Stability in open strings with broken supersymmetry

Hervé Partouche

CNRS and Ecole Polytechnique

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Based on work done in collaboration with

S. Abel, E. Dudas and D. Lewis [arXiv:1812.09714].

String theory from a worldsheet perspective, GGI, Firenze

Introduction

■ Important properties of String Theory (dualities, branes,...) have been discovered in presence of exact supersymmetry in flat space.

Susy guaranties stability of flat backgrounds from weak to strong coupling.

■ For Phenomonology and Cosmology, susy must be broken

"In a worldsheet perspective", **we work at string weak coupling.**

■ We can **start classically with AdS or flat :**

Perturbative loop corrections cannot make AdS nearly flat.

$\Longrightarrow$ We start from a **Minkowski background.**

■ **If "hard" breaking of susy,** the susy breaking scale and effective potential are

$$M = M_s \qquad \Longrightarrow \qquad \mathcal{V}_{\text{quantum}} \sim M_s^d \quad \text{in string frame}$$

$\Longrightarrow$ Minkowski destabilized

■ **If susy spontaneously broken in flat space classically = "No-scale model" :** [Cremmer, Ferrara, Kounnas, Nanopoulos,'83]

• **$\mathcal{V}_{\text{classical}}$ is positive, with a minimum at 0, and a flat direction parameterized by M, which is a field**

• String loop corrections $\Longrightarrow$ $\mathcal{V}_{\text{quantum}} \sim M^d$, generically

Better, but still too large.

We need non-generic No-Scale Models.

■ **In this talk :** We try to **improve the quantum stability of flat backgrounds with spontaneously broken susy.**

• **Lower the order of magnitude of the potential at 1-loop**

This is modest : Higher loops should be included. Their consistent definition must be addressed.

• However, the quantum potential may induces instabilities for internal moduli : **Tadpoles ? And if not, tachyonic mass terms ?**

1-loop is enough to make good improvements about this issue.

• We do this in **type I string compactified on tori,** but this can be more general (heterotic).

■ Susy breaking via **stringy Scherk-Schwarz mechanism.**

• In field theory : Refined version of a Kaluza-Klein dimensional reduction of a theory in $d+1$ dimensions

If there is a symmetry with charges Q in $d+1$ dim, we can impose Q-depend boundary conditions

$$\Phi(x^\mu, y + 2\pi R) = e^{i\pi Q}\,\Phi(x^\mu, y)$$

$$\Longrightarrow \quad \Phi(x^\mu, y) = \sum_m \Phi_m(x^\mu)\, e^{i\frac{m+\frac{Q}{2}}{R}y} \quad \Longrightarrow \quad \text{mass} = \frac{|m + \frac{Q}{2}|}{R}$$

$\Longrightarrow$ A multiplet in $d+1$ dim with degenerate states have descendent which are not-degenerate.

• If Supersymmetry : $Q = F$ is the fermionic number

$$\Longrightarrow \quad \text{super Higgs} \qquad M = \frac{1}{2R}$$

• Generalized in closed string theory [Rohm,'84][Ferrara, Kounnas, Porrati,'88] and in open string theory [Blum, Dienes,'97][Antoniadis, Dudas, Sagnotti,'98]

■ Compute the 1-loop effective potential

$$\mathcal{V}_{\text{1-loop}} = -\frac{M_{\text{s}}^d}{2(2\pi)^d}\left(\mathcal{T}+\mathcal{K}+\mathcal{A}+\mathcal{M}\right) ,$$

where
$$\mathcal{T} = \int_{\mathcal{F}} \frac{d\tau_1 d\tau_2}{\tau_2^{1+\frac{d}{2}}} \,\text{Str}\, q^{L_0-\frac{1}{2}}\bar{q}^{\tilde{L}_0-\frac{1}{2}}$$

$$\mathcal{K} = \int_0^{+\infty} \frac{d\tau_2}{\tau_2^{1+\frac{d}{2}}} \,\text{Str}\, \Omega q^{L_0-\frac{1}{2}}\bar{q}^{\tilde{L}_0-\frac{1}{2}}$$

$$\mathcal{A} = \int_0^{+\infty} \frac{d\tau_2}{\tau_2^{1+\frac{d}{2}}} \,\text{Str}\, q^{\frac{1}{2}(L_0-\frac{1}{2})}$$

$$\mathcal{M} = \int_0^{+\infty} \frac{d\tau_2}{\tau_2^{1+\frac{d}{2}}} \,\text{Str}\, \Omega q^{\frac{1}{2}(L_0-\frac{1}{2})}$$

$\mathcal{V} \sim \int \text{Str}\, e^{-\pi\tau_2\mathcal{M}^2} \quad \Longrightarrow$ **The dominant contribution arises from the lightest states.**

Suppose the classical background is such that there is no mass scale between 0 and the susy breaking scale $M = \frac{1}{2R}$

———	cM_s	: large Higgs or string scale M_s
———	M	: towers of Kaluza-Klein modes of masses $\propto M$
———	0	: n_B massless bosons and n_F massless fermions

$\Longrightarrow$ In string frame, the 1-loop effective potential is dominated by the KK modes

$$\mathcal{V}_{\text{1-loop}} = (n_F - n_B)\,\xi\, M^d + \mathcal{O}\Big((cM_sM)^{\frac{d}{2}}\, e^{-cM_s/M}\Big)\,, \text{ where } \xi > 0$$

$$\mathcal{V}_{\text{1-loop}} = (n_\text{F} - n_\text{B})\,\xi\,M^d + \mathcal{O}\Big((cM_\text{s}M)^{\frac{d}{2}}\,e^{-cM_\text{s}/M}\Big)$$

The exponential terms are negligible even for moderate M

E.g. : For
$$cM_\text{s} \sim \frac{M_\text{Planck}}{10}$$

we have
$$\mathcal{O}\Big((cM_\text{s}M)^{\frac{d}{2}}\,e^{-cM_\text{s}/M}\Big) < 10^{-120}\,M_\text{Planck}^4$$

when
$$M < 10^{-3}\,M_\text{Planck}$$

NB : $\implies$ $R > 10^2 \gg$ Hagedorn radius $R_\text{H} = \sqrt{2}/M_\text{s}$,

$\implies$ No "Hagedorn-like phase transition" (no tree level tachyon).

• Deform slightly the previous background *i.e.* **switch on small moduli deformations collectively denoted "a"**

——— $cM_{\rm s}$: large Higgs or string scale

——— M : towers of KK modes of masses $\propto M$

——— $aM_{\rm s}$: some of the $n_{\rm B} + n_{\rm F}$ states get a Higgs mass $aM_{\rm s}$

——— 0

• **$n_{\mathbf{B}}(a)$ and $n_{\mathbf{F}}(a)$ interpolate between different integer values**, reached in distinct regions in moduli space.

$\Longrightarrow$ **Expand them in "a" to find $\mathcal{V}_{\textbf{1-loop}}$ around the initial background.**

■ Because we compactify on a torus ($\mathcal{N}=4$ in 4 dim), **all moduli are Wilson lines (WL) :**

$$\mathcal{V}_{\text{1-loop}} = \mathcal{V}_{\text{1-loop}}\Big|_{a=0} + M^d \sum_{\substack{\text{massless}\\ \text{spectrum}}} \sum_{\substack{\text{their KK}\\ \text{modes}}} \sum_{r,I} Q_r a_r^I + \cdots$$

• $\boldsymbol{a_r^I}$ **is the WL along the internal circle $\boldsymbol{I}$ of the $\boldsymbol{r}$-th Cartan $\boldsymbol{U(1)}$.**

• $\boldsymbol{Q_r}$ **is the charge of the massless spectrum** (and Kaluza-Klein towers).

• combining states Q_r and $-Q_r \implies 0$: **No Tadpole**

All points in moduli space where there is no mass scale between 0 and $\boldsymbol{M}$ are local extrema.

• This is reminiscent of an argument of Ginsparg and Vafa ('87) in the non-susy $O(16) \times O(16)$ heterotic compactified on tori :

At enhanced gauge symmetry points, $U(1)^{26-d} \to$ Non-Abelian, there are additional non-Cartan massless states, with non-trivial Q_r.

$\Longrightarrow \quad Q_r \to -Q_r$ is an exact symmetry (underlying gauge symmetry) of the partition function at any genus $\Longrightarrow$ extremum.

• In the Scherk-Schwarz case : The non-existence of tadpoles should be exact (including the exponentially suppressed terms) and at any genus.

But the massless states may not contain gauge bosons. In a non-Cartan vector mutiplet, we can keep massless the fermions and give a mass to the bosons. So $U(1)$'s are still allowed, with charged fermions.

■ At quadratic order [Kounnas, H.P,'16][Coudarchet, H.P.,'18]

$$\mathcal{V}_{\text{1-loop}} = \xi(n_{\rm F} - n_{\rm B})M^d + M^d\Big(\sum_{\substack{\text{massless}\\ \text{bosons}}} Q_r^2 - \sum_{\substack{\text{massless}\\ \text{fermions}}} Q_r^2\Big) \sum_{\substack{\text{their KK}\\ \text{modes}}} \left(a_r^I\right)^2 + \cdots$$

$\Longrightarrow$ **The higher $\mathcal{V}_{\text{1-loop}}$ is, the more tachyonic it is.**

■ **We are interested in models where $n_{\rm F} = n_{\rm B}$ and tachyon free at 1-loop** to preserve flatness of spacetime (at this order).

[Abel, Dienes, Mavroudi,'15][Kounnas, H.P.,'15][Florakis, Rozos,'16]

= **"Super No-scale Models in String Theory" :** The no-scale structure exact at tree level is preserved at 1-loop, up to exponentially suppressed terms

i.e. the 1-loop potential is locally positive, with minimum at 0, and with a flat direction M.

NB: $n_{\rm F}$, $n_{\rm B}$ count observable and hidden sectors d.o.f.

■ **In this talk : We show that tachyon free models with $\mathcal{V}_{\text{1-loop}} = 0$ (or > 0) exist at 1-loop, for $d \leq 5$.**

• **In 9 dimensions :** We find the models stable with respect to the open string Wilson lines.

$$\Longrightarrow \quad \mathcal{V}_{\text{1-loop}} < 0 \quad \Longrightarrow \quad \text{runaway of } M$$

• **In d dimensions :** We have

- Open string Wilsons lines

- Closed string moduli (which also WLs) : NS-NS metric G_{IJ} and RR 2-form C_{IJ}

■ Note that in Type II and orientifold theories, there exist non-susy models with

$$\mathcal{V}_{\text{1-loop}} = 0 \qquad i.e. \qquad N_F = N_B \text{ are any mass level !}$$

[Kachru, Kumar, Silverstein,'98]
[Harvey,'98]
[Shiu, Tye,'98] [Blumenhagen, Gorlich,98]
[Angelantonj, Antoniadis, Forger,'99]
[Satoh, Sugawara, Wada,'15]

However

- Moduli stability has not been studied (⇒ tachyonic at 1-loop).
- There are no exponentially suppressed terms at 1-loop, but this does not change the fact that $\mathcal{V}_{\text{2-loops}}$ has no reason to vanish.
  [Iengo, Zhu,'00][Aoki, D'Hoker, Phong,'03]
- When a perturbative heterotic dual is known, it only has $n_\text{F} = n_\text{B}$.
  [Harvey,'98][Angelantonj, Antoniadis, Forger,'99]

In 9 dimensions

■ **Type I compactified on $S^1(R_9)$ with Sherk-Schwarz susy breaking**

• **Closed string sector :** The states with non-trivial winding n_9 are heavier than the string scale $\Longrightarrow$ exponentially suppressed

$$\text{For } n_9 = 0, \quad \text{the momentum} \quad \frac{m_9}{R_9} \longrightarrow \frac{m_9 + \frac{F}{2}}{R_9}$$

• **Open string sector :** 32 D9-branes generate $SO(32)$ on their world volume. Switch on generic Wilson lines (=Coulomb branch)

$$\mathcal{W} = \text{diag}\big(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \ldots, e^{2i\pi a_{16}}, e^{-2i\pi a_{16}}\big)$$

$$\text{momentum} \quad \frac{m_9}{R_9} \longrightarrow \frac{m_9 + \frac{F}{2} + a_r - a_s}{R_9}$$

(The Chan-Paton charges are absorbed in the WLs : $Q_r a_r \to a_r$)

■ **T-duality $R_9 \to \tilde{R}_9 = \frac{\alpha'}{R_9}$ yields a geometric picture in Type I', where WLs become positions along $\tilde{X}^9$:**

• $S^1(R_9)$ becomes $S^1(\tilde{R}_9)/\mathbb{Z}_2$ i.e. a segment with 2 O8-orientifold planes at $\tilde{X}^9 = 0$ and $\tilde{X}^9 = \pi \tilde{R}_9$.

• The D9-branes become 32 D8 "half"-branes :
16 at $\tilde{X}^9 = 2\pi a_r \tilde{R}_9$ and 16 mirror $\frac{1}{2}$-branes at $\tilde{X}^9 = -2\pi a_r \tilde{R}_9$.

• $\frac{1}{2}$-branes and mirrors $\frac{1}{2}$-branes can be coincident on an O8-plane, $a_r = 0$ or $\frac{1}{2} \implies$ ***SO(p), p* even**

• Elsewhere, a stack of q $\frac{1}{2}$-branes and the mirror stack $\Longrightarrow$ ***U(q)***

■ We look for **stable brane configurations.**

- **A sufficient condition for $\mathcal{V}_{\text{1-loop}}$ to be extremal** with respect to the a_r is that **there is no mass scale between 0 and M.**

Thus, we may concentrate on **$a_r = 0$ or $\frac{1}{2}$ only,** i.e. no brane in the bulk.

- Moreover, **this special case yields massless fermions** because

$$m_9 + \frac{F}{2} + a_r - a_s = m_9 + \frac{1}{2} + \frac{1}{2} - 0 \quad \text{can vanish}$$

(where m_9 is a winding number in the T-dual picture)

i.e. Super-Higgs and Higgs compensate

This is a good point to have $n_{\mathrm{F}} - n_{\mathrm{B}} \geq 0$.

■ We may also consider the configurations with some $\boldsymbol{a_r = \pm\frac{1}{4}}$.

This introduces a mass scale $= \dfrac{M}{2} \implies$ Such a background does not yield automatically an extremum of $\mathcal{V}_{\text{1-loop}}$

These WLs are special because :

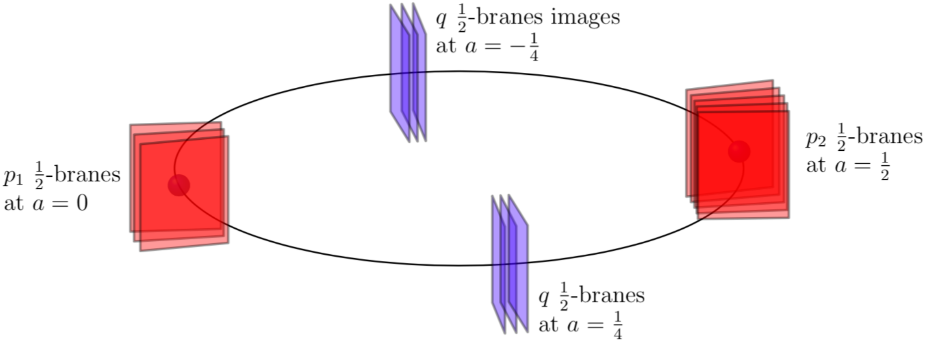
- $m_9 + \frac{1}{2} + \frac{1}{4} - (-\frac{1}{4})$ can vanish $\implies$ **massless fermions**

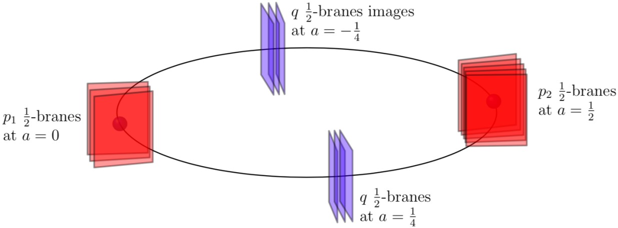
NB : WL's $= 0, \frac{1}{2}, \pm\frac{1}{4}$ are the only ones that can yield massless fermions.

- Bosons $m_9 + 0 + \frac{1}{4} - 0$ and Fermions $m_9 + \frac{1}{2} + 0 - \frac{1}{4}$ have degenerate masses $M/2$. They cancel exactly in

$$\mathcal{V} \propto \int \frac{d\tau_2}{\tau_2^{1+\frac{d}{2}}}\, \mathrm{Str}\, \frac{1+\Omega}{2}\, e^{-\pi\tau_2\mathcal{M}^2} = (n_{\mathrm{F}} - n_{\mathrm{B}})\, \xi\, M^d + \text{exp. suppressed}$$

This formula remains true in such a background, but the argument for extremality does not apply, and we have to see.


q $\frac{1}{2}$-branes images
at $a = -\frac{1}{4}$
p_1 $\frac{1}{2}$-branes
at $a = 0$
p_2 $\frac{1}{2}$-branes
at $a = \frac{1}{2}$
q $\frac{1}{2}$-branes
at $a = \frac{1}{4}$

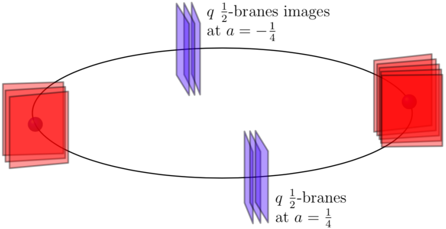


- $SO(p_1) \times SO(p_2) \times U(q) \qquad \times U(1)^2$ for $G_{\mu 9}$, RR-2-form $C_{\mu 9}$

$$n_{\rm B} = 8\left(8 + \frac{p_1(p_1-1)}{2} + \frac{p_2(p_2-1)}{2} + q^2\right)$$

- Closed string sector : dilaton, G_{MN} in NS-NS sector + RR 2-form C_{MN}

- The open strings start and end at the same stack of $\frac{1}{2}$-branes.



• **The massless fermions arise from open string only, which are stretched between "opposite stacks"** (WLs and Scherk-Schwarz compensate)

$$n_{\rm F} = 8\left(p_1 p_2 + \frac{q(q-1)}{2} + \frac{q(q-1)}{2}\right)$$

Bifundamental $\boldsymbol{(p_1, p_2)}$ and antisymmetric $\oplus$ $\overline{\text{antisymmetric}}$

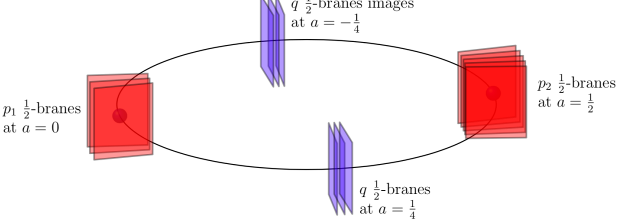
■ Compute $n_{\mathrm{F}} - n_{\mathrm{B}}$, with $p_1 + p_2 + 2q = 32$

$$\Longrightarrow \quad \boldsymbol{n_{\mathrm{F}} - n_{\mathrm{B}}} \textbf{ is minimal for } \boldsymbol{p_1 = 32,\ p_2 = 0,\ q = 0}$$

• This suggests that this configuration (which yields an extremum of $\mathcal{V}_{\text{1-loop}}$ because there are no brane in the bulk) yields an absolute minimum.

This will be seen by explicit computation of $\mathcal{V}_{\text{1-loop}}$.

• Moreover, we will see that the other extrema with higher $n_{\mathrm{F}} - n_{\mathrm{B}}$ are not minima.



■ We have described the moduli space where p_1, p_2 are even.

The moduli space admits a second, disconnected part, where p_1, p_2 are odd

If one $\frac{1}{2}$-brane is frozen at $a = 0$, and another one frozen at $a = \frac{1}{2}$, the configuration is still allowed on $S^1(\tilde{R}_9)/\mathbb{Z}_2$
[Schwarz,'99]

$$\mathcal{W} = \text{diag}\left(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \dots, e^{2i\pi a_{15}}, e^{-2i\pi a_{15}}, 1, -1\right)$$

Only 15 dynamical WLs.

All previous formula remain identical, with p_1, p_2 odd.

$$\Longrightarrow \quad \boldsymbol{n_{\mathrm{F}} - n_{\mathrm{B}}} \textbf{ is minimal for } \boldsymbol{p_1 = 31,\ p_2 = 1,\ q = 0}$$

• This suggests that this brane configuration is also stable and yields an absolute minimum of $\mathcal{V}_{\text{1-loop}}$ (in its own moduli space).

• It has an open string gauge group $SO(31) \times SO(1)$, with 8 fermions in the "bifundamental" $(p_1, 1)$.

NB : In our notations, $SO(1)$ is a trivial group $\{e\}$: Not a gauge symmetry. This notation is to remind the frozen brane at $a = \pi\tilde{R}_9$ which yields stretched strings which are fermions in the fundamental of $SO(p_1)$.

■ **To demonstrate these expectations, we compute the 1-loop potential**

It involves the torus + Klein bottle + annulus + Möbius amplitudes :

$$\mathcal{T} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^{\frac{11}{2}}} \frac{\sum_{m_9,n_9}}{\eta^8\bar{\eta}^8}\Big\{\big(V_8\bar{V}_8 + S_8\bar{S}_8\big)\Lambda_{m_9,2n_9} - \big(V_8\bar{S}_8 + S_8\bar{V}_8\big)\Lambda_{m_9+\frac{1}{2},2n_9}$$

$$+ \big(O_8\bar{O}_8 + C_8\bar{C}_8\big)\Lambda_{m_9,2n_9+1} - \big(O_8\bar{C}_8 + C_8\bar{O}_8\big)\Lambda_{m_9+\frac{1}{2},2n_9+1}\Big\}$$

$$\mathcal{K} = \frac{1}{2}\int_0^{+\infty} \frac{d\tau_2}{\tau_2^{\frac{11}{2}}} \frac{1}{\eta^8}\sum_{m_9}(V_8 - S_8)P_{m_9}$$

$$\mathcal{A} = \frac{1}{2}\int_0^{\infty} \frac{d\tau_2}{\tau_2^{\frac{11}{2}}} \frac{1}{\eta^8}\sum_{m_9}\sum_{\alpha,\beta}\big(V_8 P_{m_9+a_\alpha-a_\beta} - S_8 P_{m_9+\frac{1}{2}+a_\alpha-a_\beta}\big)$$

$$\mathcal{M} = -\frac{1}{2}\int_0^{\infty} \frac{d\tau_2}{\tau_2^{\frac{11}{2}}} \frac{1}{\hat{\eta}^8}\sum_{m_9}\sum_{\alpha}\big(\hat{V}_8 P_{m_9+2a_\alpha} - \hat{S}_8 P_{m_9+\frac{1}{2}+2a_\alpha}\big)$$

$$\mathcal{V}_{\text{1-loop}} = \frac{\Gamma(5)}{\pi^{14}}\, M^9 \sum_{l_9} \frac{\mathcal{N}_{2l_9+1}(\mathcal{W})}{(2l_9+1)^{10}} + \mathcal{O}\Big((M_{\rm s} M)^{\frac{9}{2}} e^{-\pi \frac{M_{\rm s}}{M}}\Big)$$

where $\mathcal{W} = \text{diag}\big(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \dots, e^{2i\pi a_{16}}, e^{-2i\pi a_{16}}\big)$

$$\mathcal{W} = \text{diag}\big(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \dots, e^{2i\pi a_{15}}, e^{-2i\pi a_{15}}, 1, -1\big)$$

$$\begin{aligned}\mathcal{N}_{2l_9+1}(\mathcal{W}) &= 4\big(-16 - 0 - (\text{tr}\,\mathcal{W}^{2l_9+1})^2 + \text{tr}\,(\mathcal{W}^{2(2l_9+1)})\big) \\ &= -16\left(\sum_{\substack{r,s=1 \\ r\neq s}}^{N} \cos\big(2\pi(2l_9+1)a_r\big)\cos\big(2\pi(2l_9+1)a_s\big) + N - 4\right)\end{aligned}$$

where $N = 16$ or 15

$$\mathcal{V}_{\text{1-loop}} = \frac{\Gamma(5)}{\pi^{14}}\, M^9 \sum_{l_9} \frac{\mathcal{N}_{2l_9+1}(\mathcal{W})}{(2l_9+1)^{10}} + \mathcal{O}\Big((M_{\rm s}M)^{\frac{9}{2}} e^{-\pi \frac{M_{\rm s}}{M}}\Big)$$

■ **For $a_r = 0, \frac{1}{2}, \pm\frac{1}{4}$**

$$\mathcal{N}_{2l_9+1}(\mathcal{W}) = n_{\rm F} - n_{\rm B} \quad \Longrightarrow \quad \mathcal{V}_{\text{1-loop}} = \big(n_{\rm F} - n_{\rm B}\big)\, \xi\, M^d + \mathcal{O}\Big((M_{\rm s}M)^{\frac{9}{2}} e^{-\pi \frac{M_{\rm s}}{M}}\Big)$$

- **For $a_r = \pm\frac{1}{4}$**

$$\frac{\partial \mathcal{V}_{\text{1-loop}}}{\partial a_r}\bigg|_{a_r = \pm\frac{1}{4}} \propto (p_1 - p_2)$$

Tadpole if $p_1 \neq p_2$: The bulk branes are attracted to the largest of the p_1-stack or p_2-stack.

Extremum if $p_1 = p_2$: But the WL of $U(1)$ in $U(q) = U(1) \times SU(q)$ is tachyonic. One brane moves towards $\tilde{X}^0 = 0$ or $\pi \tilde{R}_9$, regenerating the tadpole.

The branes in the bulk are highly unstable (tadpoles).

- **When all branes at $a_r = 0$ or $\frac{1}{2}$** ($\Longrightarrow$ No tadpole)

For $p_1 \geq 2$, $\boldsymbol{SO(p_1)}$ has WLs. Their masses are ≥ 0 if $p_1 - p_2 \geq 2$.

For $p_2 \geq 2$, $\boldsymbol{SO(p_2)}$ has WLs. Their masses are ≥ 0 if $p_2 - p_1 \geq 2$.

Both cannot be satisfied simultaneously !

$\Longrightarrow$ one stack must not have WLs $\Longrightarrow$ **p_2 must be 0 or 1.**

■ **Conclusion in 9 dim :**

• $\boldsymbol{SO(32)}$ and $\boldsymbol{SO(31) \times SO(1)}$ are the only stable brane configurations in their respective moduli spaces.

• M is running away.

NB : $0 - n_{\rm B} = -8 \times 504$ and $n_{\rm F} - n_{\rm B} = -8 \times 442$, which is higher because

• the dimension of $SO(31)$ is lower

• the frozen $\frac{1}{2}$-brane at $a = \frac{1}{2}$ induces a fermionic bifundam $(p_1, 1)$.

NB : In lower dim, we have more O-planes on which we can freeze more $\frac{1}{2}$-branes $\Longrightarrow n_{\rm F} - n_{\rm B} \geq 0$.

In d dimensions

■ **Type I on T^{10-d}** with metric G_{IJ} and **Scherk-Schwarz along X^9**

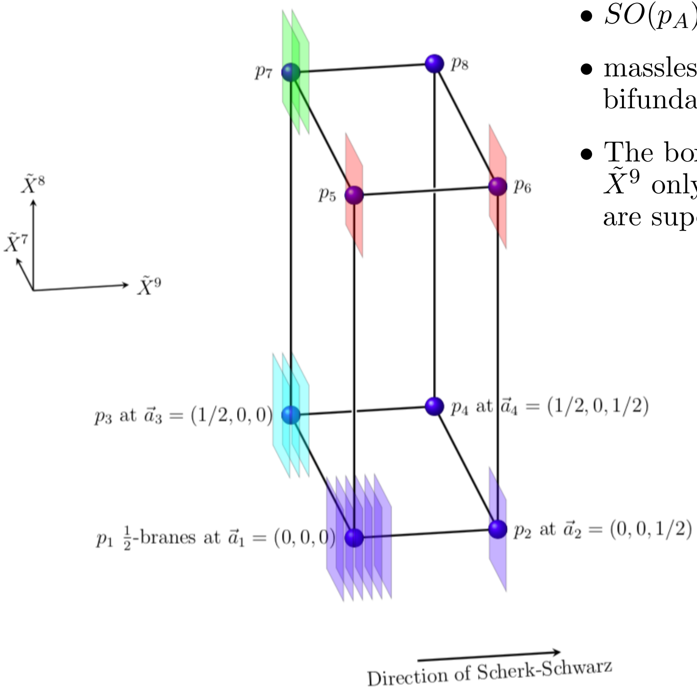
$$M = \frac{\sqrt{G^{99}}}{2}\, M_{\rm s}$$

■ **Type I'** picture obtained by **T-dualizing T^{10-d} :**

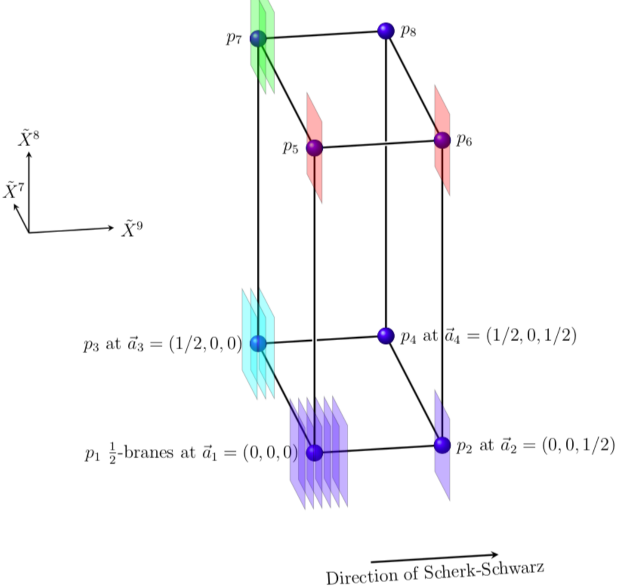
- **2^{10-d} O$(d-1)$-planes located at the corners of a $(10-d)$-dimensional box.**

- **32 "half" D$(d-1)$-branes.**

■ **$\mathcal{V}_{\text{1-loop}}$ is extremal when the 32 $\frac{1}{2}$-branes are located on the O-planes.**



- $SO(p_A)$ at corner A
- massless fermionic bifundamental (p_{2A-1}, p_{2A})
- The box is squeezed along $\tilde{X}^9$ only (the other strings are super heavy



$$n_{\rm B} = 8\left(8 + \sum_{A=1}^{2^{10-d}} \frac{p_A(p_A-1)}{2}\right), \qquad n_{\rm F} = 8 \sum_{A=1}^{2^{10-d}/2} p_{2A-1}p_{2A}$$

■ **The WL masses can be found from the potential, or**

$$\text{mass}^2 \propto \Bigg(\sum_{\substack{\text{massless}\\ \text{bosons}}} Q_r^2 - \sum_{\substack{\text{massless}\\ \text{fermions}}} Q_r^2 \Bigg) \propto T_{\mathcal{R}_\text{B}} - T_{\mathcal{R}_\text{F}}$$

where $T_\mathcal{R}$ is the Dynkin index of the representation $\mathcal{R}$ of a group G

$$T_\mathcal{R}\delta_{ab} = \frac{1}{2}\,\text{tr}\,T_a T_b, \quad {\scriptstyle (a,b=1\dots,\dim G)}$$

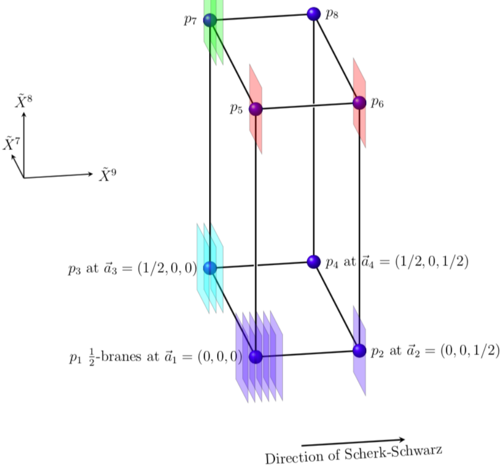
When p_{2A-1} and $p_{2A} \geq 2$, both $SO(p_{2A-1})$ and $SO(p_{2A})$ have WLs :

- For $SO(p_{2A-1})$, we have 8 bosons in the Adjoint and $8 \times p_{2A}$ fermions in the Fundamental $\Longrightarrow \text{mass}^2 \propto (p_{2A-1} - 2) - p_{2A}$

- For $SO(p_{2A})$, $\Longrightarrow \text{mass}^2 \propto (p_{2A} - 2) - p_{2A-1}$

Incompatible ! $\Longrightarrow$ One stack must not have WLs :

$SO(p_{2A-1})$ with 0 or 1 frozen $\frac{1}{2}$-brane at corner $2A$



For $SO(p_{2A-1})$, $\quad \text{mass}^2 \propto p_{2A-1} - 2 - 0 \text{ or } 1$

- It is > 0 i.e. the **WLs are stabilized**

- **Except for $SO(2)$ and $SO(3) \times SO(1)$ where mass $= 0$.**

In that case, we need to see if the quartic terms (or higher) in $\mathcal{V}_{\text{1-loop}}$ introduce instabilities.

■ **For the non-tachyonic brane configurations what is the sign of $n_F - n_B$?**

- **Many models have $n_F - n_B < 0$**

Lowest value for $SO(32)$: $n_F - n_B = -8 \times 504$

- **23 models have $n_F - n_B = 0$.** Need enough O-planes $\Longrightarrow$ $d \leq 5$

- $SO(4) \times [SO(1) \times SO(1)]^{14}$: There are 8×14 neutral fermions

- $[SO(5) \times SO(1)] \times [SO(1) \times SO(1)]^{13}$: $SO(5) + 8$ fermions in the Fundamental $+ 8 \times 13$ neutral fermions

- Other models with $SO(4)$, $SO(3)$, $SO(2)$'s.

- **The maximal value of $n_F - n_B = 8 \times 8$**

- $[SO(1) \times SO(1)]^{16}$: No gauge group, 8×16 neutral fermions

NB: All these models have an Abelian gauge group $U(1)^{10-d} \times U(1)^{10-d}$ generated by $G_{\mu J}$, $C_{\mu J}$.

■ **We can compute $\mathcal{V}_{\textbf{1-loop}}$**

- **$\mathcal{V}_{\textbf{1-loop}}$ depends on open string WLs**

$$a^I_\alpha = \langle a^I_\alpha \rangle + \varepsilon^I_\alpha, \qquad \langle a^I_\alpha \rangle \in \left\{0, \frac{1}{2}\right\}, \qquad \alpha = 1, \ldots, 32, \quad I = d, \ldots, 9$$

NB : ε^I_α are not small. $\mathcal{V}_{\text{1-loop}}$ will be the full answer.

NB : The ε^I_α go by pairs (mirrors), or are frozen to 0.

- **$\mathcal{V}_{\textbf{1-loop}}$ depends on $\boldsymbol{G_{IJ}}$**

- **$\mathcal{V}_{\textbf{1-loop}}$ does not depend on the Ramond-Ramond moduli $\boldsymbol{C_{IJ}}$** because they are also WLs, but there are no perturbative states charged under the associated $U(1)$'s, $C_{\mu I}$.

■ For the metric not to introduce a mass scale $< M$, we assume

$$\boldsymbol{G^{99} \ll |G_{ij}| \ll G_{99}, \quad |G_{9j}| \ll \sqrt{G_{99}}}, \quad i,j = d,\dots,8$$

$$\mathcal{V}_{\text{1-loop}} = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}} M^d \sum_{l_9} \frac{\mathcal{N}_{2l_9+1}(\varepsilon, G)}{|2l_9+1|^{d+1}} + \mathcal{O}\Big((cM_{\text{s}}M)^{\frac{d}{2}}\, e^{-cM_{\text{s}}/M}\Big)$$

$$\mathcal{N}_{2l_9+1}(\varepsilon,G)=4\left\{-16 - \sum_{(\alpha,\beta)\in L}(-1)^F \cos\left[2\pi(2l_9+1)\left(\varepsilon^9_\alpha-\varepsilon^9_\beta+\frac{G^{9i}}{G^{99}}(\varepsilon^i_\alpha-\varepsilon^i_\beta)\right)\right]\right.$$

$$\times\,\mathcal{H}_{\frac{d+1}{2}}\left(\pi|2l_9+1|\frac{(\varepsilon^i_\alpha-\varepsilon^i_\beta)\hat{G}^{ij}(\varepsilon^j_\alpha-\varepsilon^j_\beta)}{\sqrt{G^{99}}}\right)$$

$$\left.+\sum_\alpha \cos\left[4\pi(2l_9+1)\left(\varepsilon^9_\alpha+\frac{G^{9i}}{G^{99}}\,\varepsilon^i_\alpha\right)\right]\mathcal{H}_{\frac{d+1}{2}}\left(4\pi|2l_9+1|\frac{\varepsilon^i_\alpha\,\hat{G}^{ij}\,\varepsilon^j_\alpha}{\sqrt{G^{99}}}\right)\right\}$$

where $\hat{G}^{ij} = G^{ij} - \frac{G^{i9}}{G^{99}}\,G^{99}\,\frac{G^{9j}}{G^{99}}$ and $\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)}\,z^\nu K_\nu(2z)$

$$\mathcal{V}_{\text{1-loop}} = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}}\, M^d \sum_{l_9} \frac{\mathcal{N}_{2l_9+1}(\varepsilon, G)}{|2l_9+1|^{d+1}} + \mathcal{O}\Big((cM_{\text{s}}M)^{\frac{d}{2}}\, e^{-cM_{\text{s}}/M}\Big)$$

- From $\mathcal{N}_{2l_9+1}(\varepsilon, G)$, we recover the masses $\propto p_{2A-1} - 2 - p_{2A}$ of the ε^I_α

- The ***SO*(2) and *SO*(3) × *SO*(1) WLs are massless**

$\mathcal{N}_{2l_9+1}(\varepsilon, G)$ turns out to be totally independent of these WLs !
They are flat directions at 1-loop (up to exp. supp. terms)

- Setting the massive ones at $\boldsymbol{\varepsilon^I_\alpha = 0}$

$$\mathcal{N}_{2l_9+1}(0, G) = n_{\text{F}} - n_{\text{B}} \quad \Longrightarrow \quad \mathcal{V}_{\text{1-loop}} = \big(n_{\text{F}} - n_{\text{B}}\big)\, \xi\, M^d + \cdots$$

Independent of $G_{IJ} \Longrightarrow$ flat directions !

(Except $M = M_{\text{s}}\sqrt{G^{99}}/2$ unless $n_{\text{F}} - n_{\text{B}} = 0$)

• The fact that the NS-NS moduli G_{IJ} are massless was obvious because
i) they are WLs of the $G_{\mu J}$'s which generate $U(1)^{10-d}$
ii) there are no charged perturbative states :

$$\Longrightarrow \sum_{\substack{\text{massless}\\ \text{bosons}}} Q_r^2 - \sum_{\substack{\text{massless}\\ \text{fermions}}} Q_r^2 = 0$$

• Same thing for the RR moduli C_{IJ}, which are WLs of the $C_{\mu J}$'s which generate $U(1)^{10-d}$

• **They should be stabilized in the heterotic dual**

$$(G+C)_{IJ}|_{\text{Type I}} = (G+B)_{IJ}|_{\text{heterotic}}$$

at enhanced gauge symmetry points, where there are additional massless states with non-trivial Q_r.
These states have winding numbers $\Longrightarrow$ they are **D-string in Type I.**

• We expect only very few WLs such as those of $SO(2)$ and $SO(3) \times SO(1)$ to require an analysis at higher genus to see if they are stabilized or not.

Conclusion

■ **In open string theory compactified on a torus,** we have found **at the quantum level but weak coupling,** backgrounds

• where **all open string moduli are stabilized.**
(Additional models with $SO(2)$ or $SO(3) \times SO(1)$ factors require 2-loops analysis)

• **If $n_{\mathbf{F}} \neq n_{\mathbf{B}}$, all closed string moduli except M are flat directions at 1-loop.**
However they are expected to be stabilized at 1-loop in an heterotic framework.

• **The "Super No-Scale Models", $n_{\mathbf{F}} = n_{\mathbf{B}}$, provide consistent Minkowski vacua at 1-loop** (up to exponentially suppressed terms).
Even if non-trivial, it is modest, since
- Higher loops constraints are expected for maintaining flatness.
- The dilaton is expected not be stabilized in perturbation theory.