Superstring Perturbation Theory Using Picture Changing Operators (PCO)

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At present superstring perturbation theory can be described using two different but related approaches

– as integration over supermoduli space

$\cdots$; D'Hoker, Phong; Donagi, Witten; Witten

– using picture changing operators (PCOs)

Friedan, Martinec, Shenker; Knizhnik; Verlinde, Verlinde; $\cdots$

The goal of this talk will be to discuss some subtleties in the second approach and their resolution.

References:

A.S., arXiv:1408.0571

A.S., E. Witten, arXiv:1504.00609

Related approach has been discussed in open string field theory

Erler, Konopka, Sachs

Plan

1. Bosonic string amplitudes

2. Superstring amplitudes with PCO

3. Problems with spurious poles

4. Resolution

Bosonic string amplitudes

Unintegrated vertex operators:

$$A_i = c\bar{c}V_i, \quad V_i : \text{dimension (1,1) primary in matter CFT}$$

g-loop, N-point amplitude

$$\mathcal{A}(A_1, \cdots A_N) = (-2\pi i)^{3g-3+N} \int_{M_{g,N}} \omega_{6g-6+N}$$

$M_{g,N}$: Moduli space of genus g Riemann surface with N punctures with coordinates $(m_1, \cdots, m_{6g-6+2N})$.

$\omega_{6g-6+2N}$: A (6g-6+2N)-form in $M_{g,N}$ defined via:

$$\omega_{6g-6+2N} = \left\langle \left\{ \prod_{i=1}^{6g-6+2N} (\eta_i|B)\, dm_i \right\} A_1 \cdots A_N \right\rangle$$

$(\eta_i|B) = \int d^2z\, \eta_{i\bar{z}}{}^z b(z) + \text{c.c.}, \quad \eta_{i\bar{z}}{}^z$: **Beltrami differential**

It is useful to define a p-form ω_p on $M_{g,N}$ via

$$\left\langle \exp\left[\sum_i (\eta_i|B)dm_i\right] A_1 \cdots A_N \right\rangle = \sum_p \omega_p$$

ω_p satisfies useful identity:

$$\omega_p(Q_B A_1, A_2, \cdots, A_N) + \cdots + \omega_p(A_1, \cdots, Q_B A_N) = (-1)^p d\omega_{p-1}(A_1, \cdots A_N)$$

– useful for proving gauge invariance.

If $A_1, \cdots, A_{N-1}$ are BRST invariant and $A_N = \lambda$ for some λ, then

$$\int_{M_{g,N}} \omega_{6g-6+2N}(A_1, \cdots A_{N-1}, Q_B\lambda) = \int_{M_{g,N}} d\omega_{6g-7+2N}(A_1, \cdots, \lambda) = 0$$

up to boundary terms.

Superstring amplitudes

We shall focus on heterotic string theory

(Generalization to type II is straightforward)

World-sheet theory has bosonic ghosts β, γ

'Bosonization' of β-γ system:

$$\beta = \partial\xi\, \mathbf{e}^{-\phi}, \quad \gamma = \eta\, \mathbf{e}^{\phi}$$

ξ, η **fermionic,** ϕ **bosonic**

Picture number:

$$\mathbf{q} \textbf{ for } \mathbf{e}^{\mathbf{q}\phi}, \quad \mathbf{1} \textbf{ for } \xi, \quad -1 \textbf{ for } \eta, \quad \mathbf{0} \textbf{ for } \beta, \gamma$$

PCO: $\mathcal{X}(\mathbf{z}) = \{\mathbf{Q_B}, \xi(\mathbf{z})\}$ **carries picture number 1**

NS sector vertex operator: $c\bar{c}e^{-\phi}V_{NS}$

V_{NS}: a dimension (1,1/2) superconformal primary in matter SCFT

R sector vertex operator: $c\bar{c}e^{-\phi/2}V_R$

V_R: a dimension (1, 5/8) primary in matter SCFT

GSO projection: built into the definition of SCFT

Sum over spin structures: built into the definition of $\int_{M_{g,N}}$

On a genus g Riemann surface, we need a total picture number 2g-2 to get non-zero result.

m NS vertex operators and n R vertex operator carries total picture number $-m - n/2$.

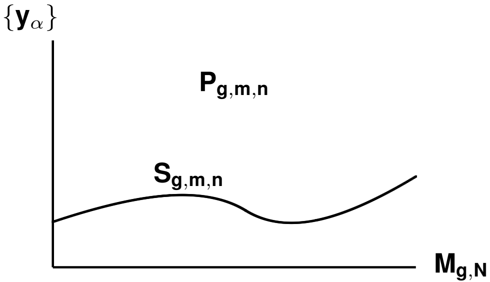
$\Rightarrow$ we need to insert $2g - 2 + m + n/2$ PCO's at $y_1, \cdots y_{2g-2+m+n/2}$.

Naive guess: Insert a factor of

$$\prod_{\alpha=1}^{2g-2+m+n/2} \mathcal{X}(y_\alpha)$$

into the bosonic string integration measure.

– true locally if the y_α's are independent of m_i, but not otherwise.



Consider a fiber bundle $P_{g,m,n}$ with base $M_{g,N}$ (N=m+n) and fiber the possible choice of PCO locations.

Fiber is (2g-2+m+n/2) complex dimensional space.

A choice of PCO locations $\Leftrightarrow$ section $S_{g,m,n}$ of $P_{g,m,n}$

Introduce p-forms $\omega_{6g-6+2N}$ in $P_{g,m,n}$ via:

$$\sum_p \omega_p = \left\langle \exp\left[\sum_i (\eta_i|B) dm_i - \sum_\alpha \partial\xi(y_\alpha) dy_\alpha \frac{1}{\mathcal{X}(y_\alpha)}\right] \left\{\prod_{a=1}^{2g-2+m+n/2} \mathcal{X}(y_a)\right\} A_1 \cdots A_N \right\rangle$$

For given choice of section $y_\alpha(m)$, the amplitude is given by

$$\mathcal{A}(A_1, \cdots A_N) = (-2\pi i)^{3g-3+N} \int_{S_{g,m,n}} \omega_{6g-6+N}$$

$$= (-2\pi i)^{3g-3+N} \int_{M_{g,N}} \left\langle \prod_{i=1}^{6g-6+2N} \left[dm_i \left\{(\eta_i|B) - \partial\xi(y_\alpha) \frac{\partial y_\alpha}{\partial m_i} \frac{1}{\mathcal{X}(y_\alpha)}\right\}\right] \left\{\prod_{a=1}^{2g-2+m+n/2} \mathcal{X}(y_a)\right\} A_1 \cdots A_N \right\rangle$$

Verlinde, Verlinde

ω_p **satisfies the identity:**

$$\omega_p(Q_B A_1, A_2, \cdots, A_N) + \cdots + \omega_p(A_1, \cdots, Q_B A_N) = (-1)^p d\omega_{p-1}(A_1, \cdots A_N)$$

– can be used to prove decoupling of pure gauge states as before

The same identity can also be used to prove the section independence of the amplitude.

For BRST invariant A_i's, $d\omega_p(A_1, \cdots A_N) = 0$.

If we have two sections S and S', consider $R \subset P_{g,,m,n}$ bounded by S and S'.

$$\int_S \omega_{6g-6+2N} - \int_{S'} \omega_{6g-6+2N} = \int_R d\omega_{6g-6+2N} = 0$$

up to contributions from the boundary of the moduli space.

Problems with spurious poles

ω_p is expected to have singularities in $P_{g,m,n}$ above the singular boundaries of $M_{g,N}$ (degenerate Riemann surfaces)

– associated with IR divergences and have well understood interpretations.

Spurious poles: Poles of ω_p above regular points in $M_{g,N}$, coming from:

1. Collision of PCOs with each other or with vertex operators

2. For $g \geq 1$ the integrand has poles where no two operators coincide

– associated with γ developing a zero mode, causing path integral over γ to diverge.

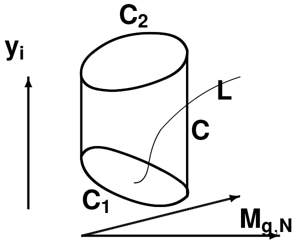
The spurious poles occur over complex codimension one subspaces of $\mathbf{P_{g,m,n}}$

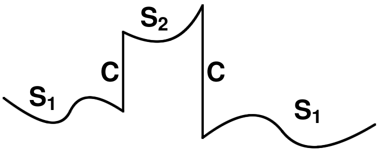
– real codimension two.

A typical section $\mathbf{S_{g,m,n}}$ will intersect loci of spurious poles.

How to integrate $\omega_{\mathbf{6g-6+2N}}$ through these singular subspaces?

Resolution via vertical integration





3-dimensional view

2-dimensional view

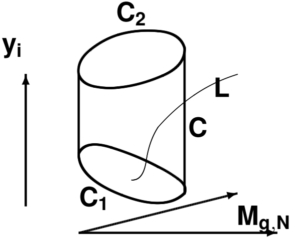
L: Spurious pole locus

Choose the integration cycle $S_{g,m,n}$ to be

$$S_1 \cup C \cup S_2$$

so that the spurious pole locus intersects $S_{g,m,n}$ along a 'vertical segment'.

A.S.



3-dimensional view

Along each fiber of the vertical segment, only y_i varies, keeping other coordinates of $P_{g,m,n}$ fixed.

The y_i dependent part of the integral:

$$\int_{y_i^{(1)}}^{y_i^{(2)}} (-\partial\xi(y_i))dy_i = \xi(y_i^{(1)}) - \xi(y_i^{(2)})$$

This has perfectly well defined correlator.

The right hand side is well defined and unambiguous even though the path may run through a pole.

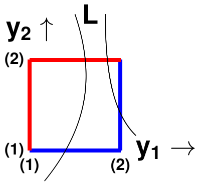
The amplitude defined this way behaves as if we have integrated along a smooth section

– satisfies the usual identities required to prove gauge invariance and section independence.

If there are multiple PCOs to be moved from one configuration to another, move them one by one as if there are multiple vertical segments.

e.g. $(y_1^{(1)}, y_2^{(1)}) \to (y_1^{(2)}, y_2^{(1)}) \to (y_1^{(2)}, y_2^{(2)})$

or $(y_1^{(1)}, y_2^{(1)}) \to (y_1^{(1)}, y_2^{(2)}) \to (y_1^{(2)}, y_2^{(2)})$



– represent two different choice of integration cycles and the difference vanishes by the usual argument.

Complete algorithm:

A.S., Witten

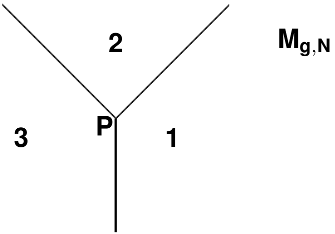
1. Divide $M_{g,N}$ into small cells.

2. Over each cell, choose PCO locations avoiding spurious poles.

3. At the boundary between two cells, moves the PCO assignment in one cell to the other using vertical integration

– corresponds to adding correction terms at each boundary between cells

4. At the intersection of boundaries, we need to add further terms due to mismatch in the vertical integration prescription.



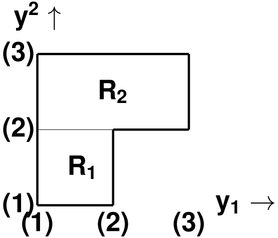
Suppose we have two PCOs.

In $1 \rightarrow 2$ we move them as $(y_1^{(1)}, y_2^{(1)}) \rightarrow (y_1^{(2)}, y_2^{(1)}) \rightarrow (y_1^{(2)}, y_2^{(2)})$

In $2 \rightarrow 3$ we move them as $(y_1^{(2)}, y_2^{(2)}) \rightarrow (y_1^{(3)}, y_2^{(2)}) \rightarrow (y_1^{(3)}, y_2^{(3)})$

In $3 \rightarrow 1$ we move them as $(y_1^{(3)}, y_2^{(3)}) \rightarrow (y_1^{(1)}, y_2^{(3)}) \rightarrow (y_1^{(1)}, y_2^{(1)})$

This leaves a gap in the integration cycle over P.



Note the hole $R_1 \cup R_2$ left behind at P.

We need to 'fill the hole' by enacting a 2D vertical segment above the codimension 2 subspace P of $M_{g,N}$.

Integration over R_1:

$$\int_{R_1} (-\partial\xi(y_1)dy_1) \wedge (-\partial\xi(y_2)dy_2) = \{\xi(y_1^{(1)}) - \xi(y_1^{(2)})\}\{\xi(y_2^{(1)}) - \xi(y_2^{(2)})\}$$

Integration over R_2:

$$\int_{R_2} (-\partial\xi(y_1)dy^1) \wedge (-\partial\xi(y_2)dy^2) = \{\xi(y_1^{(1)}) - \xi(y_1^{(3)})\}\{\xi(y_2^{(2)}) - \xi(y_2^{(3)})\}$$

Total contribution: Sum of the two

This process continues if there are more than two PCOs.

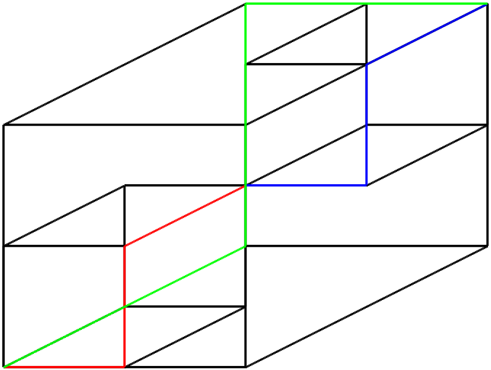
Additional corrections at the intersection of the codimension two intersections etc.

The principle remains the same: Fill the hole.

Often it is not unique.

Different choices correspond to different choices of integration cycles.

Difference between the integrands is a total derivative whose integral vanishes up to boundary terms.



An example of three PCOs

We need to ‘fill’ the region bounded by the red, blue and green lines by squares lying parallel to the coordinate axes.

Boundary terms

In order to remove possible ambiguities associated with integrals of total derivatives we need to fix the PCO arrangement near the boundary of the moduli space.

Insight from string field theory: Interacting fields carry picture numbers -1 and $-1/2$.

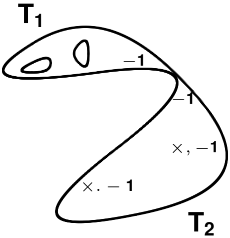
This translates to the following prescription for separating type degeneration:

The number of PCOs on each component should be such that the picture number is conserved by assigning the degenerating node to carry either picture number -1 or picture number $-1/2$.

Example: Consider genus 2 amplitude of two NS sector states.

Required number of PCOs: 2g-2+m+n/2 = 4

Now consider the degeneration associated with two loop tadpole.



Required PCO distribution: 3 on T_1, 1 on T_2

This makes working with zero picture vertex operator problematic.

Similar counting involving R degeneration shows that 1 PCO must be inserted on the neck.

Conclusion

We have a complete description of perturbative superstring amplitudes at any genus based on PCO insertions.

The description itself has ambiguities due to the freedom of choice in the PCO distribution.

However the on-shell amplitudes are free from all the ambiguities.

Even though the procedure seems complicated, once we know that there is a well defined procedure, we can manipulate the expressions to get simple results in special cases.

Example: Two loop dilaton tadpole due to Fayet-Iliopoulos term generated at one loop.

Explicit calculation using PCOs gives results in perfect agreement with effective field theory predictions.