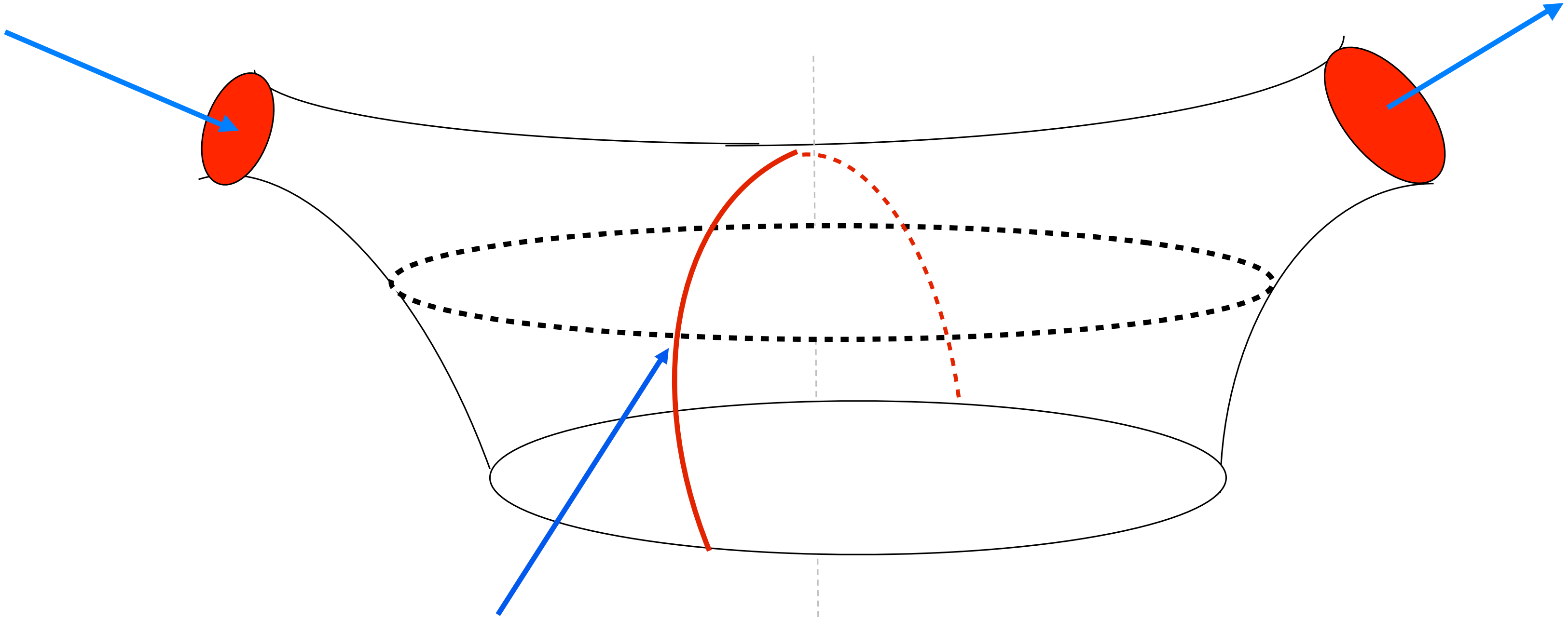
Disc(tree)-level scattering



single heavy open string produced in s-channel

String-brane scattering @ $b, R < l_s$

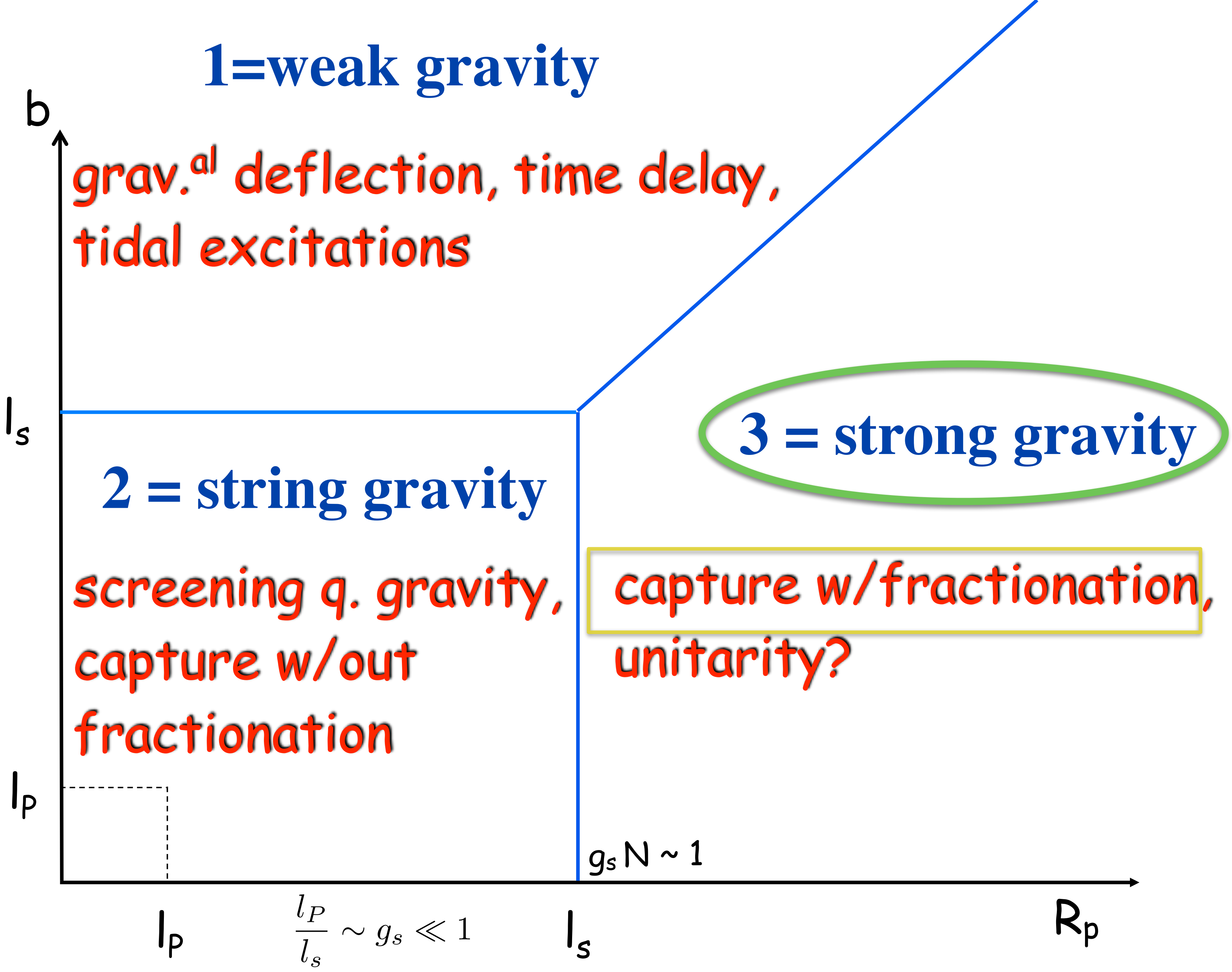
Also in this case single graviton exchange does not give a real scattering amplitude.

This is related to Regge behavior in string theory.

The imaginary part is now due to formation of <u>open</u>-strings in the s-channel.

It is exponentially damped at large impact parameter (=> irrelevant in region 1, important in region 2)

Parallels the case of the string-string collision but here we are able to describe the process at an exclusive, microscopic level (DDRV, 1510.03837).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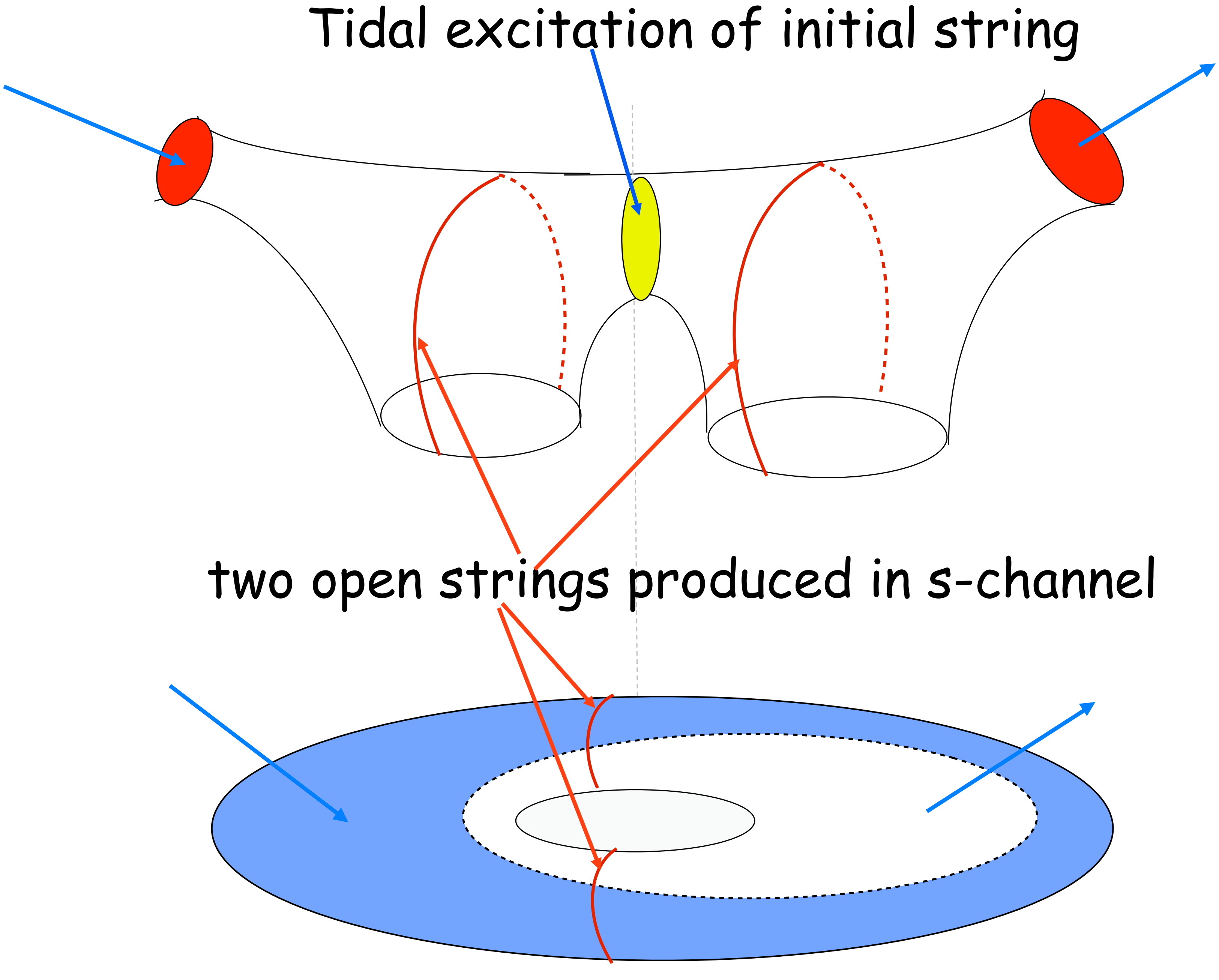
Annulus (1-loop) level scattering



another representation of the annulus diagram

Highly inelastic string-string & string-brane scattering

In string-string scattering:

$$\langle n_{closed} \rangle \sim \frac{E R_S}{\hbar} \left(\frac{R_S}{l_s} \right)^{D-4} \quad \Rightarrow \quad \langle E_{closed} \rangle \sim M_s \left(\frac{l_s}{R_S} \right)^{D-3} \sim \frac{M_s^2}{g_s^2 E}$$

If extrapolated to $R_S > l_s$ this gives only massless string modes (Hawking radiation?). Can it be trusted?

In string-brane scattering (work in progress):

$$\langle n_{open} \rangle \sim \frac{E l_s}{\hbar} \left(\frac{R_p}{l_s} \right)^{7-p} \quad \Rightarrow \quad \langle E_{open} \rangle \sim M_s \left(\frac{l_s}{R_p} \right)^{7-p} \sim M_s (g_s N)^{-1}$$

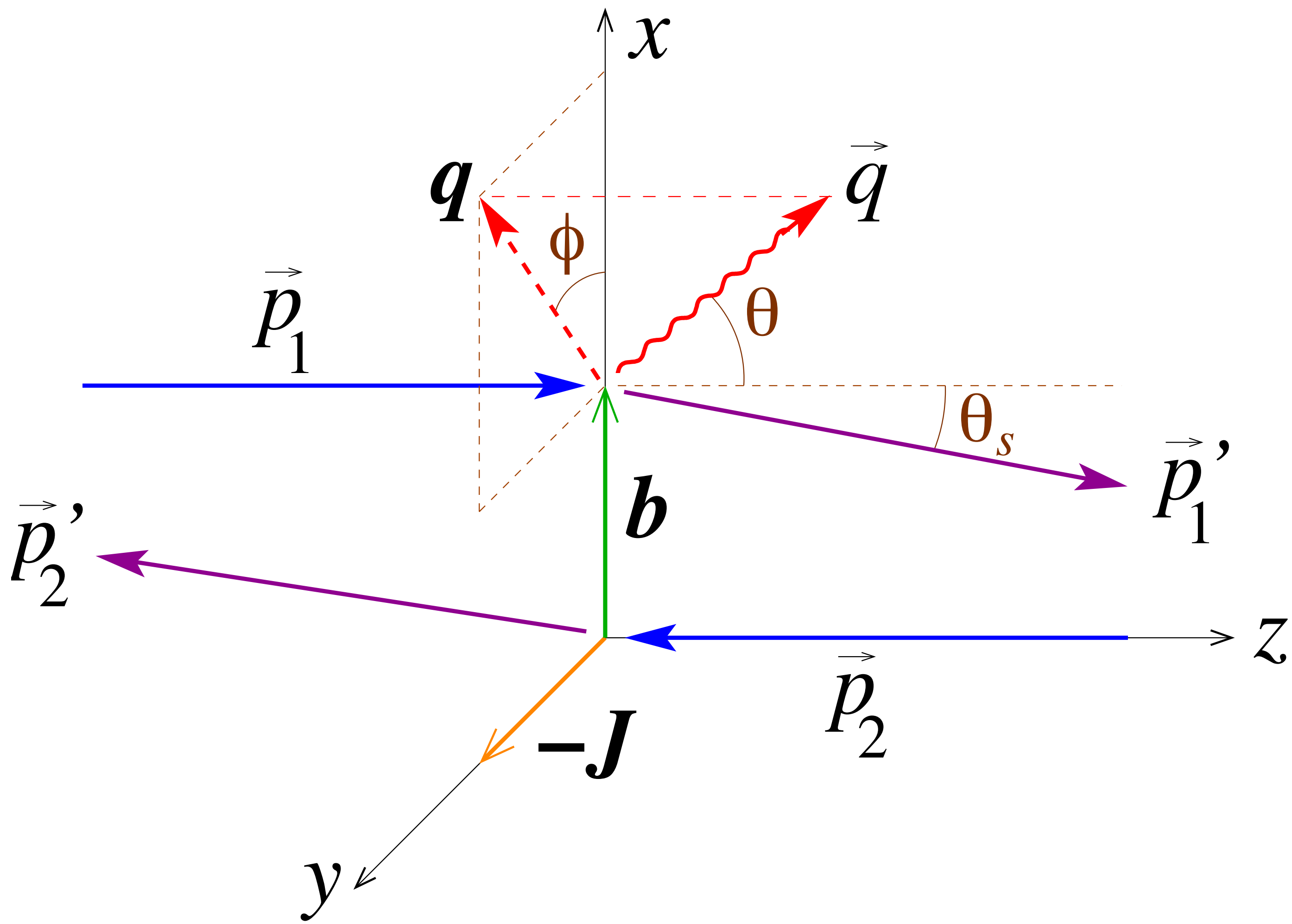
Calculation may be doable even for $R_p \gg l_s$ (~ SUGRA limit in AdS/CFT!). Can we make contact with a CFT living on the brane system?

Can we construct a unitary S-matrix describing the absorption + fractionation regime?

Challenge # 3

III: Gravitational bremsstrahlung from ultra-relativistic collisions

The process at hand



Three methods

1. A classical GR approach
   (A. Gruzinov & GV, 1409.4555)
2. A quantum eikonal approach
   (CC&Coradeschi & GV, 1512.00281, Ciafaloni, Colferai & GV, 1812.08137)
3. A soft-theorem approach (see Bianchi's talk)
   (Laddha & Sen, 1804.09193; Sahoo & Sen 1808.03288, Addazi, Bianchi & GV, 1901.10986)

Comments:

a. #2 goes over to #1 in the classical limit

b. They agree with #3 in the overlap of their respective domains of validity

Domains of validity

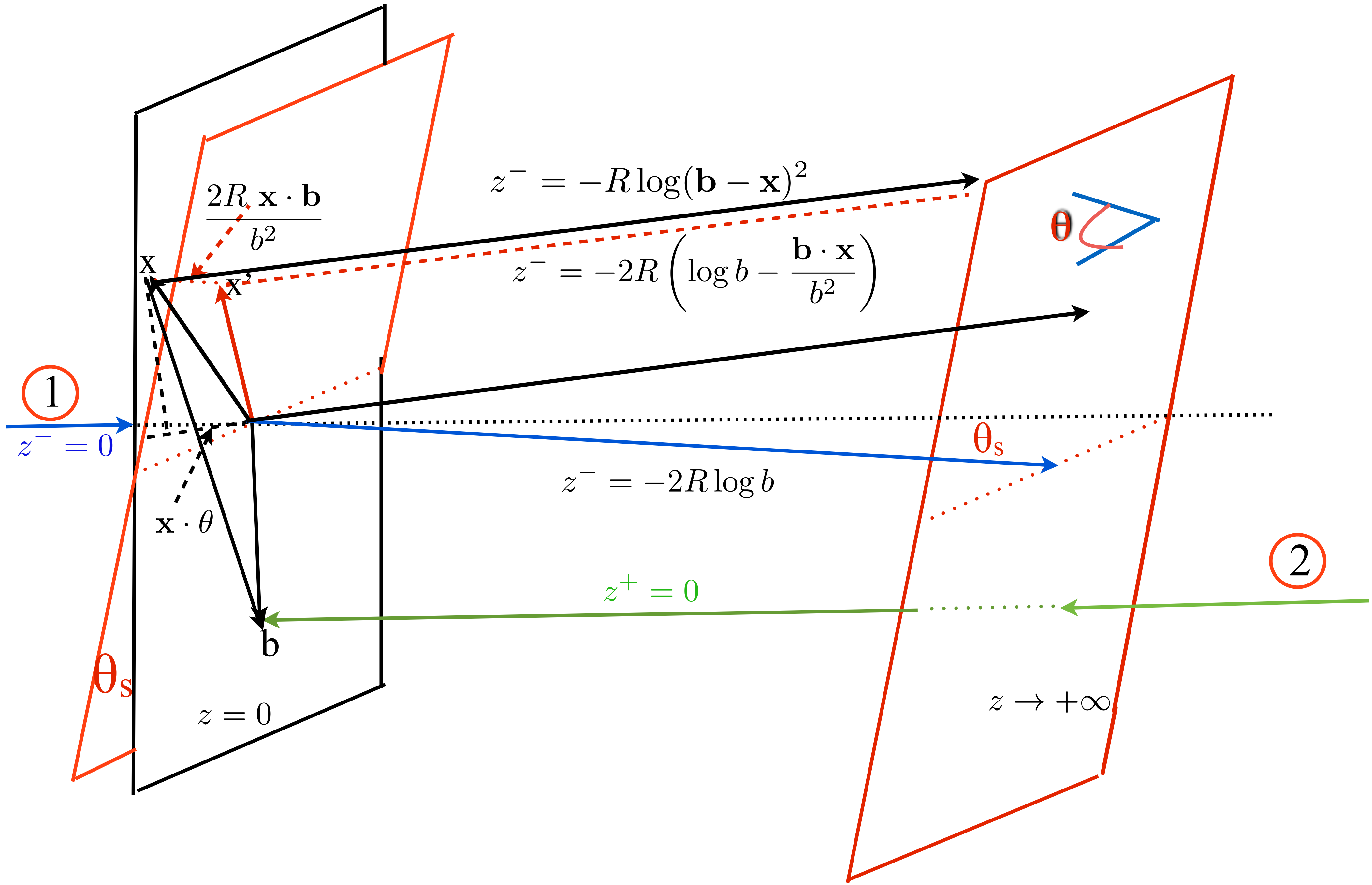
- The CGR and quantum eikonal approaches are limited to small-angle scattering but cover a wide range of GW frequencies.
- The soft-theorem approach is not limited to small deflection angles but is only valid in a much smaller frequency region.

A classical GR approach

Based on Huygens superposition principle.

For gravity this includes in an essential way the gravitational time delay in AS's shock-wave metric.

In pictures



$z^- = -R \log(\mathbf{b}-\mathbf{x})^2$
$\frac{2R\,\mathbf{x}\cdot\mathbf{b}}{b^2}$
x
x'
$z^- = -2R\left(\log b - \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}\right)$
θ
1
$z^- = 0$
θs
$z^- = -2R\log b$
$\mathbf{x}\cdot\theta$
2
$z^+ = 0$
b
θs
$z = 0$
$z \to +\infty$

A quantum treatment in eikonal approach

Emission from external and internal legs throughout the whole ladder (with its suitable phase) has to be taken into account for not so soft gravitons.

One should also take into account the (finite) difference between the (infinite) Coulomb phase of the final 3-particle state and that of an elastic 2-particle state.

When this is done, the classical result of G+V is exactly recovered for $h\omega/E \to 0$!

The classical result

Frequency + angular spectrum (s = 4E^2, R= 4GE)

$$\frac{dE^{GW}}{d\omega \; d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2 \; ; \; \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s \; ; \; \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x \; \zeta^2}{|\zeta|^4} \; e^{-i\omega \mathbf{x}\cdot\tilde{\boldsymbol{\theta}}} \left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy \qquad\qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

$$c(\omega, \boldsymbol{\theta}) = \int \frac{d^2x \; \zeta^2}{|\zeta|^4} e^{-i\omega\mathbf{x}\cdot\boldsymbol{\theta}} \left[e^{-iR\omega\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2}} - e^{+2iR\omega\frac{\mathbf{b}\cdot\mathbf{x}}{b^2}}\right]$$

Re ζ^2 and Im ζ^2 correspond to the usual (+,x) GW polarizations, ζ^2, ζ^{*2} to the two circular ones (not the cc of each other).

Subtracting the deflected shock wave (cf. P. D'Eath) is crucial!

Analytic results: a Hawking knee & an unexpected bump

For $b^{-1} < \omega < R^{-1}$ the GW-spectrum is almost flat in ω

$$\boxed{\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi}\theta_s^2 E^2 \log(\omega R)^{-2}}$$

Below $\omega = b^{-1}$ it "freezes" reproducing the ZFL

$$\boxed{\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi}\ \theta_s^2 E^2\ \log(\theta_s^{-2})}$$

Above $\omega = R^{-1}$ drops, becomes "scale-invariant"

Hawking knee!

$$\boxed{\frac{dE^{GW}}{d\omega} \sim \theta_s^2 \frac{E}{\omega}}$$

This gives a log ω^* in the "efficiency" for a cutoff at ω^*

At $\omega \sim R^{-1}\theta_s^{-2}$ the above spectrum becomes $O(Gs\,\theta_s^4)$ i.e. of the same order as terms we neglected.
Also, if continued above $R^{-1}\theta_s^{-2}$, the so-called "Dyson bound" ($dE/dt < 1/G$) would be violated. Using $\omega^* \sim R^{-1}\theta_s^{-2}$ we find (to leading-log accuracy):

$$\boxed{\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi}\,\theta_s^2\,\log(\theta_s^{-2})}$$

For $\omega > \omega^*$ G+V argued for a $G^{-1}\omega^{-2}$ spectrum which (extrapolating to $\theta_s \sim 1$) turns out to be that of a time-integrated BH evaporation!

Challenge #4: ω^* & spectrum above

the fine spectrum below 1/b

$$\boxed{\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi}\, \theta_s^2 E^2\, \log(\theta_s^{-2}) \qquad (\omega b \ll 1)}$$

$$\boxed{\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi} \theta_s^2 E^2 \log(\omega R)^{-2} \quad (\omega b \gg 1 \gg \omega R)}$$

suggest naive (monotonic) interpolation around

$\omega b \sim 1$, e.g.

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi}\theta_s^2 E^2 \log\left(\frac{b^2}{R^2(1+\omega^2 b^2)}\right) \sim \frac{4G}{\pi}\theta_s^2 E^2 \left[\log\left(\frac{b^2}{R^2}\right) - O(\omega^2 b^2)\right]$$

This appears not to be the case...

A careful study of the region $\omega R < 1$, but with ωb generic, shows that:

- At $\omega b < (\ll) 1$ there are corrections of order $(\omega b)\log(\omega b)$, $(\omega b)^2\log^2(\omega b)$ (higher logs suppressed).

- First noticed by Sen et al. in the context of soft thrms in $D=4$. Here they come from the mismatch between the two- and three-body Coulomb phase.

- These logarithmically enhanced sub and sub-sub leading corrections disappear at $\omega b > 1$ so that the previously found $\log(1/\omega R)$ behavior (for $\omega b > 1 > \omega R$), as well as the Hawking knee, remain valid.

- The ωb (both w/ and w/out $\log(\omega b)$) correction only appears for circularly polarized (definite helicity) GWs but disappear either for the (more standard) + and x polarizations, or after summing over them, or finally after integration over the azimuthal angle.

- They $(\omega b)\log(\omega b)$ terms are in complete agreement with what had been previously found by A. Sen and collaborators using soft-graviton theorems to sub-leading order (see Bianchi's talk).

- The leading $(\omega b)^2 \log^2(\omega b)$ correction to the total flux is positive and produces a bump at $\omega b \sim 0.5$.

- Could not be compared to Sen et al. who only considered $\omega b \log(\omega b)$ corrections.

- Now confirmed by Sahoo(private comm. by AS) but there are still questions about $O(\omega b)$.

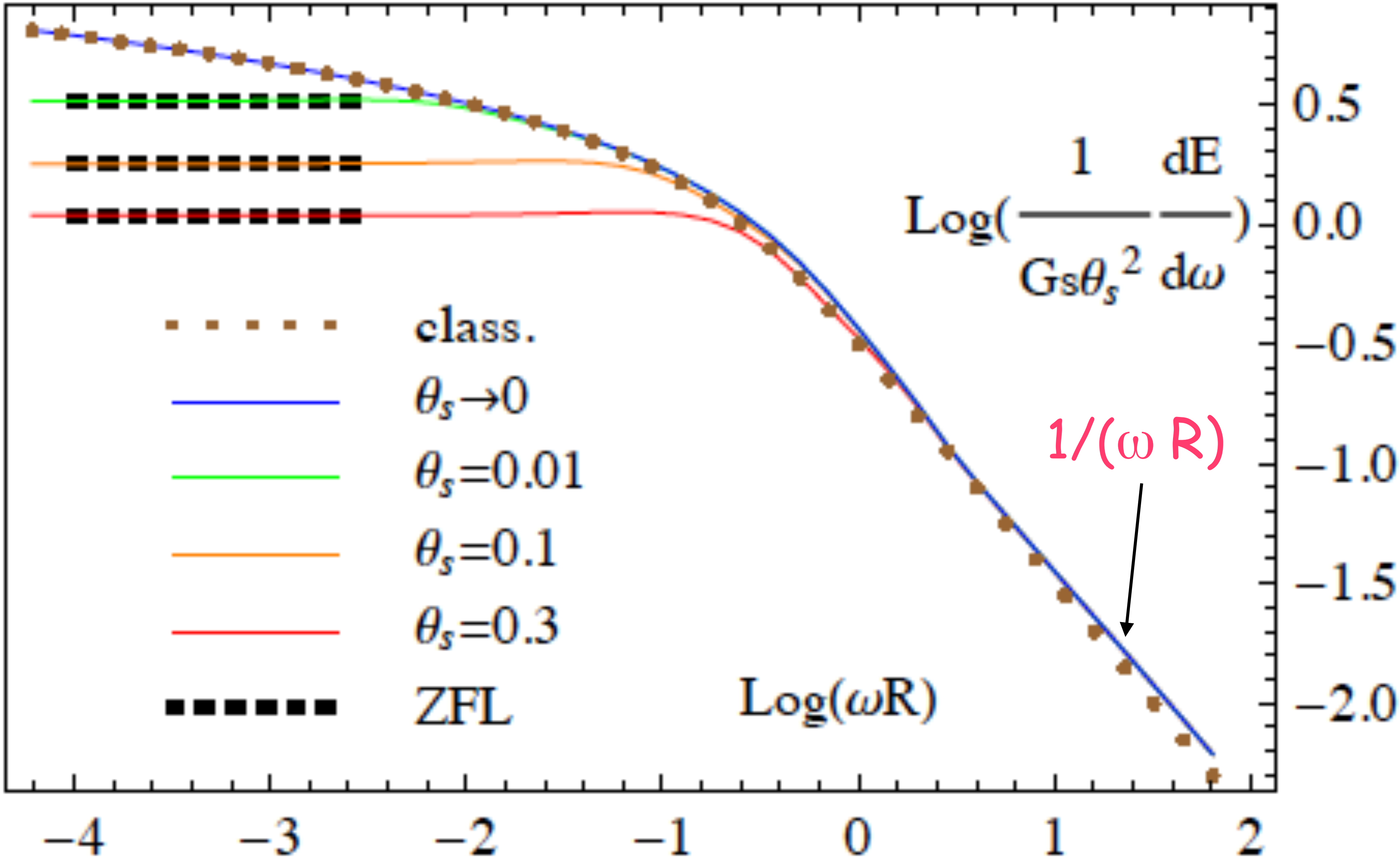
- Can be compared successfully with ABV-19 if Sen et al. recipe is adopted to $O(\omega^2)$.

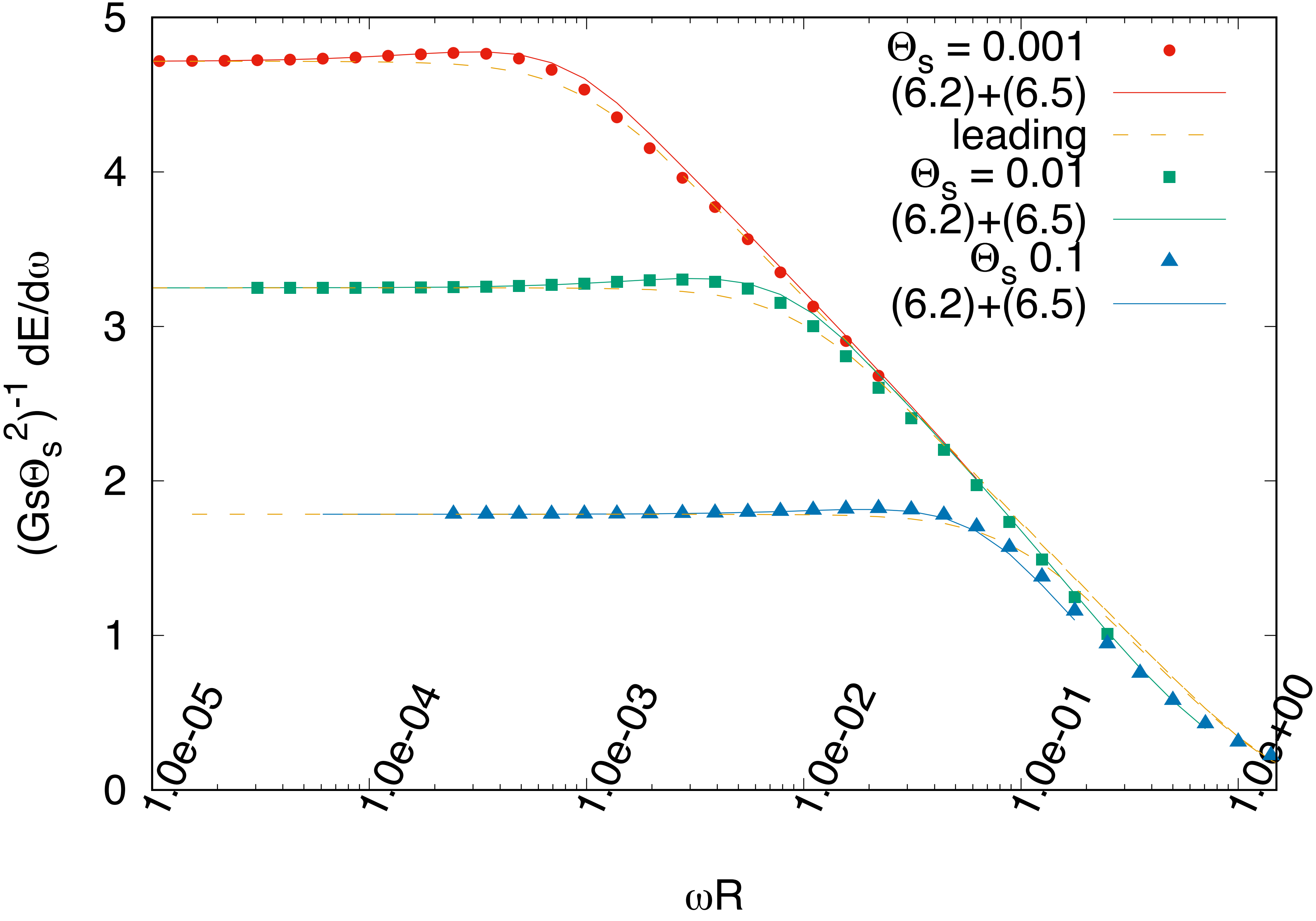
Numerical results

Ciafaloni, Colferai, Coradeschi & GV-1512.00281

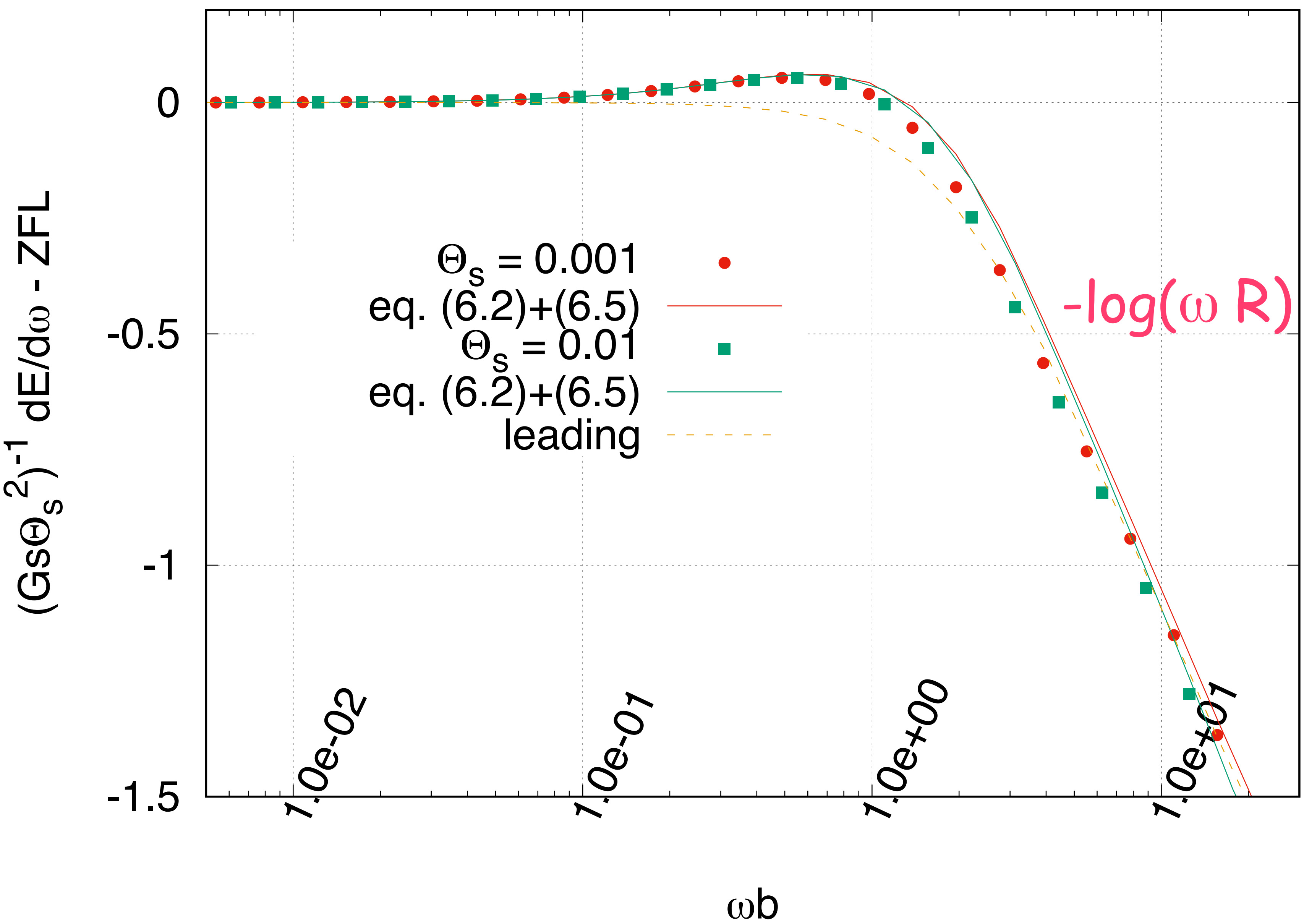
Ciafaloni, Colferai & GV-1812.08137

(CCCV 1512.00281)

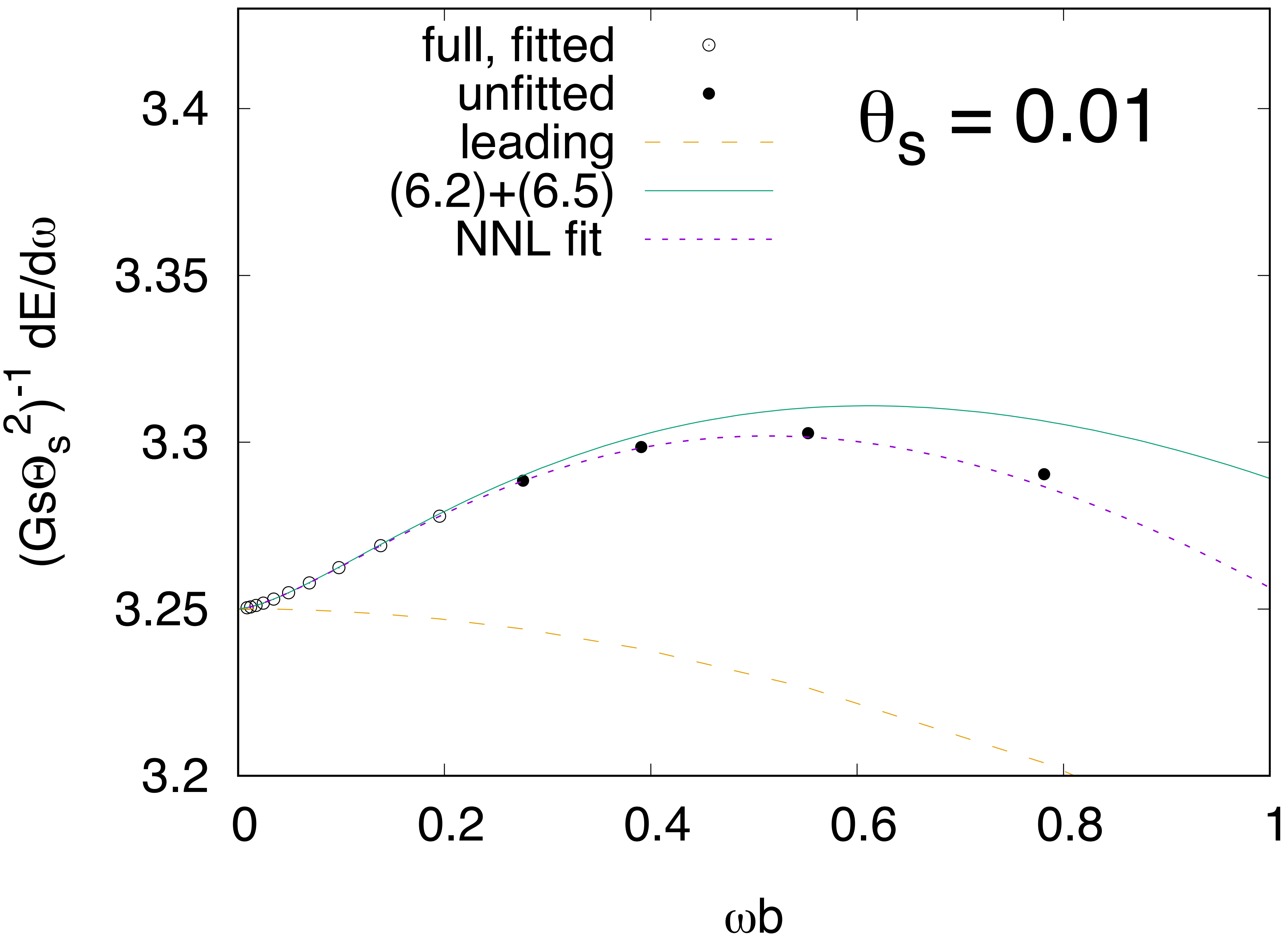




Θ_s = 0.001
(6.2)+(6.5)
leading
Θ_s = 0.01
(6.2)+(6.5)
Θ_s 0.1
(6.2)+(6.5)
$(Gs\Theta_s^2)^{-1}$ dE/dω
ωR
0
1
2
3
4
5
1.0e-05
1.0e-04
1.0e-03
1.0e-02
1.0e-01
1.0e+00

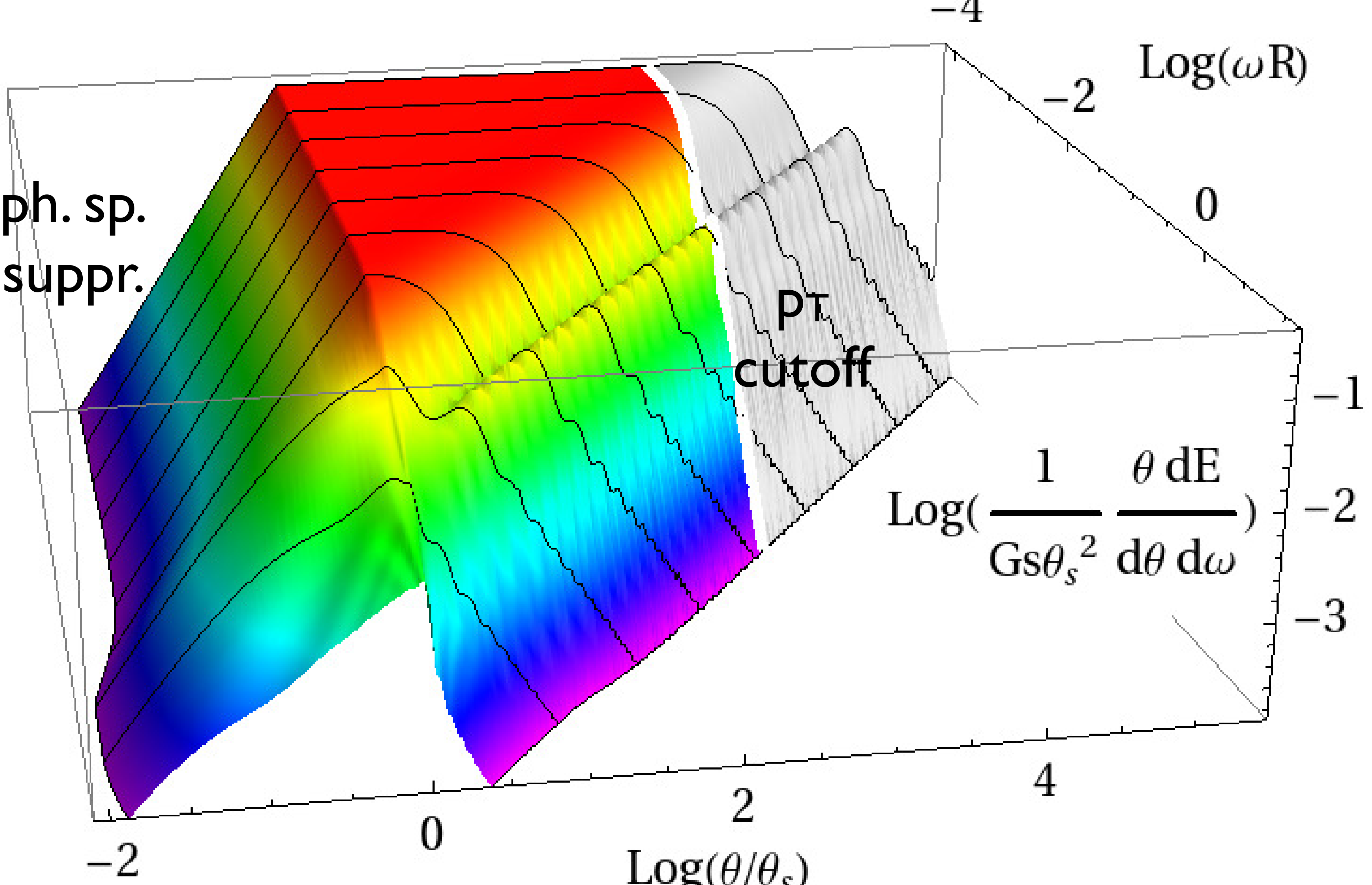


$(Gs\Theta_s{}^2)^{-1}$ dE/dω - ZFL
ωb
0
-0.5
-1
-1.5
1.0e-02
1.0e-01
1.0e+00
1.0e+01
Θ_s = 0.001
eq. (6.2)+(6.5)
Θ_s = 0.01
eq. (6.2)+(6.5)
leading
-log(ω R)



full, fitted
unfitted
leading
(6.2)+(6.5)
NNL fit
θ_s = 0.01
(GsΘ_s^2)^-1 dE/dω
ωb
3.2
3.25
3.3
3.35
3.4
0
0.2
0.4
0.6
0.8
1

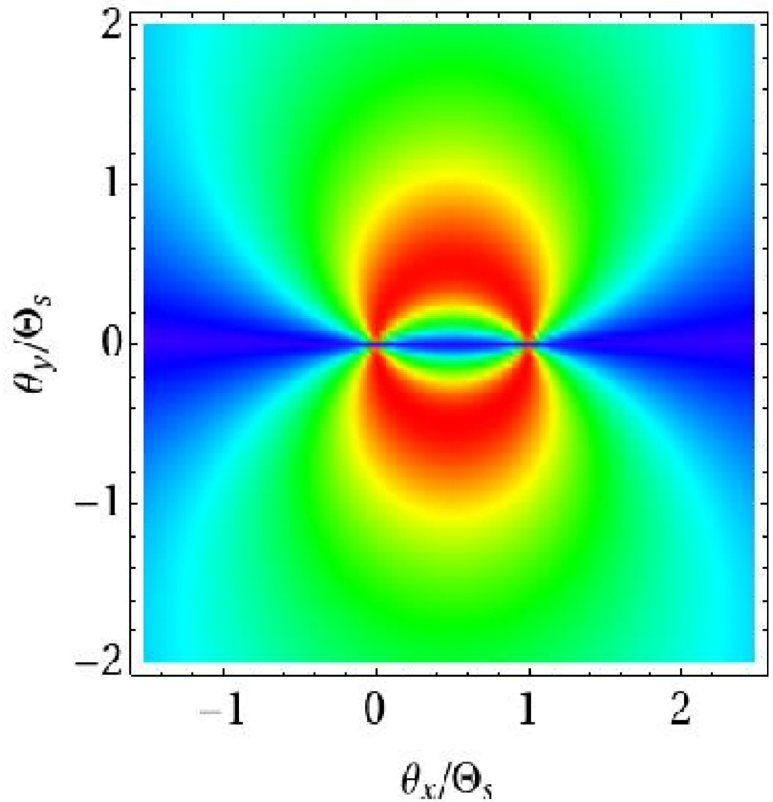
$\theta_s = 10^{-3}$

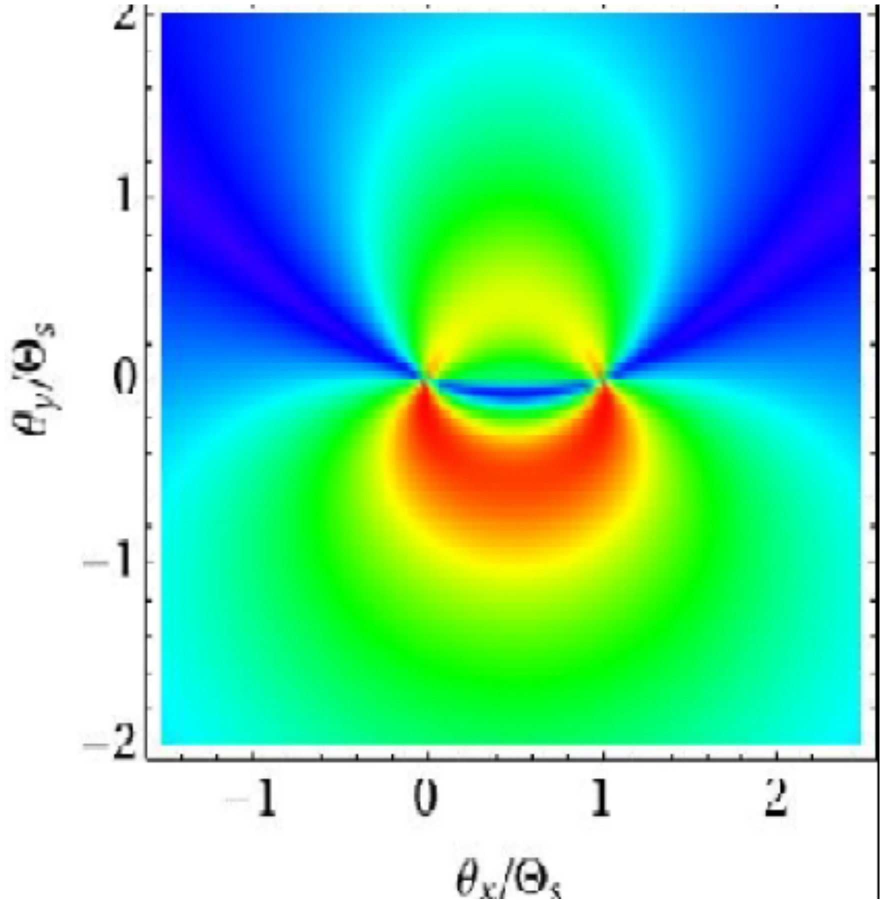


M. Ciafaloni, D. Colferai & GV, 1505.06619

Angular (polar and azimuthal) distribution

$\omega R = 10^{-3}$ $\omega R = 0.125$

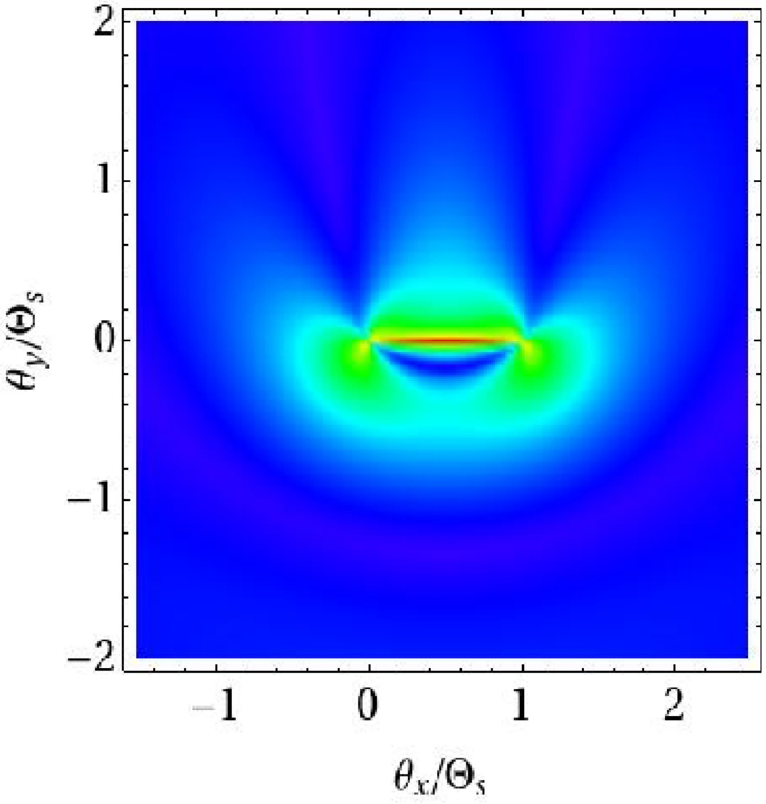




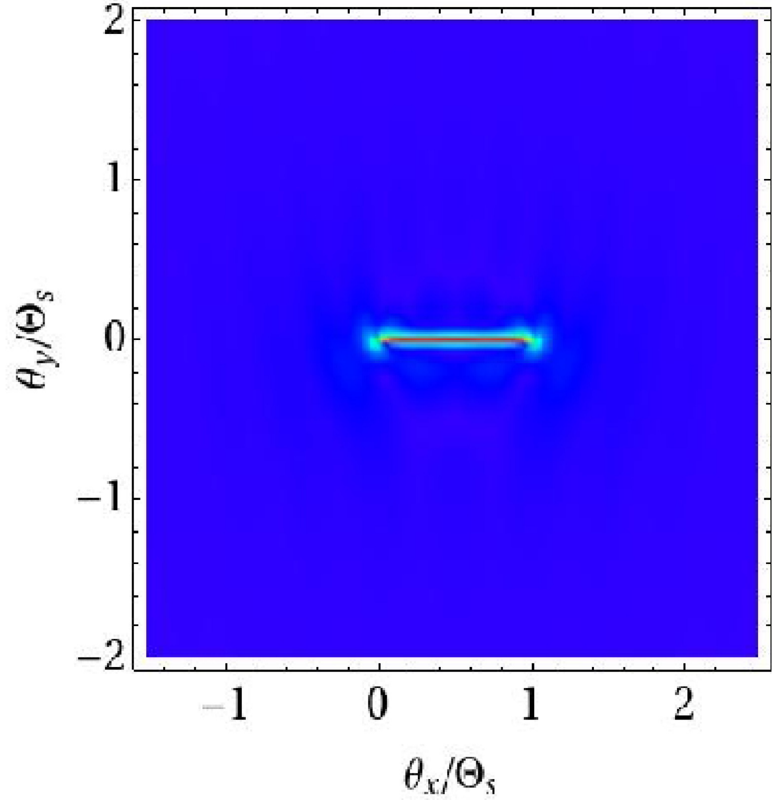
M. Ciafaloni, D. Colferai, F. Coraldeschi & GV, 1512.00281

Angular (polar and azimuthal) distribution

ωR = 1.0



ωR = 8.0



Selected for PRD’s picture gallery…

Complementarity w/ other calculations

- Grav.^{al} bremss. from a gravit^{al} collision occurs @ $O(G^3)$; same as a recent calculation of the 3PM conservative potential/deflection angle (Bern et al. 1901.04424, applied to EOB by Buonanno et. al. 1901.07102)
- Eventually, one would like to extend our method to arbitrary masses and kinematics leading hopefully to a full understanding of gravitational scattering <u>and</u> radiation at that level.
- With such a motivation in mind I'm pleased to announce:

Workshop on

Gravitational scattering, inspiral, and radiation

(GGI, May 18-July 5, 2020)