We start with a simple remark about amplitudes

The 1→ 1 amplitude in String Theory

The 1→ 1 amplitude in String Theory

$$
\begin{aligned}
\mathcal{A}_{1\to 1} &= \int \frac{DX}{vol(PSL(2))}\int d^2zV\int d^2zVe^{-S} = \\
&= \int \frac{DX}{vol(R\times U(1))}V(0)V(\infty)e^{-S} \\
&= \frac{\int dX_0}{\int d\tau}(2\pi)^{D-1}\delta(\vec{k}_1-\vec{k}_2)\langle V(0)V(\infty)\rangle'
\end{aligned}
$$

Energy is automatically conserved if we have on shell operators V.

$$X^0 = \alpha' k^0 \tau$$

$$\mathcal{A}_{1\to 1} = 2k^0(2\pi)^{D-1}\delta^{D-1}(\vec{k}_1-\vec{k}_2)$$

Similar story for two point functions in AdS.

Now to the topic of the talk

Symmetries Near the Horizon

Juan Maldacena

Institute for Advanced Study

Based on work with:

Henry Lin

Ying Zhao

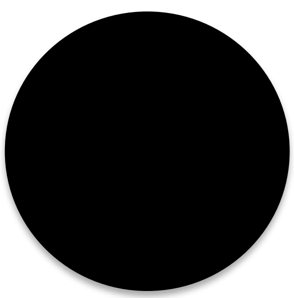
Preliminaries

Black holes and quantum systems

Basic Assumption

(central dogma)

- A black hole seen from the outside can be described as a quantum system with order S degrees of freedom (qubits) (coupled to the rest of spacetime)



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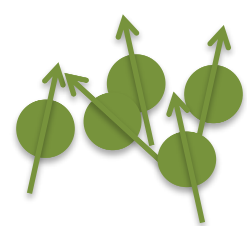
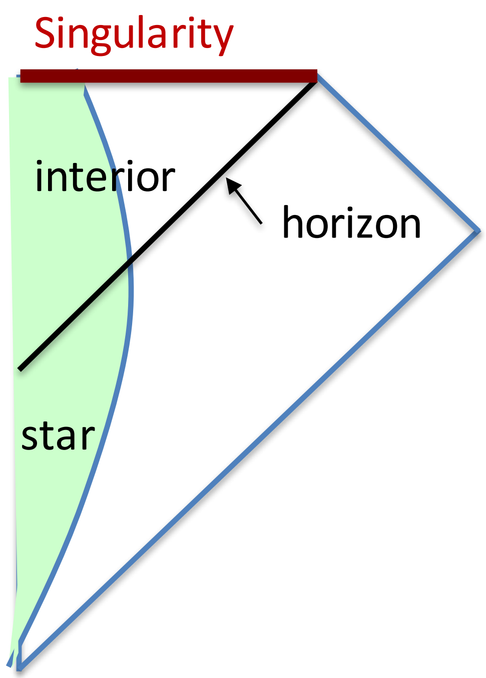


$$S = \frac{\text{Area}}{4G_N}$$

$$T_{\text{Hawking}}$$

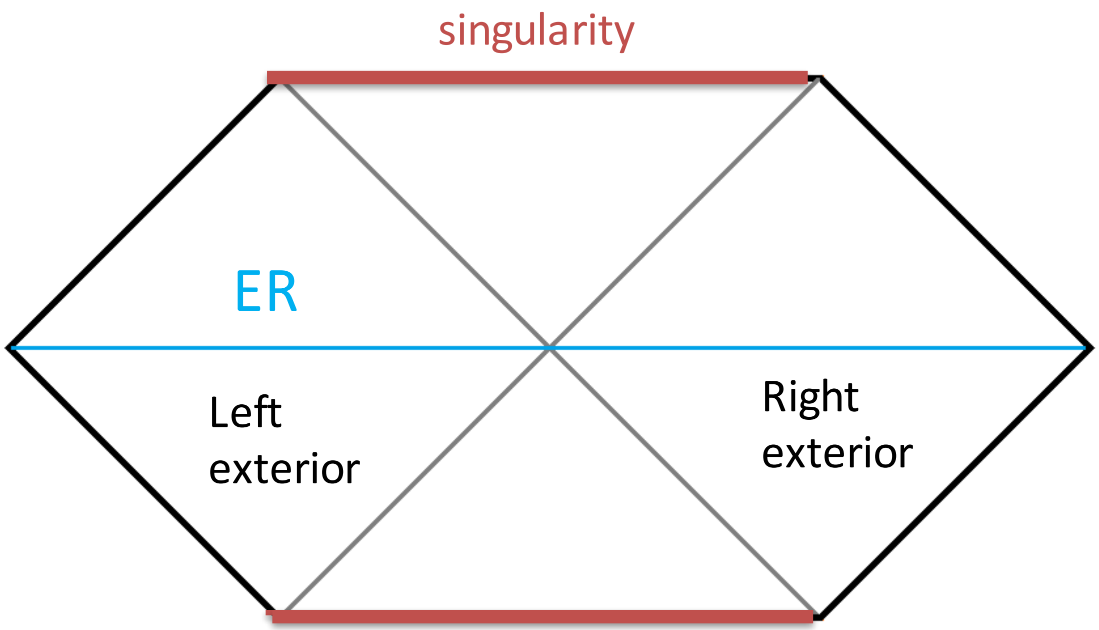
Geometry of a Black Hole made from collapse



Oppenheimer Snyder 1939

One exterior, one interior.

Full Schwarzschild solution



Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal

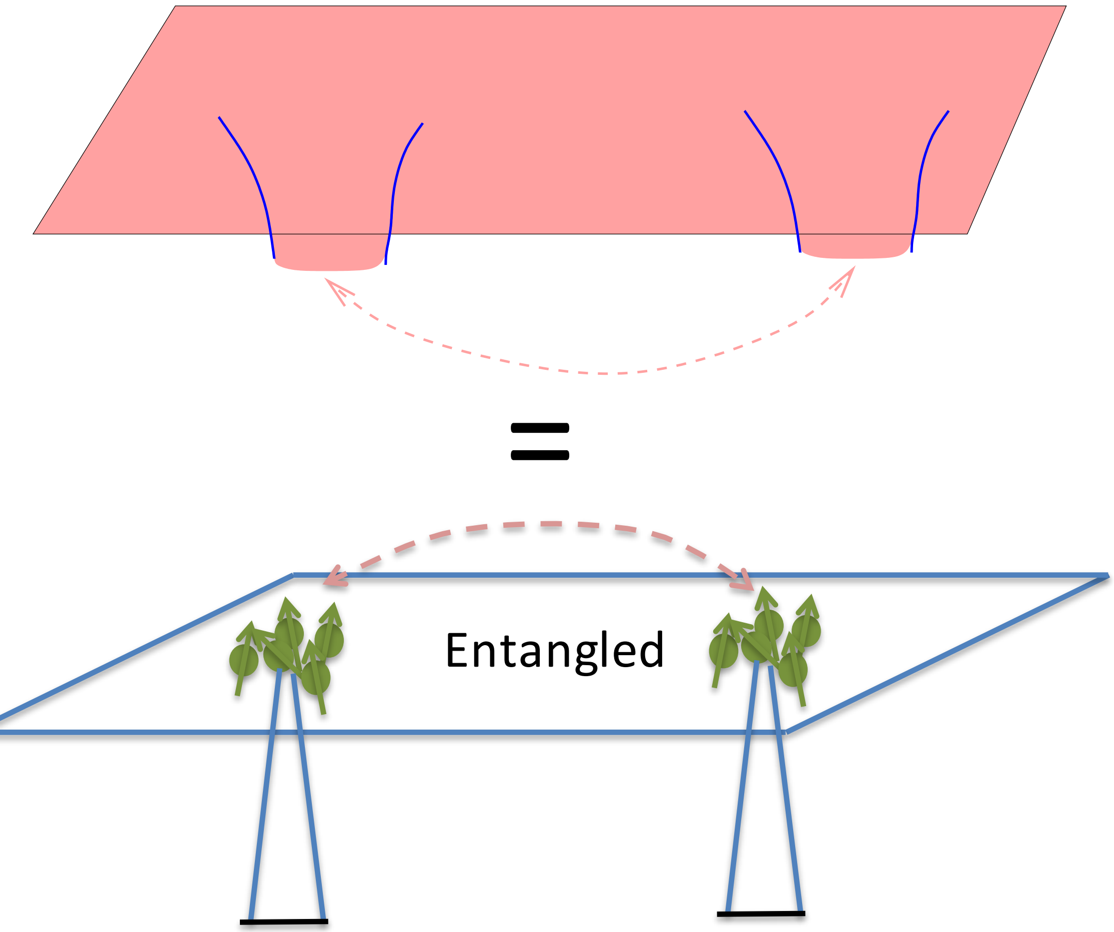
Vacuum solution. No exotic matter.
Two exteriors, sharing the interior.

If one black hole = quantum system,

What do these two connected black holes

correspond to ?

Wormhole and entangled states

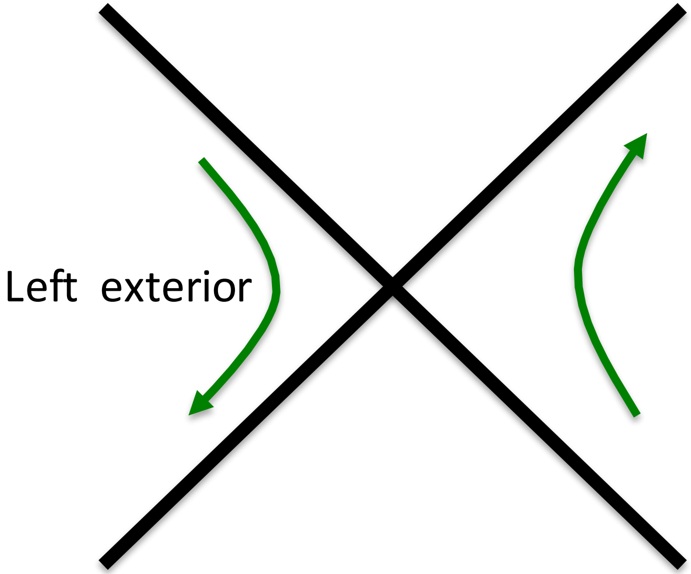


Connected through the interior

W. Israel
J.M.

In a particular entangled state

The near horizon region

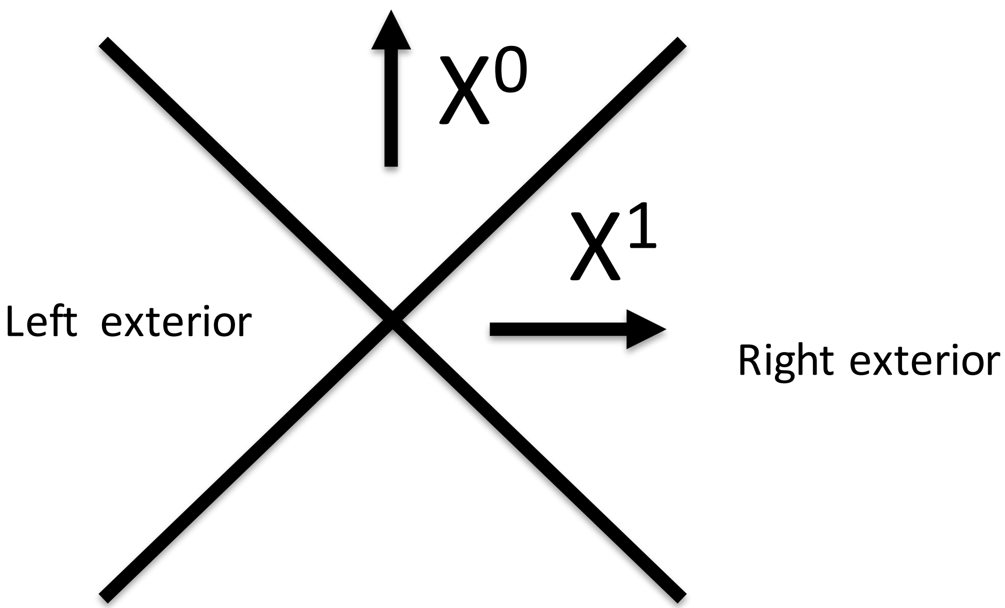


$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Boost = <u>exact</u> symmetry = $\frac{\beta}{2\pi}(H_r - H_l)$

(No simple bulk lattice discretizations in gravity)

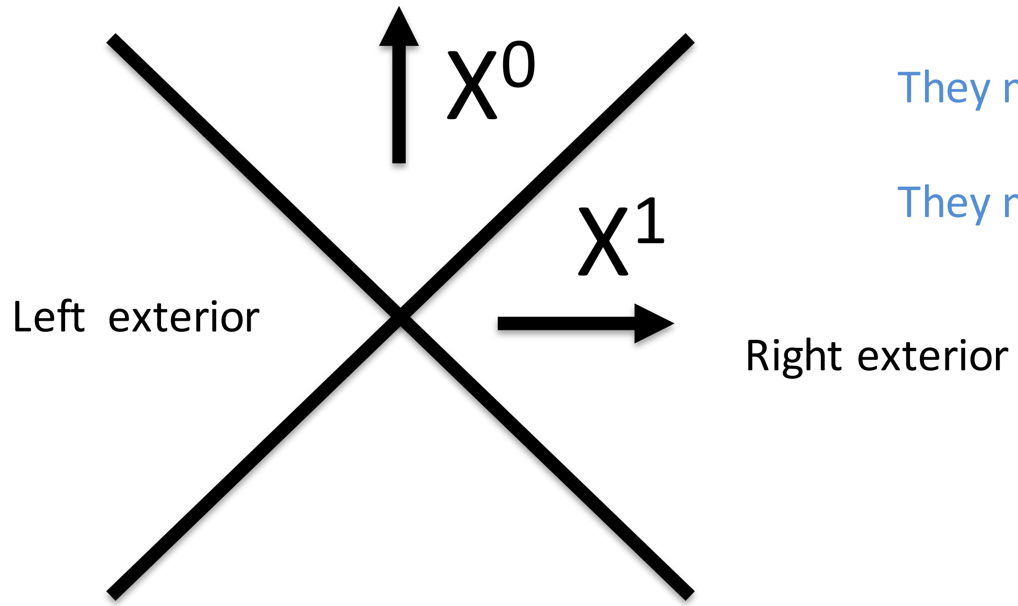
Two other approximate symmetries



E = global time translation symmetry $$X^0 \to X^0 + \text{constant}$$

P = Spatial translation $$X^1 \to X^1 + \text{constant}$$

This talk will be about these two other (approximate) symmetries



They move us behind the horizon

They move us from one side to the other

- We will discuss this for near extremal black holes.
- These are described by Nearly-AdS_2 gravity.
- A similar structure appears in the SYK model.

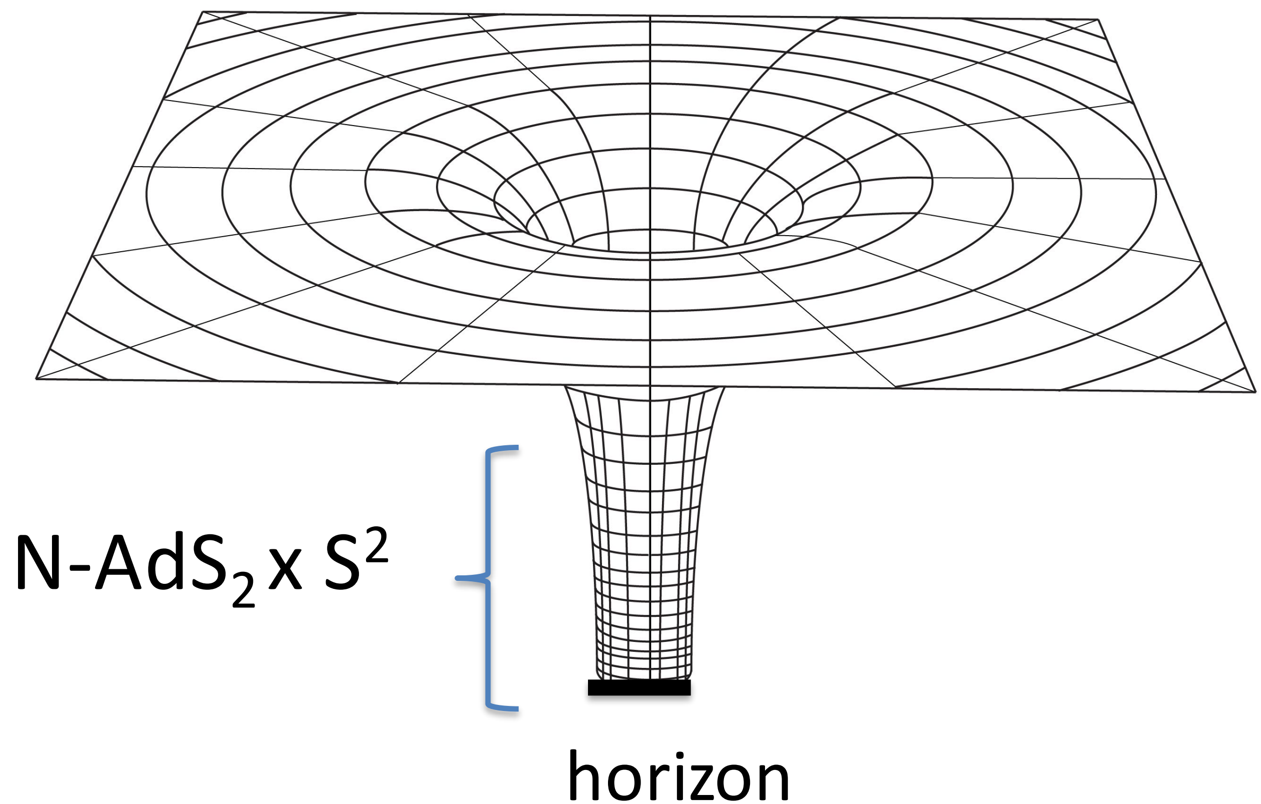
- We will now review both cases.

Review of nearly AdS_2 gravity

Near extremal black holes

$$M \geq Q$$

$$M \sim Q$$



Nearly-AdS_2 gravity

Jackiw-Teitelboim
Almheiri-Polchinski

$$S = \phi_0 \left[\int R + 2 \int K \right] + \int \phi (R+2) + 2\phi_b \int K + S_m[g_{\mu\nu}, \chi]$$

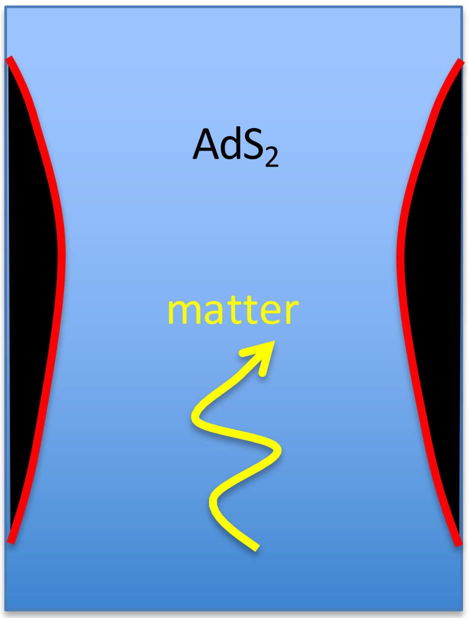
Only extermal entropy S_0

Fixes
Metric to AdS_2

Matter moves in a rigid
AdS_2 spacetime

Boundary becomes dynamical = boundary graviton = only physical degree of freedom for gravity

The surprisingly simple gravitational dynamics of N-AdS_2

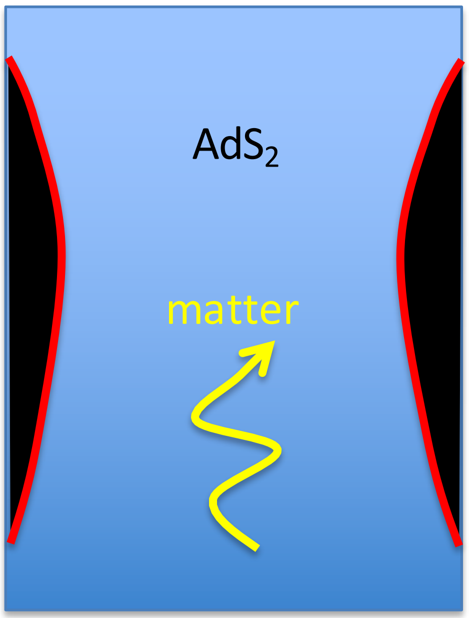


$NAdS_2 = AdS_2$ + location of boundary

$$ds^2 = \frac{-d\tau^2 + d\sigma^2}{\sin^2 \sigma}$$

Dynamics of the boundary is SL(2) invariant.

Proper time along the boundary = time of the asymptotically flat region = time of the quantum system



$$ds^2 = \frac{-d\tau^2 + d\sigma^2}{\sin^2 \sigma}$$

Simplest case = Boundaries are infinitely far away. Matter moves effectively in all of AdS$_2$ and is not affected by the motion of the boundary.

The boundary is still dynamical and tells us how to translate to the physical boundary time = time of the dual quantum system.

Then the Hilbert space splits as:

$$(\mathcal{H}_l \times \mathcal{H}_m \times \mathcal{H}_r)/SL(2)_g$$

Common motion of the three of them is not physical

- Each of the three systems is described by an $SL(2)_g$ invariant action.
- The matter moves in a rigid AdS_2
- Each boundary is like a massive particle with spin (or in an electric field in AdS_2)

$\eta_{ab} Y^a Y^b = -1$ Embedding coordinates

Rescaled embedding coordinates

$$X^a(u) \ , \qquad X \cdot X = 0 \qquad \dot{X} \cdot \dot{X} = -1$$

Proper time along the boundary = time of the dual quantum system.

SL(2) gauge constraint:

$$Q_l^a + Q_m^a + Q_r^a = 0 \qquad\qquad a = \text{gauge index}$$

Now one would be tempted to say that the symmetries we want are $Q^a{}_m$, the matter generators, since they are the ones that move the matter in AdS_2 .
But these do not have a translations to the boundary theory since they are not gauge invariant.
So, we will write gauge invariant ones by "gravitationally dressing" them.

Short review of SYK

SYK

Sachdev-Ye-Kitaev

- SYK: N Majorana fermions with all to all interactions.
- The theory has a simple large N limit, with an effective action which is a function of two times: $G(u_1, u_2)$
- This becomes the fermion two point function when we impose the equations of motion.
- At low energies, it develops an approximate conformal symmetry.

- Scaling solution.

$$G_c = |t_1 - t_2|^{-2\Delta}$$

- In the IR there is a family of solutions that are obtained by applying a time reparametrization to the above one, u → f(u).

$$G_f = [f'(u_1)f'(u_2)]^{\Delta} G_c(f(u_1) - f(u_2)) = \left[\frac{f'(u_1)f'(u_2)}{(f(u_1)-f(u_2))^2}\right]^{\Delta}$$

Kitaev

- This is only an approximate symmetry and it is explicitly broken.

$$G = G_f + G_\perp = [f'(u_1)f'(u_2)]^{\Delta}\left[\frac{1}{|f_1-f_2|^{2\Delta}} + G_\perp(f(u_1), f(u_2))\right]$$

Low energy SYK action

$$S = -\frac{N\alpha_s}{\mathcal{J}} \int \{f_l(u), u\} - \frac{N\alpha_s}{\mathcal{J}} \int \{f_r(u), u\} + S_{\rm conf}[\delta_\perp G]$$

$$\{f, u\} = \frac{f'''}{f'} - \frac{3}{2}\frac{f''^2}{f'^2}$$

$$X^a = \frac{1}{f'}(1, \cosh f, \sinh f)$$

Same as the action of the boundaries in the gravity theory

Analogous to the action of matter for the gravity theory. Independent of f(u).

$$(\mathcal{H}_l \times \mathcal{H}_m \times \mathcal{H}_r)/SL(2)_g$$

$$Q_l^a + Q_m^a + Q_r^a = 0$$

Becomes exact in the limit:

$$N \to \infty, \qquad (\beta\mathcal{J}) \to \infty\ , \qquad \frac{N}{\beta\mathcal{J}} = \text{fixed} \qquad S_0 \to \infty$$

Construction of gauge invariant generators

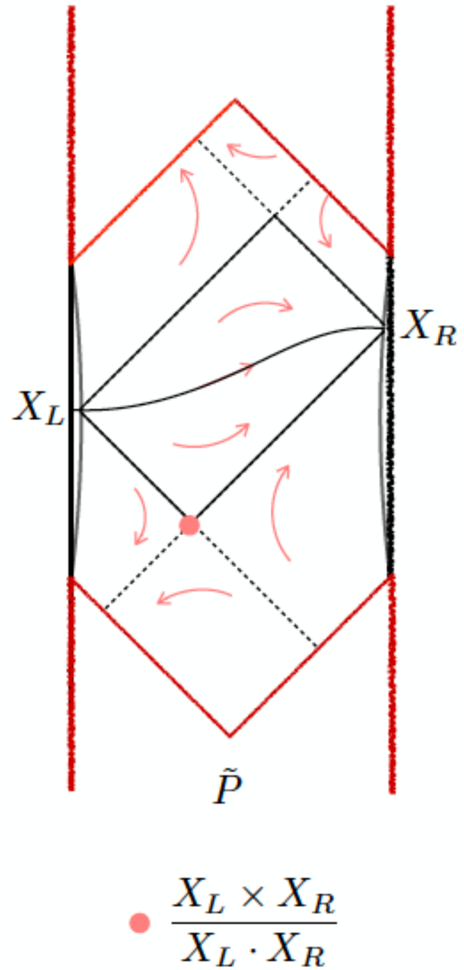
Momentum generator

Vectors: $X_l^a, \quad X_r^a$

Construct gauge invariant generators. First

$$\tilde{P} \propto (X_l \times X_r).Q_m$$

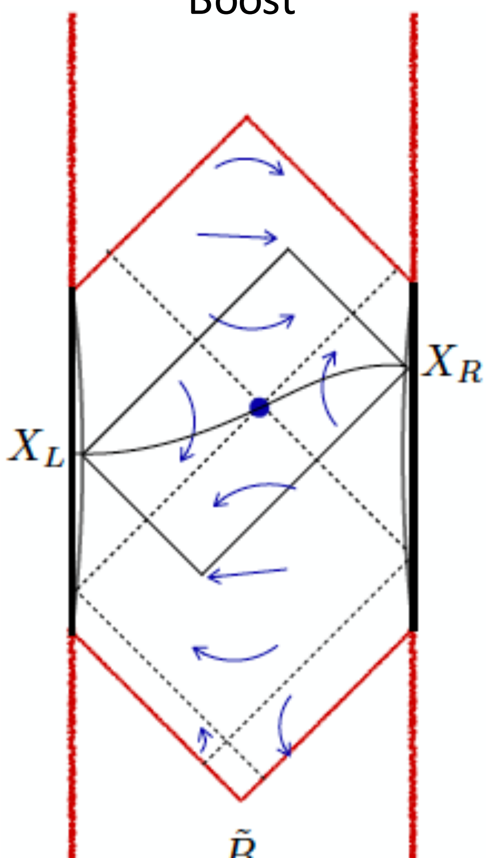
Translation along the geodesics joining the left and right points



Two others: $X_l . Q_m \ , \qquad X_r . Q_m$

Linear combinations give

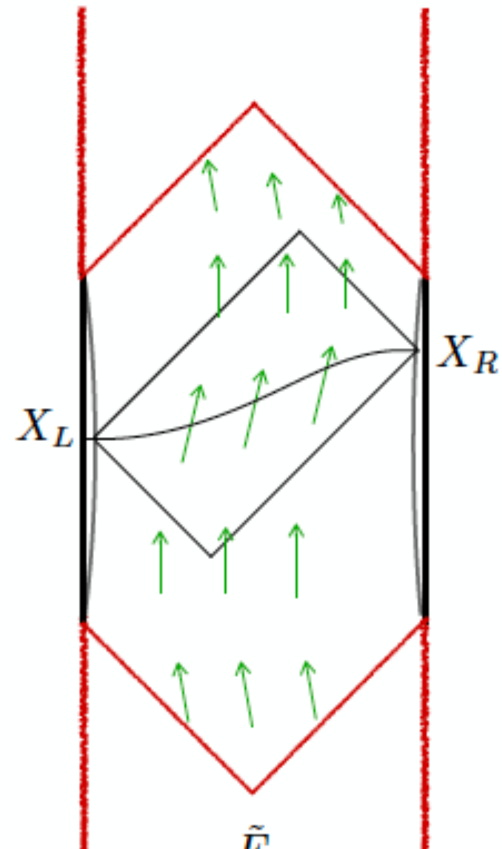
Boost



$\tilde{B}$

• $\dfrac{X_R + X_L}{\sqrt{-2X_R \cdot X_R}}$

Energy



$\tilde{E}$

- These are expressions for three gauge invariant SL(2) generators,

$$G^A = e^A_a(X_l, X_r)\, Q^a_m$$

- They act on the physical Hilbert space.
- → Hilbert space is infinite dimensional
- Do not confuse them with the $SL(2)_g$ constraints.
- They include quantum gravity corrections (finite Schwarzian coupling).
- Do not include effects of other topologies ($S_0 \to \infty$)
- The X dependence can be viewed as a gravitational dressing.

- They do not commute with the Hamiltonian since $X_l(u_l)$, $X_r(u_r)$ depend on time.
- Only depend on time due to the boundary positions. $G^A(u_l, u_r)$
- To the extent that we can solve for the boundary positions → we can write

$$G^A(0,0) = \Lambda^A_B G^B(u_l, u_r)$$

Conserved charges

Operator in the boundary theory (Schwarzian)

Expressions purely in terms of boundary quantities.

- Use the gauge constraints to write

$$Q^a_m = -(Q^a_l + Q^a_r)$$

- Rewrite them all in terms of boundary quantities, and their derivatives.

$$\tilde{P} = (\partial_{u_l} - \partial_{u_r}) \log[-2X_l.X_r] = (\partial_{u_l} - \partial_{u_r})\ell$$

$$0 = \dot{x}_l + p_m + \dot{x}_r \to p_m = -\dot{x}_r - \dot{x}_l = (\partial_{u_l} - \partial_{u_r})(x_r(u_r) - x_l(u_l))$$

- Similar expression for the other two.
- It involves the distance between the two boundaries.

Distance between the two boundaries

$$\frac{1}{(-2X_l.X_r)^{\Delta}} \propto \langle \psi_l^j \psi_r^j \rangle$$

- Could be extracted by taking the logarithm of the correlator.

$$\ell = \log[-2X_l.X_r] = -\frac{1}{\Delta}\log[\psi_l^j.\psi_r^j]$$

- Well defined as long as the operator does not vanish.
- Is OK in the scaling limit we defined before.

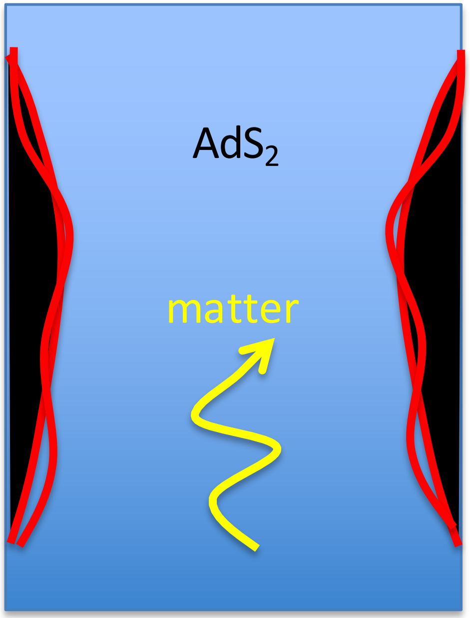
$$N \to \infty, \qquad (\beta\mathcal{J}) \to \infty \ , \qquad \frac{N}{\beta\mathcal{J}} = \text{fixed}$$

- It is a quantity that is well defined only around this “wormhole” phase. Like the phase of a superconductor.
- Previously thought of in terms of “complexity” Susskind et al

- Now we will now work to get more explicit, but approximate, forms for the generators.
- These will have the advantage of being well defined in the finite N theory.

Semiclassical limit

$$S - S_0 \sim \frac{r_e^3 T}{G_N} \sim \frac{N}{\beta \mathcal{J}} \gg 1$$



- Gravity (Schwarzian) is weakly coupled and close to classical.
- Boundaries follow classical trajectories + small fluctuations.

$$t_l = \tilde{u}_l + \epsilon_l(\tilde{u}_l) \ , \qquad t_r = \tilde{u}_r + \epsilon_r(\tilde{u}_r) \ , \qquad \tilde{u} \equiv \frac{2\pi u}{\beta}$$

- We can now calculate the distance in terms of classical solutions + small fluctuations.
- The generators G^A have a simple approximation in terms of ε

$$\tilde{B} \sim \epsilon_r' - \epsilon_r''' - (\epsilon_l' - \epsilon_l''')$$
$$\tilde{P} \sim \epsilon_r'' - \epsilon_l''$$
$$\tilde{E} \sim -(\epsilon_r''' - \epsilon_l''')$$

All evaluated at $u_l = u_r = 0$

- We can also expand correlators in ε and, as long as we get the same combination → we are getting these generators.

$$\left[\frac{t_l' t_r'}{\cosh^2(\frac{t_l - t_r}{2})} \right]^\Delta$$

$$t_l = \tilde{u}_l + \epsilon_l(\tilde{u}_l) \ , \qquad t_r = \tilde{u}_r + \epsilon_r(\tilde{u}_r) \ , \qquad \tilde{u} \equiv \frac{2\pi u}{\beta}$$

- The gauge constraints set these combinations of ε equal to the matter charges. (In the appropriate gauge).

$$Q_l^a + Q_m^a + Q_r^a = 0$$

Global energy

JM, Qi

- Two identical coupled SYK models

$$H_{\text{coupled}} = H_r + H_r + i\mu\psi_l^j\psi_r^j$$

- Ground state is very close to the thermofield double state at some temperature given by a combination of μ and J.

$$\tilde{E} \sim \frac{\beta}{2\pi}\left[H_{\text{coupled}} - \langle H_{\text{coupled}}\rangle_0\right]$$

β dependent through μ .

Empty wormhole

Redshift factor, makes it equal to unit normalized global AdS_2 energy

$$\tilde{E} \sim -(\epsilon_r''' - \epsilon_l''') \sim H_{\mathrm{coupled}} - \langle H_{\mathrm{coupled}} \rangle_0$$

$$H_{\text{coupled}} = H_r + H_r + \underline{i\mu\psi_l^j\psi_r^j}$$

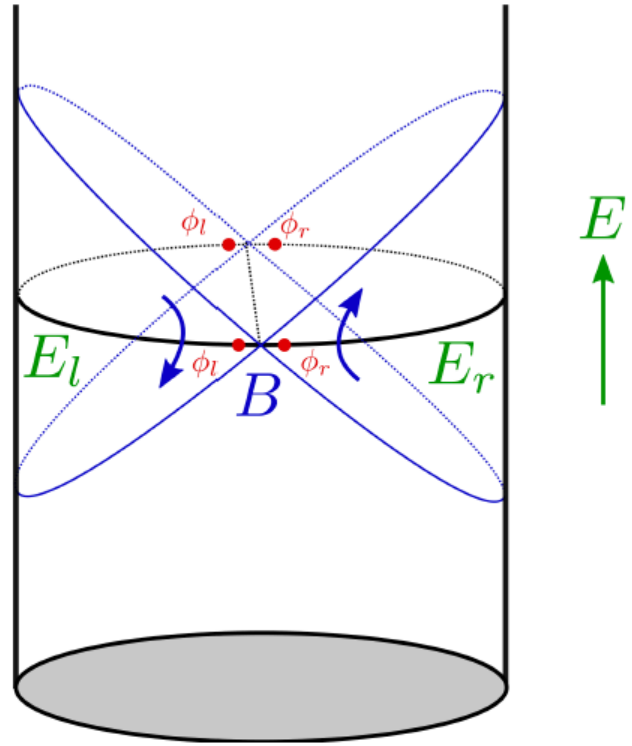
Involves a coupling between the left and right systems

Related to the traversable wormhole construction

Gao, Jafferis, Wall

Side Comment

Rindler coordinates.



Global energy is

$$E = \int_{\text{Sphere}} T_{00} = E_l + E_r + \sum_i \phi_l^i \phi_r^i \, , \qquad E_l = \int_{l-\text{Hemisphere}} T_{00}$$

Similar to: $$H_{\text{coupled}} = H_r + H_r + i\mu \psi_l^j \psi_r^j$$

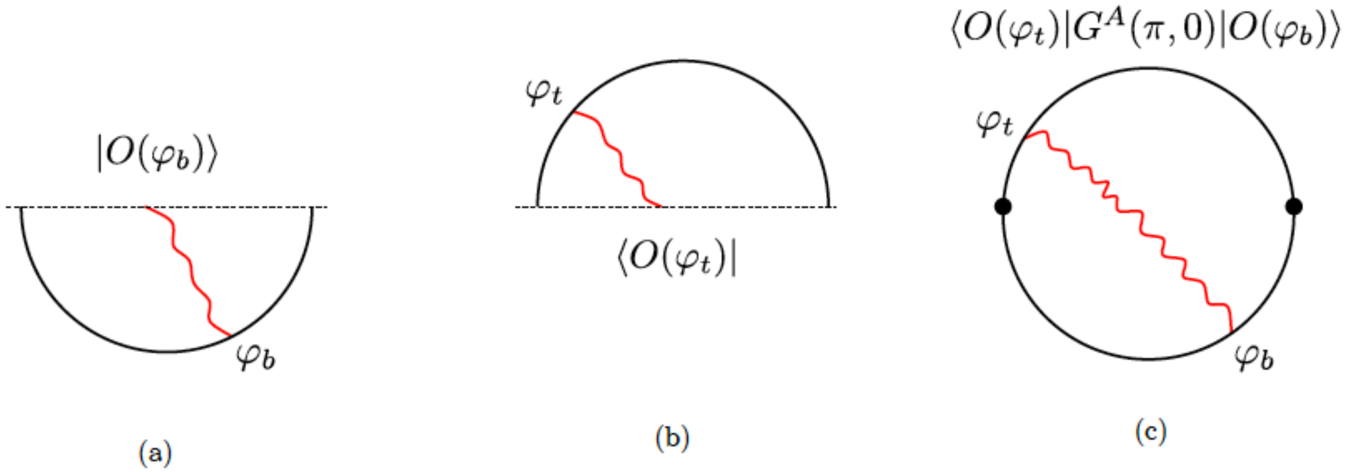
End of side comment

Operator-State map in NCFT$_1$

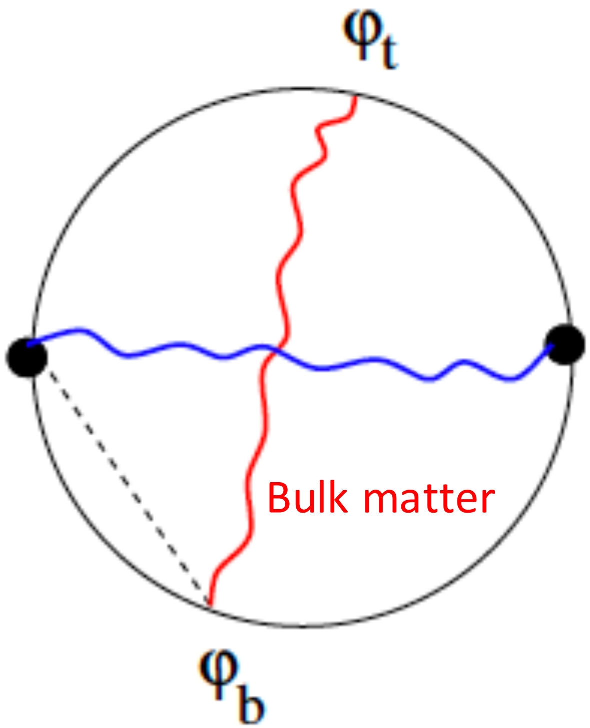
Operators are mapped to states of the two sided system.

Vacuum → TFD.

Matrix elements of correlators → Related to OTOC



Related to out of time order correlators or OTOC



Measuring distance via correlators

OTOC → simple bulk displacements.

Here we simply invert the logic.

Relation to “size”

Some papers have been proposing a relation between the momentum of bulk particles to “size”. It was proposed as a conjecture, or ``experimental’’ relationship, with some rescaling factors, etc.

Roberts, Stanford, Streicher;
Susskind; Brown, Gharibyan, Streicher, Susskind, Thorlacius, Zhao; Qi, Streicher

Size is the number of operators that we have to apply to the infinite temperature TFD to create the state we care about.
The infinite temperature TFD is a very simple state consisting of essentially Bell pairs of fermions

Size can be measured by the left-right correlator.

$$\hat{S} = i\psi_l \psi_r$$

$$H_{\rm coupled} = H_r + H_r + i\mu \psi_l^j \psi_r^j$$

Gives a precise relation

Bulk momentum is related to the time derivative of size

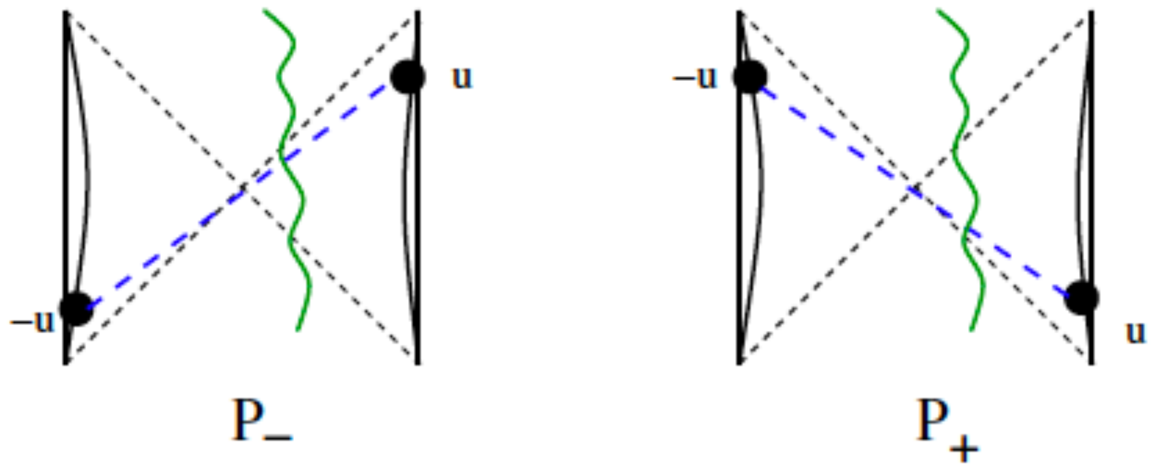
$$P \propto [B, E] \propto [H_r - H_r, H_r + H_l + \mu S] \propto \dot{S}$$

This relates ``size'' to symmetry generators and ``explains'' its previously observed properties.

Order from Chaos

$$e^{i\tilde{u}\hat{B}}\hat{E}e^{-i\tilde{u}\hat{B}} = \frac{\beta}{2\pi}\left[H_r + H_l - \tilde{\eta}\sum_j O_l^j(-u)O_r^j(u) - \langle\cdots\rangle_{\mathrm{TFD}}\right] \simeq \cosh\tilde{u}\hat{E} - \sinh\tilde{u}\hat{P}$$

$$e^{i\tilde{u}\hat{B}}\hat{P}e^{-i\tilde{u}\hat{B}} = \partial_{\tilde{u}}\left(\frac{\beta\tilde{\eta}}{2\pi}\sum_j O_l^j(-u)O_r^j(u)\right) \simeq -\sinh\tilde{u}\hat{E} + \cosh\tilde{u}\hat{P}$$



$$-P_+ = \frac{E-P}{2} = -\lim_{u\to-\mathrm{large}} e^{\frac{2\pi u}{\beta}}\eta\sum_j\left[O_l^j(-u)O_r^j(u) - \langle O_l^j(-u)O_r^j(u)\rangle_0\right]$$

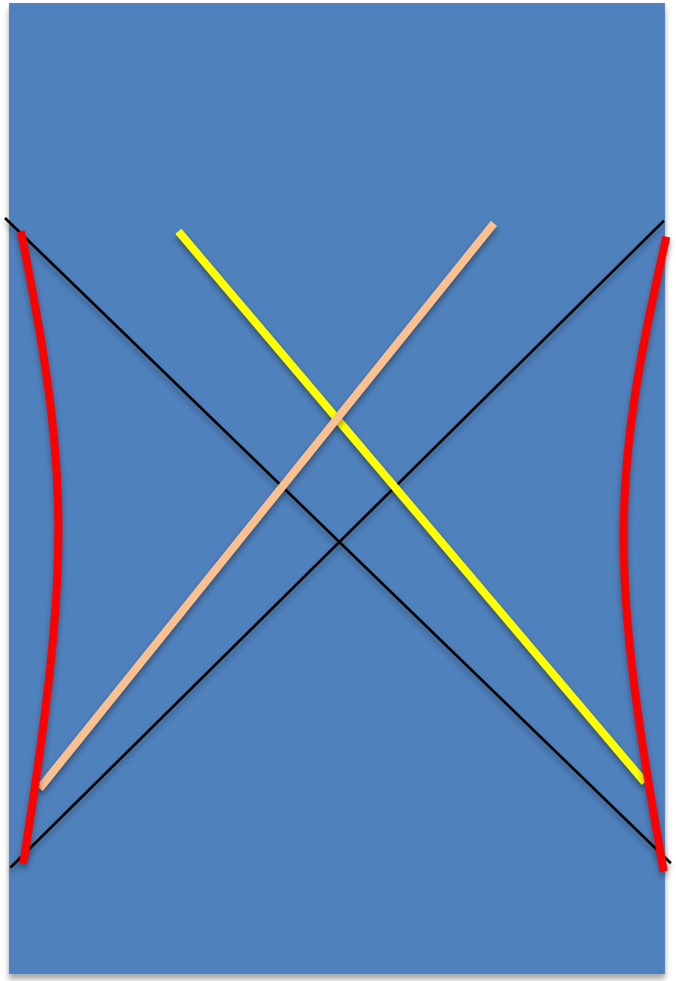
$$-P_- = \frac{E+P}{2} = -\lim_{u\to+\mathrm{large}} e^{-\frac{2\pi u}{\beta}}\eta\sum_j\left[O_l^j(-u)O_r^j(u) - \langle O_l^j(-u)O_r^j(u)\rangle_0\right]$$

$$H_{\mathrm{coupled}} = H_r + H_r + i\mu\psi_l^j\psi_r^j$$ Only this term grows

Comments about exploring the interior

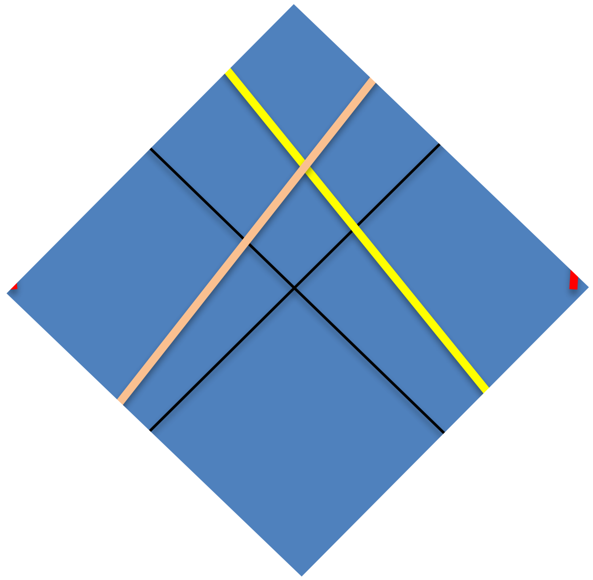
Interactions behind the horizon

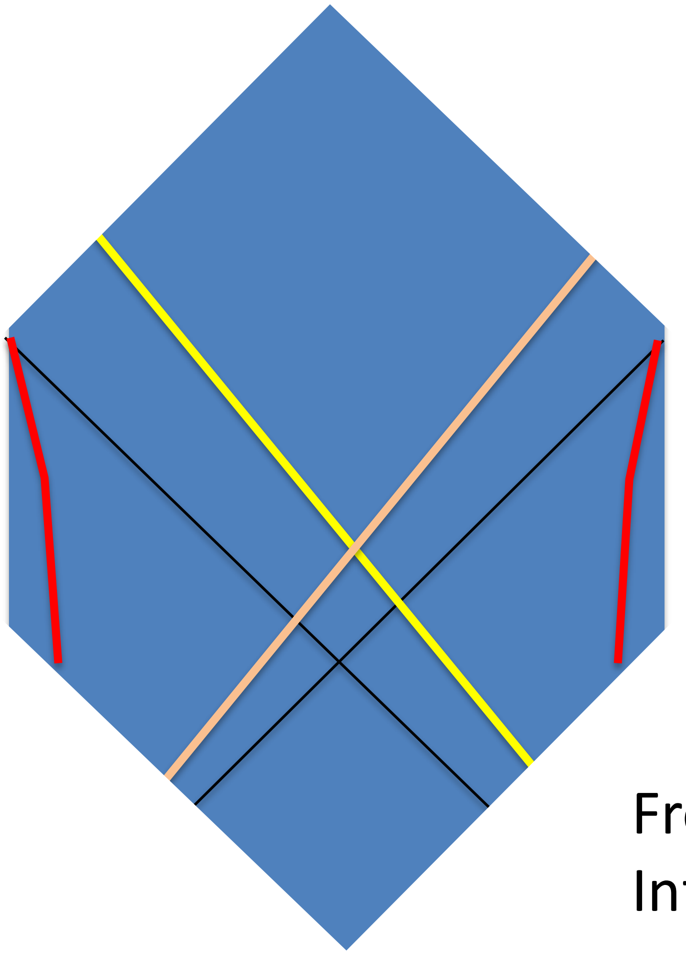
$$H = H_L + H_R$$



Evolve the TFD, backwards, insert some excitations with unitary operators.

We expect that the initial state is describing the Wheeler de Wit patch



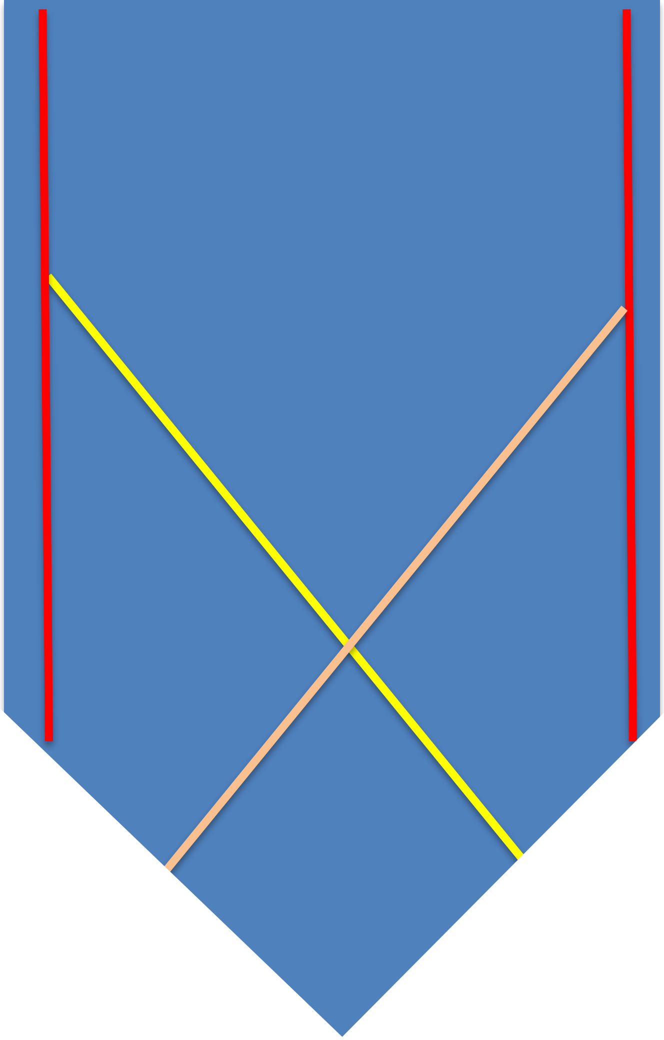


We can evolve it with the decoupled hamiltonian

$$H = H_L + H_R$$

From the boundary theory they should NOT Interact in any way.

Fortunately, their interaction is behind the horizon so we cannot see it from either boundary.



We can evolve it with the coupled hamiltonian

$$H = H_L + H_R + H_{int}$$

We can now see the interaction.

It is OK because the underlying Hamiltonian has an interaction between the two sides.

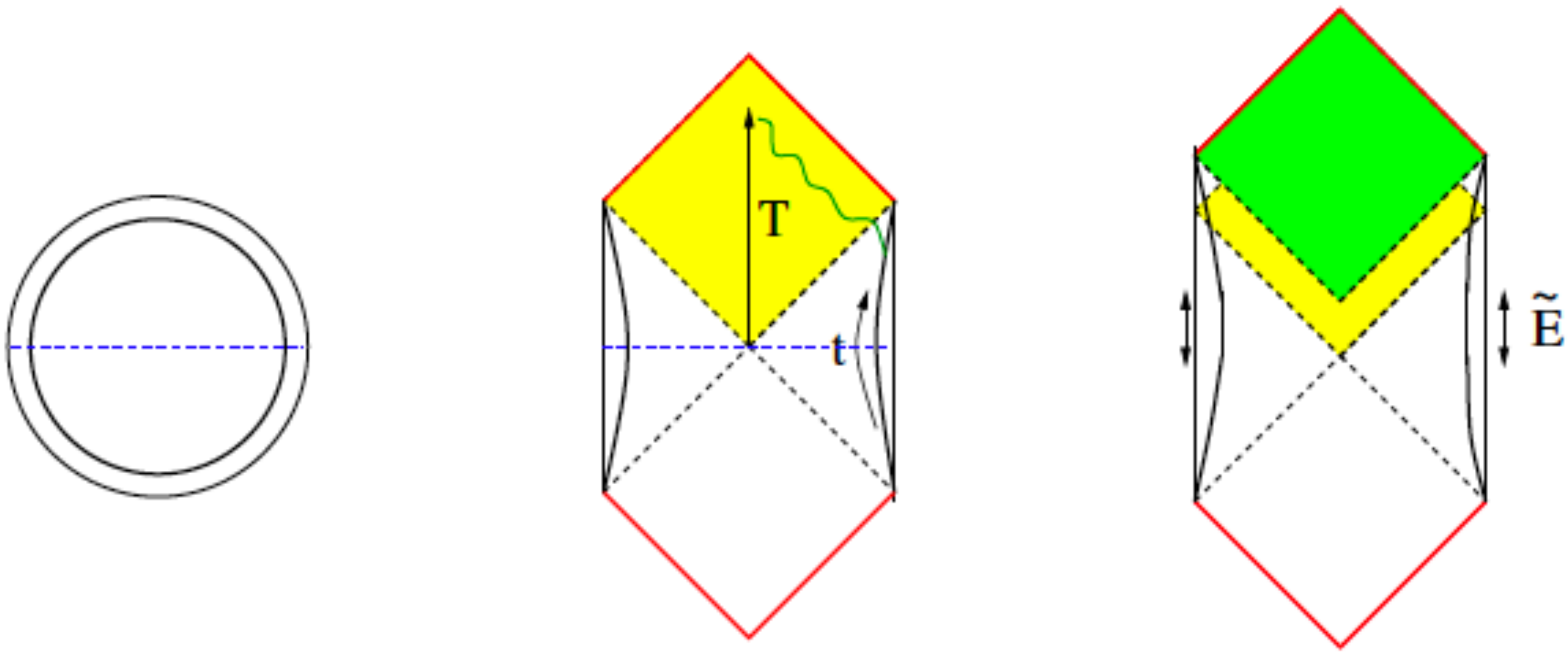
Make that interaction behind the horizon more real. But always through the lens of a particular evolution.

Acting with two sided operators we can ~~see~~ create the interior

A black hole is not a ``state’’. It is a state + a particular time evolution.

It is a space-time after all

Bulk near inner horizon?



1) We do not know the boundary conditions for the bulk matter beyond the region covered by the physical boundaries.

2) Similar to Coleman de Luccia decays to AdS. We expect that corrections due to irrelevant operators give a divergence at the inner horizon.

Some more conceptual comments

Analogy between two coupled SYK models and ``superconductors''.

- ``Superconductor'' = System with a spontaneously broken U(1) global symmetry.

SYK model for charged fermions.

Sachdev

$$H = \sum_{ij;kl} \psi^i \psi^j \bar{\psi}^k \bar{\psi}^l$$

Two copies plus boundary interaction. (q >4 is simpler)

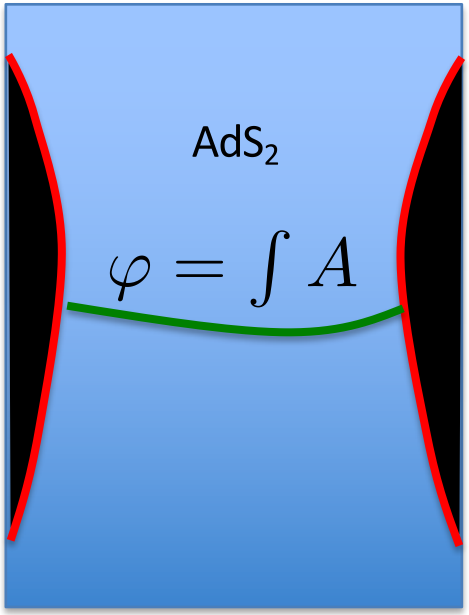
$$H_{\rm coupled} = H_l + H_r + \eta \psi_l^i \bar{\psi}_r^i \bar{\psi}_l^j \psi_r^j$$

Preserves $U(1)_l x U(1)_r$

Wormhole solution or TFD → breaks spontaneously $U(1)_l - U(1)_r$.
But preserves $U(1)_l + U(1)_r$

Goldstone mode

$$\varphi = \int A$$



If instead we had the interaction:

$$H_l + H_r + i\mu(\psi_l^j \bar{\psi}_r^j + \bar{\psi}_l^j \psi_r^j)$$

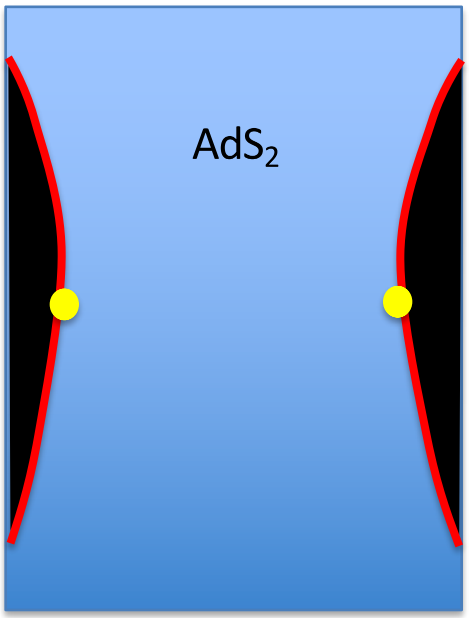
Symmetry would also be explicitly broken, and one value of $\varphi = \int A$ would be selected.

State displays spontaneous + explicit breaking of this symmetry.

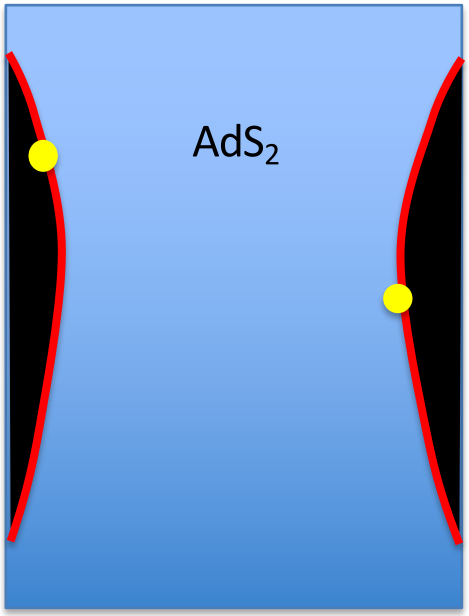
Time superfluids

- We start with two time translation symmetries: H_l and H_r
- The wormhole, or the TFD state break spontaneously $H_l + H_r$. But preserves $H_l - H_r$
- The Goldstone mode is the relative time shift between the two sides. This is one of the physical modes of the wormhole (the other is the mass of both black holes).

$$|\Delta u\rangle = e^{-iH_l \Delta u}|TFD\rangle \propto \sum_n e^{-i\Delta u E_n - \beta E_n/2}|\bar{E}_n\rangle|E_n\rangle$$



Time shifted one.



Ordinary evolution is just a simple motion in the space of time shifted wormhole:

$$\Delta u = u_r + u_l$$

Goldstone is linear in time because the Hamiltonian is the broken symmetry.

Space-time superfluids

The time-superfluid picture is valid for any wormhole or TFD.

When the wormhole or TFD are those of a Nearly AdS_2 or CFT_1

Then on each side we have more symmetries, including and approximate SL(2) conformal symmetry.

The TFD is breaking this to a common SL(2) symmetry.

The Goldstones are: The mass and the time shift.

In addition the symmetries are explicitly broken by the Hamiltonian.

Conclusions

- We explored the symmetries of the near horizon region of near extremal black holes or SYK modes.
- Constructed ``exact'' SL(2) generators.
- Discussed approximate expression in the semiclassical limit.
- Explained why and how ``size'' is connected to energy and momentum.

Slogans

- Order from chaos.
- Time superfluids
- Spacetime superfluids.