

Direct Deflection of Particle Dark Matter

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FONDS NATIONAL SUISSE SCHWEIZERISCHER NATIONALFONDS FONDO NAZIONALE SVIZZERO SWISS NATIONAL SCIENCE FOUNDATION Based on 1908.06982 with: A. Berlin, R. T. D'Agnolo, P. Schuster, N. Toro





Arcetri, September 3, 2019

Outline

- The dark matter landscape
- Contrasting experimental techniques
 - $m_{DM} > keV$
 - m_{DM} < keV
- Bridging the gap presenting a general approach
- · A specific model: (pseudo-)millicharge
- Implications of (pseudo-)millicharge
 - New observables
- · An experimental proposal
- Outlook

The Landscape

Corn Hill, E. Hopper (1930)

Arcetri, September 3, 2019

DARK MATTER CANDIDATES: MeV meV eV KeV MeV GeV TeV 10 ⁻¹⁸ kg ng mg g kg ton 10 ⁶ kg	, 10 ¹² kg 10 ¹⁸ kg 10 ²⁴ kg 10 ³⁰⁰ kg
AXIONS STERILE NEUTRINOS ELECTRONS PAINTED WITH SPACE CAMOUFLAGE AXIONS NEUTRINOS ELECTRONS PAINTED WITH SPACE CAMOUFLAGE BEES NO-SEE-UMS 8-BALLS NONOLITHS PYRAMIDS	GAMMA NEUTRON SOLAR SYSTEM RAYS STAR DATA STABILITY MAYBE THOSE ORBIT LINES IN SPACE DIAGRAMS ARE REAL AND VERY HEAVY
	Munroe (xkcd/2035)

4









The experimental landscape



Experimental techniques > keV: particle effects



Experimental techniques > keV: particle effects



Experimental techniques > keV: particle effects



Experimental techniques « keV: collective effects



Experimental techniques « keV: collective effects



• Coupling to EM:

E.g. ADMX, DM Radio, ABRACADABRA

Sikivie (1983, 1984) Chaudhuri, Graham, Irwin, Mardon, Rajendran, Zhao (2015) Kahn, Safdi, Thaler (2016) Chaudhuri, Graham, Irwin, Mardon (2019)



Bridging the gap



Sub-keV particle DM — low recoil Assume interacts via long-range mediator



Interaction-dependent



Deflector

Coherent effect induced



Deflector

Coherent effect induced



Deflector

Effect magnitude set by deflector, not DM

Coherent flow into detector







Makes use of large number density at low DM mass

Advantage: requires sensitivity to small energy transfer

Health warning: requires low-mass mediator

A Concrete Example: (Effectively) Millicharged DM



Millicharges & pseudo-millicharges



Violin & Candlestick, Georges Braque (1910), SFMOMA

Dark Matter coupled to a Dark Photon

Kinetically mixed Dark Photon

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{\epsilon}{2} F_{\mu\nu} F'_{\mu\nu} + \frac{m_{A'}^2}{2} A'_{\mu} A'^{\mu} + e A_{\mu} J^{\mu}_{\rm EM} + e_D A'_{\mu} J^{\mu}_{\rm D}$$

• Dark Current:

$$J_{\rm D}^{\mu} = \bar{\chi} \gamma^{\mu} \chi, \quad \left(\varphi^{\dagger} \partial^{\mu} \varphi - (\partial^{\mu} \varphi)^{\dagger} \varphi\right)$$

Fermion

Scalar

Freeze-In Sub-MeV

McDonald (2001) Hall, Jedamzik, March-Russell, West (2009)

- DM never in thermal equilibrium
- Slow leakage into dark sector

- At low masses, frozen in by electron annihilation

 (and plasmon decay see 1902.08623 Dvorkin, Lin & Schutz)



Self-Interactions via a Light Mediator

• Ellipticity Constraints:

- Substructure mergers:
- Bullet cluster:





Wittman, Golovich, Dawson (2017)



Carnegie-Irvine Galaxy Survey

Randall, Markevitch, Clowe, Gonzalez, Bradac (2007)

$$rac{\sigma}{m_{\chi}} \lesssim rac{0.7 \ \mathrm{cm}^2}{\mathrm{g}}$$



Chandra X-Ray Observatory

Feng, Kaplinghat, Tu, Yu (2009) Agrawal, Cyr-Racine, Randall, Scholtz (2016)

 $\alpha_D \lesssim 10^{-10} \left(\frac{m_{\chi}}{\text{MeV}}\right)^{3/2}$

Self-Interactions via a Light Mediator

• Ellipticity Constraints:

Substructure mergers:

• Bullet cluster:

Review

Tulin & Yu (2017)

 $\frac{\sigma}{m_{\chi}} \lesssim \frac{1~{\rm cm}^2}{{\rm g}}$ Harvey et al (2015) Wittman, Golovich, Dawson (2017)

Peter, Rocha, Bullock, Kaplinghat (2012)

$$\frac{\sigma}{m_{\chi}} \lesssim \frac{2 \text{ cm}^2}{\text{g}}$$

Randall, Markevitch, Clowe, Gonzalez, Bradac (2007)

 $\frac{\sigma}{m_{\chi}} \lesssim \frac{0.7 \text{ cm}^2}{\text{g}}$



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 $\alpha_D \lesssim 10^{-10} \left(\frac{m_{\chi}}{\mathrm{MeV}}\right)^{3/2}$











Quo pseudo-millicharge:

Kinetically mixed A'

Self-interaction constraints

Requirements for Freeze-In

Quo pseudo-millicharge:



Quo pseudo-millicharge:



DM effectively MQ on experimental scales!
$$A_{\mu} \to A_{\mu} + \epsilon A'_{\mu} \& A'_{\mu} \to \frac{A'_{\mu}}{\sqrt{1 - \epsilon^2}} \text{ rotation:}$$
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{m_{A'}^2}{2(1 - \epsilon^2)} A'_{\mu} A'^{\mu} + \frac{e(A_{\mu} + \epsilon A'_{\mu}) J^{\mu}_{\text{EM}}}{\sqrt{1 - \epsilon^2}} + \frac{e_D}{\sqrt{1 - \epsilon^2}} A'_{\mu} J^{\mu}_{\text{D}}$$

$$A_{\mu} \to A_{\mu} + \epsilon A'_{\mu} \& A'_{\mu} \to \frac{A'_{\mu}}{\sqrt{1 - \epsilon^2}} \text{ rotation:}$$
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{m_{A'}^2}{2(1 - \epsilon^2)} A'_{\mu} A'^{\mu} + \frac{e(A_{\mu} + \epsilon A'_{\mu}) J^{\mu}_{\text{EM}}}{\sqrt{1 - \epsilon^2}} + \frac{e_D}{\sqrt{1 - \epsilon^2}} A'_{\mu} J^{\mu}_{\text{D}}$$

When SM charges set up a visible EM field, also set up a macroscopic hidden field

$$A_{\mu} \to A_{\mu} + \epsilon A'_{\mu} \& A'_{\mu} \to \frac{A'_{\mu}}{\sqrt{1 - \epsilon^2}} \text{ rotation:}$$
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{m_{A'}^2}{2(1 - \epsilon^2)} A'_{\mu} A'^{\mu} + \frac{e(A_{\mu} + \epsilon A'_{\mu}) J^{\mu}_{\text{EM}}}{\sqrt{1 - \epsilon^2}} A'_{\mu} J^{\mu}_{\text{D}}}$$

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When SM charges set up a visible EM field, also set up a macroscopic hidden field

c.f. true milliQ:
$$\mathcal{L} \supset eA_{\mu} \left(J_{\rm EM}^{\mu} + q_{\rm eff} J_{\rm D}^{\mu} \right)$$
 $q_{\rm eff} = \frac{\epsilon e_D}{e}$

New Observables

Bend/Trap Dark Matter
$$r_g = \frac{m_\chi v_\chi}{qB} \longrightarrow \frac{m_\chi v_\chi e}{\epsilon e_D B}$$

 $r_g \sim \text{meter} \times \left(\frac{m_\chi}{\text{keV}}\right)^{3/2} \left(\frac{10 \text{ T}}{B}\right)$

Accelerate/Stop Dark Matter

$$m_{\chi} v_{\chi}^2 \sim q_{\rm eff} e \,\Delta V$$

$$\Delta V \sim \mathrm{MV} \times \left(\frac{m_{\chi}}{\mathrm{keV}}\right)^{3/2}$$

An Experimental Concept

Deflecting and Detecting Millicharged* Dark Matter



Inducing Dark Matter Waves



Charge Density Calculation

Debye Screening of a potential in a thermal plasma: $T \equiv (m_{\chi}/3) \langle v^2 \rangle \simeq (m_{\chi}/2) v_0^2$



But, potential is shielded — need exact computation of this effect

Charge Density Calculation w/ Shield

Charge density as sum of charges:

$$\rho_{\chi}(\mathbf{x},t) = eq_{\text{eff}} \sum_{j=0}^{1} (-1)^{j} \int d^{3}\mathbf{v} \ f_{j}(\mathbf{x},\mathbf{v},t)$$
$$= \frac{1}{2} eq_{\text{eff}} \ n_{\chi} \sum_{j=0}^{1} (-1)^{j} \int d^{3}\mathbf{x}_{i} \ d^{3}\mathbf{v}_{i} \ f(\mathbf{v}_{i}) \ \delta^{(3)}(\mathbf{x} - \mathbf{x}_{\text{def}}(t;\mathbf{x}_{i},\mathbf{v}_{i}))$$

Treat effect of deflector as small perturbation:

 $\mathbf{x}_{def} \equiv \mathbf{x}_{free} + \Delta \mathbf{x}_{def} , \ \mathbf{v}_{def} \equiv \mathbf{v}_{free} + \Delta \mathbf{v}_{def} \qquad \mathbf{x}_{free}(t) \equiv \mathbf{x}_i + \mathbf{v}(t - t_0) , \ \mathbf{v}_{free}(t) \equiv \mathbf{v}_i$

$$\Delta \mathbf{x}_{def}(t) \simeq (-1)^j \; \frac{eq_{eff}}{m_{\chi}} \; \iint_{t_0 < t' < t'' < t} \; dt' \, dt'' \; \boldsymbol{E}_{def}(\mathbf{x}_{free}(t')) \; e^{i\omega t'}$$

EM force, neglecting v_x-suppressed B-field effect

Charge Density Calculation w/ Shield

Resultant charge density:

$$\rho_{\chi}(\mathbf{x},t) \simeq -\frac{(eq_{\text{eff}})^2 \rho_{\text{DM}}}{m_{\chi}^2} \ e^{i\omega t} \ \int dv \ d^3 \mathbf{x}' \ f(v \, \hat{\mathbf{v}}) \ \frac{\rho_{\text{def}}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \ e^{-i\omega|\mathbf{x} - \mathbf{x}'|/v} \qquad \hat{\mathbf{v}} = \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|}$$

Expand in multipole moments — first non-zero is charge radius

$$\rho_{\chi}(\mathbf{x},t) \simeq -\frac{(eq_{\text{eff}})^2 \rho_{\text{DM}}}{m_{\chi}^2} e^{i\omega t} \left(\rho_{\chi}^{(1)} + \rho_{\chi}^{(2)} + \rho_{\chi}^{(3)} + \cdots \right)$$

$$\rho_{\chi}(\mathbf{x}) \simeq -\frac{(eq_{\text{eff}})^2 \rho_{\text{\tiny DM}} \mathcal{R}_{\text{def}}^2}{6m_{\chi}^2} \int dv \ \nabla^2 \frac{f(v \, \hat{\mathbf{x}})}{|\mathbf{x}|}$$

Comparison w/ Debye estimate

$$\rho_{\chi}^{\text{Debye}}(\mathbf{x}) \sim -\frac{(eq_{\text{eff}})^2 \rho_{\text{DM}}}{m_{\chi}^2 v_0^2} \frac{eq_{\text{def}}(\mathbf{x})}{|\mathbf{x}|}$$

Suppression due to charge radius: further x² suppressed:

$$\rho_{\chi}(\mathbf{x}) \propto \rho_{\chi}^{\text{Debye}}(R) \left(\frac{R}{|\mathbf{x}|}\right)^{3}$$

Effect vanishes in limit where $v_{wind} \rightarrow 0$

$$\rho_{\chi}(\mathbf{x}) \sim \rho_{\chi}^{\text{Debye}}(R) \left(\frac{v_{\text{wind}}}{v_0}\right)^2 \left(\frac{R}{|\mathbf{x}|}\right)^3$$



Current density

Calculation proceeds in same manner as for charge density

$$j_{\chi}(\mathbf{x}) \sim \rho_{\chi}(\mathbf{x}) v_{\text{wind}}$$

Compare with Debye estimate:

$$j_{\chi}^{\text{Debye}} \equiv \rho_{\chi}^{\text{Debye}} v_{\text{wind}}$$

Current density velocity-suppressed

B-field signal therefore suppressed w.r.t. **E**-field signal



Effect of the Dark Matter Wind



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Effect of the Dark Matter Wind

 $\xi \equiv \left(\frac{v_{\rm wind}}{v_0}\right)$ Recall that charge and current density zero without wind Far-field limit $\rho_{\chi}(\mathbf{x},t) \simeq \frac{2}{9} e^{i\omega t} \rho_{\chi}^{\text{Debye}} \left(\frac{R}{|\mathbf{x}|}\right)^{3} \xi e^{-\xi^{2}} \left[2\pi^{-1/2}c_{w}(1-s_{w}^{2}\xi^{2}) + e^{c_{w}^{2}\xi^{2}}\xi \left(2c_{w}^{2}(1-s_{w}^{2}\xi^{2}) - s_{w}^{2}\right) \operatorname{erfc}(-c_{w}\xi)\right]$ 0.0001 $= 1.5 \times 220$ km/s -0.0001 $v_0 = 220 \text{ km/s}$ 0.0003 -0.0003 $= 0.5 \times 220$ km/s 0.0007 -0.0007 $\rho_{\chi}(t) \, / \, \rho_{\chi}(0)$ deflector y/Rwind 0.015 0.006 0.002 0 $v_0 = 1.5 \times 220 \text{ km/s}$ 0.5 0 -22 -40 4 x/R $t \, [day]$

51



Effect of the Dark Matter Wind



Detecting Dark Matter Waves



Oscillation of deflector induces oscillation of charge and current densities in detector:

$$\rho_{\chi}(t) \simeq \rho_{\chi} e^{i\omega t} , \ \boldsymbol{j}_{\chi}(t) \simeq \boldsymbol{j}_{\chi} e^{i\omega t}$$

Recall requirement

 $\omega \lesssim \pi v_{\chi}/R \sim \mathrm{MHz} \times (R/\mathrm{meter})^{-1}$

Solution: Lumped LC Resonator

 $\omega_{\rm LC} = \frac{1}{\sqrt{LC}}$

Ring up signal over Q cycles

Detecting Dark Matter Waves

Since E-field signal dominant, capacitative pickup optimal



$$U_{\rm s} = \int_V \frac{1}{2} \epsilon \mathbf{E}^2$$

Effective volume of capacitor/antenna bounded by shielded volume

DM Radio being built for B-field signal — large effective inductor volume



 $U_{\rm s} = \frac{1}{2}LI^2 = \int_V \frac{1}{2} \frac{\mathbf{B}^2}{\mu}$ Effective volume of inductor — many coils

Signal to Noise

$$\mathrm{SNR} \simeq \frac{\omega \, Q \, t_{\mathrm{int}}}{4 \, T_{\mathrm{LC}}} \, \int_{\mathrm{det}} d^3 \mathbf{x} \, \left(E_{\chi}^2 \text{ or } B_{\chi}^2 \right) \propto \left(\frac{q_{\mathrm{eff}}}{m_{\chi}} \right)^4$$

Unpack this expression

Thermal (Johnson-Nyquist) noise limited power spectral density

$$\langle V_{\rm LC} \rangle^2 \simeq \frac{1}{C_{\rm LC}} \int_{\rm det} d^3 \mathbf{x} \ E_{\chi}^2$$

 $4R_{\rm LC}T_{\rm LC}$

Signal voltage power spectral density (E-field)

$$\mathrm{SNR} = \frac{\langle V_{\mathrm{LC}} \rangle^2}{4R_{\mathrm{LC}}T_{\mathrm{LC}}}$$

SNR is ratio of PSDs

 $Q_{\rm LC} \equiv \frac{1}{\omega C_{\rm LC} R_{\rm LC}}$

Signal to Noise

$$\mathrm{SNR} \simeq \frac{\omega \, Q \, t_{\mathrm{int}}}{4 \, T_{\mathrm{LC}}} \, \int_{\mathrm{det}} d^3 \mathbf{x} \, \left(E_{\chi}^2 \text{ or } B_{\chi}^2 \right) \propto \left(\frac{q_{\mathrm{eff}}}{m_{\chi}} \right)^4$$

Unpack this expression

 $\begin{array}{|c|c|c|c|c|} \hline \propto Q & \mbox{resonant detector allows ring-up of signal over Q cycles} \\ & \mbox{e.g. AURIGA searching for Grav. Waves — achieved Q~10^6} \\ \hline \mbox{DM Radio planning on Q$$>$10^6} \\ \hline \mbox{requires coherence time > integration time} \\ & \mbox{achieved by phase-locking deflector to e.g. NIST atomic clock} \\ & \mbox{phase can drift small amounts: $P_{\rm s} \propto (1 - \mathcal{O}(\delta \phi^2))$ \\ \hline \end{array}$









Outlook

- For DM masses < MeV Non-Thermal History
- Such models may be accompanied by an ultralight mediator
- Active Direct Detection through two-step process:



Outlook

- An example model: (effectively) millicharged DM
- Detectable freeze-in for KeV MeV requires ultralight A' $m_{A'} \lesssim 10^{-9} \ {\rm eV}$
- Range of force: $1/m_{A'} \gtrsim 10^3 \text{ m}$
- Borrow intuition from lower mass DM searches instead of extending WIMP-style searches
- Possible new signals at LSW experiments, Direct Detection

BACKUP

Poisson Fluctuations

Poisson variation of local DM number density

Leads to time-variation of amplitude of driving term:

 $\rho_{\chi}(t) \simeq \rho_{\chi} e^{i\omega t}$

$$\boldsymbol{j}_{\chi}(t) \simeq \boldsymbol{j}_{\chi} e^{i\omega t}$$



For $m_\chi \lesssim 1~{
m GeV},~\delta n_\chi \lesssim 10^{-3} n_\chi$

High-frequency fluctuations — effect averages to zero

Low-frequency fluctuations — negligible modulation of amplitude

Thermal Noise

$$\mathcal{R} \qquad dP = \langle E \rangle = k_B T d\omega = \frac{\bar{V}^2}{4R}$$

$$\bar{V}^2 = \lim_{t_{\rm int}\to\infty} \frac{1}{t_{\rm int}} \int_0^{t_{\rm int}} V(t) V^*(t) dt \qquad \tilde{\bar{V}}(\omega) = \frac{1}{\sqrt{t_{\rm int}}} \int_0^{t_{\rm int}} \bar{V}(t) e^{-i\omega t} dt$$

$$\int |\tilde{\bar{V}}(\omega)|^2 d\omega = \bar{V}^2$$

$$|\tilde{\bar{V}}(\omega)|^2 = 4Rk_BT$$

Noise Power Spectral Density

Phase knowledge

Measured:

$$B(t) = B_0 \sin (\omega_{em} t + \varphi(t)) + B_n(t)$$

$$\tilde{B}(\omega) = \frac{1}{\sqrt{t_{int}}} \int_0^{t_{int}} B(t) \sin(\omega t) dt = \tilde{B}_0(\omega) + \tilde{B}_n(\omega)$$

$$t_{\rm int} < \tau \qquad \qquad |\tilde{B}_0(\omega)|^2 = B_0^2 t_{\rm int}$$

 $t_{\rm int} > \tau$ $|\tilde{B}_0(\omega)|^2 = \sqrt{N_{\rm meas}} B_0^2 \tau$ $N_{\rm meas} = t_{\rm int}/\tau$

Electric Field with Massive Dark Photon

EoM for scalar potential:
$$\left(\partial^2+m_{A'}^2\right)\phi'=
ho'+\epsilon
ho$$

Potential sourced by SM charges:

$$\phi' = \frac{\epsilon e}{4\pi} \frac{e^{-m_{A'}r}}{r}$$

Potential sourced by dark charges:

$$\phi' = \frac{e'}{4\pi} \frac{e^{-m_{A'}r}}{r}$$

massless A' limit equivalent to r << 1/m

Dark Current With Massive Dark Photon



Equations of motion:

$$\left(\nabla^2 - \partial_t^2\right) \mathbf{E} = \nabla \rho_{\rm SM} + \partial_t \mathbf{j}_{\rm SM} ,$$
$$\left(\nabla^2 - \partial_t^2 - m_{A'}^2\right) \mathbf{E'} = \nabla \left(\rho_D + \epsilon \rho_{\rm SM}\right) + \partial_t \left(\mathbf{j}_{\rm D} + \epsilon \mathbf{j}_{\rm SM}\right)$$

Dark Current With Massive Dark Photon



Equations of motion:

$$\left(\nabla^2 - \partial_t^2\right) \mathbf{E} = \nabla \rho_{\rm SM} + \partial_t \mathbf{j}_{\rm SM} ,$$
$$\left(\nabla^2 - \partial_t^2 - m_{A'}^2\right) \mathbf{E}' = \nabla \left(\rho_D + \epsilon \rho_{\rm SM}\right) + \partial_t \left(\mathbf{j}_{\rm D} + \epsilon \mathbf{j}_{\rm SM}\right)$$

Solutions:

$$\mathbf{E}_{\mathbb{r}} = a_{\mathbb{r}} J_0(r\omega_{\text{em}}) + b_{\mathbb{r}} Y_0(r\omega_{\text{em}}) ,$$
$$\mathbf{E}'_{\mathbb{r}} = \frac{i\mathbf{j}_{\mathbf{D}}\omega_{\text{em}}}{k^2} \theta(D-r) + c_{\mathbb{r}} J_0(rk) + d_{\mathbb{r}} Y_0(rk)$$

 $k = \sqrt{\omega_{\rm em}^2 - m_{A'}^2}$

Receiving the Signal

Solutions: $E_{\Gamma} = a_{\Gamma} J_0(r\omega_{em}) + b_{\Gamma} Y_0(r\omega_{em}) ,$ $E'_{\Gamma} = \frac{i \mathbf{j}_{\mathbf{D}} \omega_{em}}{k^2} \theta(D - r) + c_{\Gamma} J_0(rk) + d_{\Gamma} Y_0(rk) \qquad k = \sqrt{\omega_{em}^2 - m_{A'}^2}$ Basis: $E_{\text{vis}} = E + \epsilon E' \qquad E_{\text{inv}} = E' - \epsilon E$

Boundary Conditions:

- Well-behaved at r=0:
$$b_I = d_I = 0$$

Receiving the Signal

Solutions: $E_{\Gamma} = a_{\Gamma} J_0(r\omega_{em}) + b_{\Gamma} Y_0(r\omega_{em}) ,$ $E'_{\Gamma} = \frac{i \mathbf{j}_{D} \omega_{em}}{k^2} \theta(D - r) + c_{\Gamma} J_0(rk) + d_{\Gamma} Y_0(rk) \qquad k = \sqrt{\omega_{em}^2 - m_{A'}^2}$ Basis: $E_{vis} = E + \epsilon E' \qquad E_{inv} = E' - \epsilon E$

Boundary Conditions:

- Well-behaved at r=0: $b_I = d_I = 0$ - Outgoing E_{vis} in region III: $b_{III} = -ia_{III}$ $d_{III} = -ic_{III}$

Receiving the Signal

Solutions: $E_{r} = a_{r}J_{0}(r\omega_{em}) + b_{r}Y_{0}(r\omega_{em}) ,$ $E'_{r} = \frac{i\mathbf{j}_{D}\omega_{em}}{k^{2}}\theta(D-r) + c_{r}J_{0}(rk) + d_{r}Y_{0}(rk) \qquad k = \sqrt{\omega_{em}^{2} - m_{A'}^{2}}$ Basis: $E_{vis} = E + \epsilon E' \qquad E_{inv} = E' - \epsilon E$

Boundary Conditions:

- Well-behaved at r=0:

$$b_I = d_I = 0$$

- Outgoing E_{vis} in region III:

 $b_{III} = -ia_{III} \qquad d_{III} = -ic_{III}$

- Conducting wall at r=R
Solutions: $\mathbf{E}_{\mathbb{r}} = a_{\mathbb{r}} J_0(r\omega_{\text{em}}) + b_{\mathbb{r}} Y_0(r\omega_{\text{em}}) ,$ $\mathbf{E}'_{\mathbb{r}} = \frac{i\mathbf{j}_{\mathbf{D}}\omega_{\text{em}}}{k^2} \theta(D-r) + c_{\mathbb{r}} J_0(rk) + d_{\mathbb{r}} Y_0(rk)$

Basis:

$$E_{\rm vis} = E + \epsilon E'$$
 $E_{\rm inv} = E' - \epsilon E$

Boundary Conditions:

- Well-behaved at r=0:

$$b_I = d_I = 0$$

- Outgoing E_{vis} in region III: b_{j}

 $b_{III} = -ia_{III}$ $d_{III} = -ic_{III}$

- Conducting wall at r=R
- Continuity at r=D

 $k = \sqrt{\omega_{\rm em}^2 - m_{A'}^2}$

Solutions:

$$\mathbf{E}_{\mathbb{F}} = a_{\mathbb{F}} J_0(r\omega_{\rm em}) + b_{\mathbb{F}} Y_0(r\omega_{\rm em}) ,$$

$$\mathbf{E}'_{\mathbb{F}} = \frac{i\mathbf{j}_{\mathbf{D}}\omega_{\rm em}}{k^2} \theta(D-r) + c_{\mathbb{F}} J_0(rk) + d_{\mathbb{F}} Y_0(rk) \qquad \qquad k = \sqrt{\omega_{\rm em}^2 - m_{A'}^2}$$

Basis:

$$E_{\rm vis} = E + \epsilon E'$$
 $E_{\rm inv} = E' - \epsilon E$

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Boundary Conditions:

- Well-behaved at r=0:

$$b_I = d_I = 0$$

- Outgoing E_{vis} in region III:

 $b_{III} = -ia_{III}$ $d_{III} = -ic_{III}$

- Conducting wall at r=R
- Continuity at r=D
- Continuity of derivative at r=D

Solutions:

$$\mathbf{E}_{\mathbb{r}} = a_{\mathbb{r}} J_0(r\omega_{\rm em}) + b_{\mathbb{r}} Y_0(r\omega_{\rm em}) ,$$

$$\mathbf{E}'_{\mathbb{r}} = \frac{i\mathbf{j}_{\mathbf{D}}\omega_{\rm em}}{k^2} \theta(D-r) + c_{\mathbb{r}} J_0(rk) + d_{\mathbb{r}} Y_0(rk) \qquad \qquad k = \sqrt{\omega_{\rm em}^2 - m_{A'}^2}$$

Basis:

$$E_{\rm vis} = E + \epsilon E'$$
 $E_{\rm inv} = E' - \epsilon E$

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Boundary Conditions:

- Well-behaved at r=0:

$$b_I = d_I = 0$$

- Outgoing E_{vis} in region III:
- $b_{III} = -ia_{III}$ $d_{III} = -ic_{III}$

- Conducting wall at r=R
- Continuity at r=D
- Continuity of derivative at r=D
- Continuity of Einv at r=R

Solutions:

$$\mathbf{E}_{\mathbb{r}} = a_{\mathbb{r}} J_0(r\omega_{\rm em}) + b_{\mathbb{r}} Y_0(r\omega_{\rm em}) ,$$

$$\mathbf{E}'_{\mathbb{r}} = \frac{i\mathbf{j}_{\mathbf{D}}\omega_{\rm em}}{k^2} \theta(D-r) + c_{\mathbb{r}} J_0(rk) + d_{\mathbb{r}} Y_0(rk) \qquad \qquad k = \sqrt{\omega_{\rm em}^2 - m_{A'}^2}$$

Basis:

$$E_{\rm vis} = E + \epsilon E'$$
 $E_{\rm inv} = E' - \epsilon E$

Boundary Conditions:

- Well-behaved at r=0:

$$b_I = d_I = 0$$

- Outgoing E_{vis} in region III:

 $b_{III} = -ia_{III}$ $d_{III} = -ic_{III}$

- Conducting wall at r=R
- Continuity at r=D
- Continuity of derivative at r=D
- Continuity of E_{inv} at r=R
- Continuity of derivative of Einv at r=R

Received Signal

E-Field observed in Cavity:

$$\mathbf{E}_{visI} = |\mathbf{j}_{\mathbf{D}}| \frac{\epsilon}{\omega_{em}} \left(i - \frac{iJ_0(r\omega_{em})}{J_0(R\omega_{em})} \right) e^{i\omega_{em}t} \mathbf{\hat{x}}$$

Received Signal

E-Field observed in Cavity:

$$\mathbf{E}_{visI} = |\mathbf{j}_{\mathbf{D}}| \frac{\epsilon}{\omega_{em}} \left(i - \frac{iJ_0(r\omega_{em})}{J_0(R\omega_{em})} \right) e^{i\omega_{em}t} \mathbf{\hat{x}}$$

B-Field observed in Cavity:

$$\mathbf{B}_{\mathrm{vis}\mathbf{I}} = -|\mathbf{j}_{\mathbf{D}}| \frac{\epsilon}{\omega_{\mathrm{em}}} \frac{J_1(r\omega_{\mathrm{em}})}{J_0(R\omega_{\mathrm{em}})} e^{i\omega_{\mathrm{em}}t} \hat{\phi}$$

Additional Signals: Light Shining through Walls

Usual light shining through walls — jiggle SM charges to produce $A + \epsilon A'$:



$$\left(\nabla^2 - \partial_t^2\right) \mathbf{E} = \nabla \rho_{\rm SM} + \partial_t \mathbf{j}_{\rm SM} ,$$
$$\left(\nabla^2 - \partial_t^2 - m_{A'}^2\right) \mathbf{E}' = \nabla \left(\rho_D + \epsilon \rho_{\rm SM}\right) + \partial_t \left(\mathbf{j}_{\rm D} + \epsilon \mathbf{j}_{\rm SM}\right)$$



Usual light shining through walls — jiggle SM charges to produce $A + \epsilon A'$:



$$B_{\rm rec} = 0.39Q\epsilon^2 \frac{m_{A'}^4}{\omega_{\rm em}^4} \frac{L}{d} B_{\rm em}$$

Transverse

Longitudinal dark photon emission wins

In the presence of DM, the story changes



$$\left(\begin{smallmatrix} & - & \\$$

$$\mathcal{O}(e_D^2\epsilon^2m_{A^\prime}^2\omega_{
m em}^{-2})~$$
 L, T

In the presence of DM, the story changes



In the presence of DM, the story changes



In the presence of DM, the story changes





