AdS$_3$ Holography and Black Hole Microstates

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References

Based on

1507.009445, 1606.01119, 1607.03908, 1705.09250, 1710.06820, 1711.10474, 1812.06479, 1904.xxxxx

with I. Bena, A. Bombini, A. Galliani, E. Martinec, E. Moscato, S. Rawash, R. Russo, M. Shigemori, D. Turton, N. Warner, C. Wen

Overview and Motivations

Black holes and 2D CFT

- In some situations, a black hole is dual to an ensemble in a 2D CFT

$$\text{Black hole} \xrightarrow{\text{decoupling}} \text{AdS}_3 \xleftrightarrow{\text{holography}} \text{2D CFT}$$

- A b.h. microstate is dual to a "heavy" operator ($\Delta_H \sim c \gg 1$) and (in some cases) is described by a 10D classical geometry

(Lunin, Mathur)

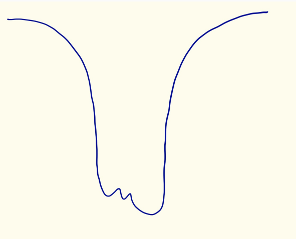
$g_s^2 N \gg 1$ $g_s^2 N \ll 1$

$\mathbb{R}^{4,1} \times S^1$

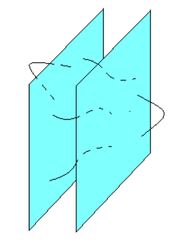
$\text{AdS}_3 \times S^3$

$r \sim R_{\text{Hor}} \longrightarrow$

no horizon!



$\longleftrightarrow$



ds_H^2 O_H

Holographic probes

- Microstates can be probed by "light" operators ($\Delta_L \sim O(c^0)$)
- 3-point correlators

$$\langle O_L(1)\rangle_{ds^2_H} \longleftrightarrow \langle \bar{O}_H(\infty)O_H(0)O_L(1)\rangle$$

  - They are extracted from the asymptotic expansion of ds^2_H
  - If O_H and O_L are susy they do not depend on the moduli
  - They can be used to test the map between gravity and free CFT

- 4-point correlators

$$\langle O_L(z)\bar{O}_L(1)\rangle_{ds^2_H} \longleftrightarrow \langle \bar{O}_H(\infty)O_H(0)O_L(z)\bar{O}_L(1)\rangle$$

  - They are derived by solving the wave equation in ds^2_H
  - They generically depend non-trivially on the moduli
  - Unitarity requires that they do not vanish at large Lorentzian time

The D-brane system

Extremal near-horizon limits: 4D

- The extremal 4-charge black hole in type II on $\mathbb{R}^{3,1} \times T^6$

$$\begin{array}{ccc} \mathrm{D6}_{123456}\,\mathrm{D2}_{12}\,\mathrm{D2}_{34}\,\mathrm{D2}_{56} & \xrightarrow{\text{decoupling}} & \mathrm{AdS}_2 \times S^2 \times T^6 \longleftrightarrow \mathrm{1D\,CFT} \\ \Big\downarrow \text{U-duality} & & \\ \mathrm{KKM}_{12345(6)}\,\mathrm{D1}_5\,\mathrm{D5}_{12345}\,\mathrm{P}_5 & \xrightarrow{\text{decoupling}} & \mathrm{AdS}_3 \times S^2 \times T^5 \longleftrightarrow \mathrm{2D\,CFT} \end{array}$$

- The 2D CFT is the $(4, 0)$ MSW theory; it is not well-understood
- The system simplifies if KKM $\to 0$ and $R_6 \to \infty$

$\Rightarrow$

Extremal near-horizon limits: 5D

(Strominger, Vafa)

- The extremal 3-charge black hole in type IIB on $\mathbb{R}^{4,1} \times S^1 \times T^4$

$$\mathrm{D1}_5\,\mathrm{D5}_{12345}\,\mathrm{P}_5 \xrightarrow{\text{decoupling}} \mathrm{AdS}_3 \times S^3 \times T^4 \longleftrightarrow \mathrm{2D\,CFT}$$

with $\mathrm{vol}(T^4) \sim \ell_s^4$ and $R(S^1) \gg \ell_s$

- The 2D CFT is the $(4,4)$ D1D5 CFT with $c = 6n_1 n_5 \equiv 6N \gg 1$
- The CFT has a 20-dim moduli space:
  - free orbifold point $\longleftrightarrow$ $R_{AdS} \ll \ell_s$
  - strong coupling point $\longleftrightarrow$ $R_{AdS} \gg \ell_s$

The D1-D5 CFT

The D1-D5 CFT

- Symmetries:
  (4,4) SUSY with $SU(2)_L \times SU(2)_R$ R-symmetry $\longleftrightarrow$ S^3 rotations
- The orbifold point: sigma-model on $(T^4)^N/S_N$
  The elementary fields are 4 bosons, 4 fermions and twist fields
- Chiral primary operators: $O_{(j,\bar{j})}$ with $h = j$, $\bar{h} = \bar{j}$ (and their descendants with respect to L_{-n}, J^-_{-n}, $G^-_{-n-1/2}$) are protected
- Spectral flow:
  - NS $\longrightarrow$ R
  - $j \longrightarrow j + \frac{N}{2}$, $h \longrightarrow h + j + \frac{N}{4}$
  - (anti)CPO $\longrightarrow$ RR ground states with $h = \bar{h} = \frac{N}{4}$

A simple D1D5 state

(Maldacena-Maoz et al.)

- States carrying D1-D5 charges are RR ground states ($h = \bar{h} = \frac{N}{4}$)
  Note: $h, \bar{h} \sim c \;\Rightarrow$ "heavy" states $\Rightarrow$ classical geometry
- A simple example:

$$|0\rangle_{\rm NS} \xrightarrow{\text{spectral flow}} |N/2, N/2\rangle_R$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\text{AdS}_3 \times S^3 \qquad \phi \to \phi - \tau \,,\; \psi \to \psi - \sigma \qquad \text{AdS}_3 \times' S^3$$

with (ϕ, ψ) S^3 coordinates and (τ, σ) AdS_3 coordinates

- The geometry dual to the maximally rotating RR ground state $|N/2, N/2\rangle_R$ is $\text{AdS}_3 \times' S^3$ with S^3 non-trivially fibered over AdS_3

The graviton gas

- If O_k is a (anti)CPO of dimension k one can consider its descendants

$$O_{k,m,n,q} \equiv (J_0^+)^m (L_{-1})^n (G^{+1}_{-\frac{1}{2}} G^{+2}_{-\frac{1}{2}})^q \, O_k$$

- Spectral flow maps $O_{k,m,n,q}$ to a D1-D5-P state with $h > \bar{h} = \frac{N}{4}$
- "Semi-classical" states are coherent states

$$|B_1, B_2, \ldots\rangle_{\mathrm{NS}} \equiv \sum_{p_1, p_2, \ldots} (B_1 O_{k_1,m_1,n_1,q_1})^{p_1} (B_2 O_{k_2,m_2,n_2,q_2})^{p_2} \ldots |0\rangle_{\mathrm{NS}}$$

- When $B_i^2 \sim N \gg 1$ the p_i-sum is peaked for $p_i \approx B_i^2/k$

What is the gravitational description of $|B_1, B_2, \ldots\rangle_{\mathrm{NS}}$?

The gravity side

Superstrata: construction

- Holography associates to O_k a sugra field ϕ_k : $O_k \longleftrightarrow \phi_k$
- At linear order in B_i $|B_1, \ldots\rangle_{\rm NS}$ is a perturbation of the vacuum

$$|0\rangle_{\rm NS} + B_i\, O_{k_i,m_i,n_i,q_i}\, |0\rangle_{\rm NS} \longleftrightarrow {\rm AdS}_3 \times S^3 + B_i\, \phi_{k_i,m_i,n_i,q_i}$$

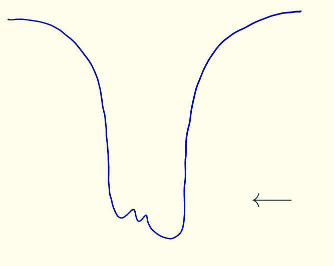
where ϕ_{k_i,m_i,n_i,q_i} solves the linearised sugra eqs. around $\mathrm{AdS}_3 \times S^3$

$$\phi_{k,m,n,0} = \frac{\rho^n}{(\rho^2+1)^{\frac{n+k}{2}}} \sin^{k-m}\theta \, \cos^m\theta \, e^{i\,[(k-m)\phi - m\psi + (k+n)\tau + n\sigma]}$$

- One can extend the linearised solution to an exact solution of the sugra eqs. valid for $B_i^2 \sim N$
- The non-linear extension is non-unique: ambiguities are fixed by imposing regularity

Superstrata: result

- The non-linear solutions are smooth and horizonless
- The solutions are asymptotically $\mathrm{AdS}_3 \times S^3$ but in the interior AdS_3 and S^3 are non-trivially mixed
- There is a continuous family of solutions, parametrised by B_i, for fixed values of the global D1, D5, P charges



$\mathbb{R}^{4,1} \times S^1$

$\mathrm{AdS}_3 \times S^3$

$\longleftarrow$ $r \sim R_{\mathrm{Hor}}$ no horizon!

(Bena, Ceplak, Heidmann, SG, Martinec, Russo, Shigemori, Turton, Warner)

Holographic 3-point correlators

HHL correlators

(Kanitscheider, Skenderis, Taylor)

- We compute
$$\langle O_L \rangle_H \equiv \langle \bar{O}_H(\infty) O_H(0) O_L(1) \rangle$$
with
    - $O_H \xrightarrow{\text{spectral flow}} \sum_{p_1,\dots} (B_1 O_{k_1,m_1,n_1,q_1})^{p_1} \dots \xleftrightarrow{\text{holography}} ds^2_H$
    - $O_L = O_k \xleftrightarrow{\text{holography}} \phi_k$
- $\langle \bar{O}_H O_H O_L \rangle$ do not depend on the CFT moduli
- One can extract $\langle O_k \rangle_H$ from the geometry ds^2_H
$$\phi_k \stackrel{\rho\to\infty}{\longrightarrow} \rho^{-k} \langle O_k \rangle_H$$
- One can compare $\langle O_k \rangle_H$ with the value computed in the orbifold CFT

An example

- $O_H = \sum_{p_1,p_2} (B_1 O_1)^{p_1} (B_2 O_2)^{p_2}$

  with O_1, O_2 CPOs of dimension 1 and 2
- From the CFT

  $$\langle O_1 \rangle_H \sim B_1 \quad , \quad \langle O_2 \rangle_H \sim B_2$$

  This agrees with the gravity result as a consequence of the linear construction of ds^2_H
- There is a CPO O'_1 such that $\langle O'_1 O_1 O_2 \rangle \neq 0 \Rightarrow$

  $$\langle O'_1 \rangle_H \sim B_1 B_2$$

  This agrees with the gravity result, including the numerical coefficient, as a consequence of the non-linear terms needed for regularity of ds^2_H

A technical remark

- For operators of dimension greater than 1 the holographic map is complicated by the possible mixing between single-particle CPOs and between single-particle and multi-particle CPOs

$$O_k + O'_k + O_{k-1}O_1 + \dots \xleftrightarrow{\text{holography}} \phi_k$$

- The map has been worked out for $k \leq 2$ providing non-trivial evidence of the map

$$O_H \xleftrightarrow{\text{holography}} ds^2_H$$

at the full non-linear level

Holographic 4-point correlators

HHLL correlators

- How to compute holographically

$$\mathcal{C}_H(z,\bar z) \equiv \langle \bar O_H(\infty) O_H(0) O_L(z,\bar z) \bar O_L(1)\rangle$$

- $O_L(z,\bar z) \equiv O_k(z,\bar z) \longleftrightarrow \phi_k(\rho; z,\bar z)$
- Solve the linearised e.o.m. for ϕ_k in the background $ds^2_H \longleftrightarrow O_H$
- Pick the non-normalisable solution such that
  - at the boundary ($\rho \to \infty$)

$$\phi_k(\rho; z,\bar z) \overset{\rho\to\infty}{\longrightarrow} \delta(z-1)\,\rho^{k-2} + b(z,\bar z)\,\rho^{-k}$$

vev of $O_L(z,\bar z)$

source for $\bar O_L(1)$

  - in the interior ($\rho \to 0$) $\phi(\rho; z,\bar z)$ is regular
- The correlator is given by

$$\mathcal{C}_H(z,\bar z) = \langle O_H | O_L(z,\bar z)\bar O_L(1) | O_H\rangle = b(z,\bar z)$$

A simple example

- We take

$$O_H = \sum_p (B\, O_1)^p \quad , \quad O_L = O_1$$

- O_H flows to a RR ground state $\Rightarrow$ $P = 0$
- The ensemble of RR ground states corresponds to a “small black hole” (massless limit of BTZ)

$$\frac{ds^2}{R^2_{\mathrm{AdS}}} = \frac{d\rho^2}{\rho^2} + \rho^2(-d\tau^2 + d\sigma^2) + d\Omega_3^2$$

- The geometry ds^2_H dual to O_H approximates the small black hole geometry in the limit $B^2 \to N$
- Computing $\mathcal{C}_H$ for heavy states with $P \neq 0$ has been possible only for small B until now

Results

Gravity

$$\mathcal{C}_H = \alpha\, e^{-i\tau} \sum_{l\in\mathbb{Z}} e^{il\sigma} \sum_{n=1}^{\infty} \frac{\exp\left[-i\alpha\sqrt{(|l|+2n)^2 + \frac{(1-\alpha^2)l^2}{\alpha^2}}\tau\right]}{\sqrt{1+\frac{1-\alpha^2}{\alpha^2}\frac{l^2}{(|l|+2n)^2}}} + N(1-\alpha^2)e^{-i\tau}$$

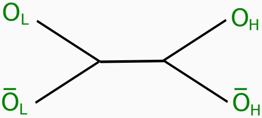
with $z = e^{i(\tau+\sigma)}, \bar{z} = e^{i(\tau-\sigma)},\ \alpha = \left(1-\frac{B^2}{N}\right)^{1/2}$

Free CFT

$$\mathcal{C}_H = \frac{1}{|z||1-z|^2} + \frac{B^2}{2N}\frac{|z|^2+|1-z|^2-1}{|z||1-z|^2} + \frac{(N-B^2)B^2}{N}\left(1-\frac{1}{N}\right)\frac{1}{|z|}$$

The OPE interpretation

- The 4-point function can be reconstructed from the $z \to 1$ OPE



$$O_L(z)\bar{O}_L(1) \sim |1-z|^{-2}(1+(1-z)J+(1-z)^2 T+\ldots)$$
$$+\sum_{n,\ell}(1-z)^{n+\ell}(1-\bar{z})^n :O_L\partial^{n+\ell}\bar{\partial}^n\bar{O}_L:$$
$$+\text{ stringy operators}$$

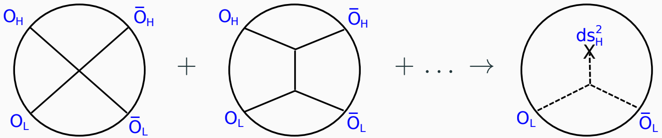
- The first line gives the affine identity block, which dominates in the light-cone limit ($\bar{z} \to 1$)
- In the second line are non-BPS double-trace operators with

$$h = 1+n+\ell+\frac{\gamma_{n\ell}}{N} \quad , \quad \bar{h} = 1+n+\frac{\gamma_{n\ell}}{N}$$

- The operators in the third line are dual to string modes and have $h, \bar{h} \to \infty$ in the sugra limit

A comment on the method

- Holographic correlators of single-trace operators (like O_L) are usually computed by summing Witten diagrams
- This technique has not been extend to correlators with multi-trace operators (like O_H)
- Even for single-trace correlators, Witten diagrams in AdS_3 are subtle: no holographic correlator in a 2D CFT has ever been computed before
- Our approach bypasses Witten diagrams:



- In a certain limit: $\langle \bar{O}_H O_H O_L \bar{O}_L \rangle \to \langle \bar{O}_L O_L O_L \bar{O}_L \rangle$

(SG, Russo, Wen)

Late-time behaviour and unitarity

The late time behaviour of the HHLL correlator

- We focus on the limit $B^2 \to N \Leftrightarrow \alpha \to 0$ in which ds^2_H approximates the "small b.h."
- In this limit the series giving $\mathcal{C}_H$ is dominated by terms with $n \gg \frac{|l|}{2\alpha}$:

$$\mathcal{C}_H \sim e^{-i\tau}\left[\frac{1}{1-e^{i(\sigma-\tau)}}+\frac{1}{1-e^{-i(\sigma+\tau)}}-1\right]\frac{\alpha}{1-e^{-2i\alpha\tau}}$$

- The time-dependence of the correlator is controlled by α:
  - for $\tau \ll \alpha^{-1}$ one has $\mathcal{C}_H \sim \tau^{-1}$;
    this is the same behaviour of the 2-point function in the "small b.h."
  - for $\tau \gtrsim \alpha^{-1}$ $\mathcal{C}_H$ stops decreasing with τ and oscillates
- Correlators in a pure or thermal state in a unitary theory with finite entropy do not vanish at late times

The late-time behaviour of $\mathcal{C}_H$ is consistent with unitarity already at large c

Summary and outlook

Summary

- The identification of black hole microstates with “heavy” states of a dual CFT provides a guide to their gravitational description
- At strong coupling (some) heavy states are described by smooth horizonless geometries
- HHL and HHLL correlators can be extracted from these geometries and can be used to substantiate the map between states and geometries and to probe the unitarity of the gravity picture
- If probed for a short time microstates are indistinguishable from the black hole, but for sufficiently long times microstates deviate from the black hole and produce correlators that are consistent with unitarity already at large c

Outlook

- Classical supergravity works well for atypical states in the black hole ensemble
- For some observables, deviations from a typical state and the classical black hole should be exponentially suppressed in the entropy
- How much of our analysis can be extended to typical states?
- And what about microstates of non-BPS black holes?
- It is possible that classical supergravity probes cannot resolve the structure of typical states
- Does one need to resort to full string theory?

Extra slides

A technical remark

- The e.o.m. for $\phi \longleftrightarrow O_L$ is complicated
- It is simpler to compute

$$\widetilde{\mathcal{C}}_H(z,\bar{z}) \equiv \langle \bar{O}_H(\infty) O_H(0) \widetilde{O}_L(z,\bar{z}) \overline{\widetilde{O}}_L(1) \rangle$$

with

$$\widetilde{O}_L \equiv G\bar{G}O_L \overset{\text{holography}}{\longleftrightarrow} \tilde{\phi} \ \underline{\text{minimally coupled scalar in 6D}}$$

- Since $GO_H = 0$, $\mathcal{C}_H$ and $\widetilde{\mathcal{C}}_H$ are related by the Ward identity

$$\widetilde{\mathcal{C}}_H(z,\bar{z}) = \partial\bar{\partial}\left[|z|\,\mathcal{C}_H(z,\bar{z})\right]$$

- The WI is a non-trivial check on the gravity computation when both $\mathcal{C}_H$ and $\widetilde{\mathcal{C}}_H$ can be computed

The small B limit

- When $B^2 \ll N$ O_H, spectrally flowed to the NS sector, is light

$$O_H \xrightarrow{\text{spectral flow}} \sum_p B^p O^p \longrightarrow O \equiv O_L \quad \text{for} \quad B^2 = 1$$

- Naively one expects $\langle \bar{O}_H O_H O_L \bar{O}_L \rangle \to \langle \bar{O}_L O_L O_L \bar{O}_L \rangle$ for $B^2 = 1$
- This is not correct! There is an order of limit problem:
  - HHLL: take $N \to \infty$ with B^2/N fixed and then $B^2/N \to 0$
  - LLLL: take $B^2 = 1$ first and then $N \to \infty$
- But it works for $z \to 1$, more precisely

the $B^2 \to 0$ limit of the HHLL correlator correctly captures all the single-trace operators exchanged between O_L and $\bar{O}_L$

Reconstructing the LLLL correlator

One can uniquely reconstruct $\mathcal{C}_L \equiv \langle \bar{O}_L(z_1) O_L(z_2) O_L(z_3) \bar{O}_L(z_4) \rangle$ from

- for $z_1 \to z_2$, $\mathcal{C}_L = \lim_{B^2 \to 1} \mathcal{C}_H$
- $\mathcal{C}_L$ is symmetric under $z_2 \leftrightarrow z_3$ exchange
- $\mathcal{C}_L$ is consistent with the flat space limit ($R_{\mathrm{AdS}} \to \infty$)
- the operator with the lowest dimension exchanged for $z_2 \to z_3$ is protected

One finds

$$\mathcal{C}_L = \left(1 - \frac{1}{N}\right)(1 + |1 - z|^{-2}) + \frac{2}{\pi N}|z|^2(\hat{D}_{1122} + \hat{D}_{1212} + \hat{D}_{2112})$$

where $\hat{D}_{i_1 i_2 i_3 i_4}$ is the Witten contact digram with operators of dimension $i_1, \ldots, i_4$

(The generalisation to generic CPOs is under construction)