Axions from Strings

Ed Hardy

Based on work with
Marco Gorghetto & Giovanni Villadoro

[arXiv:1806.04677,
ongoing]

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SM strong CP problem

$$\mathcal{L} \supset \theta_0 \frac{\alpha_S}{8\pi} G\tilde{G}$$

Neutron EDM  $d_n < 2.9 \cdot 10^{-26}\, e\, cm$

$$\theta' = \theta_0 + \arg\left(\mathrm{Det} M_q\right) \lesssim 10^{-10}$$

Strong CP Problem

Other phases in Yukawa matrices order 1

Non-decoupling contributions from new CP violating physics

Effects on large distance physics irrelevant

Begs for a dynamical explanation!

The QCD axion

Spontaneously broken anomalous global U(1)

$$\boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

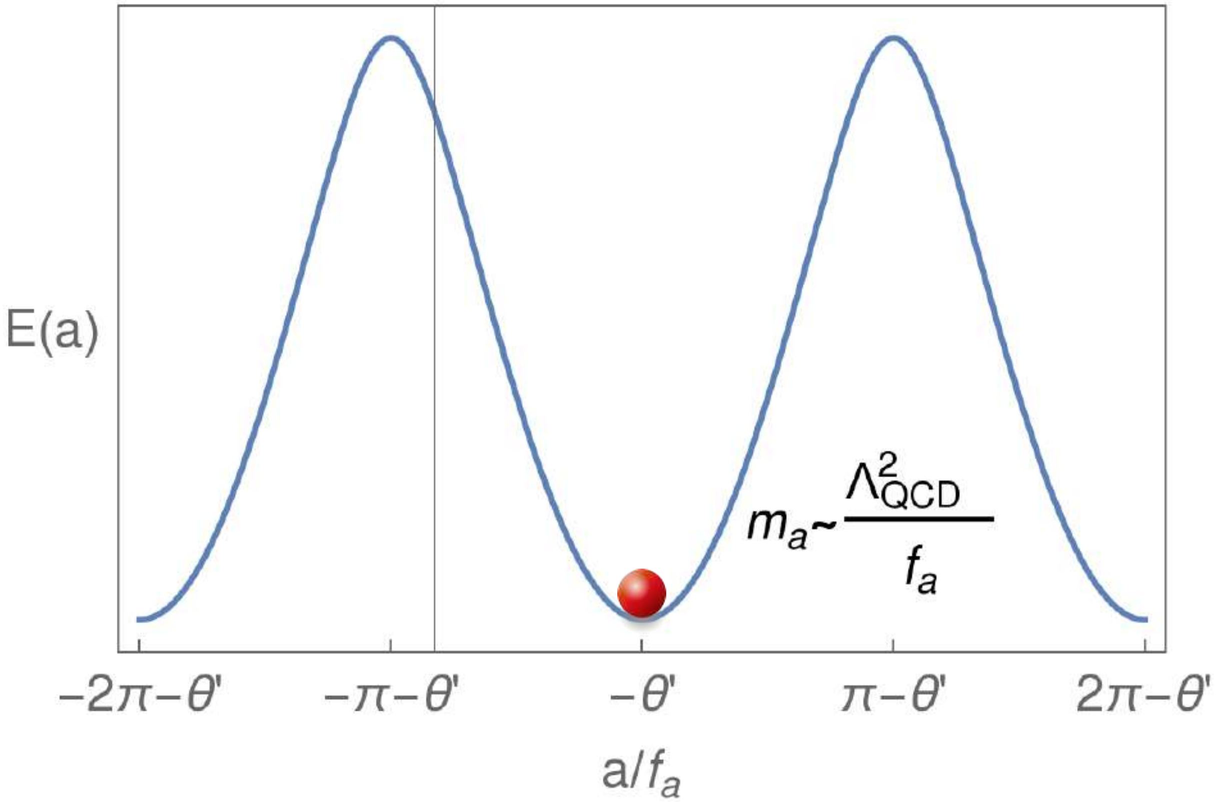
$$a \to a + \delta$$

$$f_a \gtrsim 10^8\ \mathrm{GeV}$$

QCD runs into strong coupling

→ axion potential

$$E(a) \geq E(a = -\theta')$$



Solves the SM strong CP problem $\theta_{\mathrm{tot}} = \langle a \rangle + \theta' = 0$

The QCD axion

Motivated from UV and IR perspectives

- Solves a problem with the SM
- Automatic Dark Matter candidate
- Plausible in typical string compactifications

Less explored than other possibilities, experimental progress likely

What can theory contribute?

Highlight especially well motivated parts of parameter space

Determine existing limits from e.g. astrophysical systems

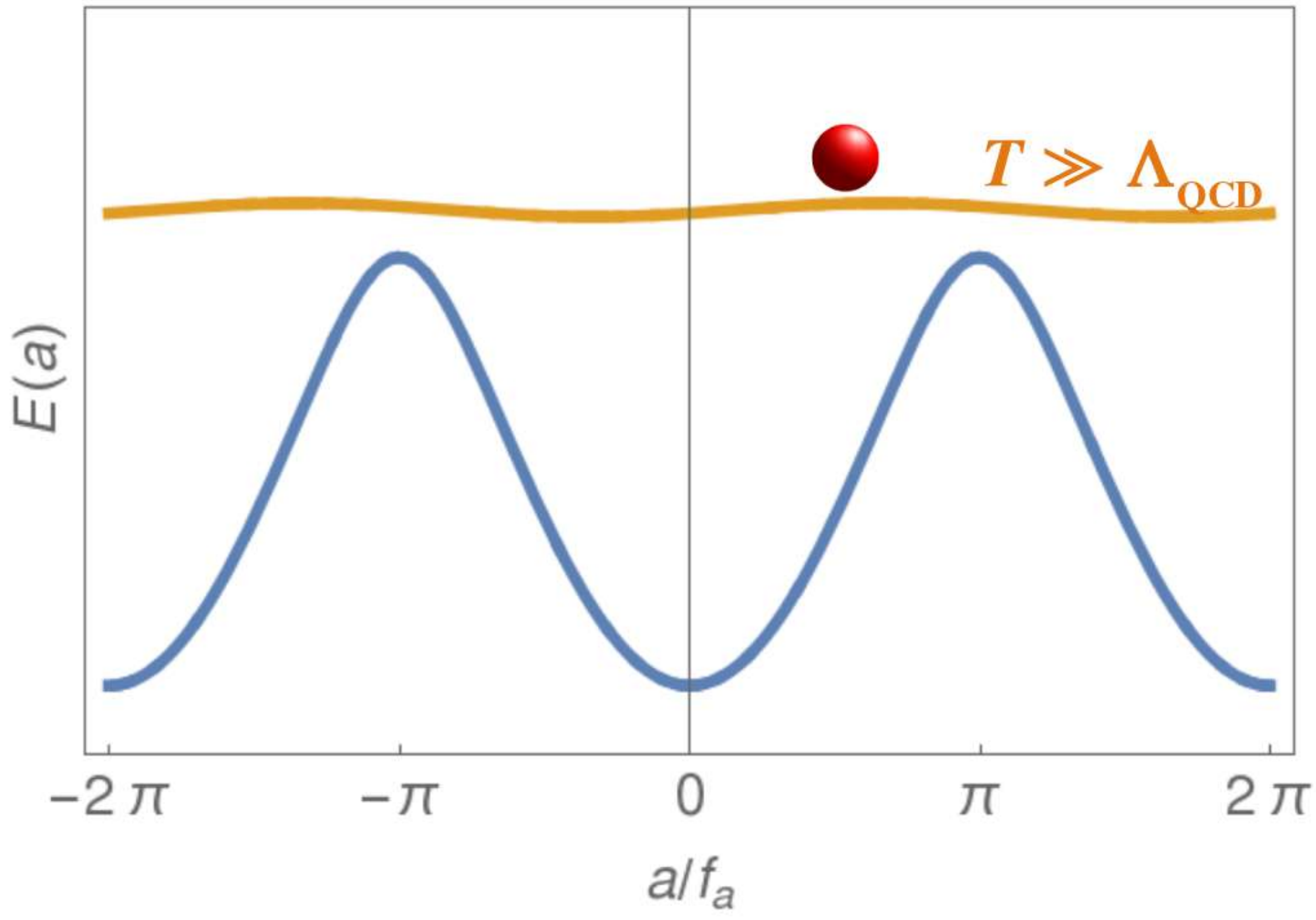
Understand physics implications of new searches

In case of an anomaly or discovery interpret what has been seen

Dark matter

Misalignment

$$\ddot{a} + 3H(T)\dot{a} + m_a^2(T)\,a = 0$$



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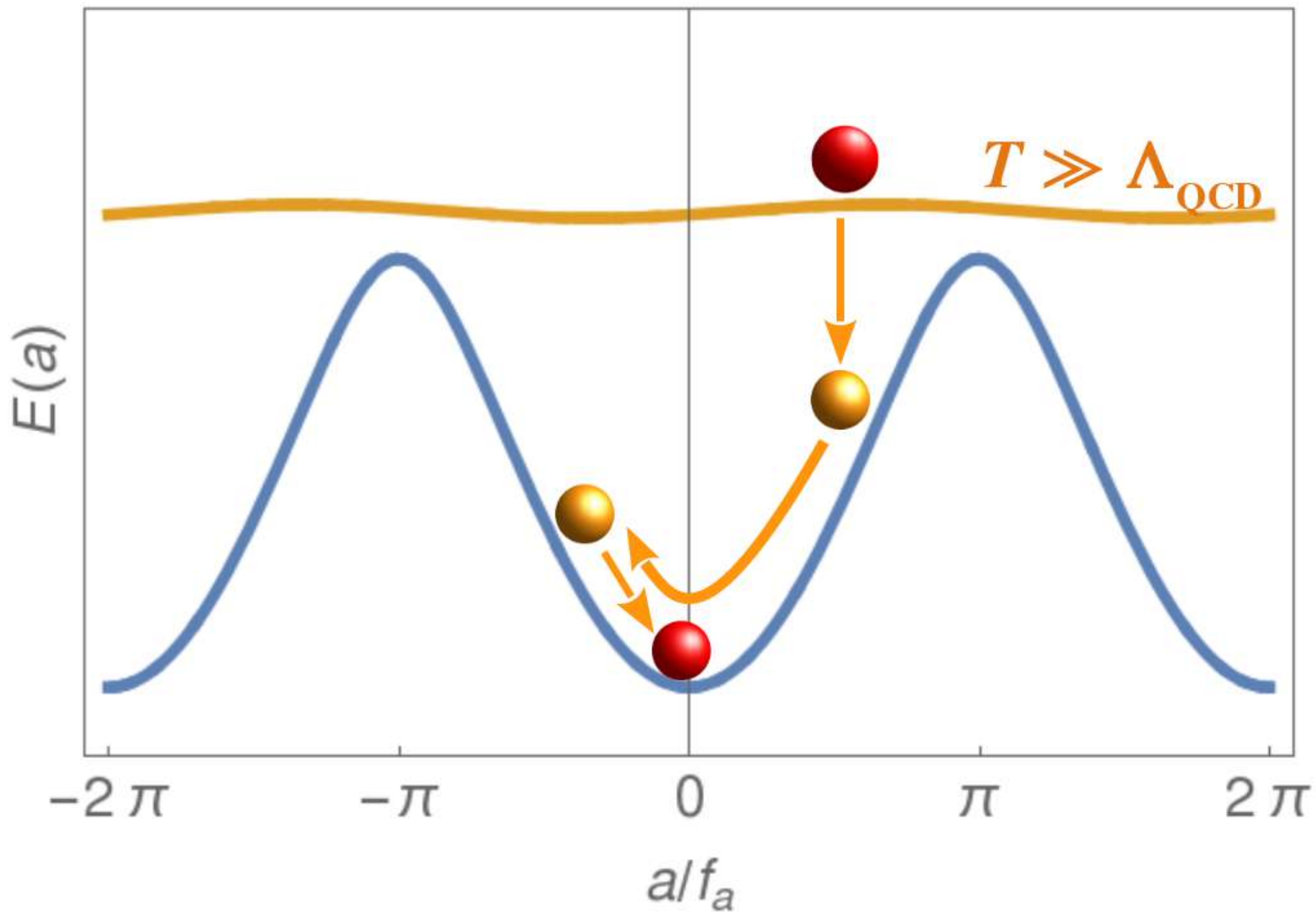
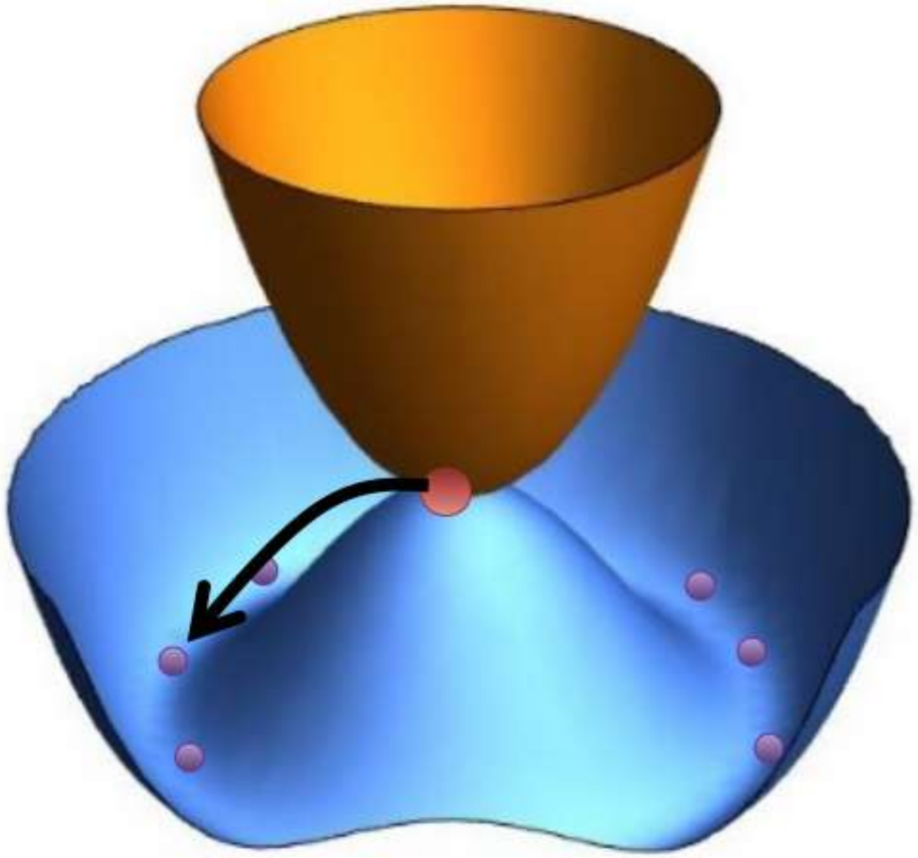


Dark matter

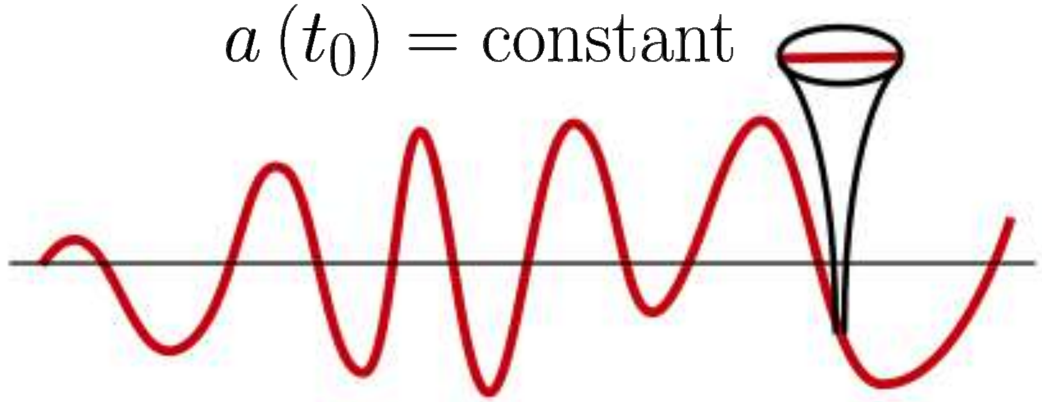
Immediately after U(1) breaking, the axion field is random over the universe:



Dark matter scenarios

PQ symmetry broken during inflation
and not subsequently restored

$a\,(t_0) = \text{constant}$



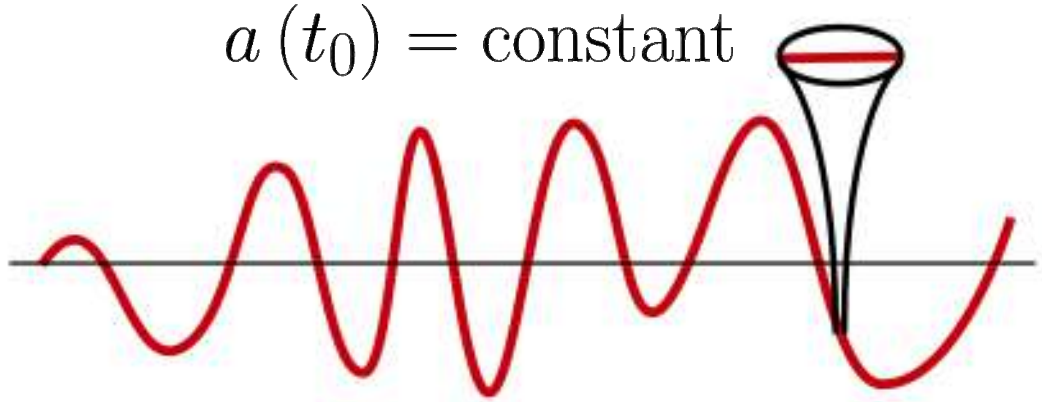
only contribution from misalignment
but not calculable

$\theta_0 < \pi \quad \Leftrightarrow \quad f_a \gtrsim 10^{12}\ \mathrm{GeV}$

(For smaller f_a , i.e. larger masses, the axion still solves the Strong CP problem, but is not DM)

Dark matter scenarios

PQ symmetry broken during inflation and not subsequently restored	PQ symmetry unbroken during inflation or subsequently restored
$a\,(t_0) = \text{constant}$	$a\,(t_0) = \text{random}$



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$$\theta_0 < \pi \quad \Leftrightarrow \quad f_a \gtrsim 10^{12}\ \text{GeV}$$

misalignment contribution fixed

$$\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$$

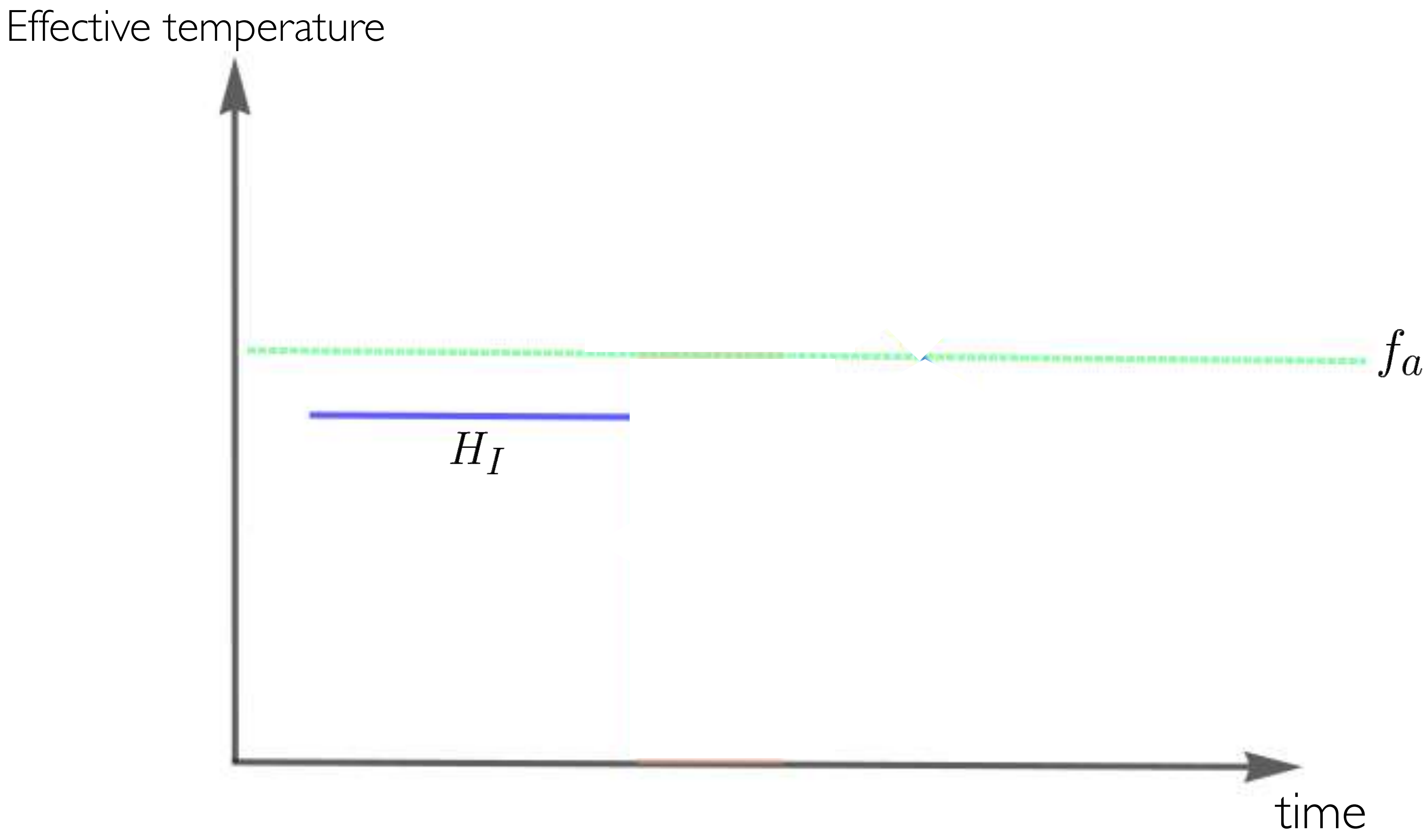
extra axion from strings + domain walls
large uncertainties

$$\Omega_a > \Omega_a^{mis} \quad \Leftrightarrow \quad f_a < f_a^{mis} \sim 10^{12}\ \text{GeV}$$

(For smaller f_a , i.e. larger masses, the axion still solves the Strong CP problem, but is not DM)

Boundary between regimes

Depends on the details of reheating, e.g. with inflaton decay rate Γ



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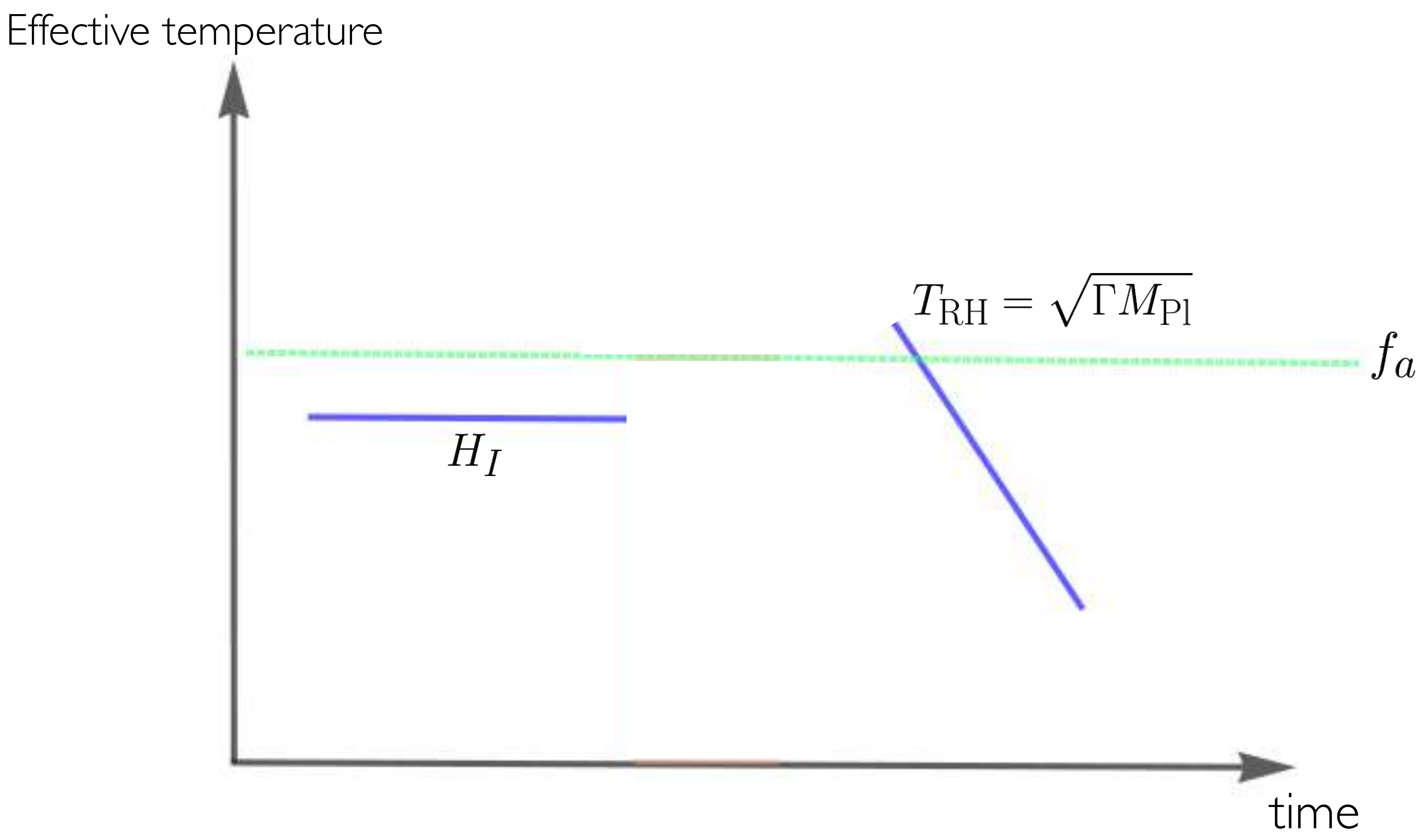

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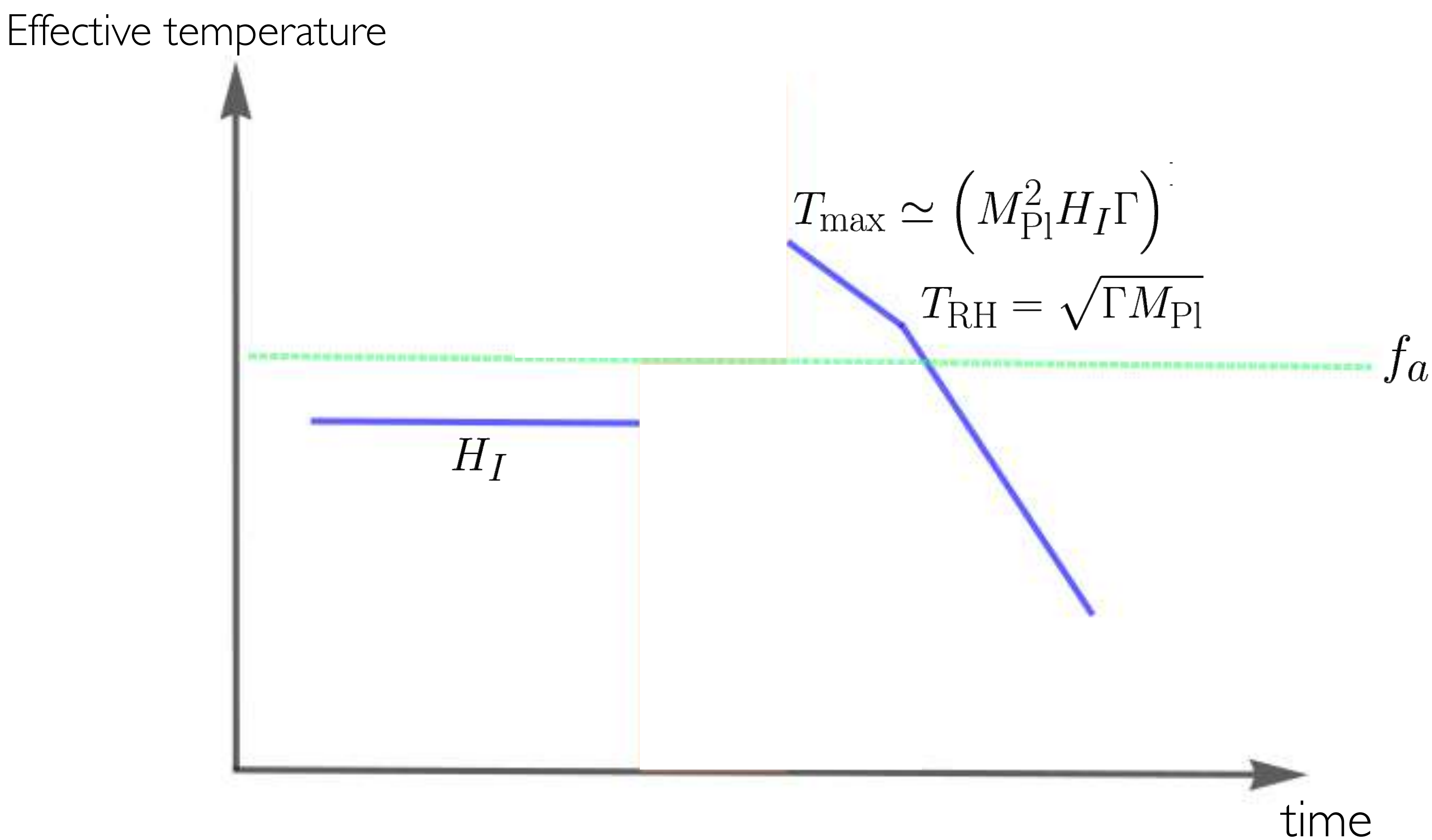

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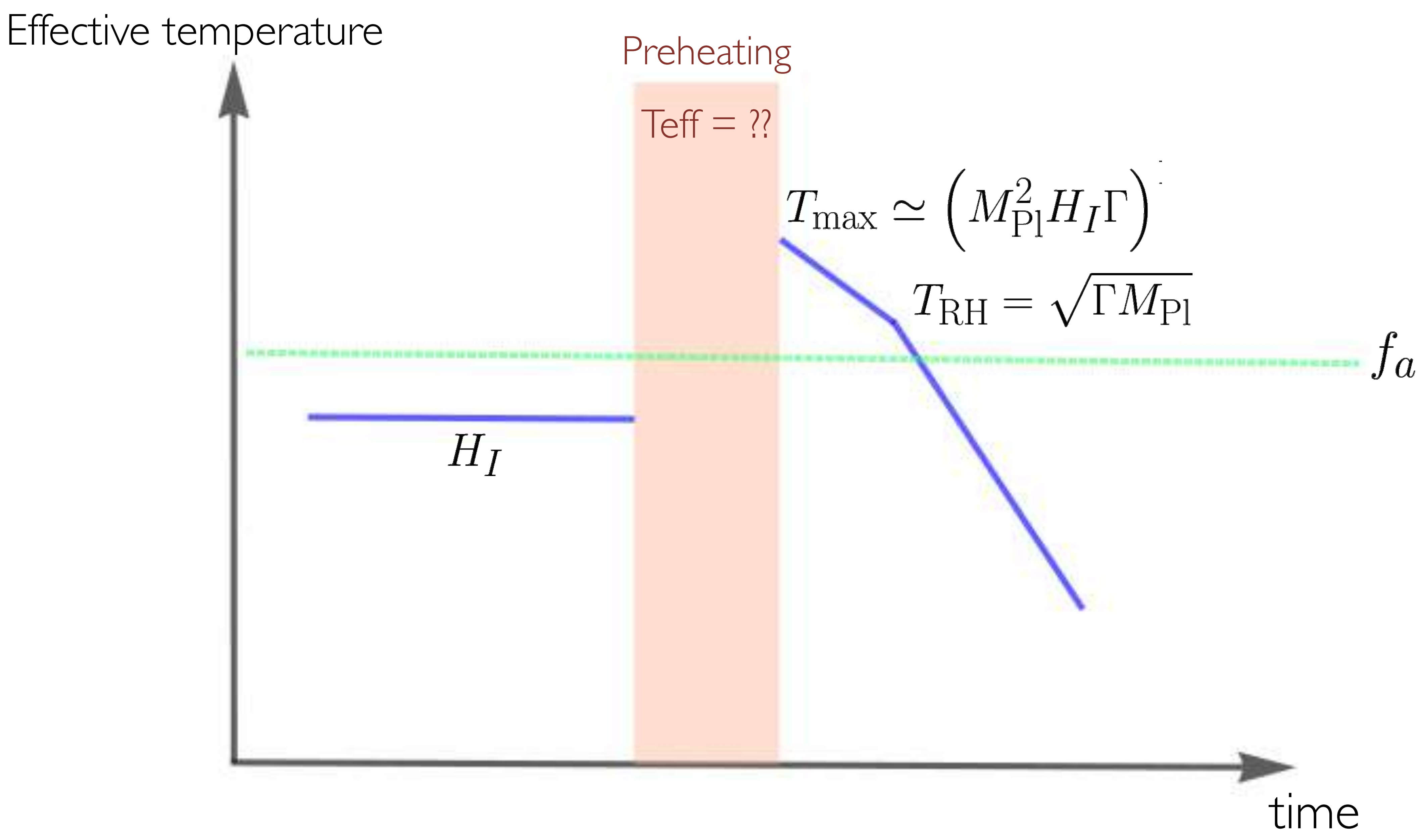
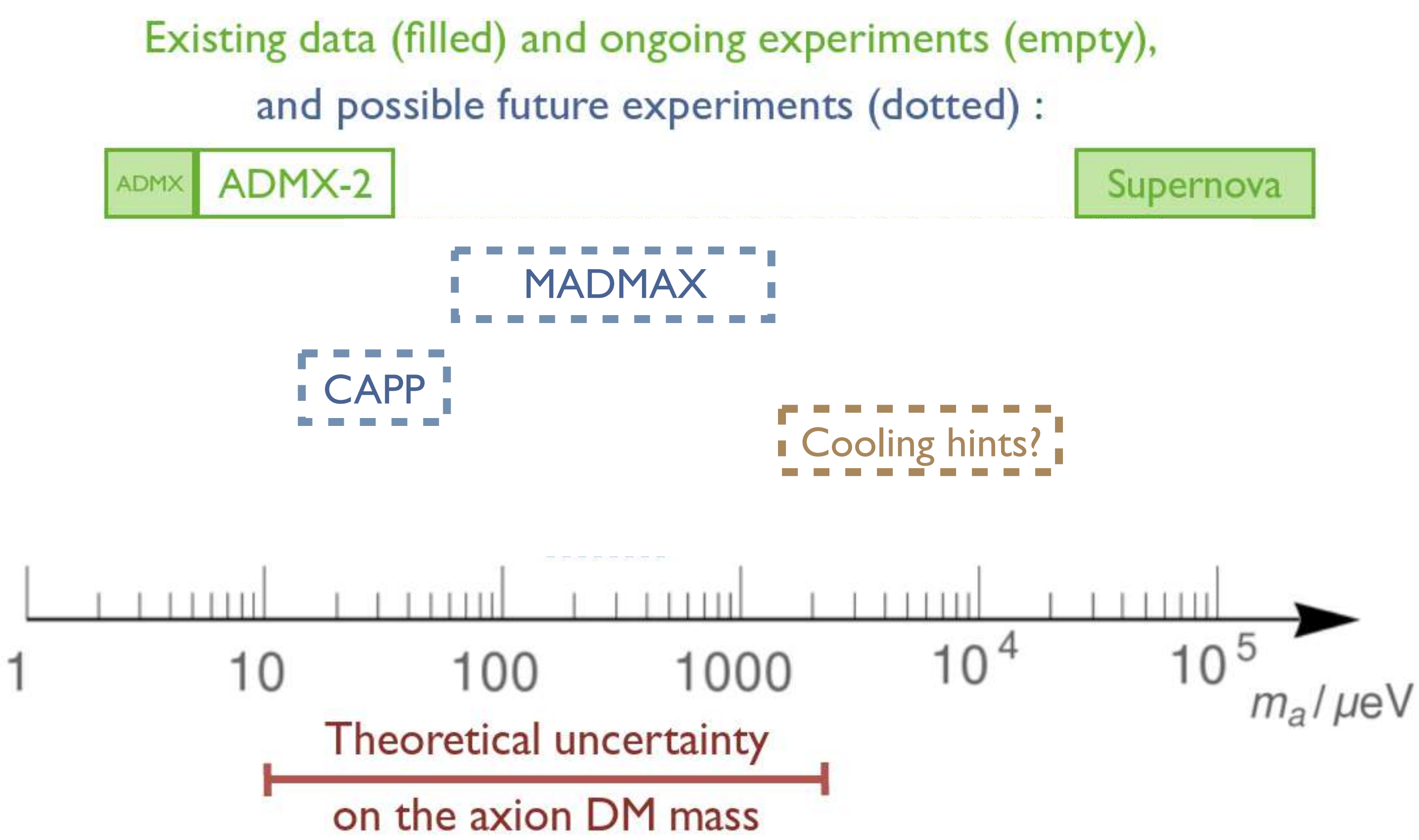


U(1) breaking after inflation

In principle extremely predictive

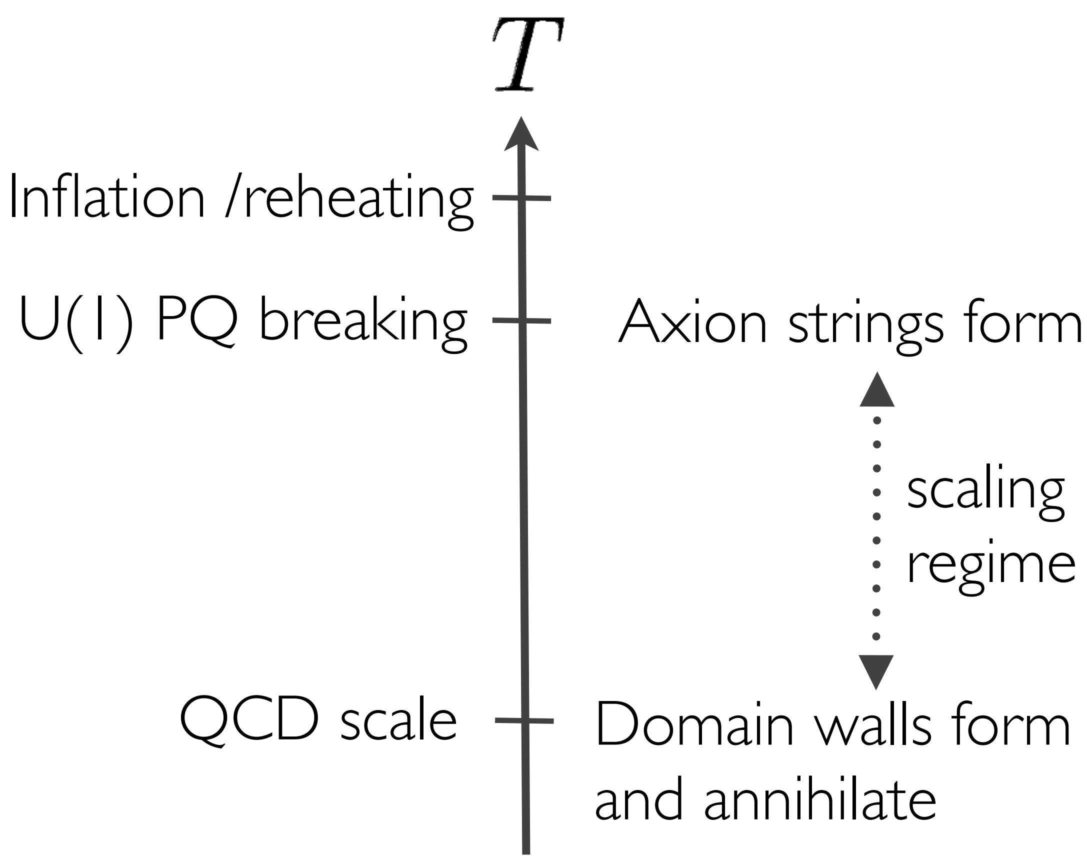
unique DM axion mass

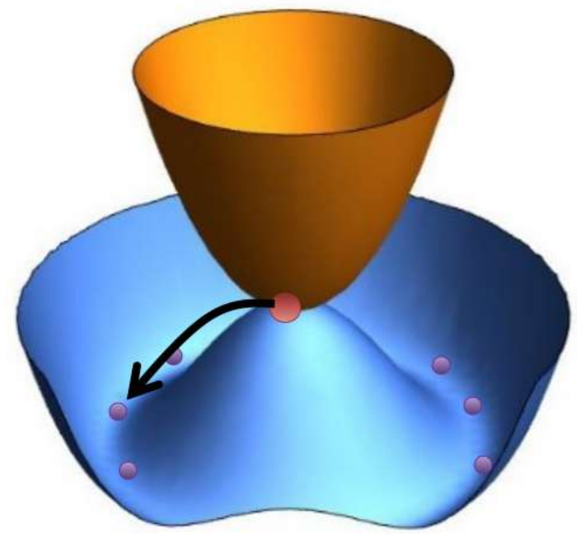


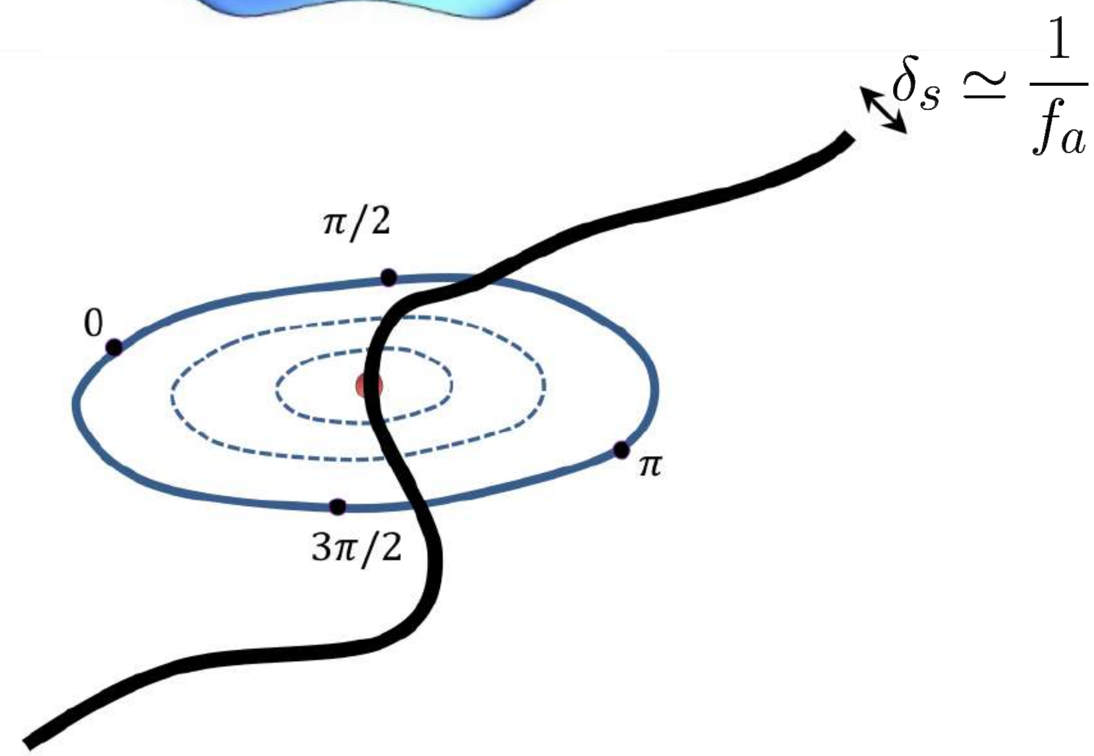
Reliable prediction: interpret ongoing experiments, design future experiments

Precise agreement with an experimental discovery → minimum inflation scale

Strings and domain walls







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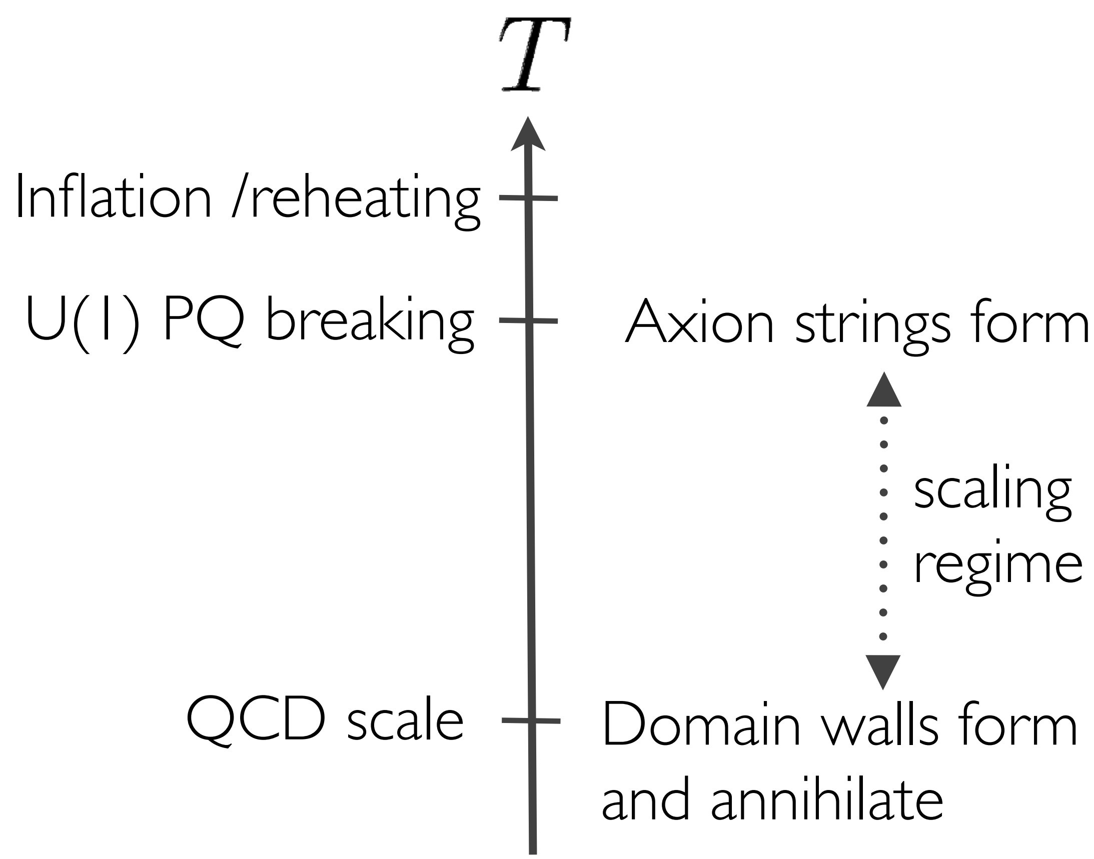
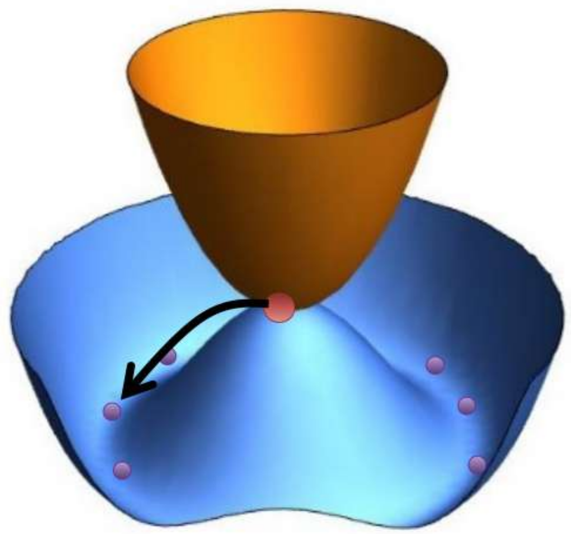
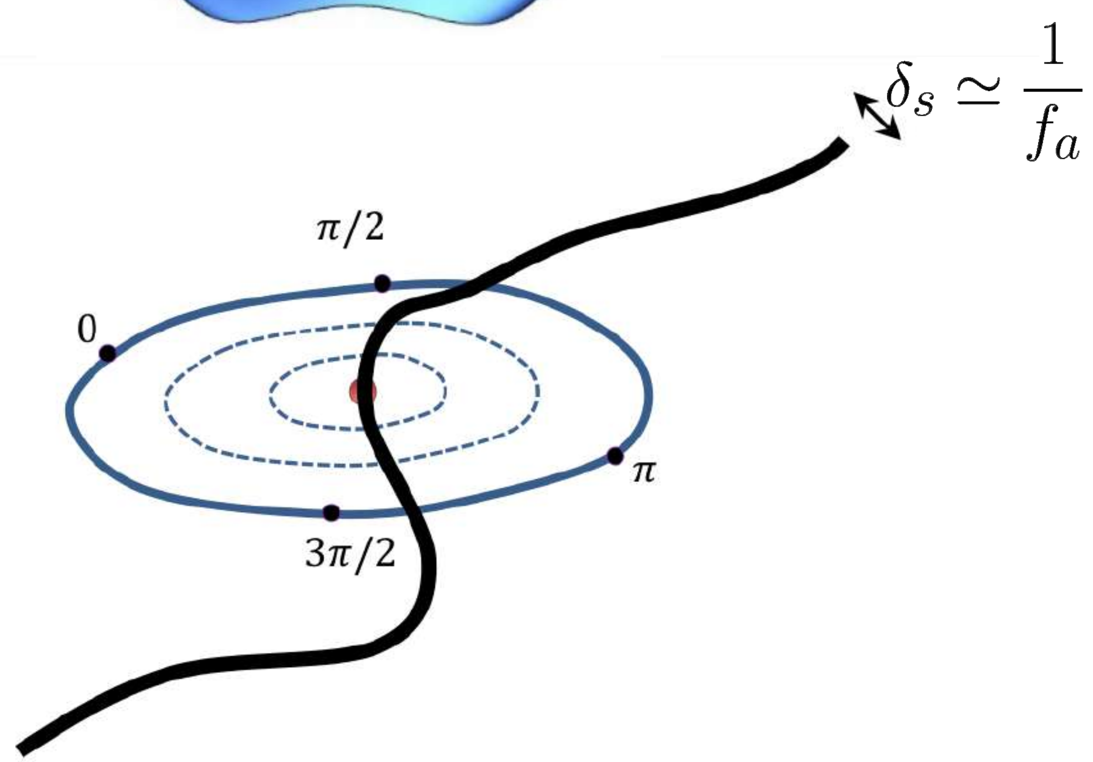






Significant proportion of DM axions produced by strings and domain walls

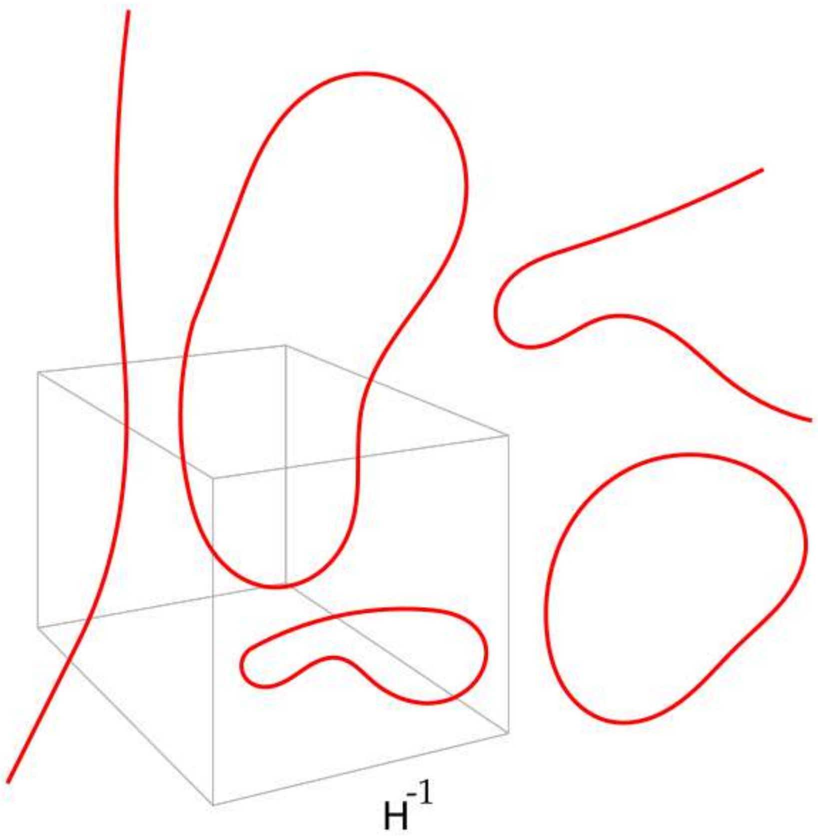
Axion emission during scaling

Parametrisation:

$$\rho_{\text{scaling}} = \frac{\xi(t)\,\mu(t)}{t^2}$$

$\xi(t)$ = Length of string per Hubble volume

$\mu(t)$ = string tension = energy per length



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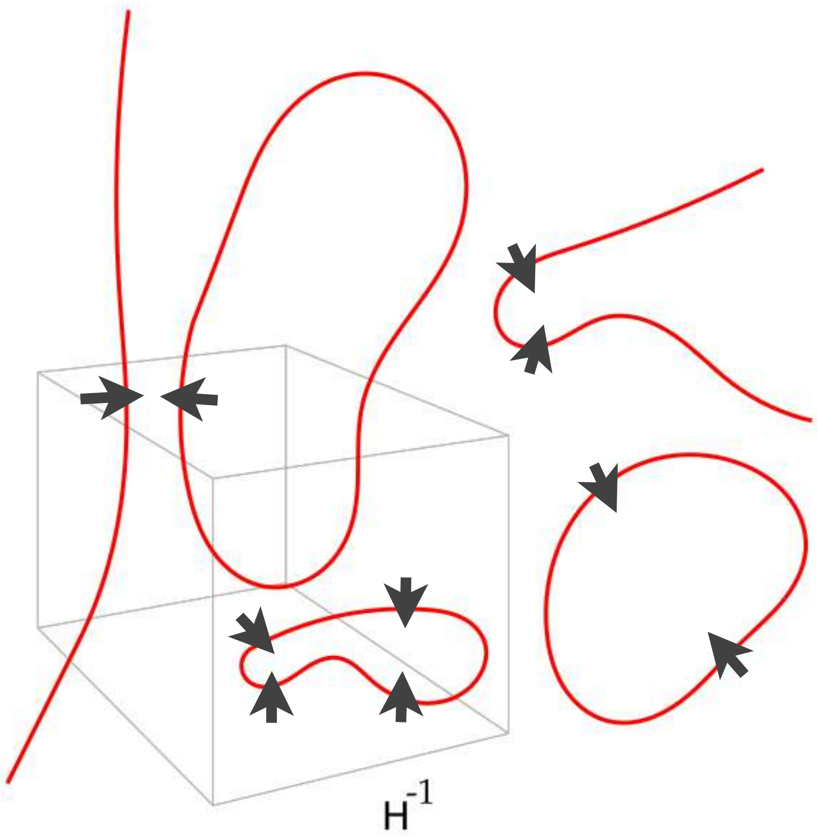

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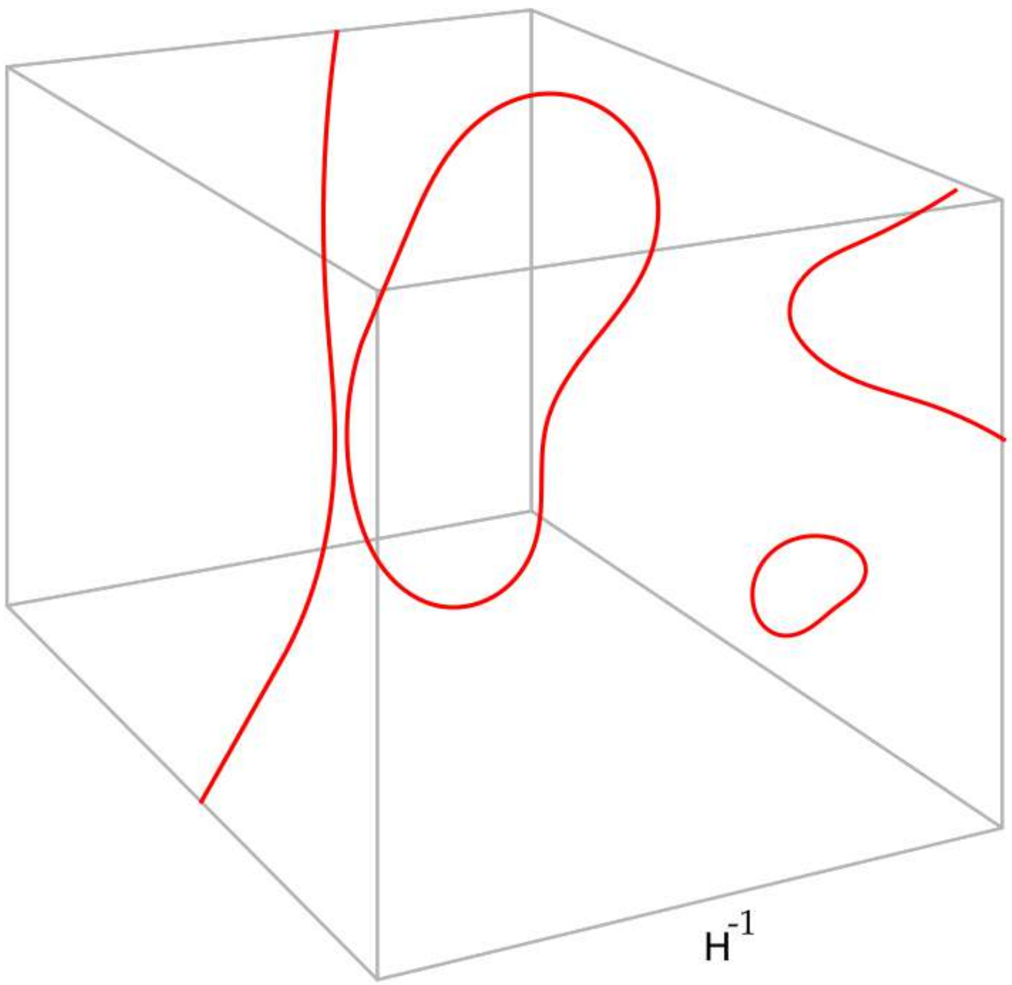


Neglecting string cores, Hubble is the only relevant scale

$\xi(t)$ & $\mu(t)$ approximately constant

Energy release:

$$P_{\text{emitted}} \simeq \frac{\xi(t)\,\mu(t)}{t^3}$$

Axion emission during scaling

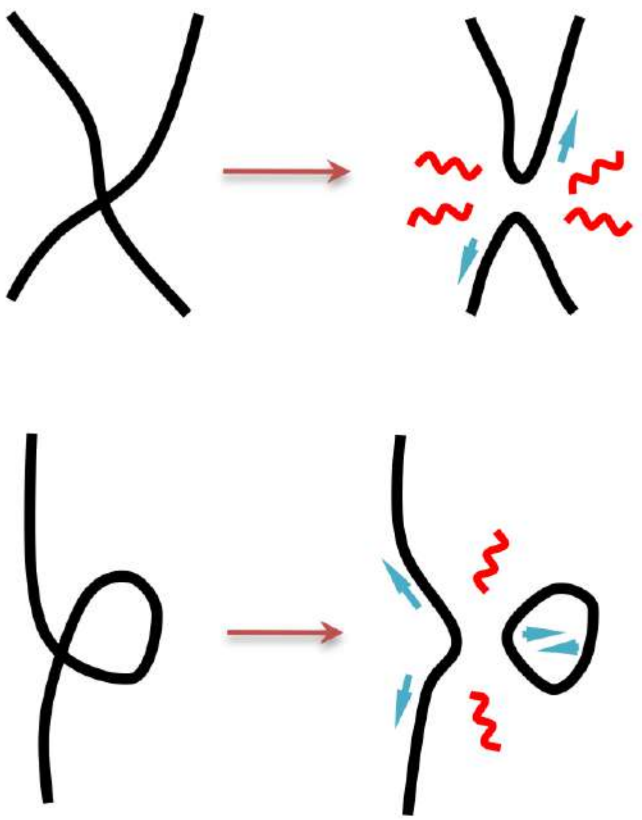
$$P_{\text{emitted}} \simeq \frac{\xi(t)\,\mu(t)}{t^3}$$

We focus on emission by string network during the scaling regime:

gives a lower bound on the DM axion mass

Also required to set the correct initial conditions for domain walls at axion mass turn on

String dynamics

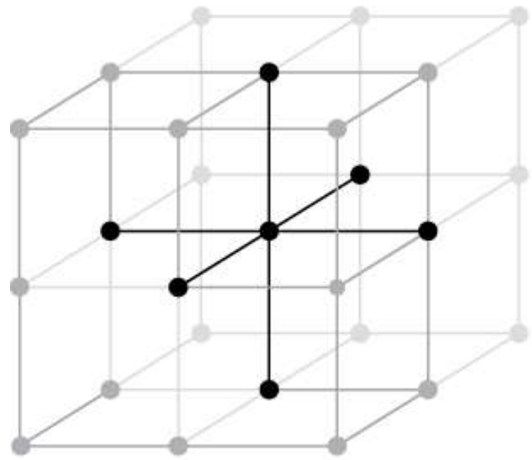


Hard to study analytically, can help with qualitative understanding, but full network has complicated interactions and dynamics

Instead resort to numerical simulations

Numerical simulation

Simulate full complex scalar field and potential on a lattice (no benefit to simulating just the axion)



Evolve using finite difference algorithm

Identify strings by looking at field change around loops in different 2D planes

 group identified lattice points

Why it's hard

Large separation of scale

- String core is very thin $\delta_s \simeq \frac{1}{f_a}$
- Hubble distance is much larger $H^{-1} \simeq \frac{M_{\rm pl}}{T^2} \simeq \frac{M_{\rm pl}}{\Lambda_{\rm QCD}^2}$

String tension depends on the ratio of string core size and Hubble scale

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String tension depends on the ratio of string core size and Hubble scale

$$\mu(t) \simeq \pi f_a^2 \log\left(\frac{H(t)^{-1}}{\delta_s}\right) =: \pi f_a^2 \log(\alpha(t))$$

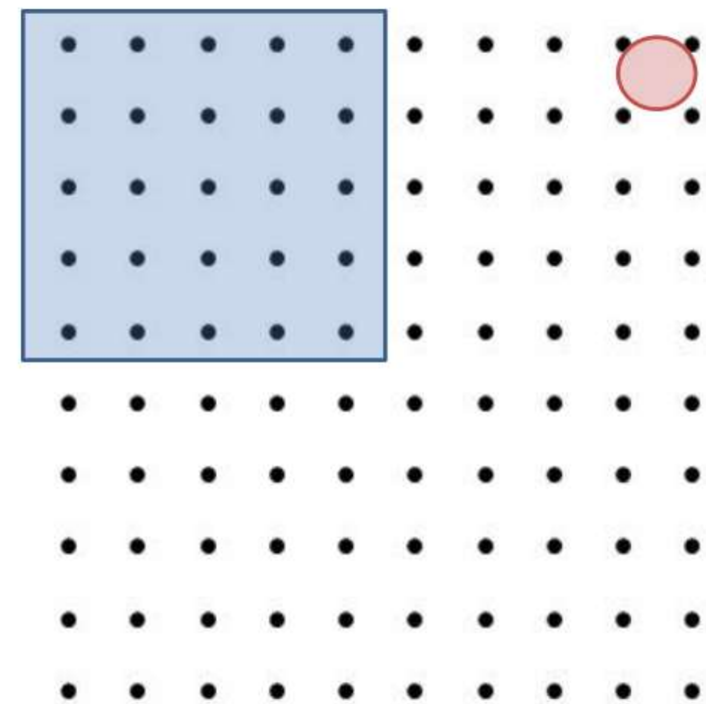
Physical scale separation $\alpha \sim 10^{30}$

$\Rightarrow$ $\log \alpha \simeq 70$

Why it's hard

Numerical simulations need

- a few lattice points per string core
- a few Hubble patches



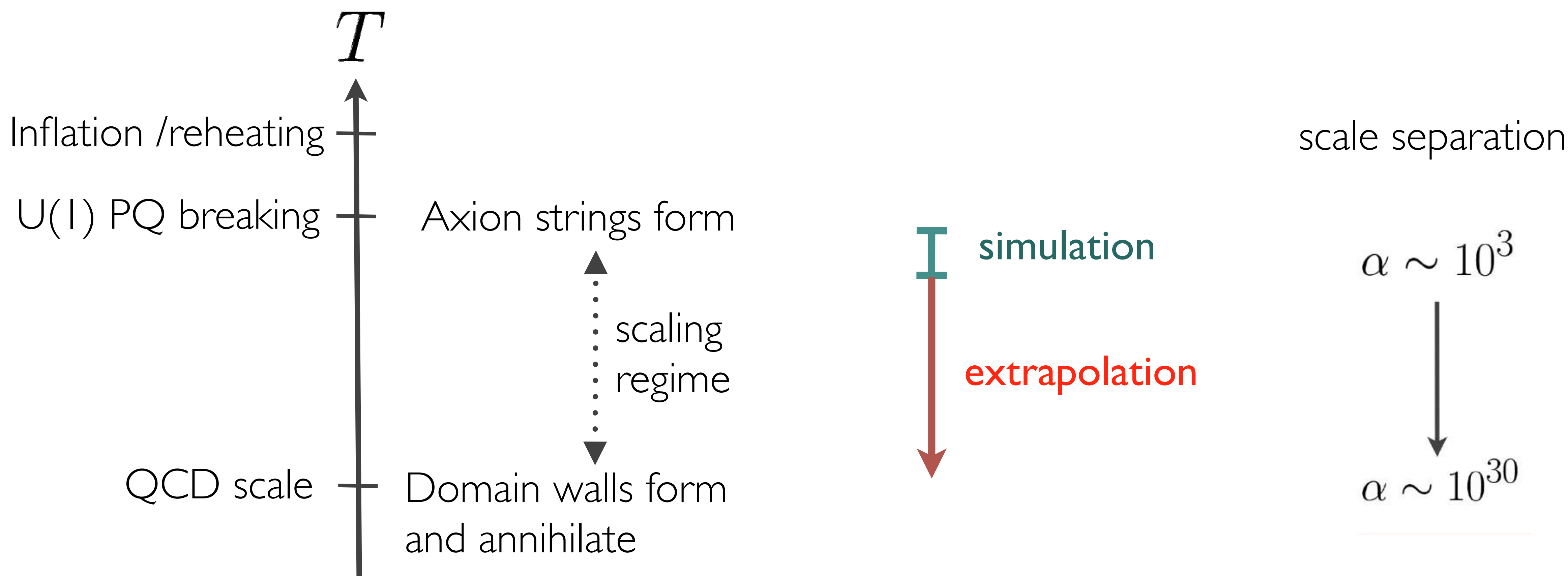
Can only simulate grids with $\sim 5000^3$ points

simulations: $\log \alpha \leq \log(\frac{\square}{\bigcirc}) \simeq 7$

physical: $\log \alpha \sim 70$

We simulate at small scale separation then extrapolate

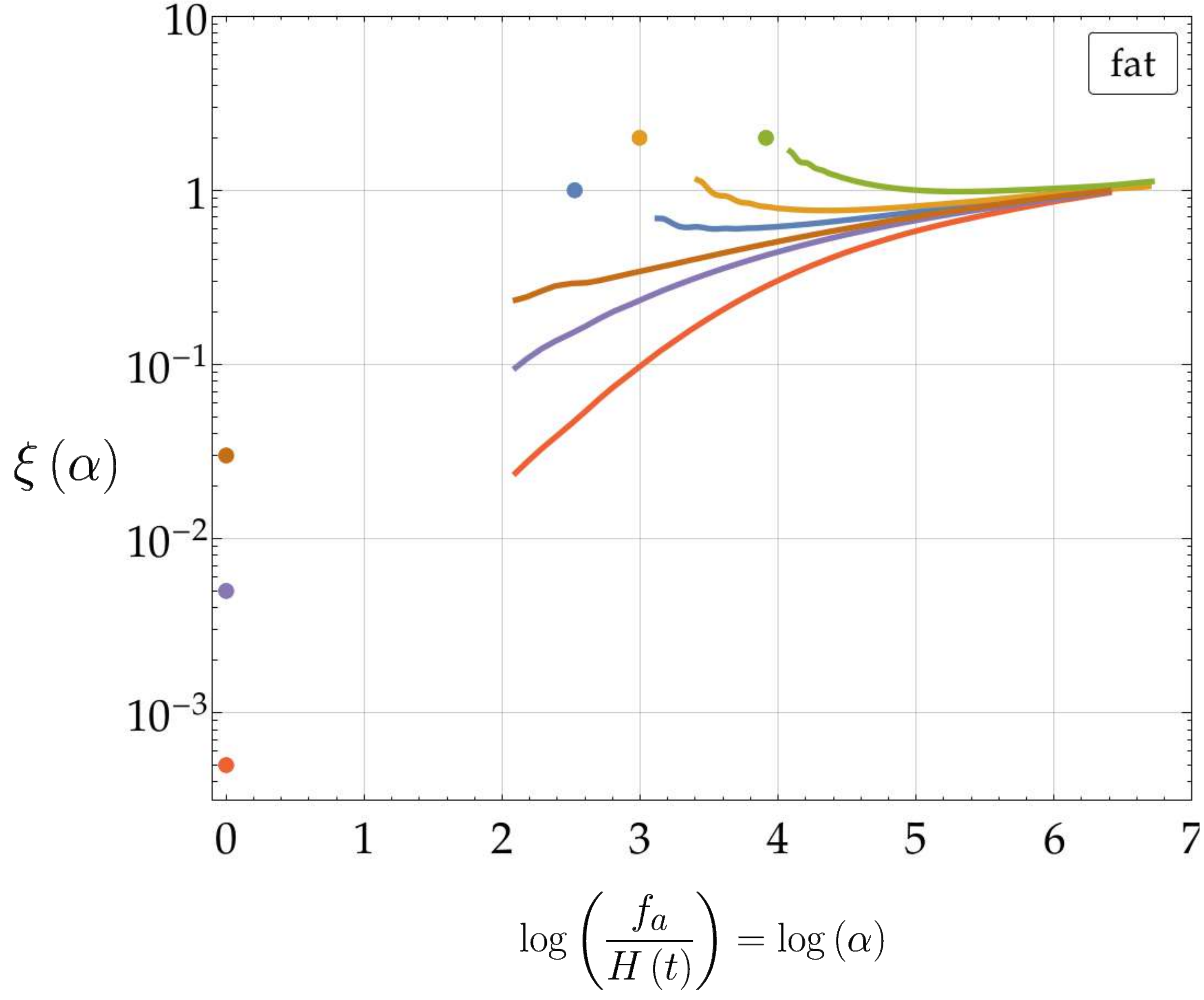
Extrapolation



Understanding the dependence of the physics on the scale separation is crucial

String length per Hubble volume

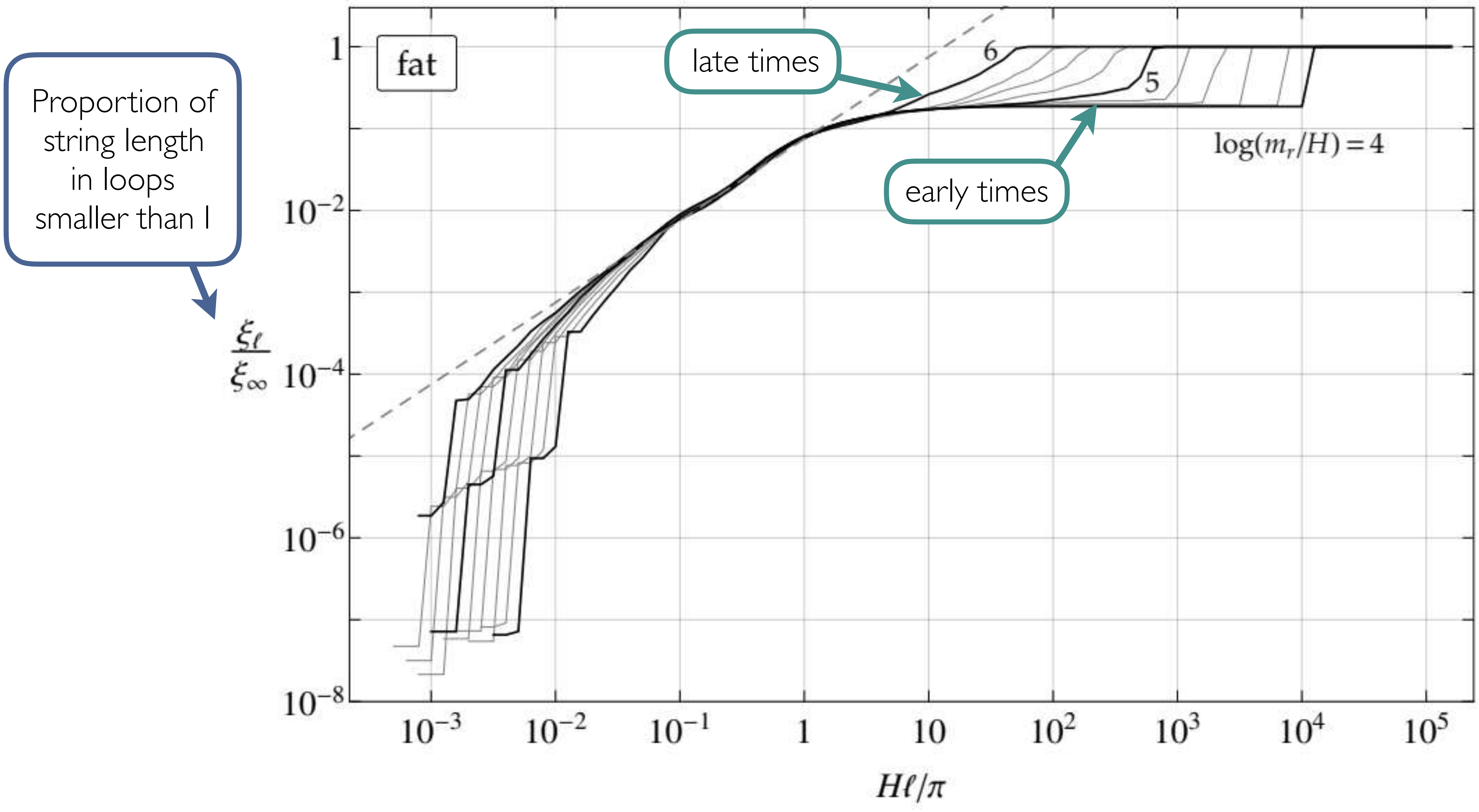
Start with overdense/ underdense, also with random field initial conditions



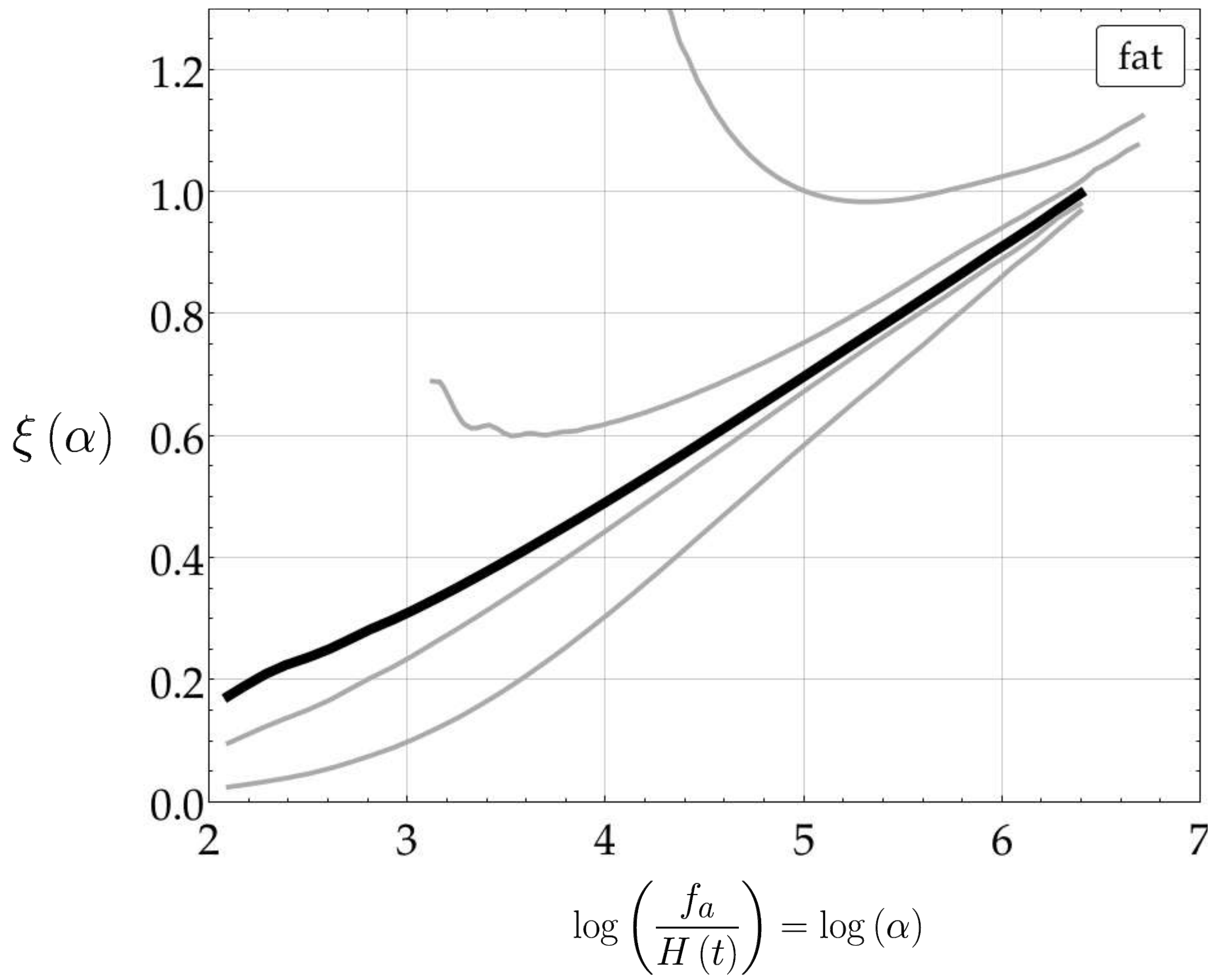
Solution is approximately scale invariant

Final result is not dependent on the details of the phase transition

Distribution of loop lengths



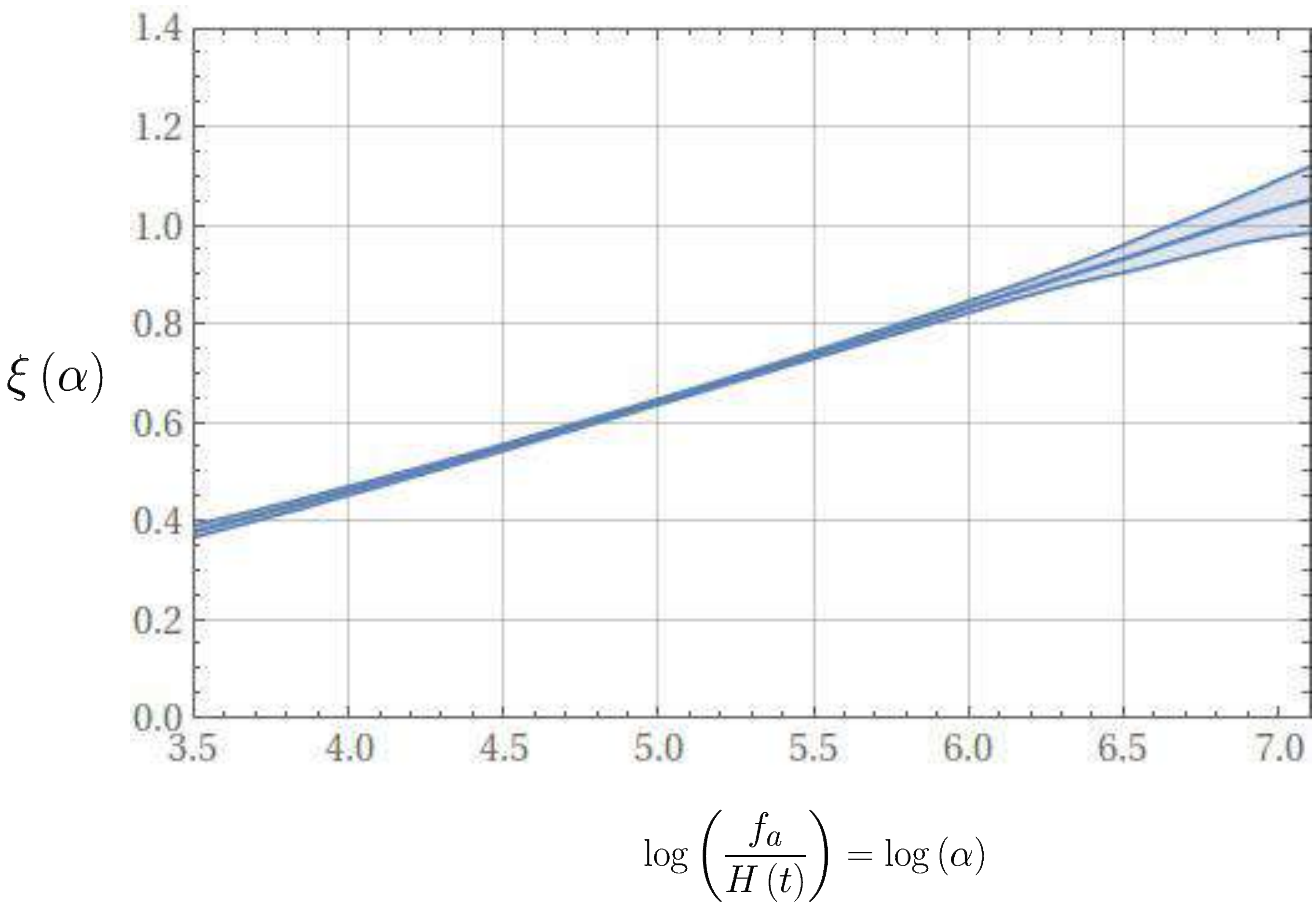
String length per Hubble volume



Find a log increase,

theoretically plausible:
tension is increasing

String length per Hubble volume



Find a log increase,

theoretically plausible: tension is increasing

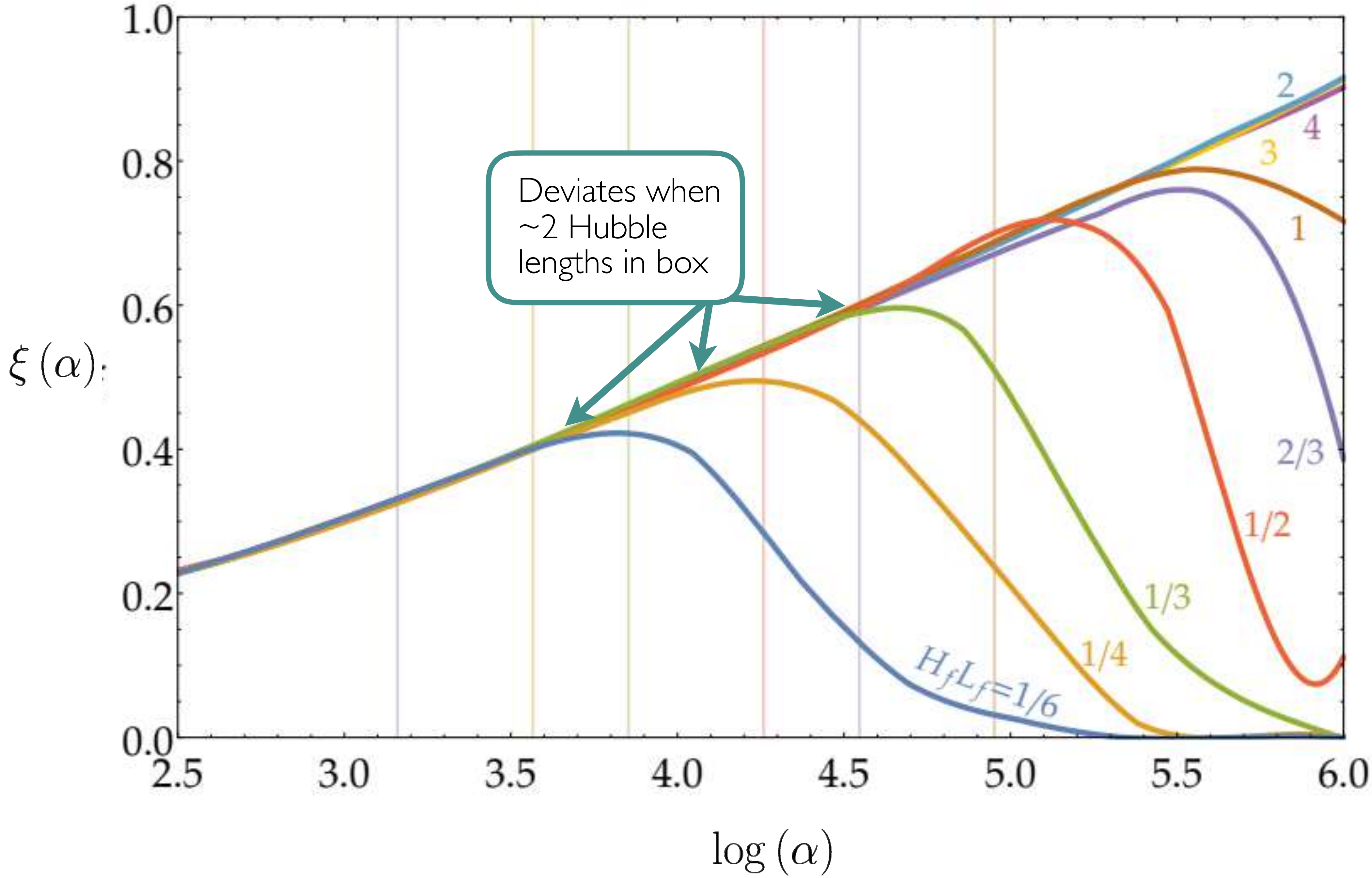
If extrapolation is valid, grows to ~10 at QCD scale

Energy release:

$$P_{\text{emitted}} \simeq \frac{\xi(t)\,\mu(t)}{t^3}$$

Numerical checks

E.g. number of Hubble patches at end of simulation



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Global strings in 2d

In 2D strings are equivalent to point charges:

Away from string cores, define a dual EM field that obeys Maxwell's equations

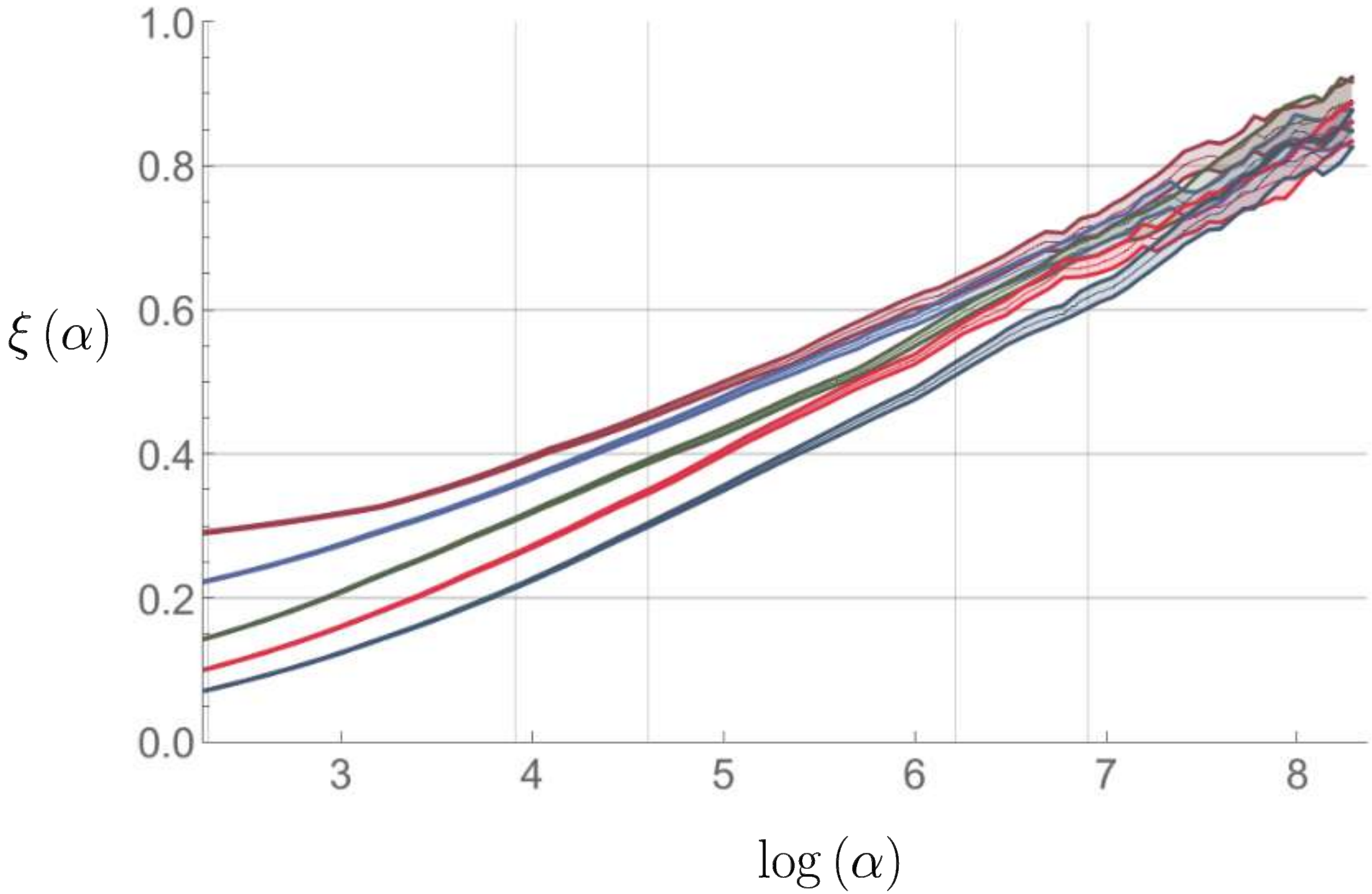
Strings source the EM field, flux through a loop is $2\pi f_a n_{\text{enclosed}}$

Potential between two strings $V(r) = -\frac{q_1 q_2}{2\pi} \log r$

Mass of equivalent charges $M \simeq \pi f_a^2 \log\left(\frac{r_0}{\delta_s}\right)$

String number density ~ log is reasonable

Global strings in 2d



3D Collapsing Loops

At large log, global string tension is large, dynamics the same as local strings up to corrections

$$\sim \frac{1}{\log \alpha}$$

Analytic solution for Nambu-Goto string:

- loop bounces many times

Alternative, coupled strongly to the axion:

- collapsing loop is overdamped

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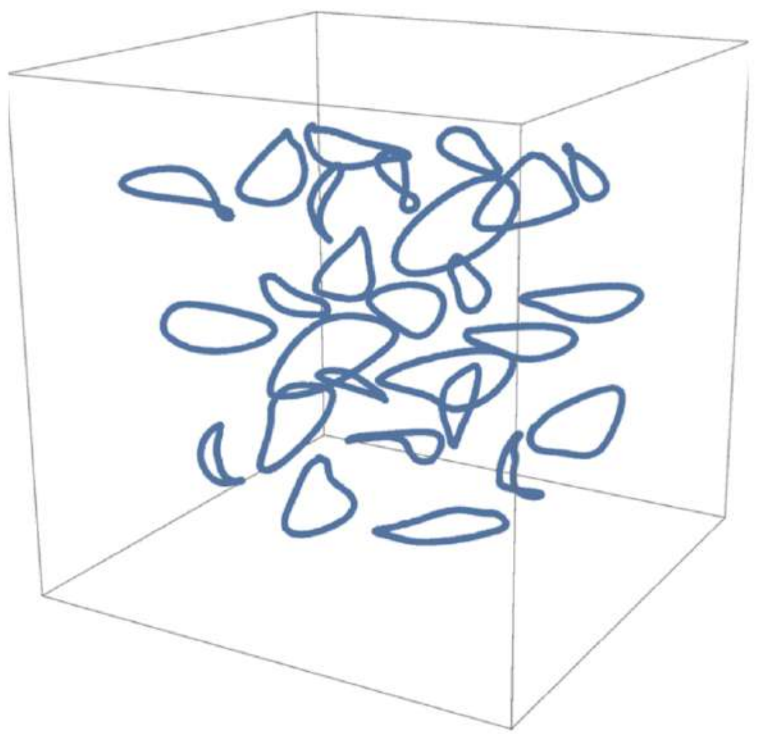

Simulate an ensemble of non-circular loops

Analytic solution for Nambu-Goto string:

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Alternative, coupled strongly to the axion:

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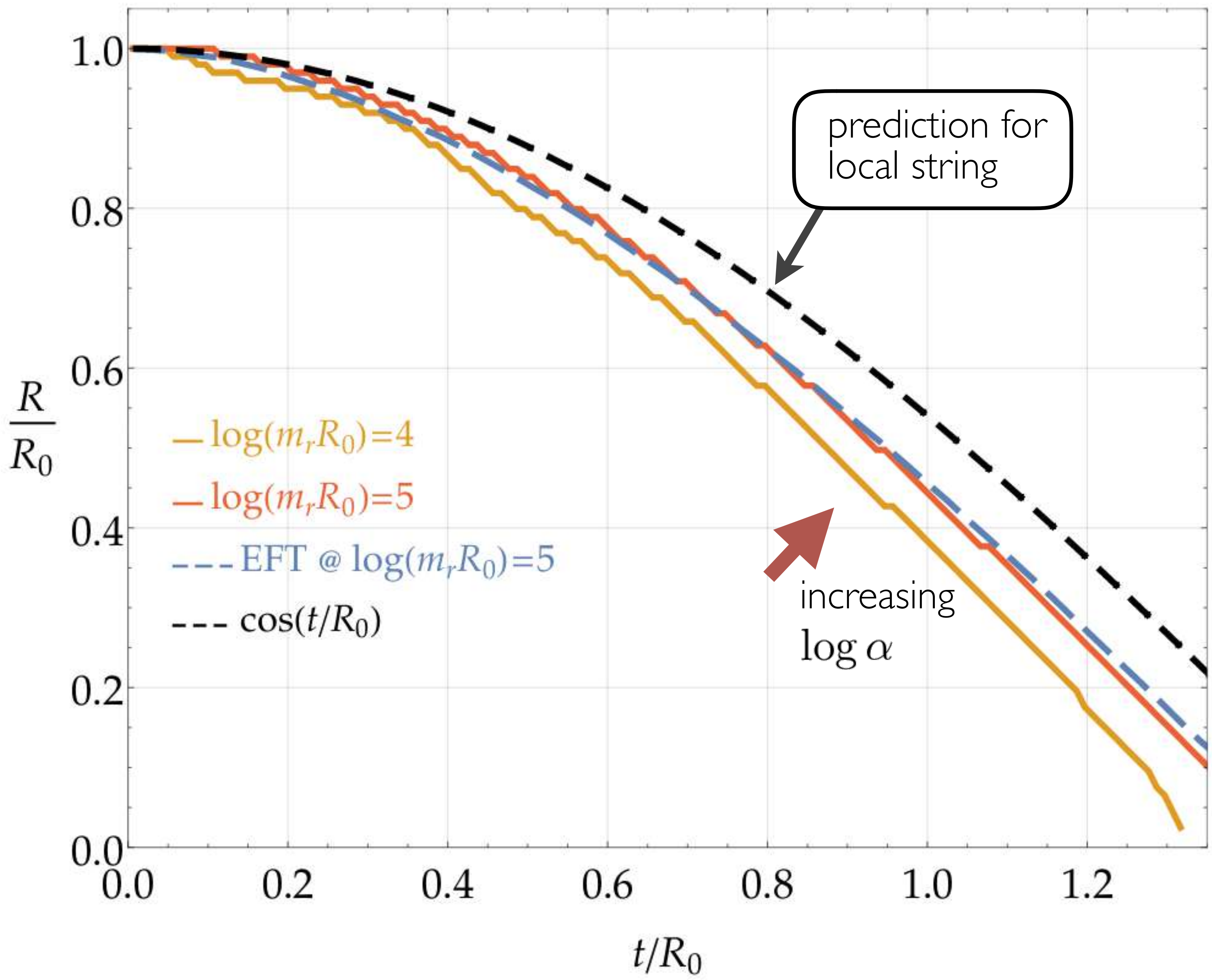
Collapsing Loops



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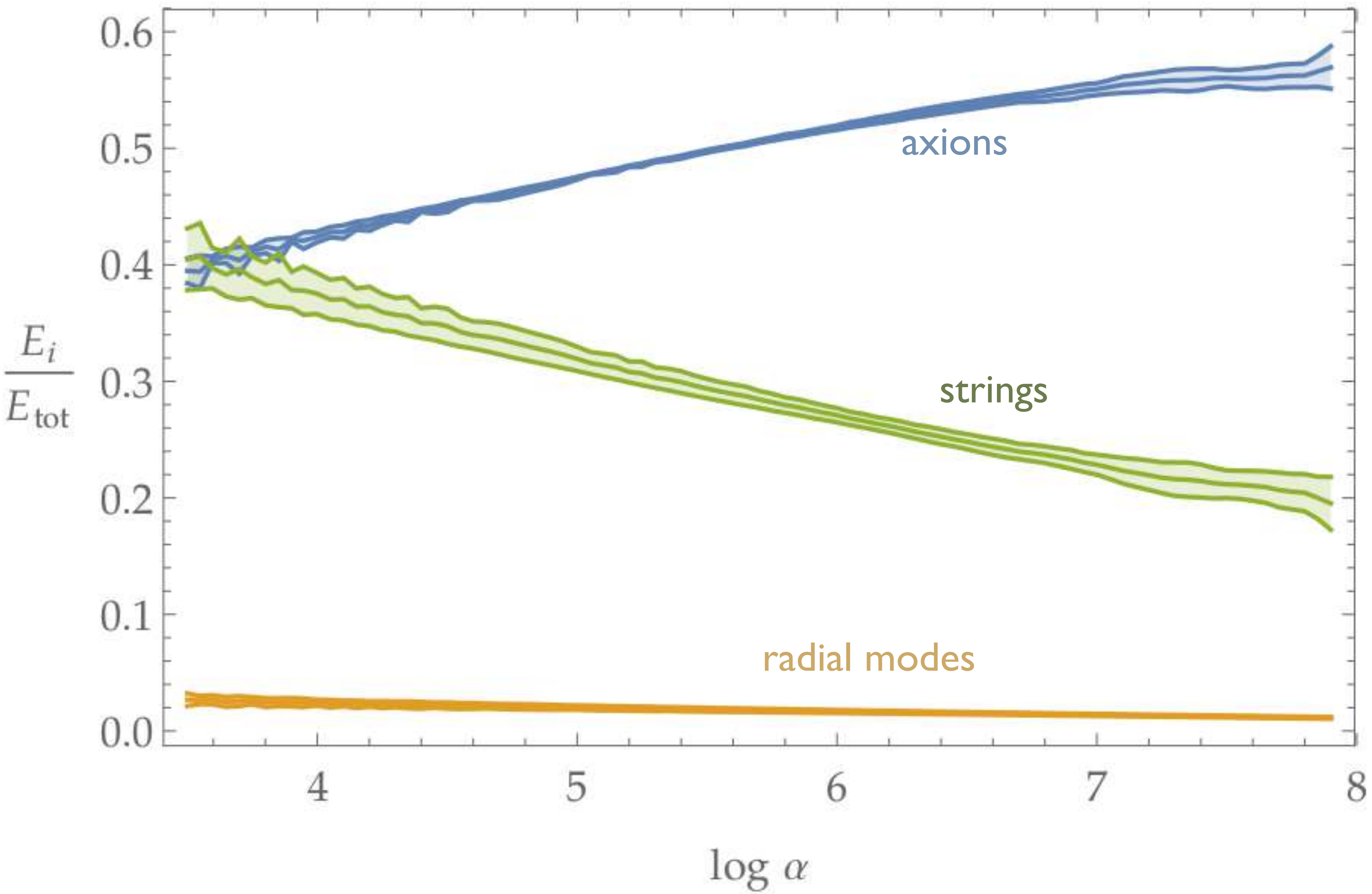
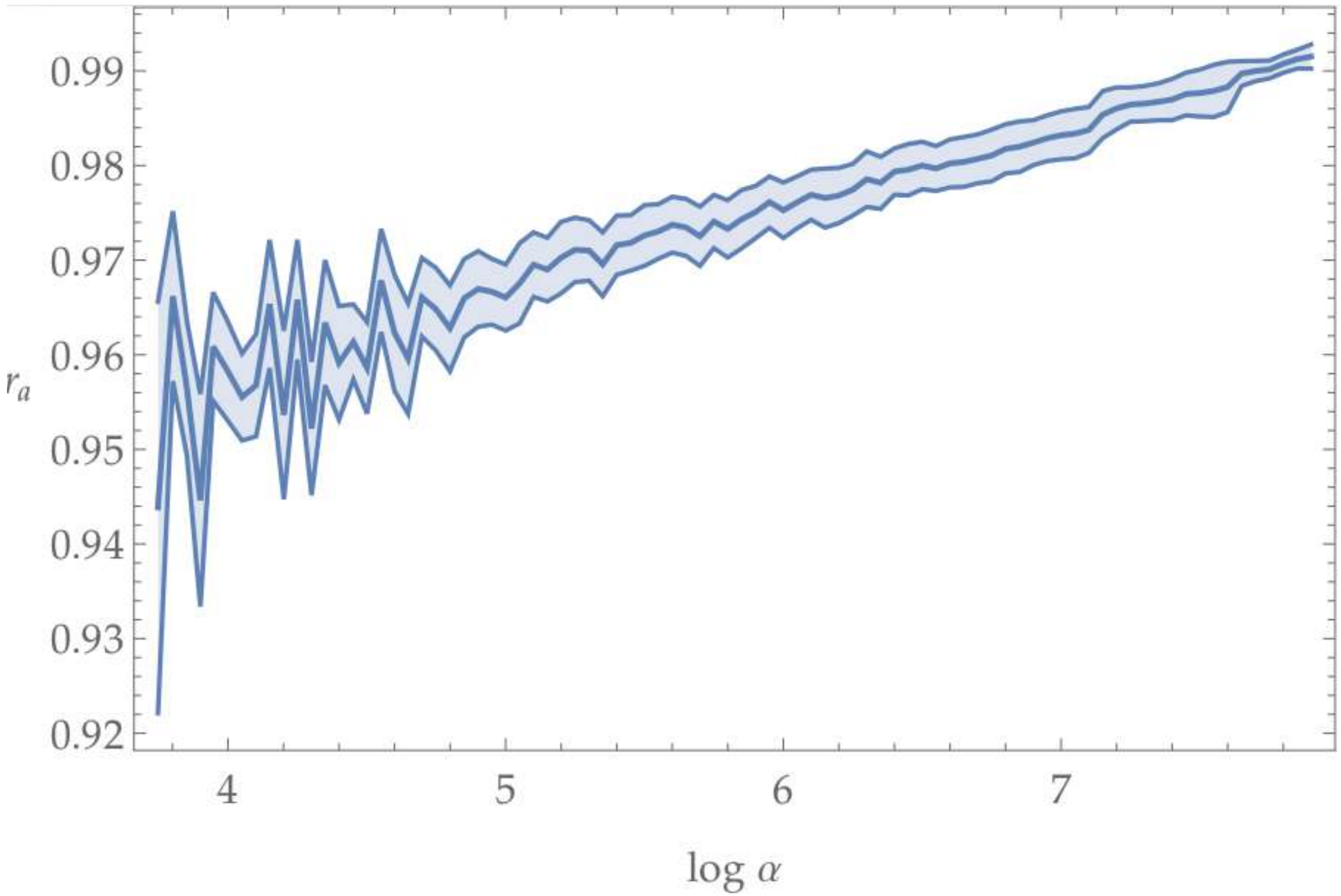
Energy distribution

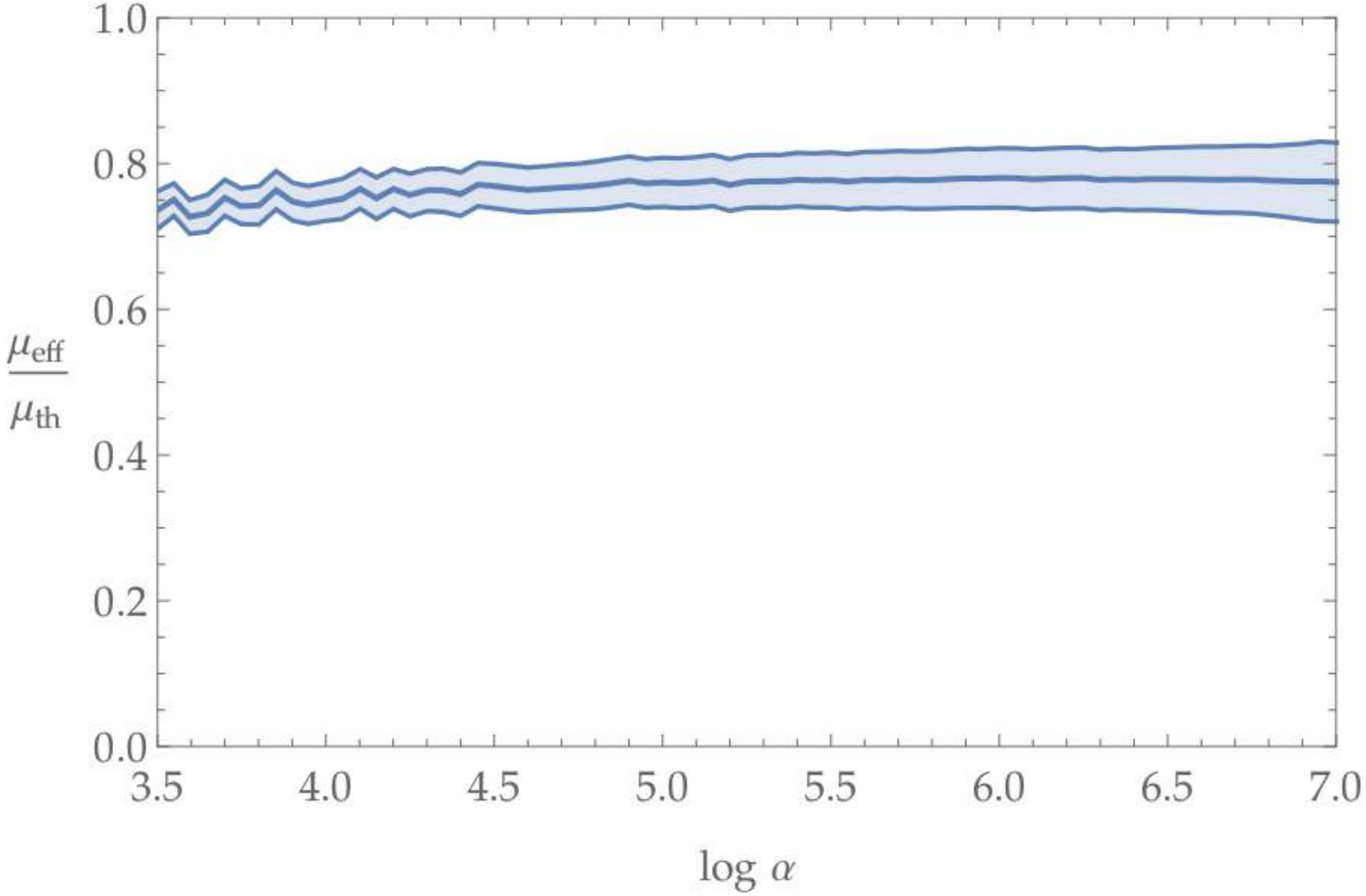


Emission ratio to axions



Effective tension

Calculate the effective string tension in simulations from string energy and $\xi(\alpha)$

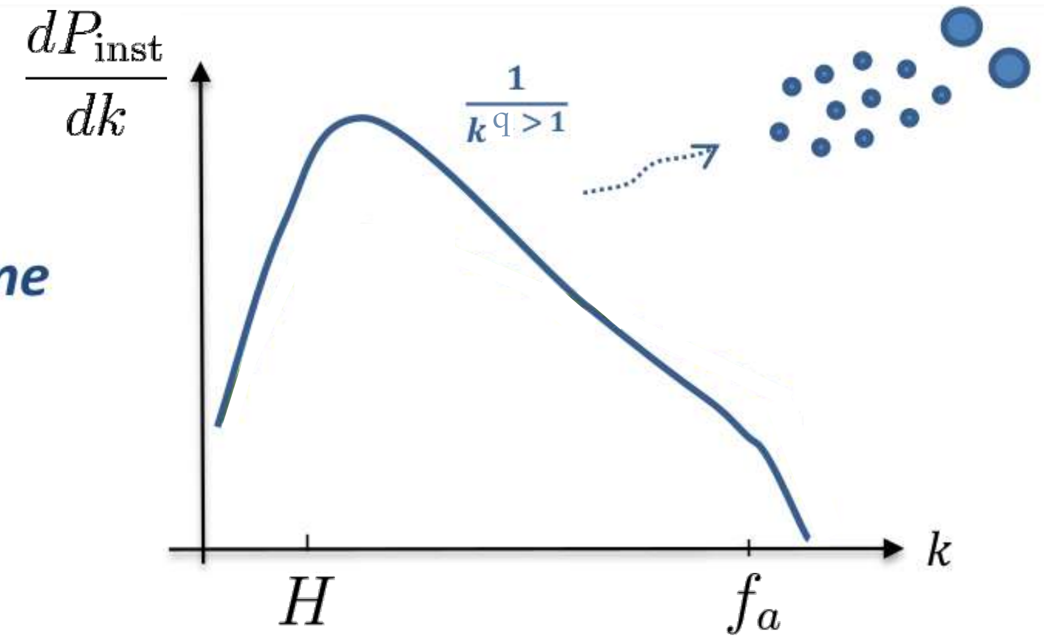


Agrees well with theoretically expected form

$$\mu_{\mathrm{th}} \simeq \pi f_a^2 \log\left(c\frac{H^{-1}}{\delta_s}\right)$$

Distribution of axion momenta

$\frac{dP_{inst}}{dk}$ $\equiv$ ***spectrum of axions radiated per unit time***



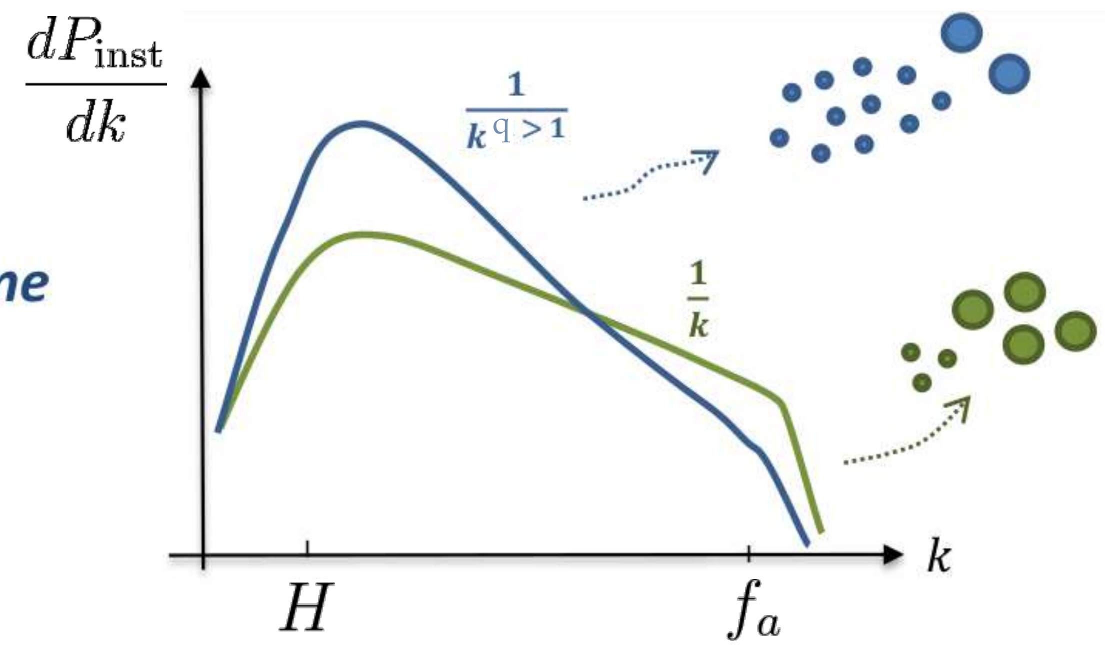
Theoretical expectations

natural cut-offs at H and f_a but:

(1) $\frac{dP_{inst}}{dk} \sim \frac{1}{k^q}$ "soft" spectrum with $\langle k^{-1} \rangle \sim H^{-1}$

$\boxed{q > 1}$

Distribution of axion momenta



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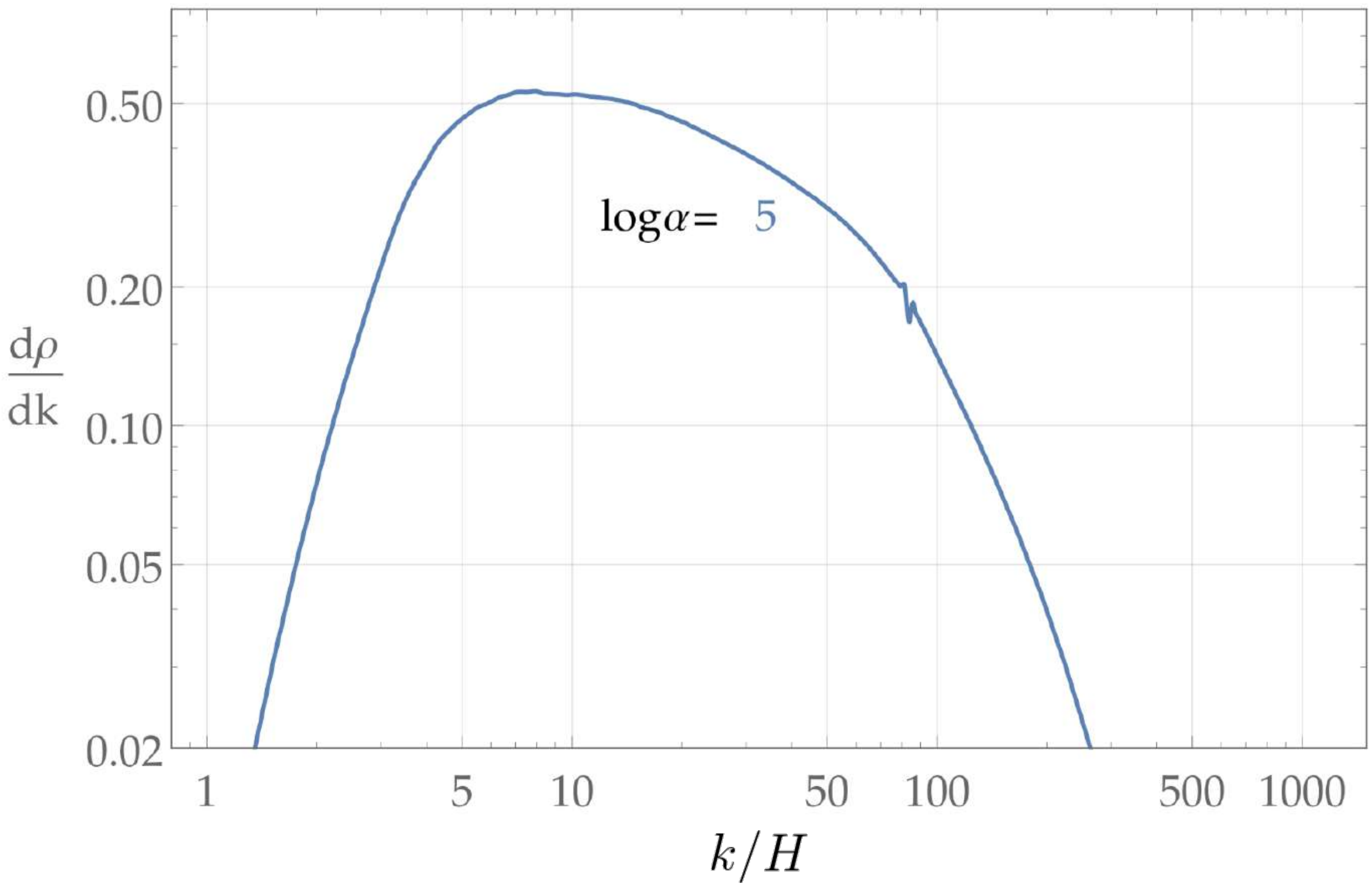
$q > 1$

(2) $\frac{dP_{inst}}{dk} \sim \frac{1}{k}$ "hard" spectrum with $\langle k^{-1} \rangle \sim \frac{H^{-1}}{\log\left(f_a/H\right)}$

Total spectrum



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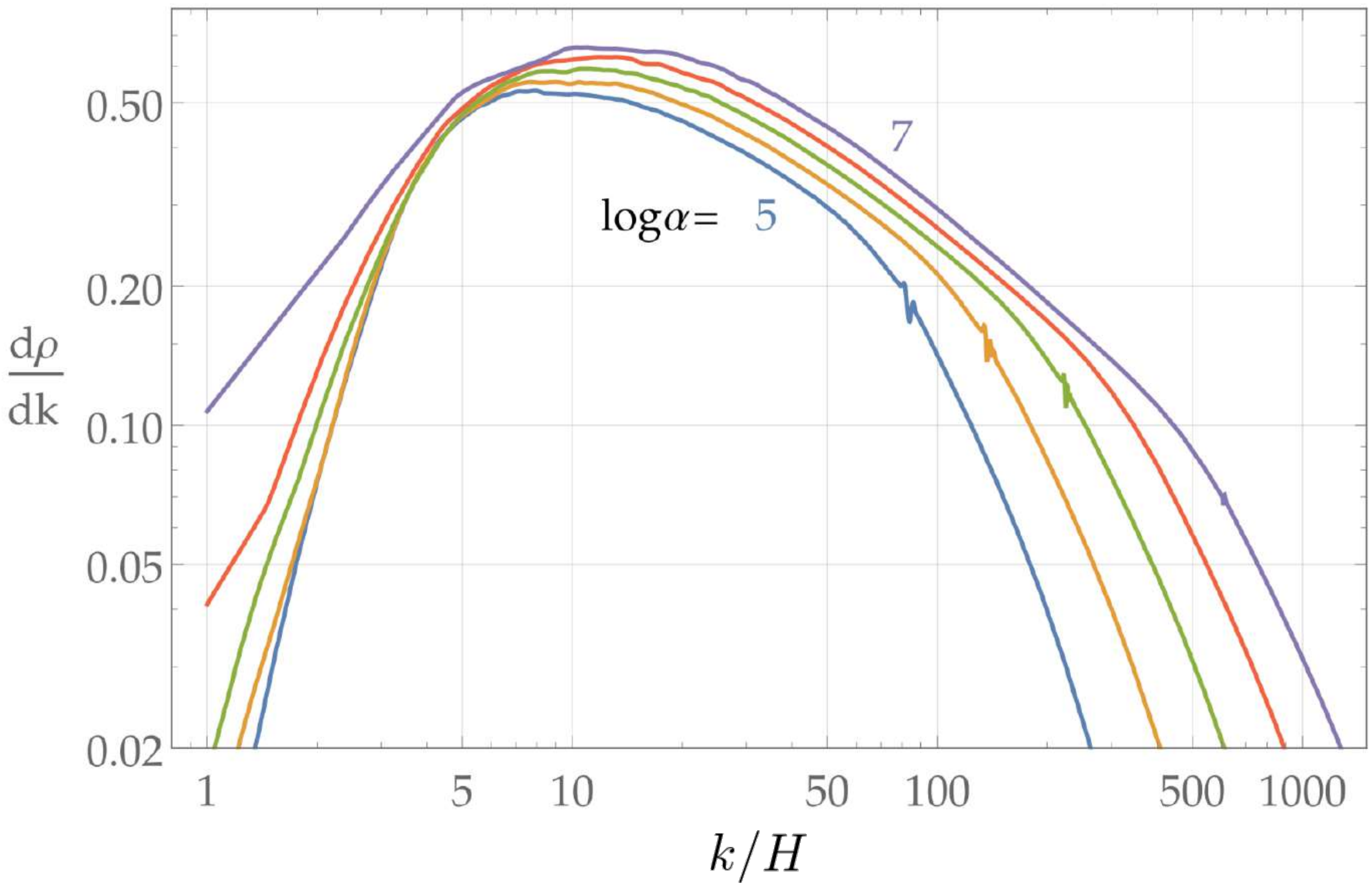
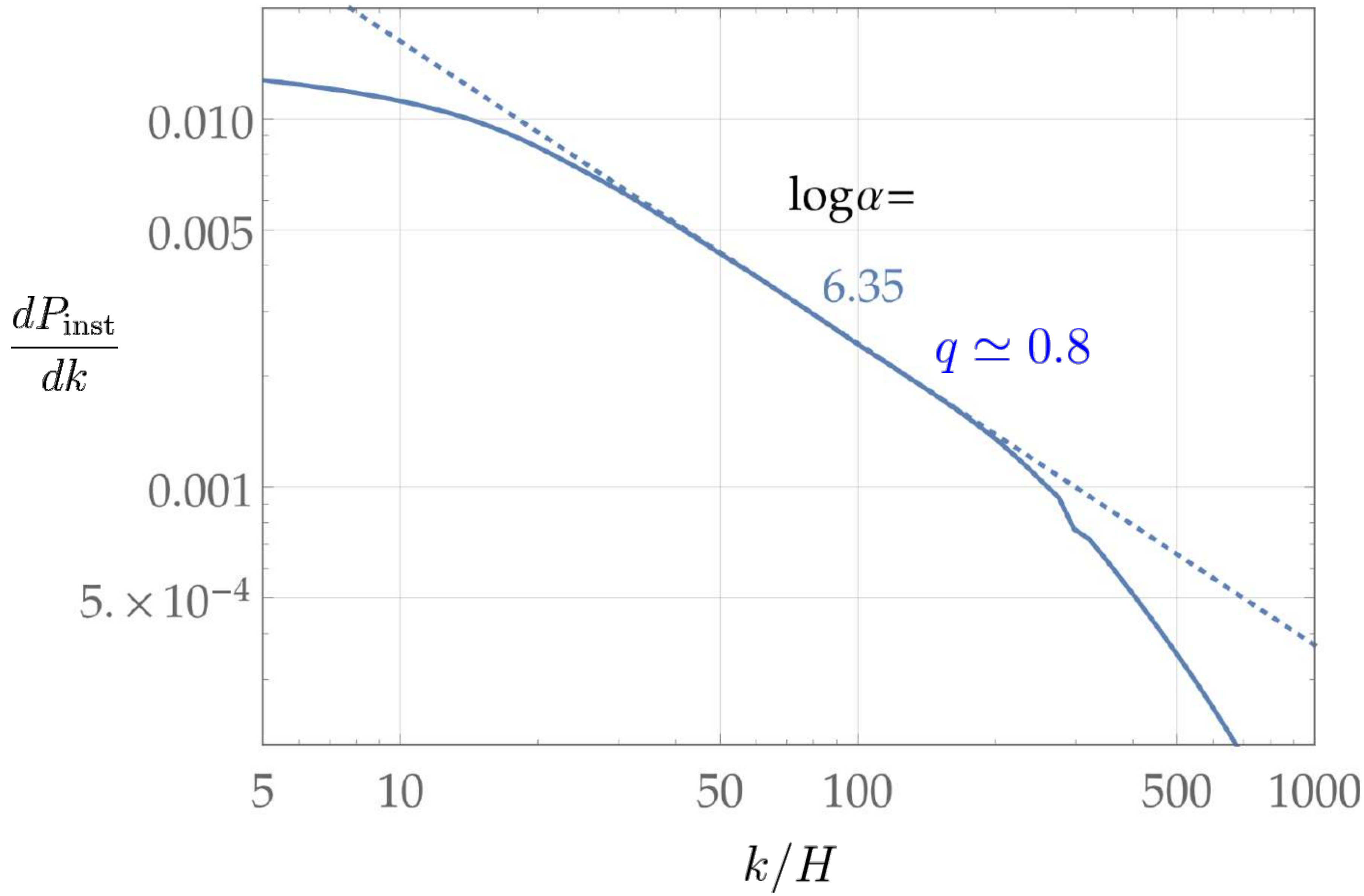


Instantaneous emission spectrum

The physically relevant thing to extrapolate



UV dominated!

Instantaneous emission spectrum

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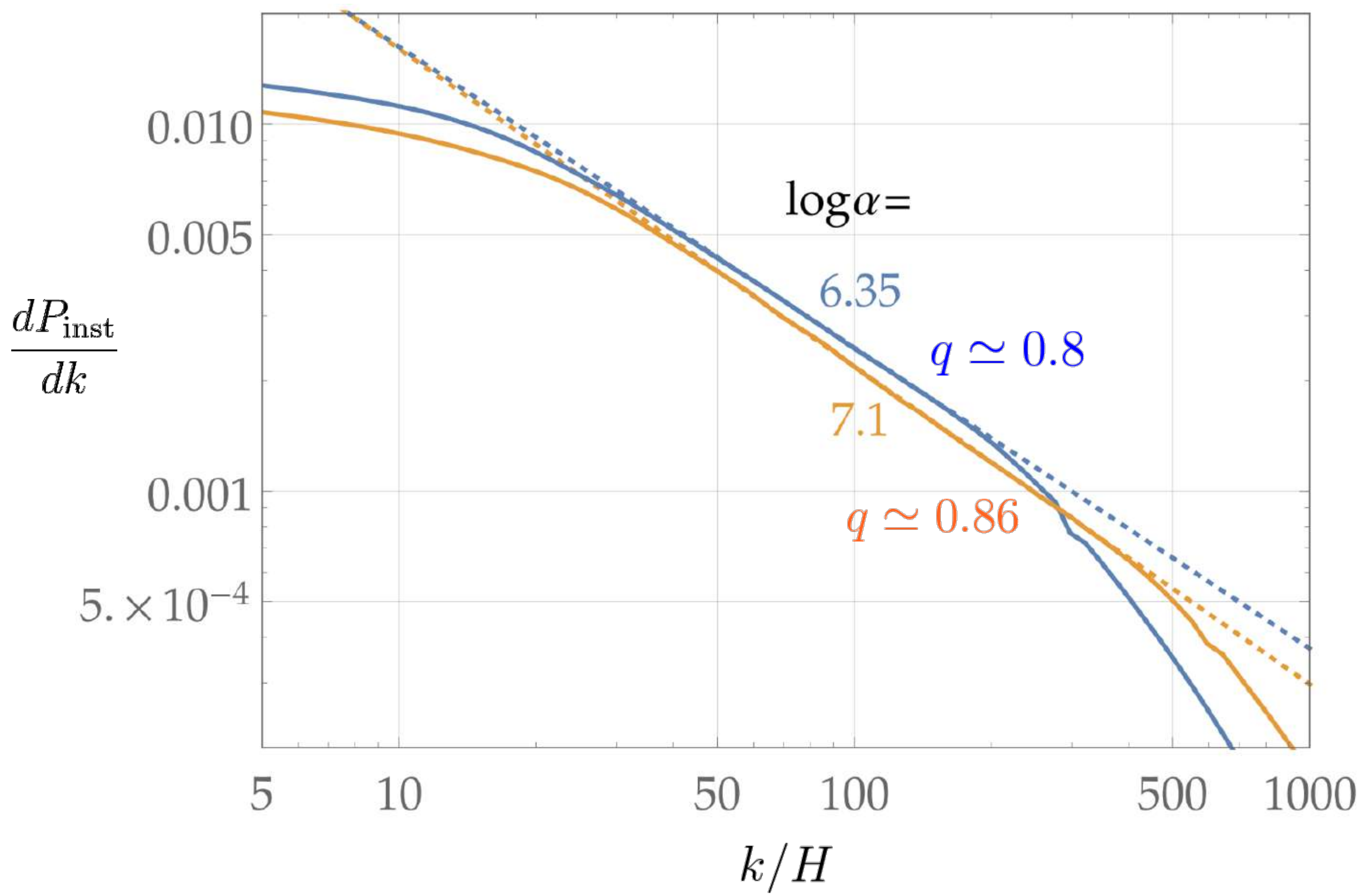
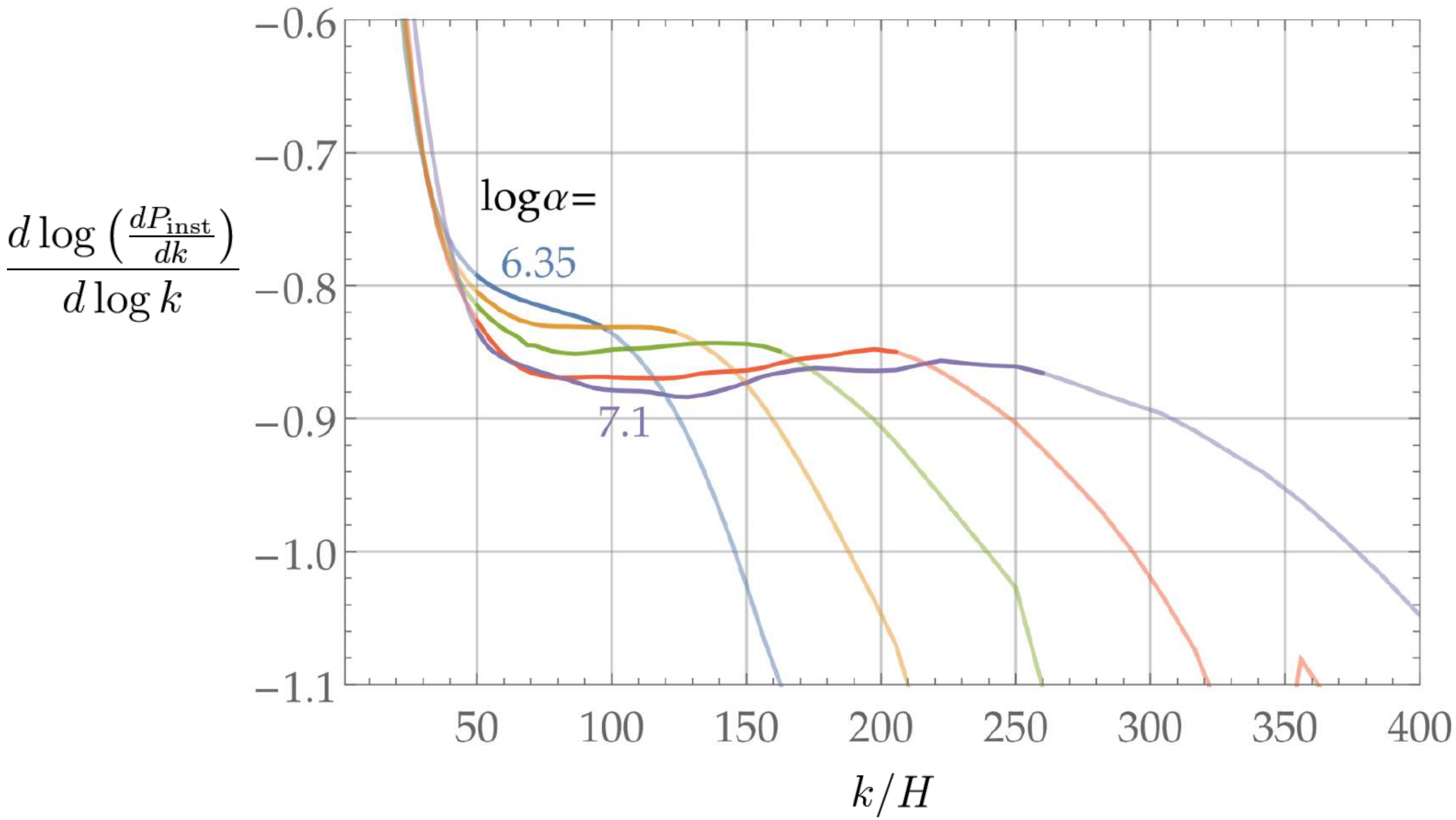


UV dominated!

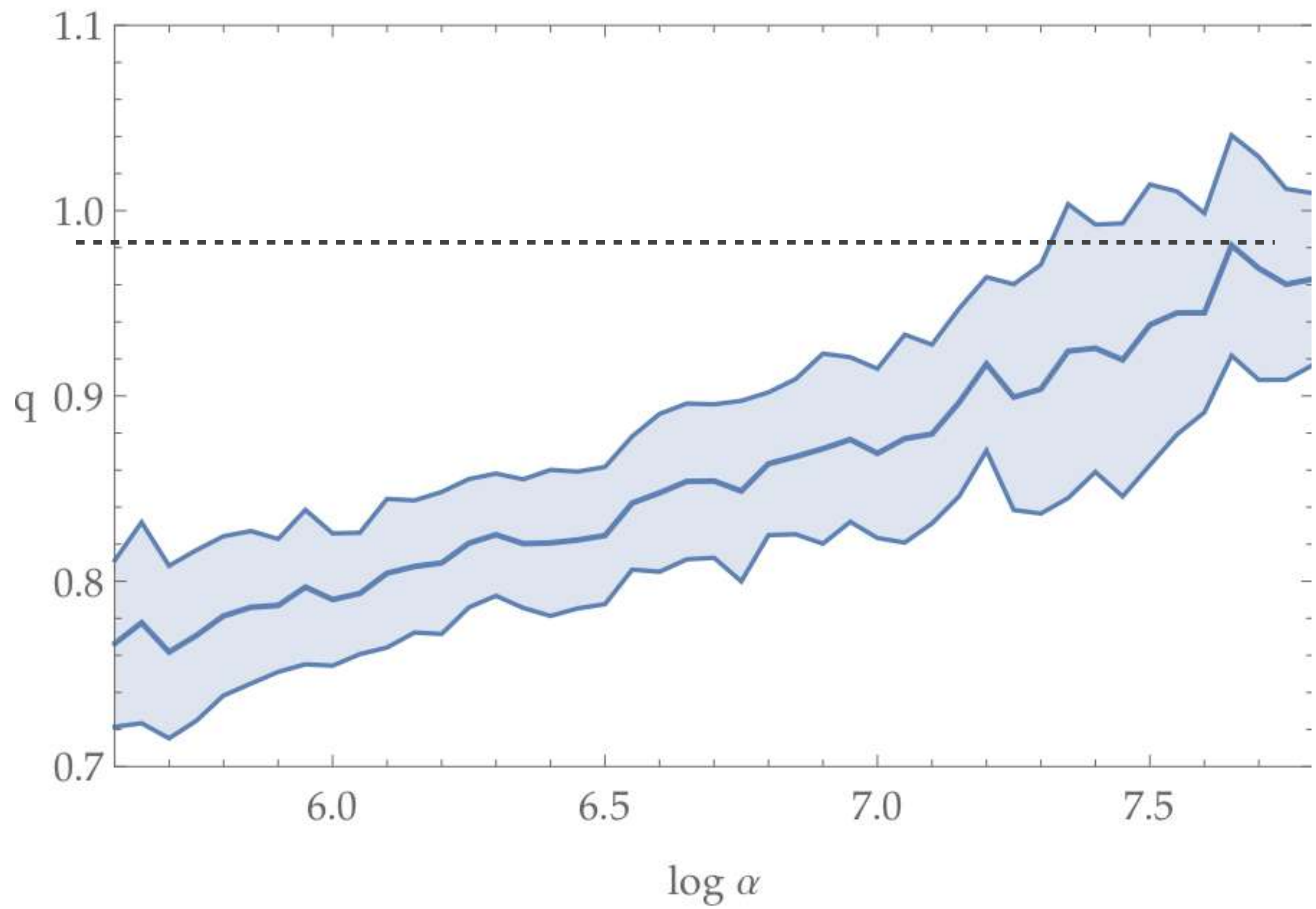
Fitting the power law



Slope of the instantaneous spectrum

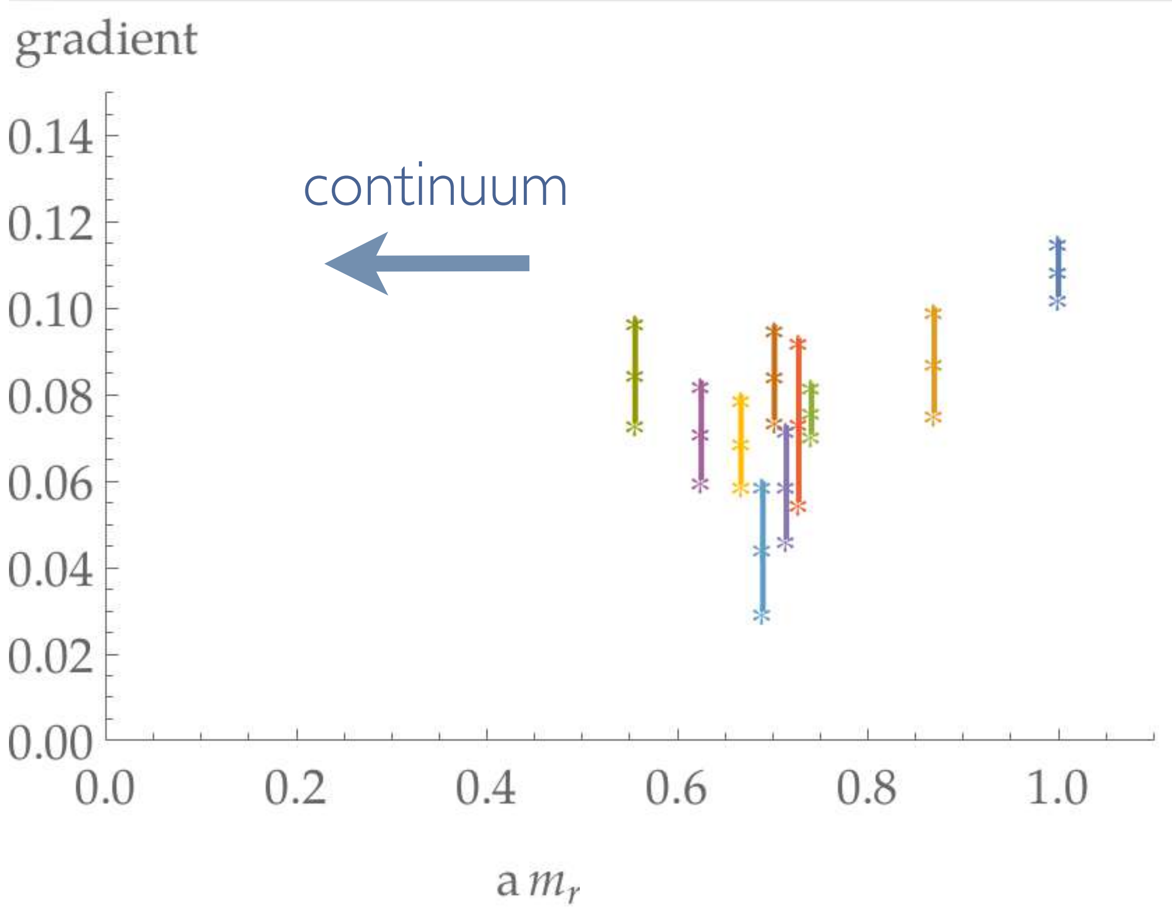
Fitting the power law

Best fit over the constant slope region:

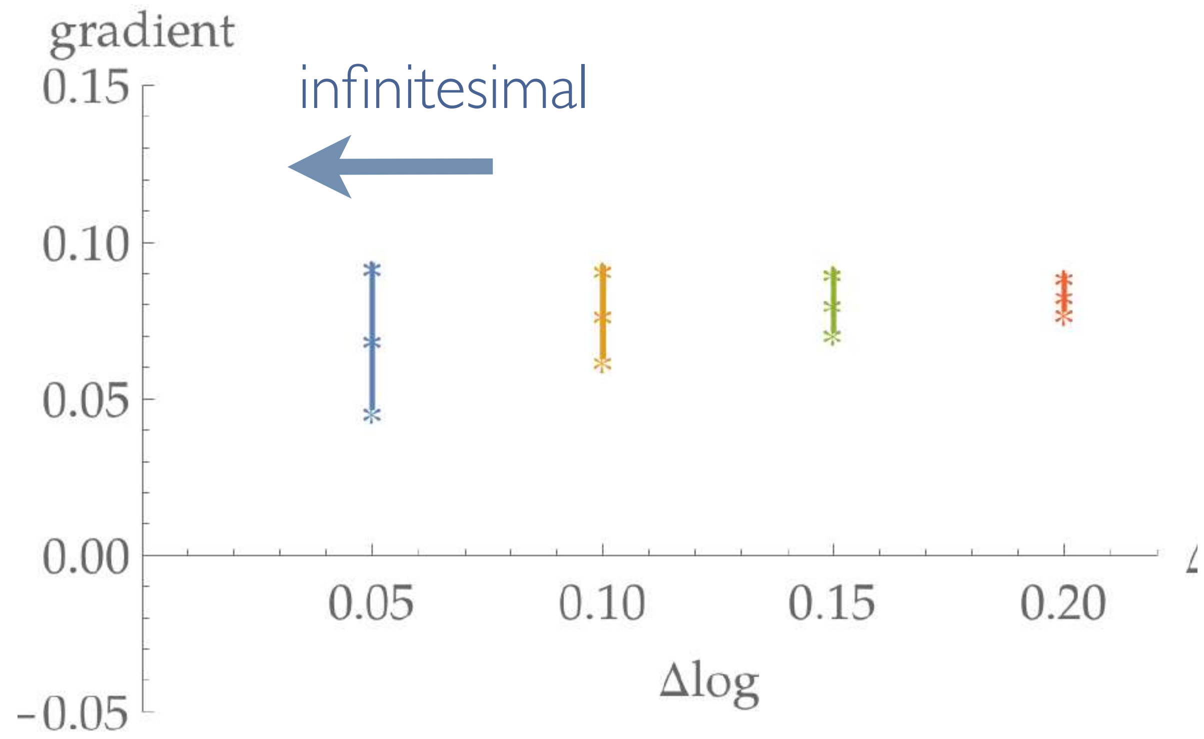


Also seems to have a log dependence

Systematics



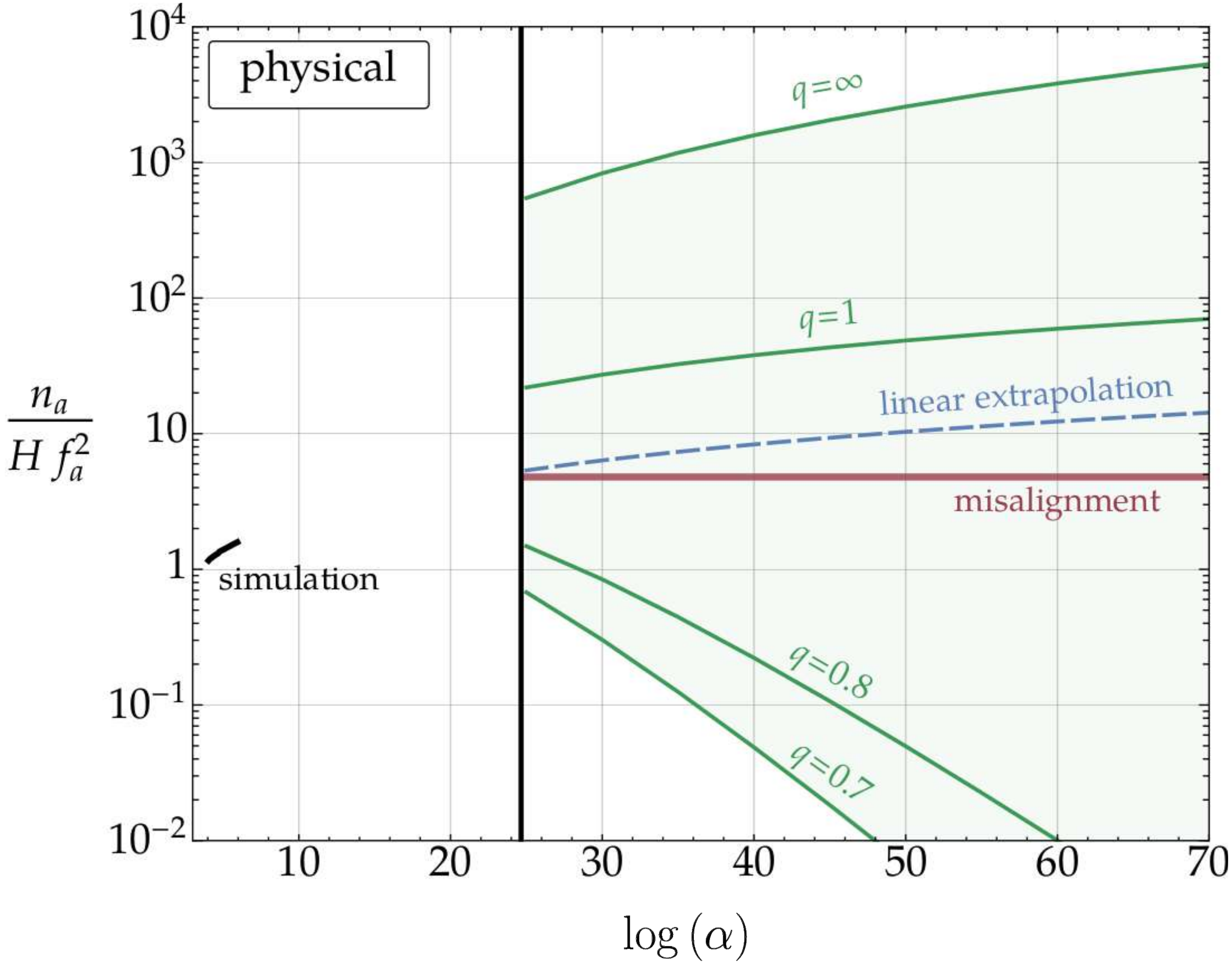
Lattice spacing



Time gap for evaluating F

Axion number density

Extrapolate all the way to large logs



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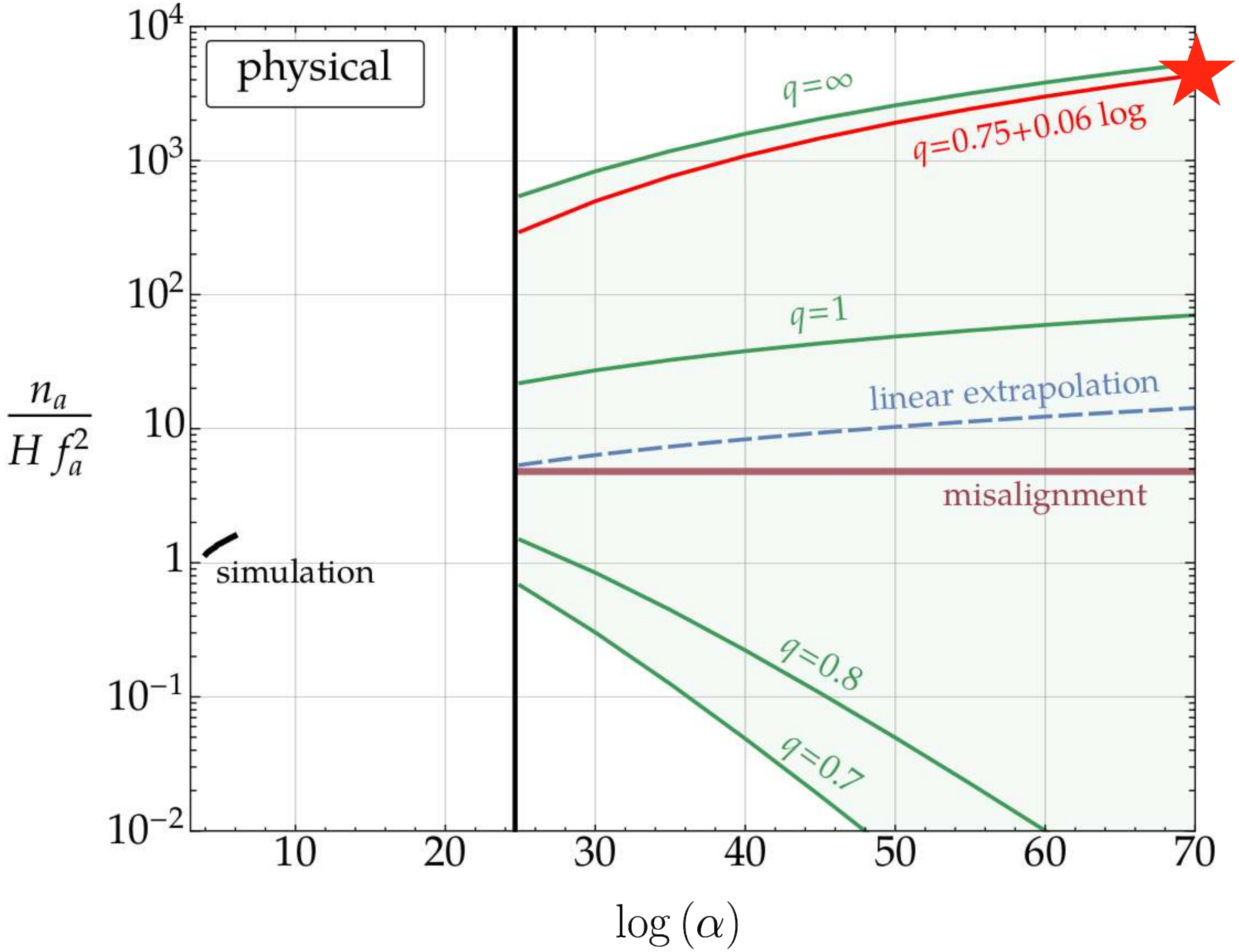
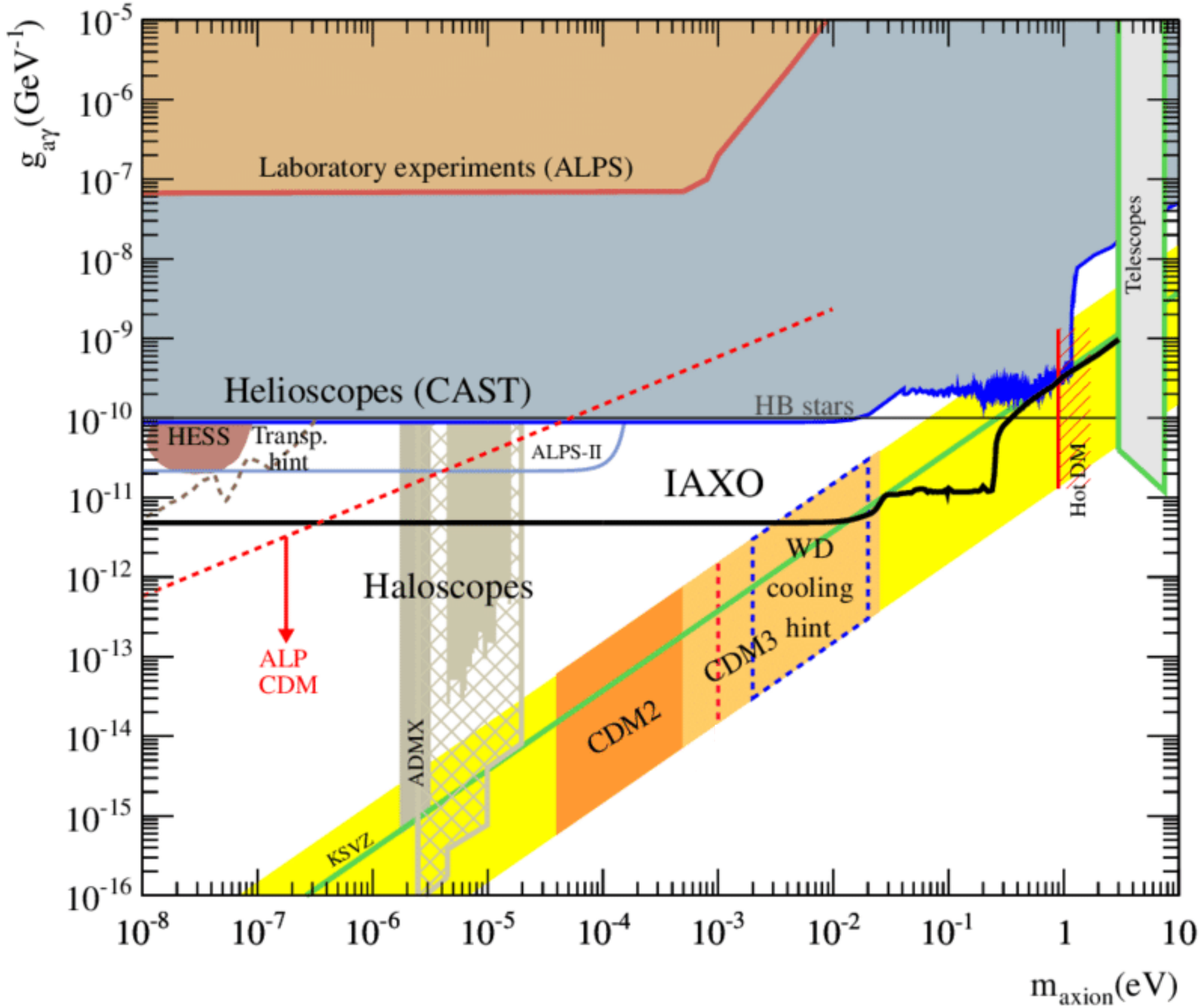


Possible impact on the relic abundance?



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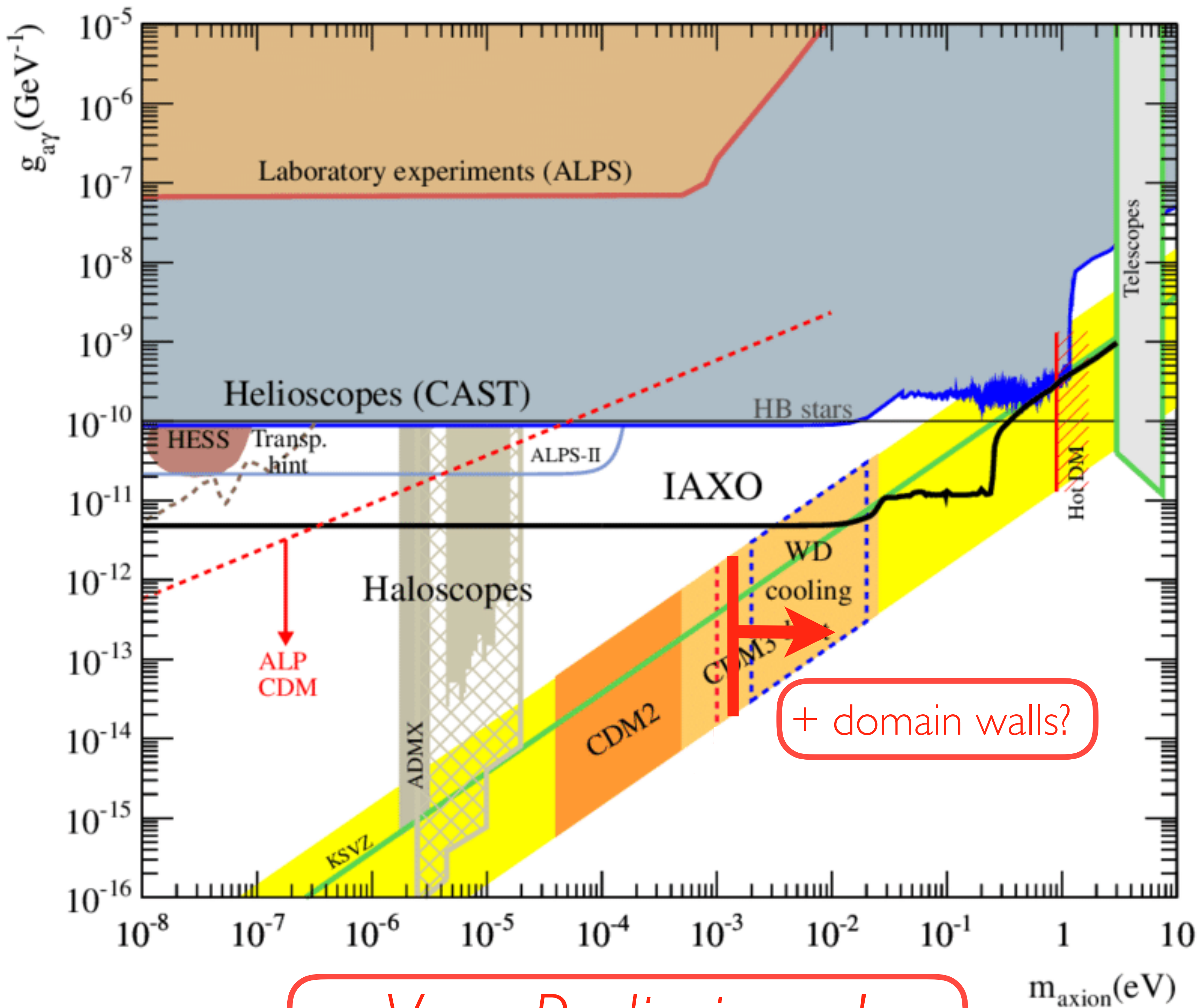

Very Preliminary!

Future Improvements?

- Bigger computers, running for longer, lead to relatively little gain
- Effective field theory approach is tempting: carry out a simulation where the degrees of freedom are evolving strings
- Might be possible to parameterise the probability of passing through, rate that curves straighten out etc. but not straightforward

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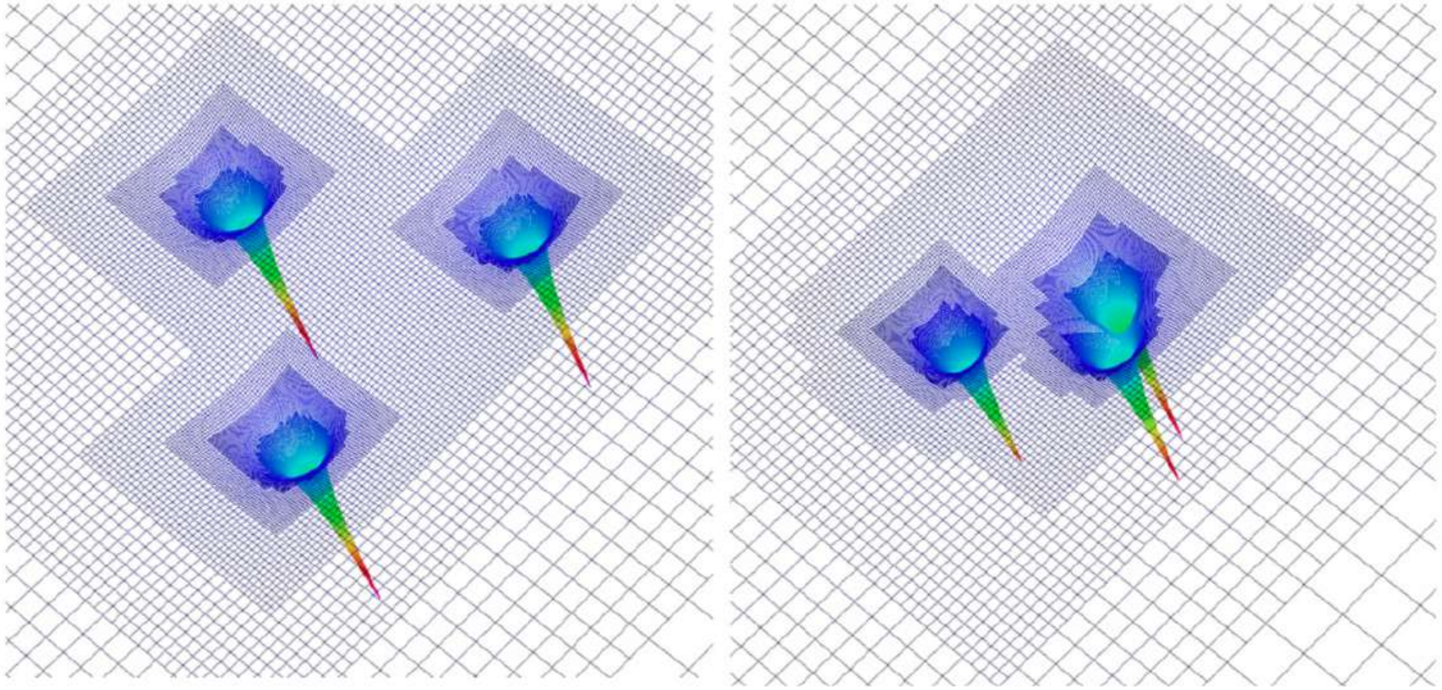
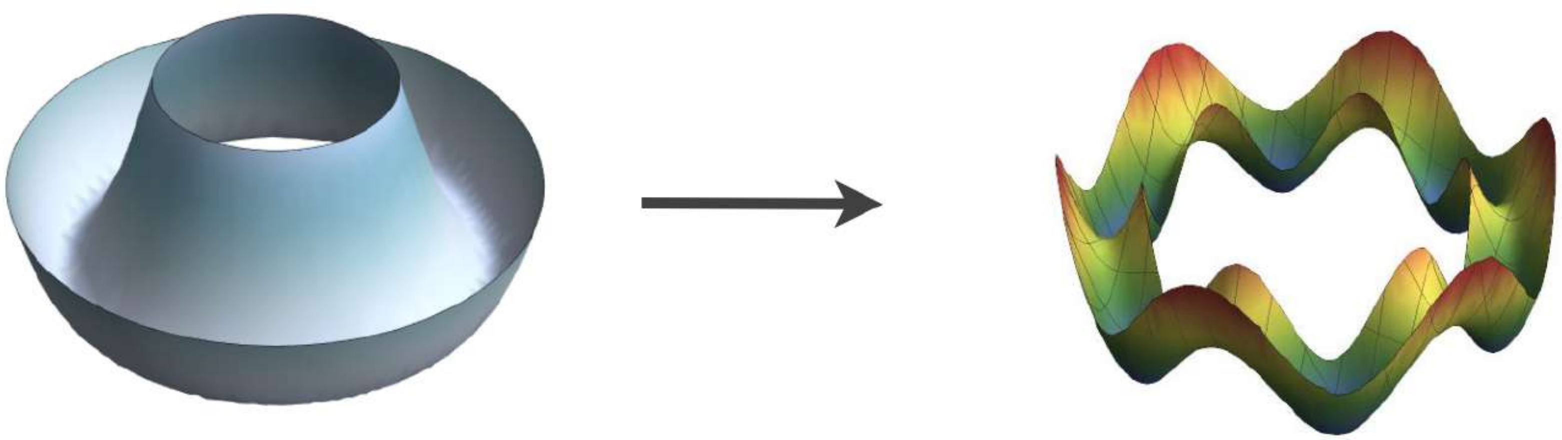
- Might be possible to parameterise the probability of passing through, rate that curves straighten out etc. but not straightforward
- Adaptive mesh, win a factor of 10?



Domain walls

To get a final result, also need to study the dynamics of domain walls



Depends on the anomaly coefficient:

- $N = 1$: unstable, automatically decay
- $N > 1$: stable in the absence of extra PQ breaking, current simulations seems marginally ruled out unless fine-tuned

Domain walls

Axion mass becomes cosmologically relevant when

$$m_a(T_0) \simeq H(T_0)$$

Subsequently it increases fast, and quickly $m_a(T) \gg H(T_0)$

But typical size of domain walls still $\sim 1/H(T_0)$, momentum of lowest harmonics $\sim H(T_0)$
emission at higher harmonics strongly suppressed

Could this delay the destruction of the domain wall network? Potentially a big effect on the relic abundance?

Conclusions

- QCD axion particularly well motivated
- PQ symmetry breaks after inflation in large classes of models
- In principle leads unique prediction for the axion dark matter mass
- Simulations are far from the physically relevant regime
- Essential to extrapolate, and to be aware of the uncertainties

Thanks

Fat string trick

Increase the string core size with time $V(t) = \lambda(t)\left(|\phi|^2 - f_a^2\right)^2$

$$\lambda(t) = \frac{\lambda_0}{a(t)^2} = \frac{\lambda_0}{t} \quad \Rightarrow \quad \delta_s \sim \frac{1}{\sqrt{\lambda} f_a} \sim t^{1/2}$$

Same maximum value of log in the two cases, but fat string trick means going from

$\alpha \sim 10$ to $\alpha \sim 1000$ takes $t/t_0 \sim 10^4$ instead of $t/t_0 \sim 10^2$

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- Can see convergence to a scaling solution more clearly
- Redshifting means that initial energy has less impact on the spectrum, more time to calculate the energy emitted between shots
- Larger separation between $k \sim H$ and $k \sim 1/\delta_s$ at early times

Look at results with and without using this trick