Axion Thoughts:

i.e. what we/I don’t yet understand about QCD Axion DM

Giovanni Villadoro

The Abdus Salam
International Centre
for Theoretical Physics

Axion Solution to the Strong CP Problem!

$$\det[y_u y_u^\dagger, y_d y_d^\dagger] \quad \Rightarrow \quad \delta \quad \approx 1.2$$

$$\det(y_u y_d) e^{i\theta_0} \quad \Rightarrow \quad \theta \quad \lesssim 10^{-10}$$

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Weinberg, Wilczek ‘78
(KSVZ, DSFZ...)

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi} G\tilde{G}$$

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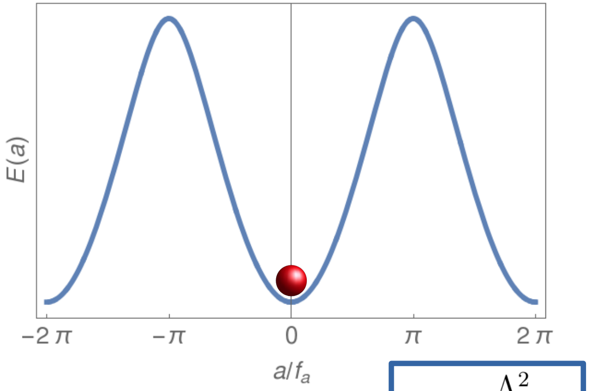

Vafa Witten '84

$$\boxed{m_a \approx \frac{\Lambda_{\mathrm{QCD}}^2}{f_a}}$$

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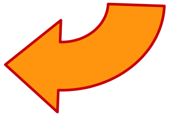
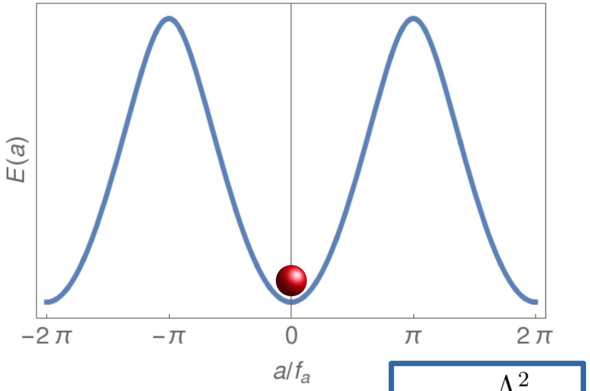


$$10^8 \lesssim \frac{f_a}{\rm GeV} \lesssim 10^{17}$$

$$10^{-10} \lesssim \frac{m_a}{\rm eV} \lesssim 10^{-1}$$



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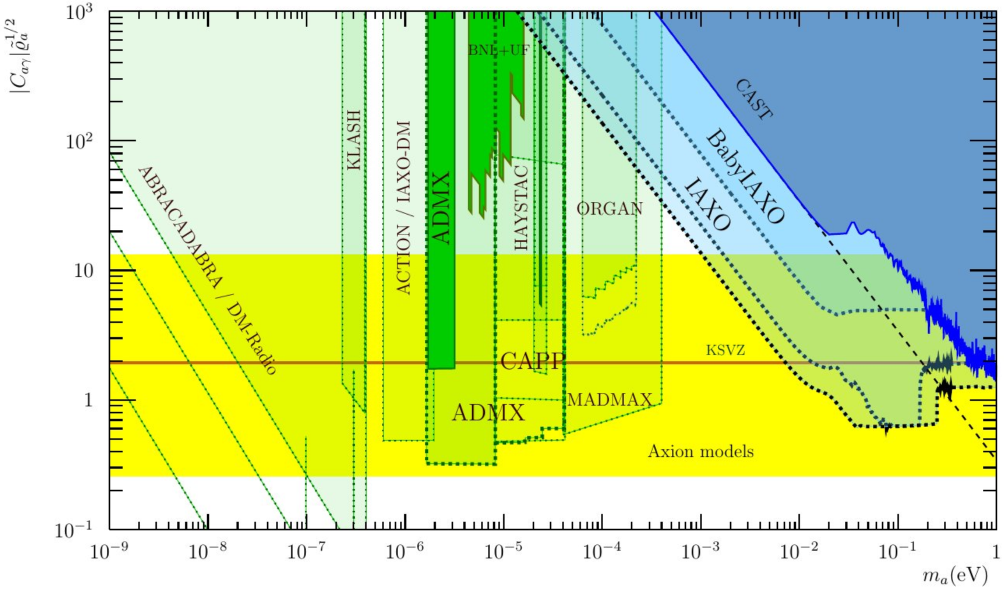
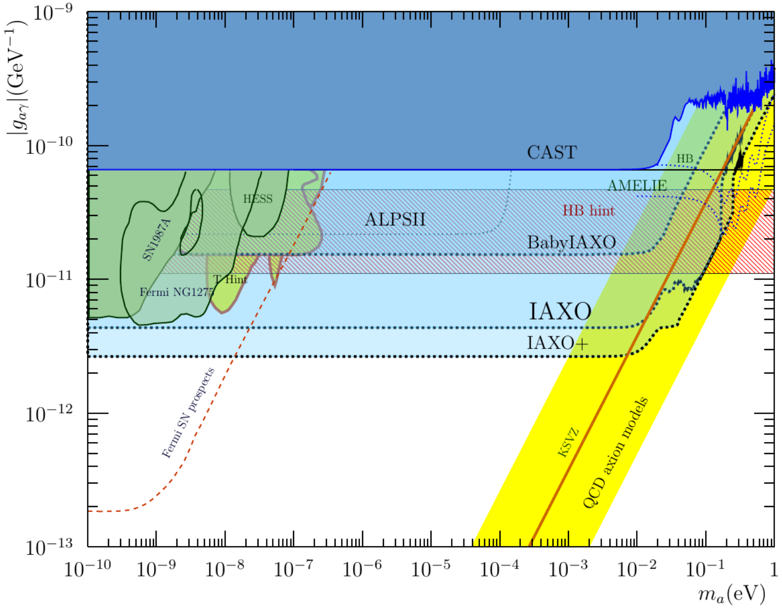
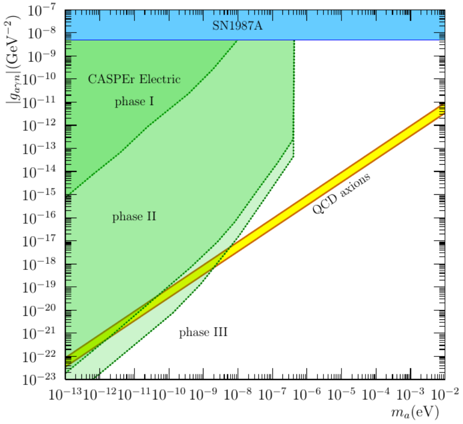
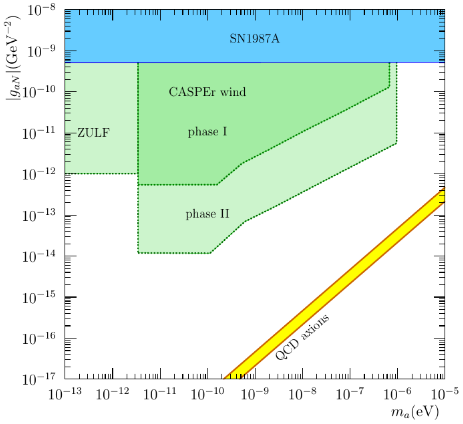
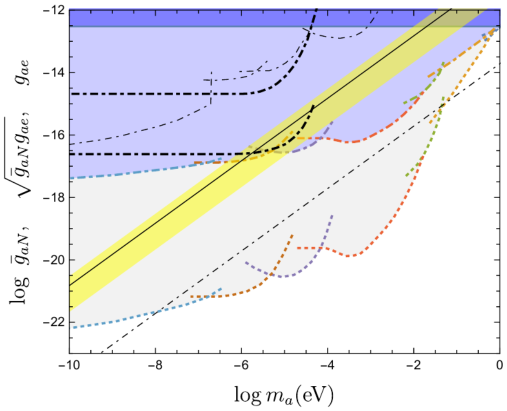










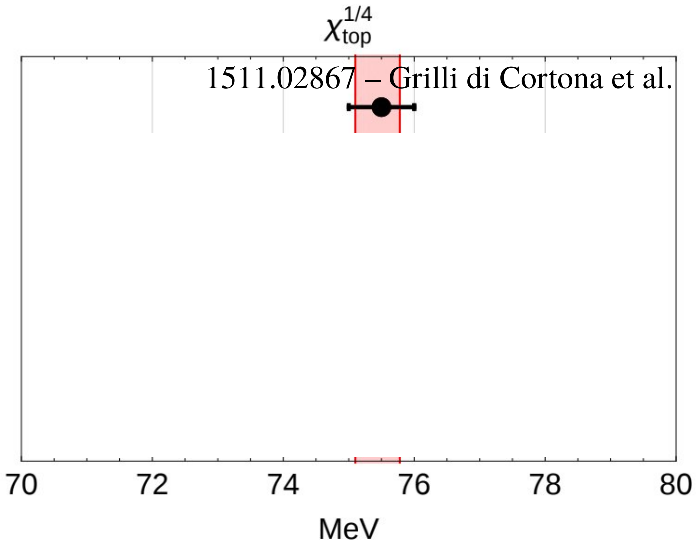
from 1801.08127
Irastorza. Redondo

QCD Axion Mass

$$m_a = \frac{\chi_{\rm top}^{1/2}}{f_a} = \frac{\sqrt{m_u m_d}}{m_u + m_d}\frac{m_\pi f_\pi}{f_a}$$

Weinberg ‘78

$$m_a = \frac{\chi_{\text{top}}^{1/2}}{f_a}$$



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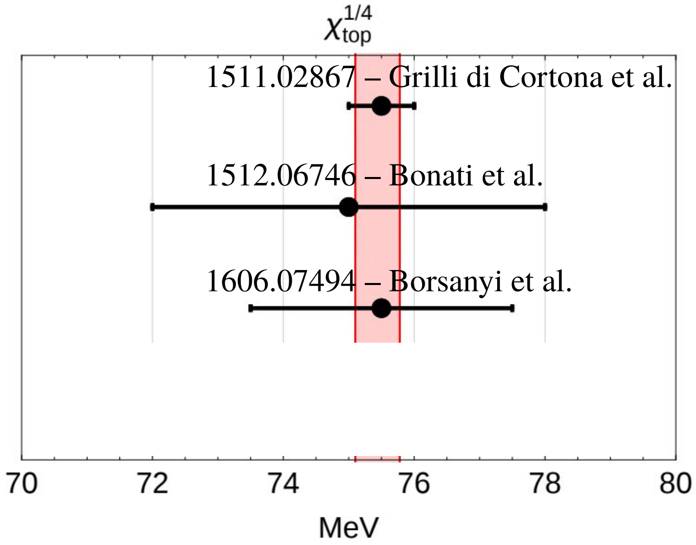

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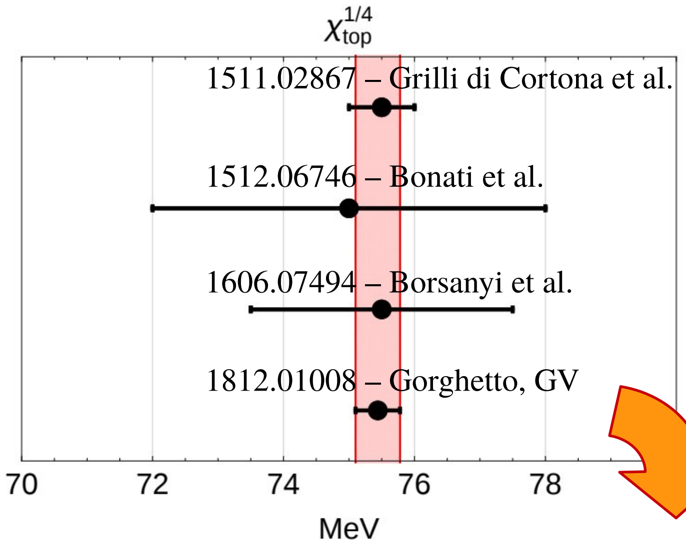


$$m_a = \Big[\underbrace{5.815(22)_z(04)_{f_\pi}}_{\boxed{\text{LO}}} \underbrace{-0.121(38)_{\ell_i^r}}_{\boxed{\text{NLO}}} \underbrace{-0.022(07)_{\ell_i^r}(05)_{c_i^r}}_{\boxed{\text{NNLO}}} \underbrace{+0.019(06)_{k_i^r}}_{\boxed{\text{EM}}}\Big]\,\mu\text{eV}\,\frac{10^{12}\text{ GeV}}{f_a}$$

$$m_a = 5.691(51)\,\mu\text{eV}\,\frac{10^{12}\text{ GeV}}{f_a}$$

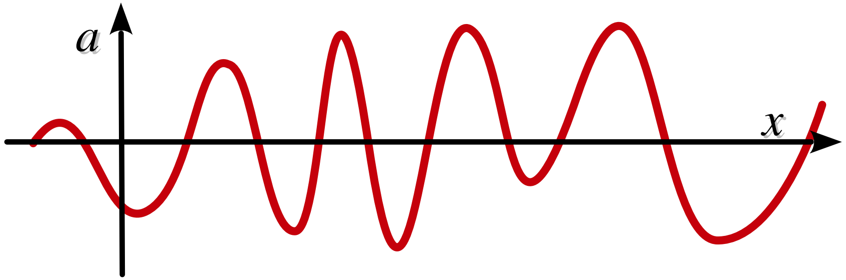
$$\chi_{\rm top}^{1/4} = 75.44(34)\text{ MeV}$$

Axion DM

Abbott, Dine, Fischler,
Preskill, Sikivie,Wise,
Wilczek ‘83

Scenario I: no PQ restoration after inflation

$f_a > \max\{H_I, T_R\}$



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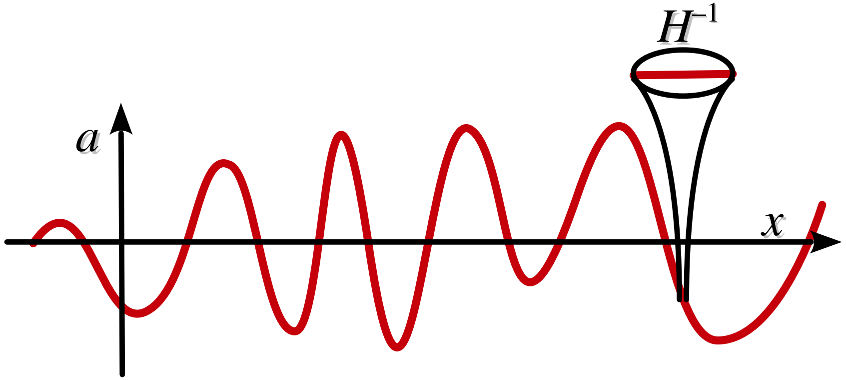


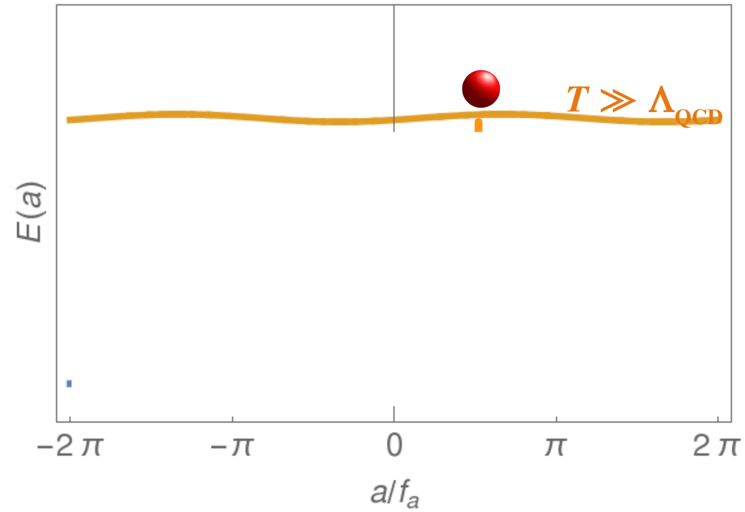
$a(t_0) = \text{const}$ (within Hubble)

after inflation

Scenario I:

$$a(t_0) = \text{const} \equiv \theta_0 f_a$$

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$



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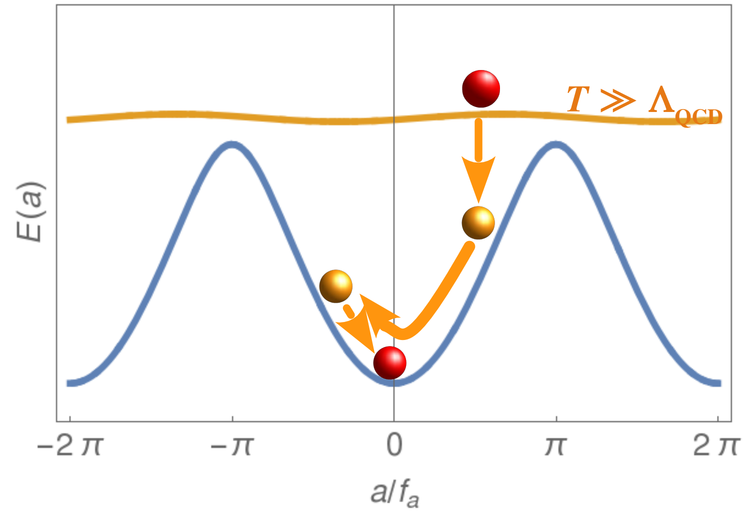
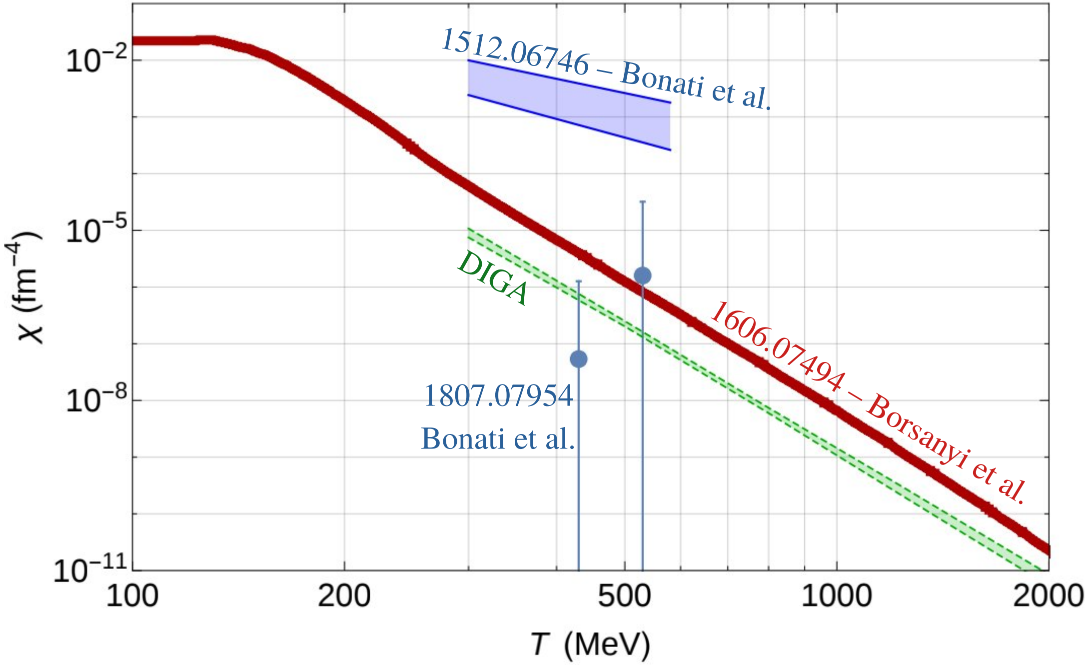


$$\rho_a = m_a^2 a^2$$

$$\Omega_a = 0.1\, k\, \theta_0^2 \left[\frac{f_a}{10^{12}\text{GeV}}\right]^{1+\epsilon}$$

Finite Temperature QCD Axion Mass



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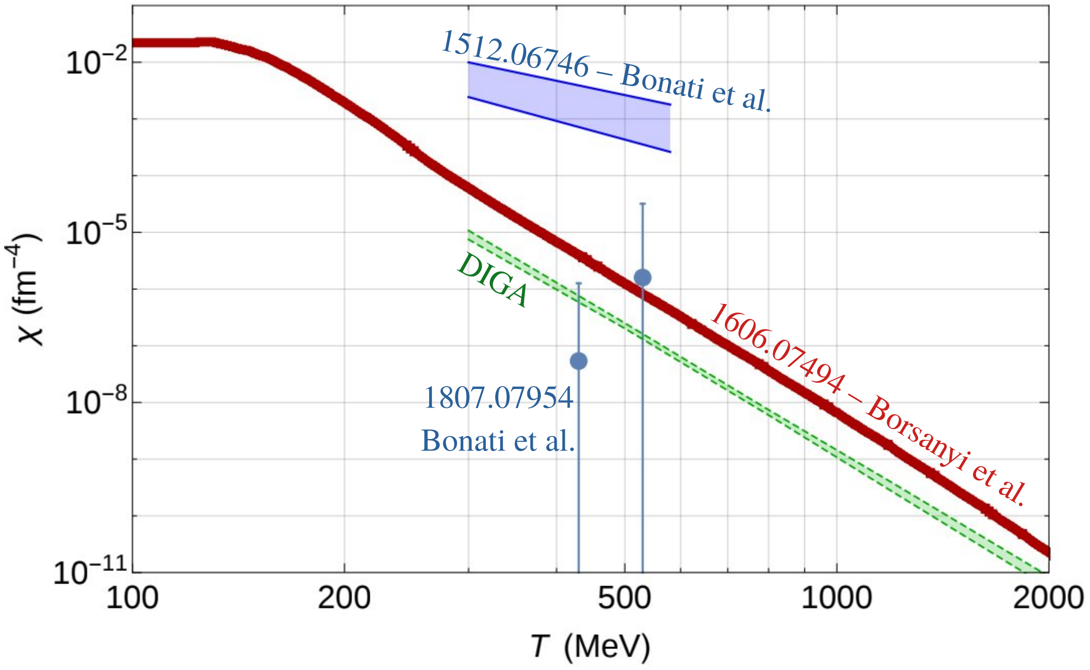

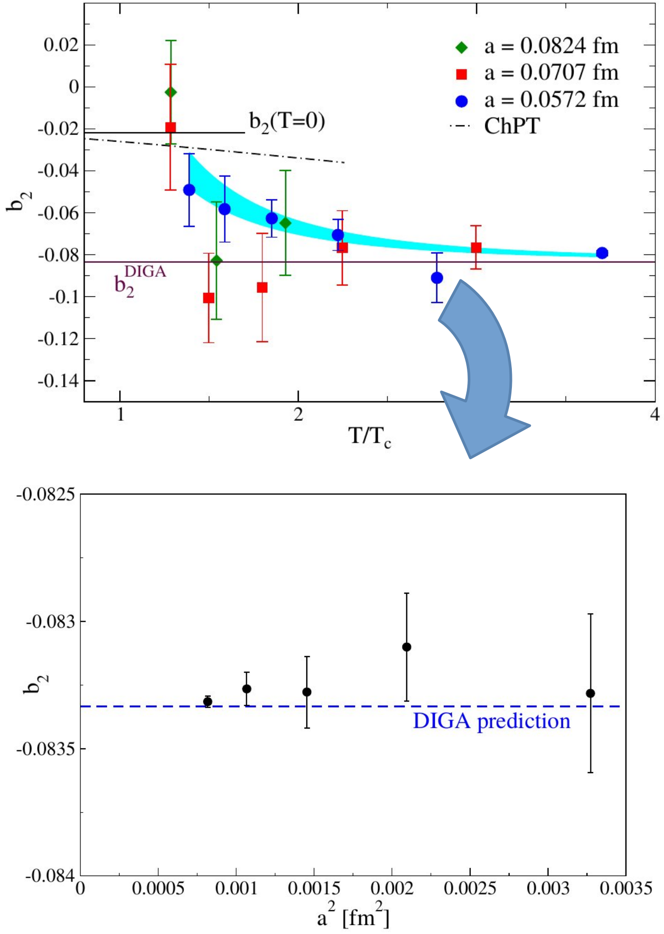

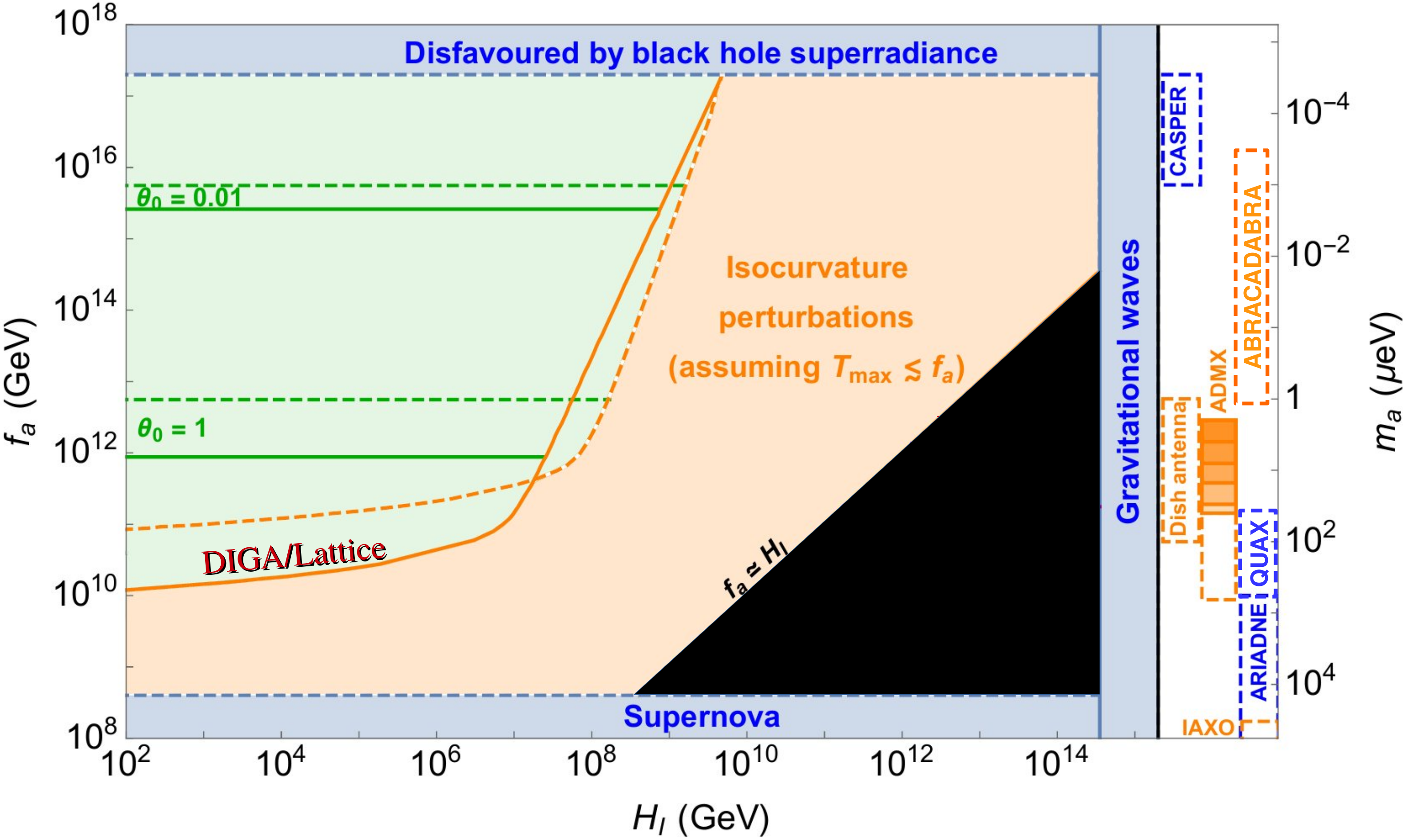
Disfavoured by black hole superradiance
Isocurvature
perturbations
(assuming $T_{\max} \lesssim f_a$)
$\theta_0 = 0.01$
$\theta_0 = 1$
DIGA/Lattice
$f_a \simeq H_I$
Gravitational waves
Supernova
f_a (GeV)
10^{8}
10^{10}
10^{12}
10^{14}
10^{16}
10^{18}
H_I (GeV)
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
m_a (μeV)
10^{-4}
10^{-2}
1
10^{2}
10^{4}
CASPER
ABRACADABRA
ADMX
Dish antenna
QUAX
ARIADNE
IAXO

Scenario II: PQ restoration after inflation

$f_a < \max\{H_I, T_R\}$

$$\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$$

no free parameters from intial conditions
abundance calculable! *(in principle)*

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Multiple contributions:

- misalignment (0-mode) $\Omega_a^{(0)} = 0.1\ k_\alpha \left[\xcancel{\frac{\theta}{2.15}}\right]^2 \left[\frac{f_a}{10^{12}\ \mathrm{GeV}}\right]^{1+\varepsilon}$

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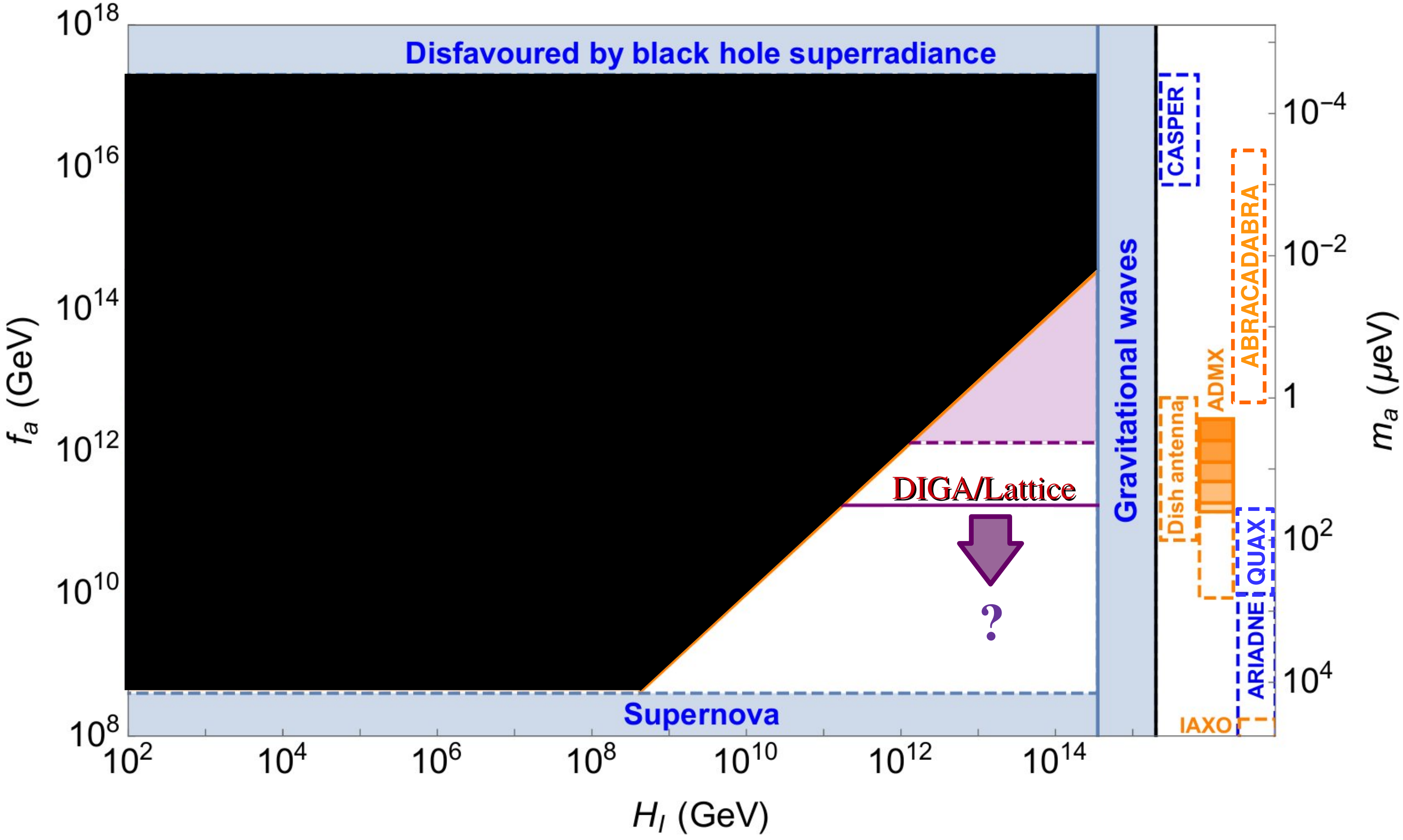
- topological defects (strings and domain walls)



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f_a (GeV)
10^{18}
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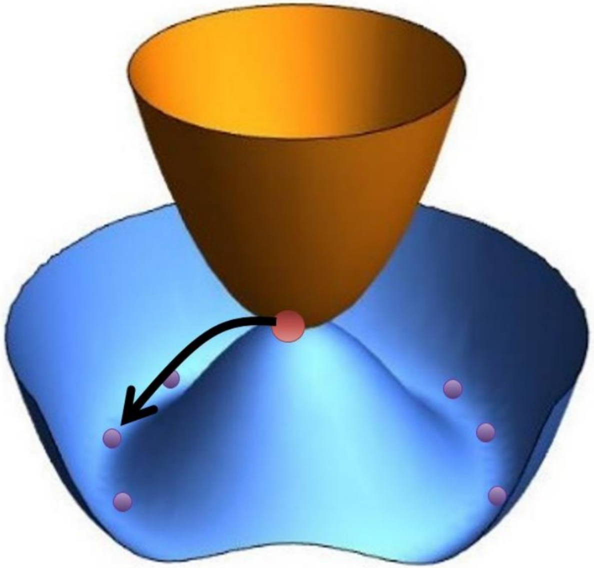
10^{10}
10^{8}
H_I (GeV)
10^{2}
10^{4}
10^{6}
10^{8}
10^{10}
10^{12}
10^{14}
m_a (μeV)
10^{-4}
10^{-2}
1
10^{2}
10^{4}

PQ phase transition

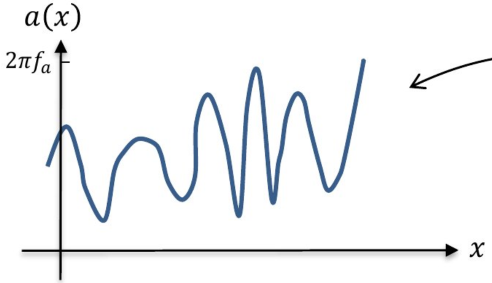
$T \gtrsim f_a$



$\phi = |\phi| e^{i\frac{a}{f_a}}$

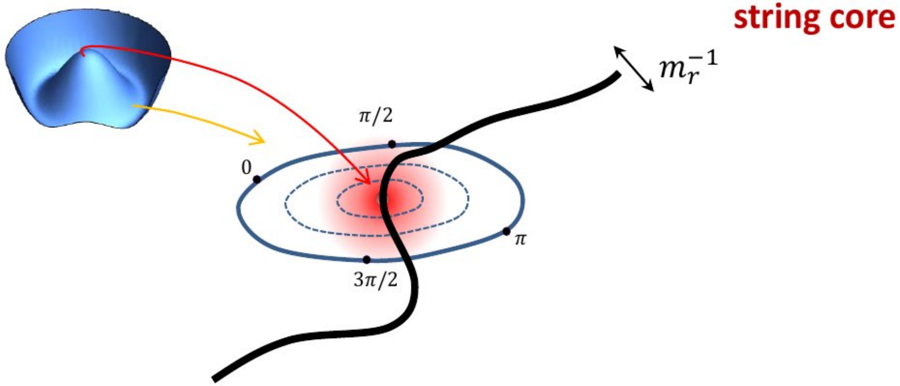
$T \lesssim f_a$

similarly if $H \gtrsim f_a$



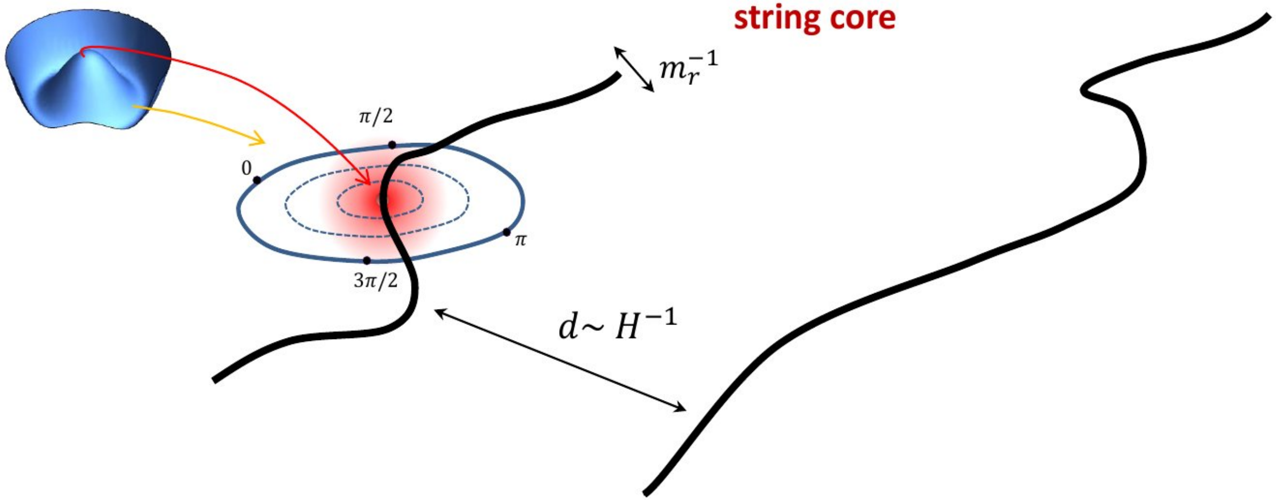
after PQ axion field has random fluctuations over the observable universe

Axionic Strings



string tension $\mu = \frac{E}{L}$

Axionic Strings



string tension

$$\mu = \frac{E}{L} \sim \pi f_a^2 \log \frac{d}{m_r^{-1}}$$

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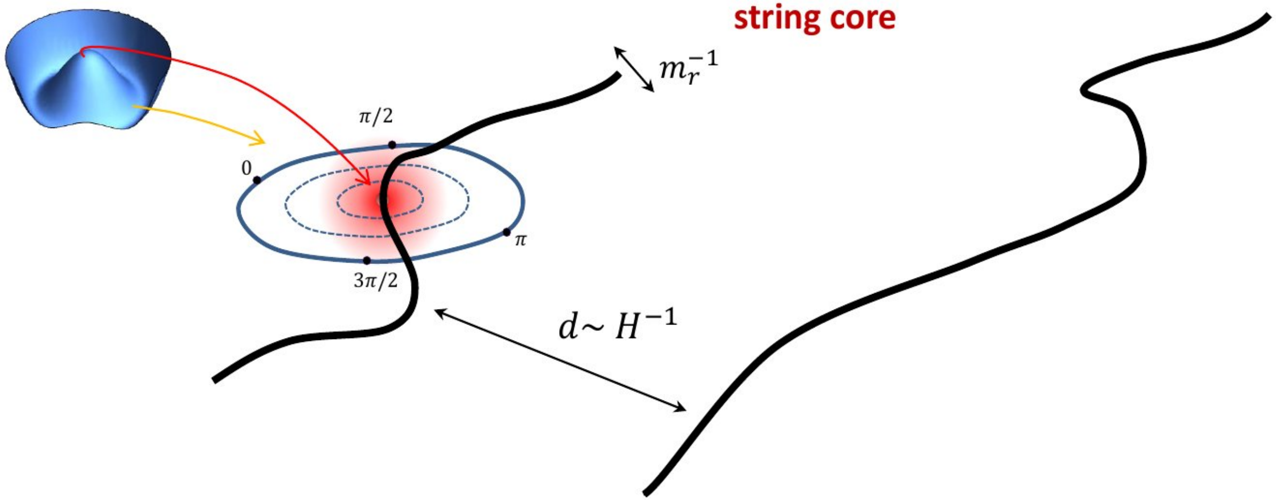

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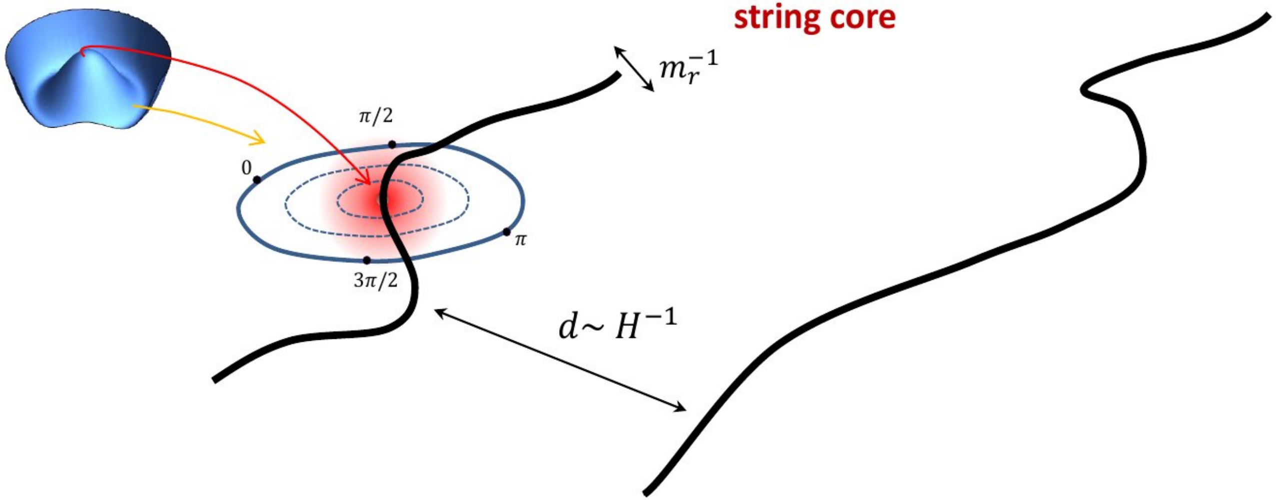
radial
core

Axionic Strings



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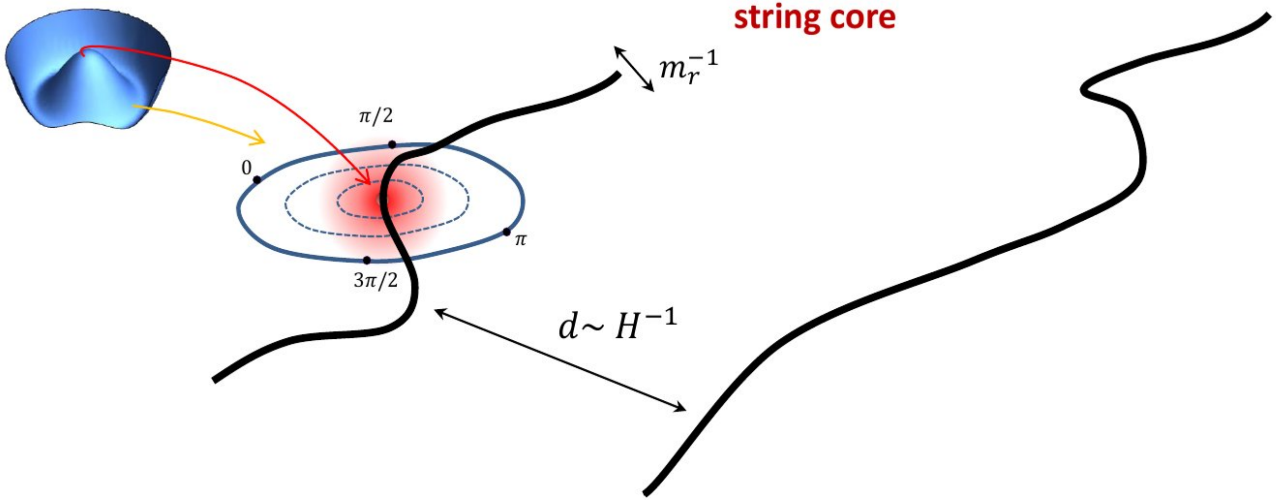
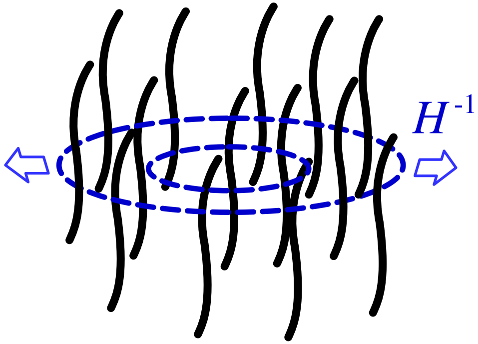


string tension

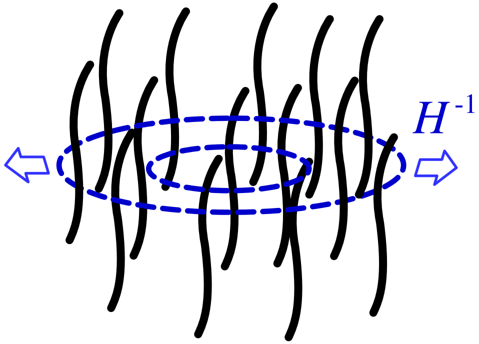
$$\mu = \frac{E}{L} \sim \underset{\text{radial core}}{\boxed{\pi f_a^2}} \underset{\text{axion field}}{\boxed{\log\frac{d}{m_r^{-1}}}} \sim \pi f_a^2 \log\frac{m_r}{H}$$

free strings



$$\rho_{\text{free}} \propto \frac{1}{R^2(t)} = \frac{1}{t}$$

free strings



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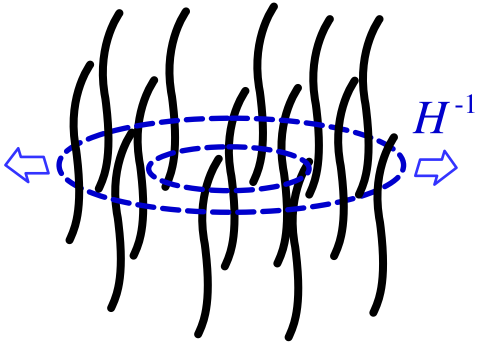

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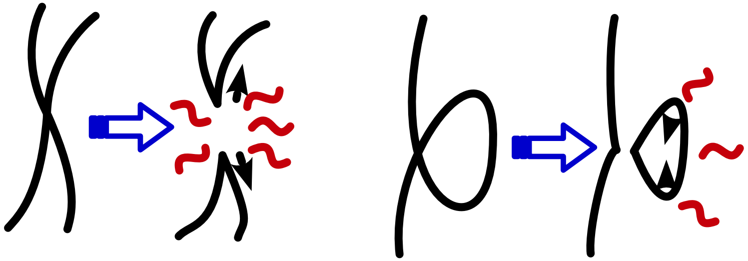
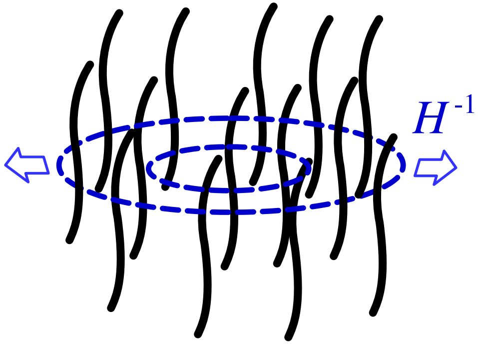
string recombination



free strings



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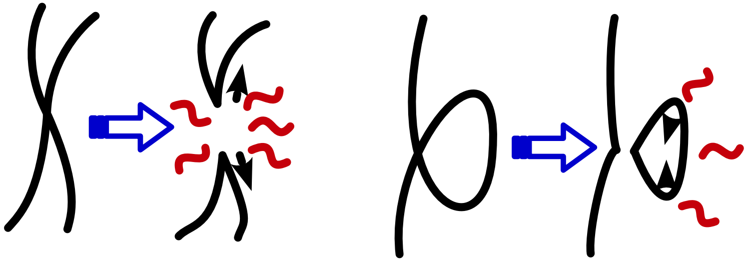
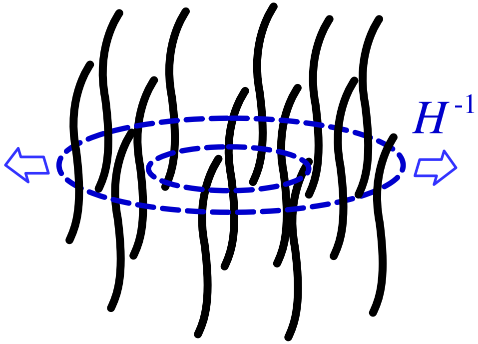


scaling solution

$$\rho_s = \xi \frac{\mu}{t^2}$$

ξ = (# strings) / (Hubble Patch)

free strings



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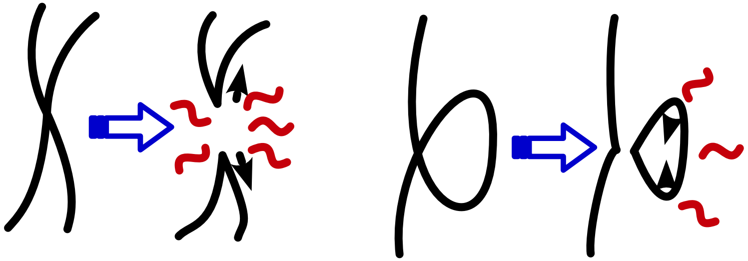

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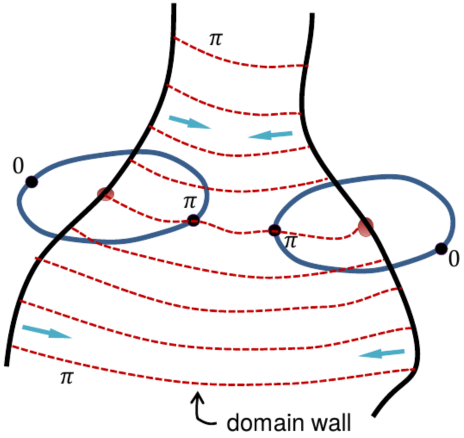
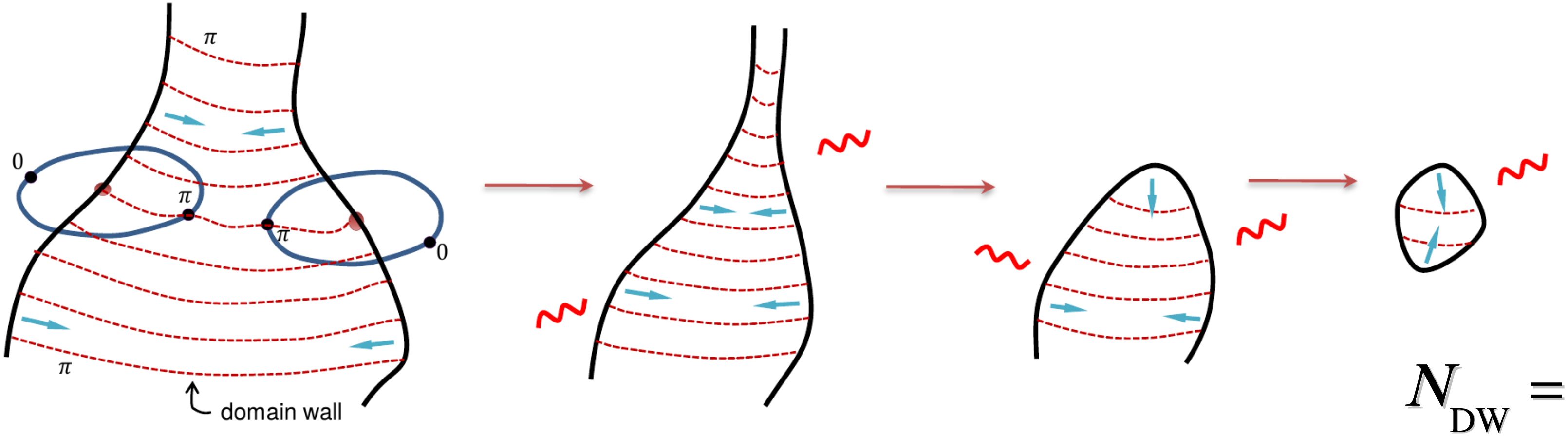
difference goes in radiation

@ H~m_a (T~Λ_{QCD})

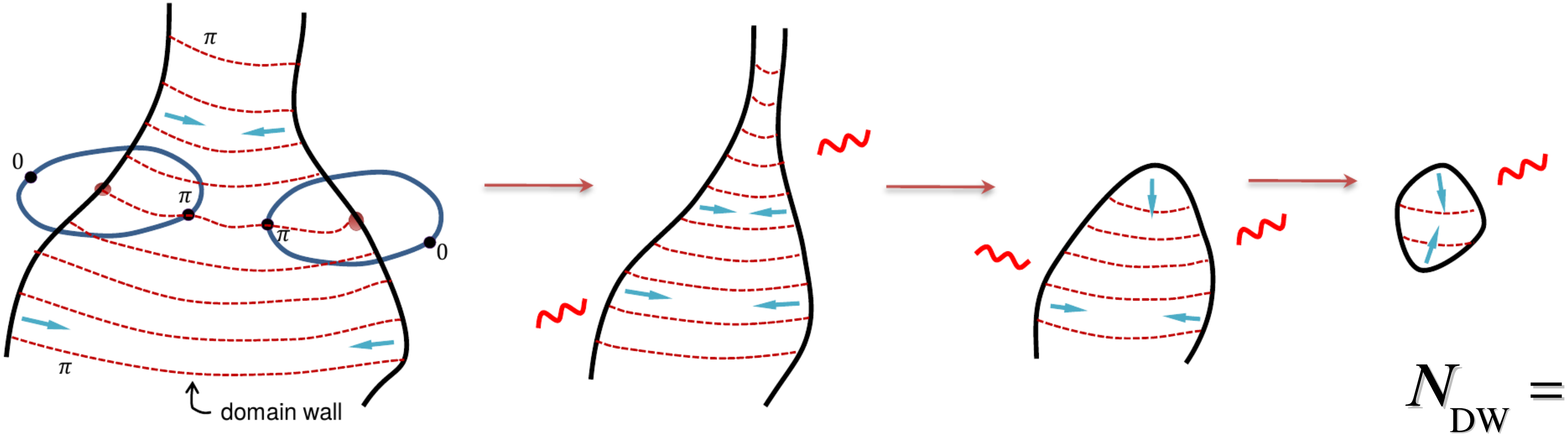


@ $H \sim m_a$ ($T \sim \Lambda_{\mathrm{QCD}}$)

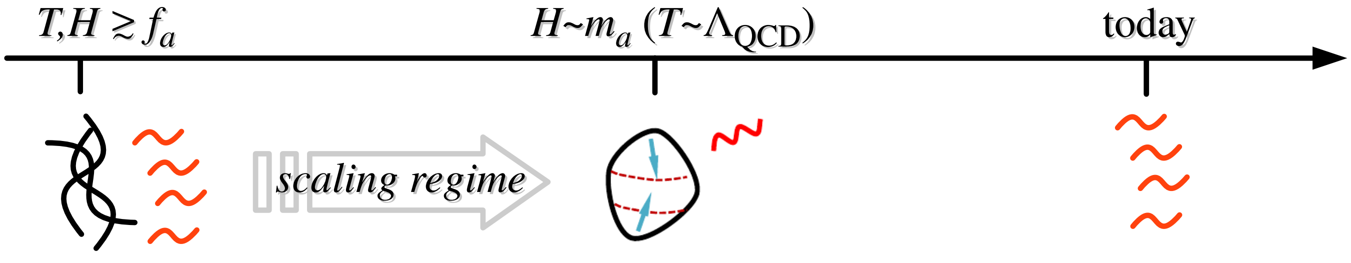


N_{DW} =1 case

@ $H\sim m_a$ ($T\sim\Lambda_{\rm QCD}$)



$N_{\rm DW}=1$ case



$T,H \gtrsim f_a$

$H\sim m_a$ ($T\sim\Lambda_{\rm QCD}$)

today

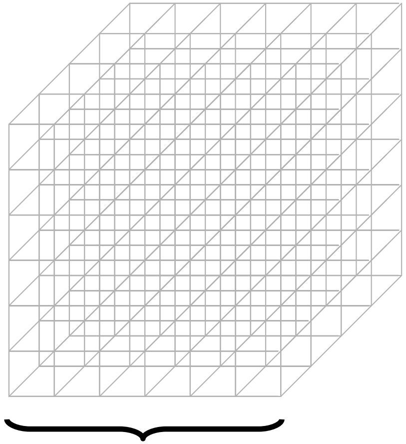
scaling regime

$\log(m_r/H)$ 1÷15 ~70

Numerical Simulation

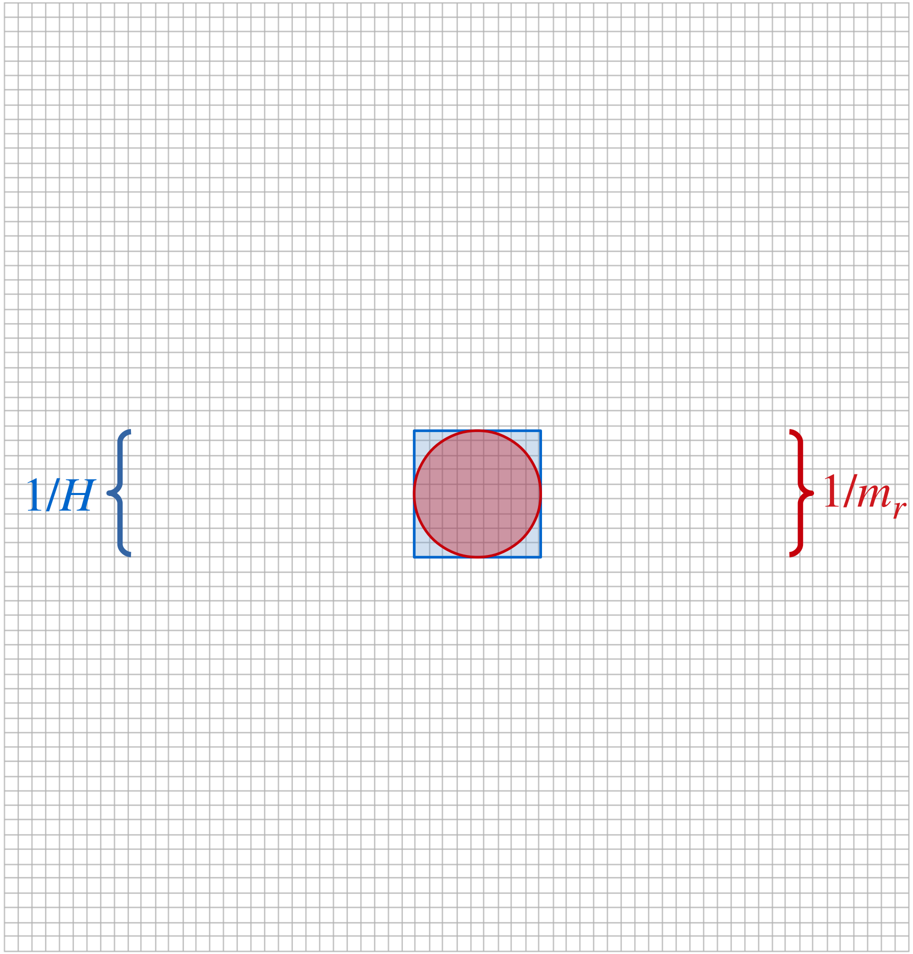
$$V(\Phi) = \frac{\lambda}{4}\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2$$

$$\Phi = \frac{f_a + r}{\sqrt{2}}\, e^{ia/f_a}$$



$N \approx 1k - 4k - 8k$

The Bottle Neck



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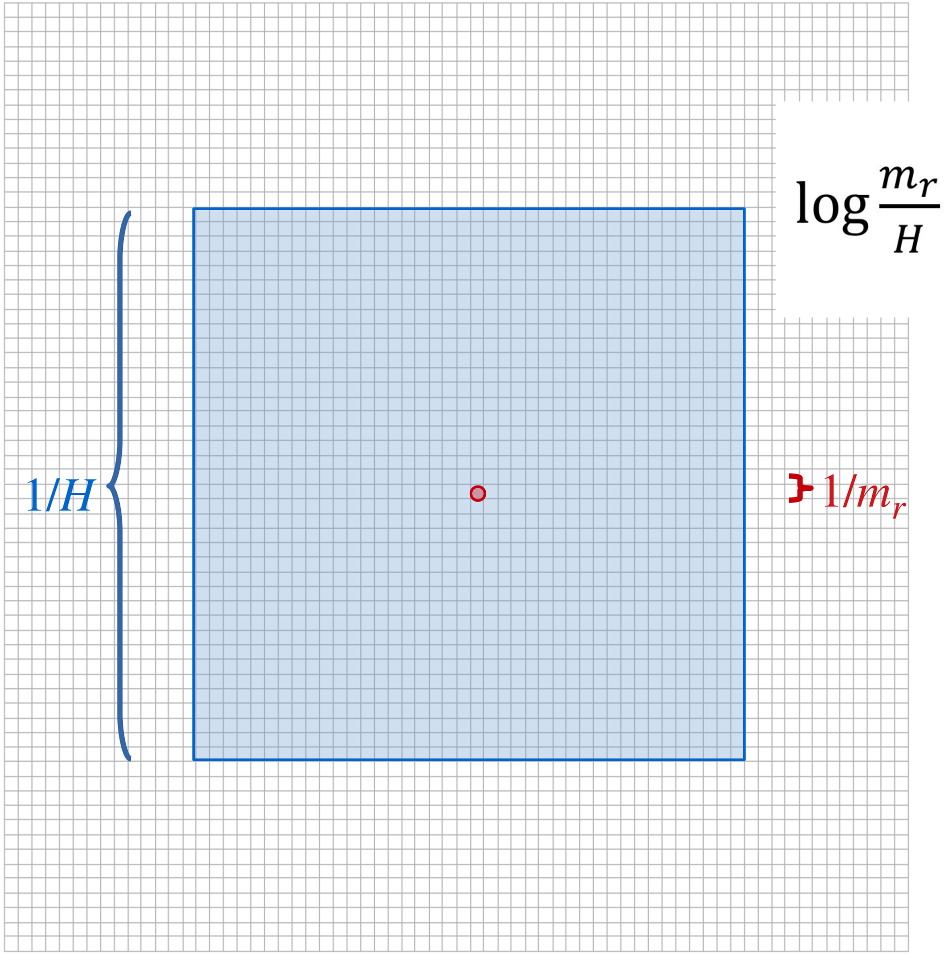
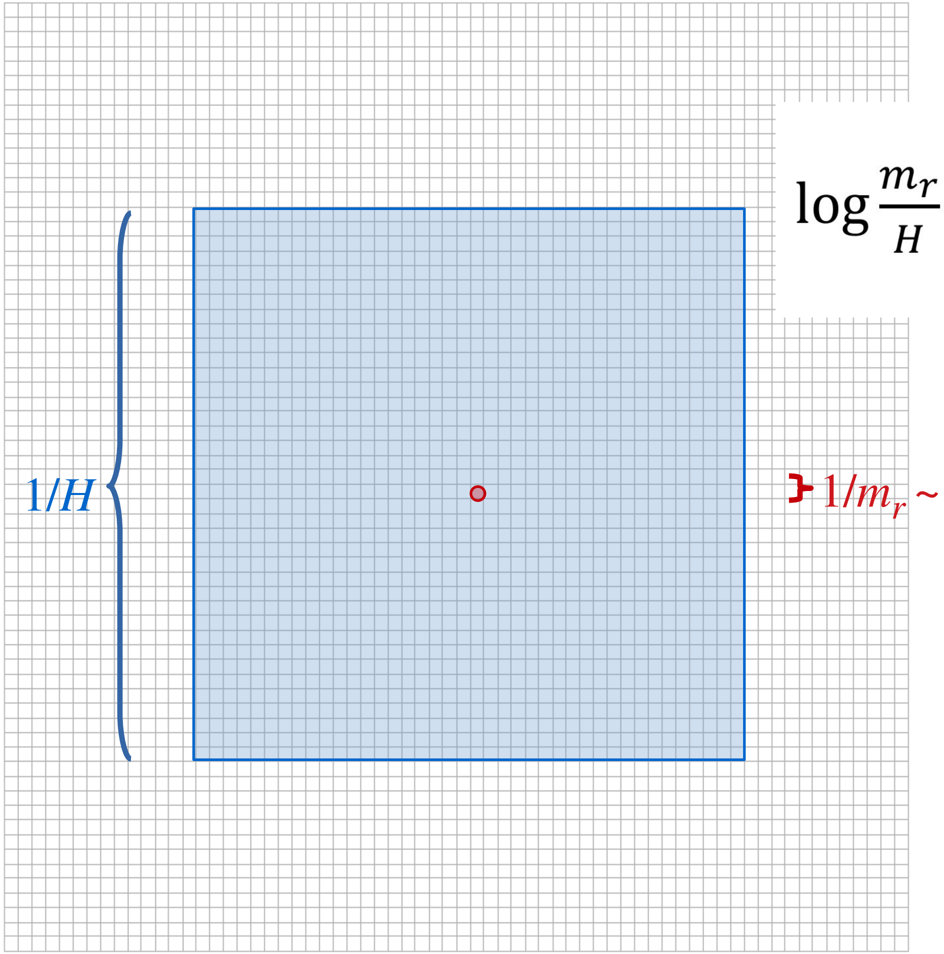


$$\log \frac{m_r}{H} \le \log\left(\frac{\square}{\bigcirc}\right) \sim 7 \ll 70$$

The Bottle Neck

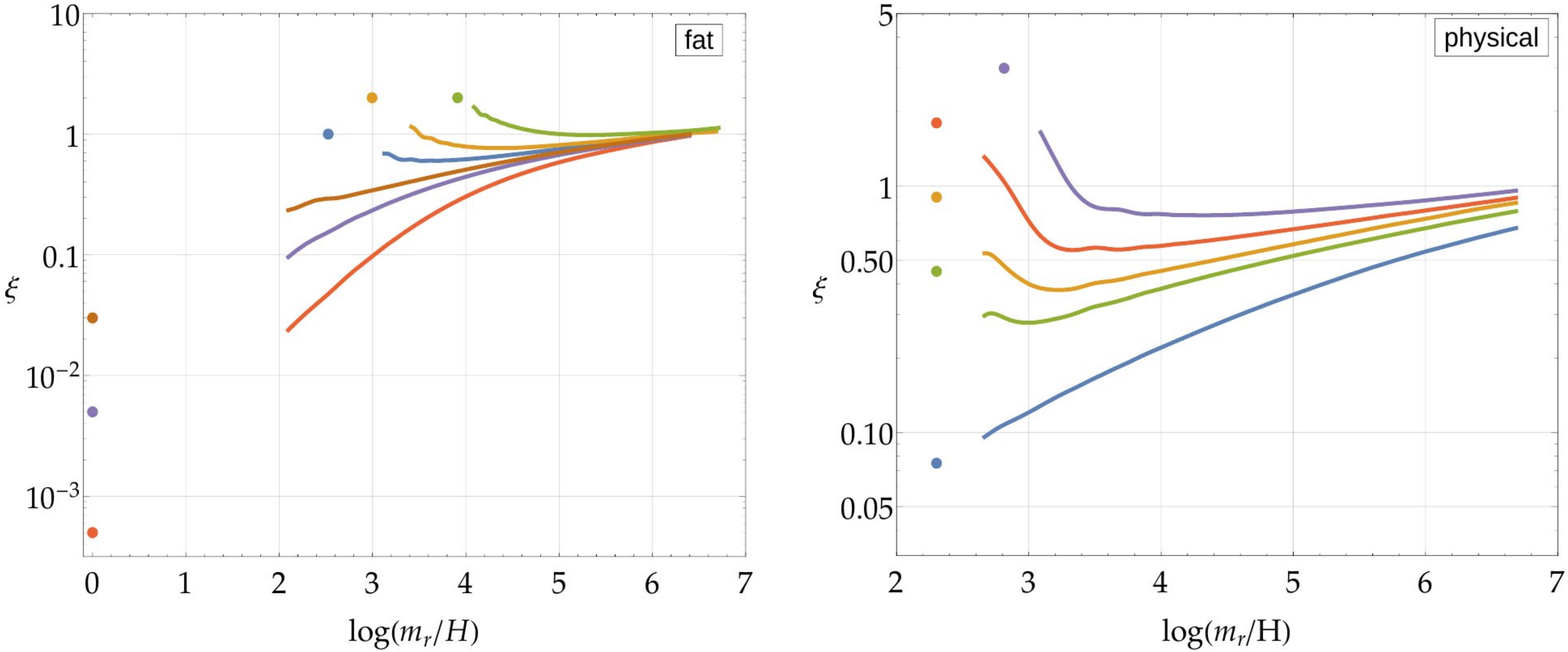


$$\log\frac{m_r}{H} \leq \log\left(\frac{\square}{\bigcirc}\right) \sim 7 \ll 70$$

fat string trick

$1/m_r \sim R \sim t^{1/2}$

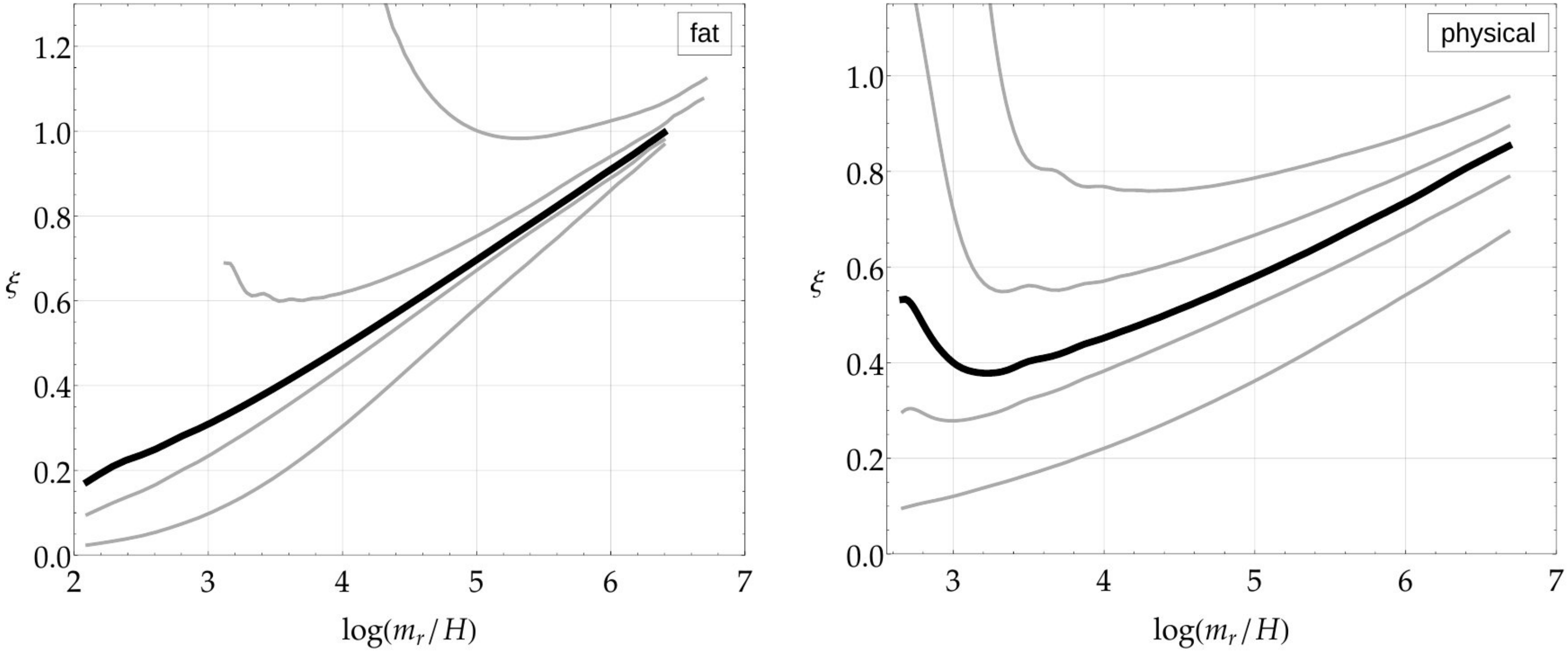
The Scaling Solution is an Attractive Solution



1806.04677 – Gorghetto, Hardy, GV

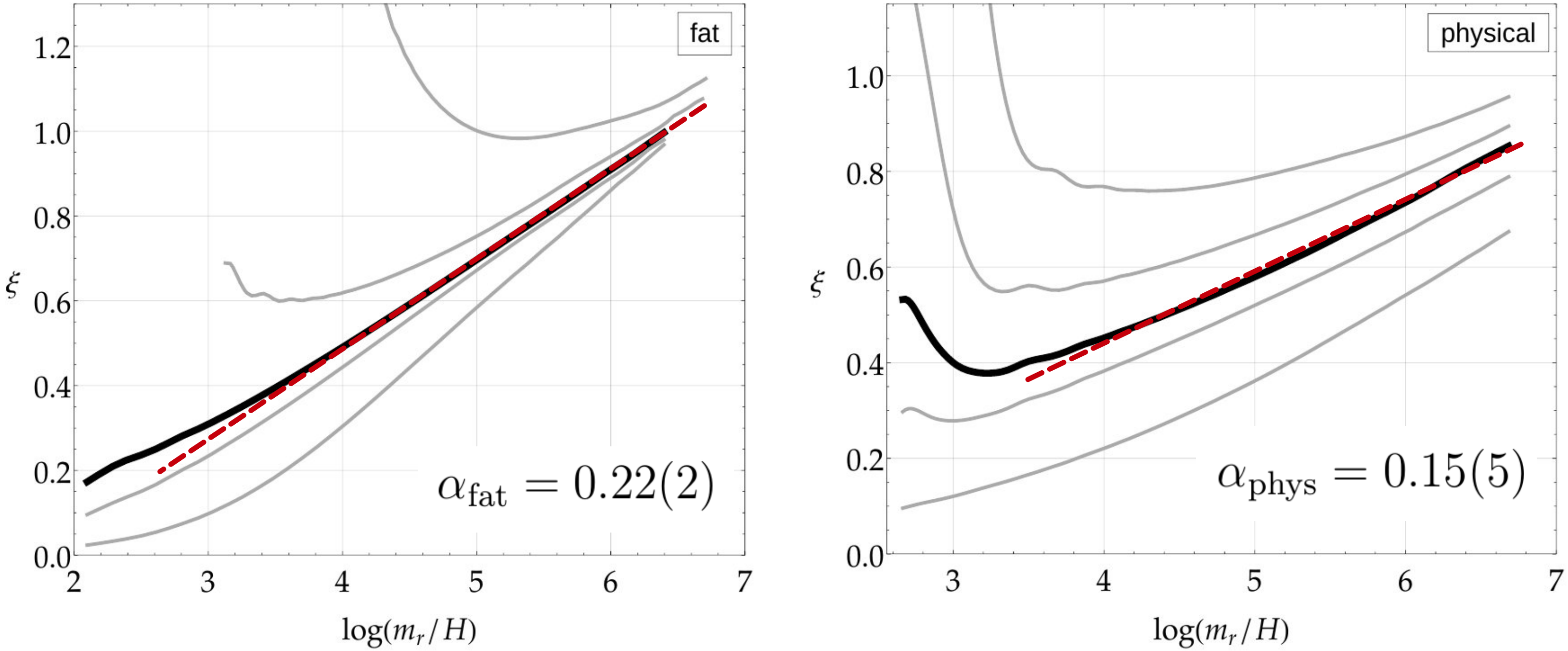
Scaling Violation

1806.04677 – Gorghetto, Hardy, GV



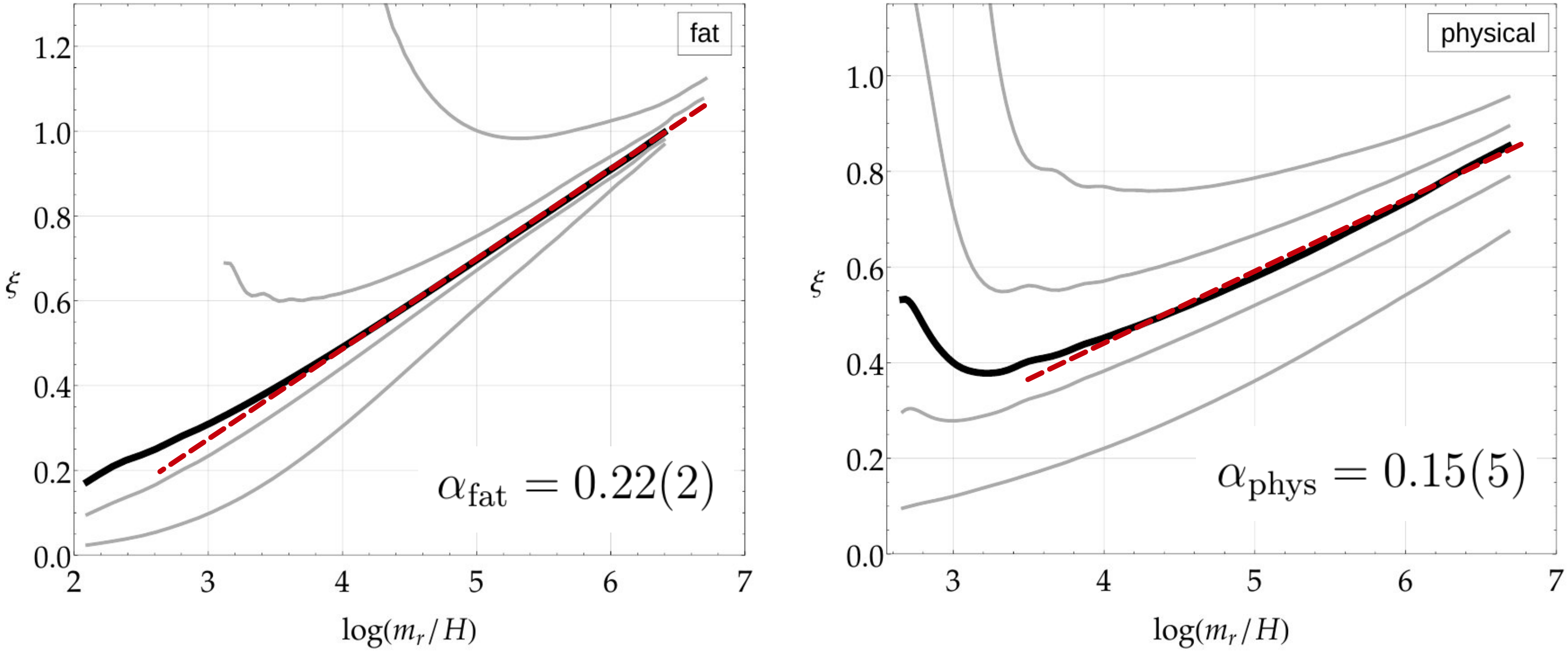
Also observed in:
1509.00026 – Fleury, Moore
1806.05566 – Kawasaki et al.
1906.00967 – Buschmann, Foster, Safdi
preliminary – Redondo, Saikawa, ...

Scaling Violation



$$\xi(t) = \alpha \log\left(\frac{m_r}{H}\right) + \beta$$

Scaling Violation

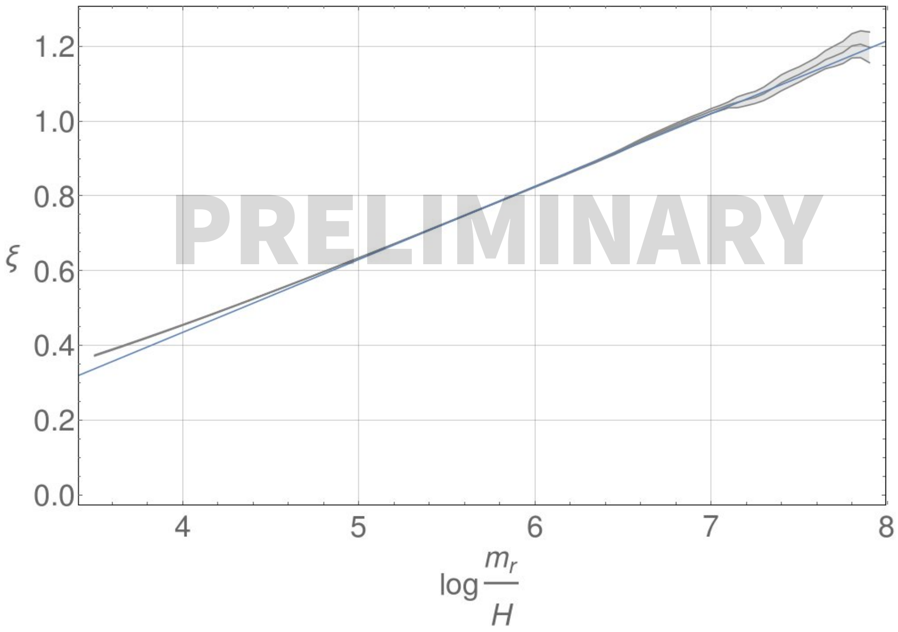


$$\xi(t) = \alpha \log\left(\frac{m_r}{H}\right) + \beta$$

$$\xi \xrightarrow{\log = 70} 10$$

Scaling Violation (4k simulation)

Gorghetto, Hardy, GV

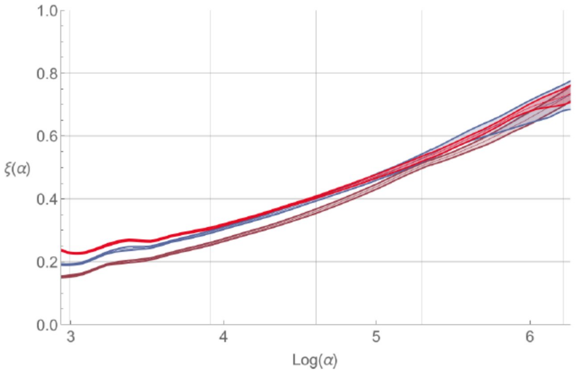


What is the large ξ behavior?

Origin of scaling violation?

Larger physical simulations suggest some curvature (log^2?)

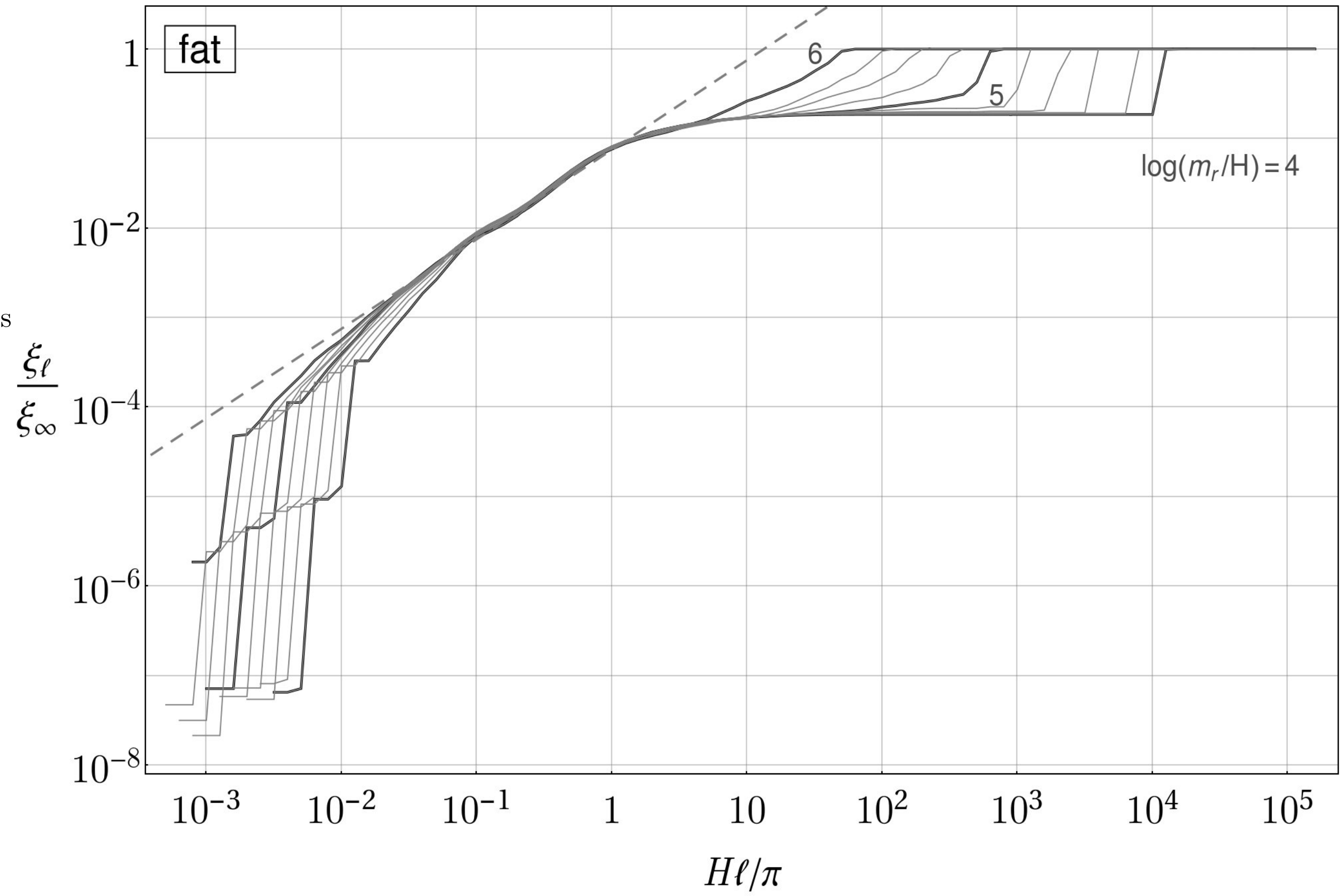
Scaling violation observed also in local U(1) string networks...



Loop Distribution

1806.04677 – Gorghetto, Hardy, GV

ξ contained in loops
of size up to ℓ



Loop Distribution

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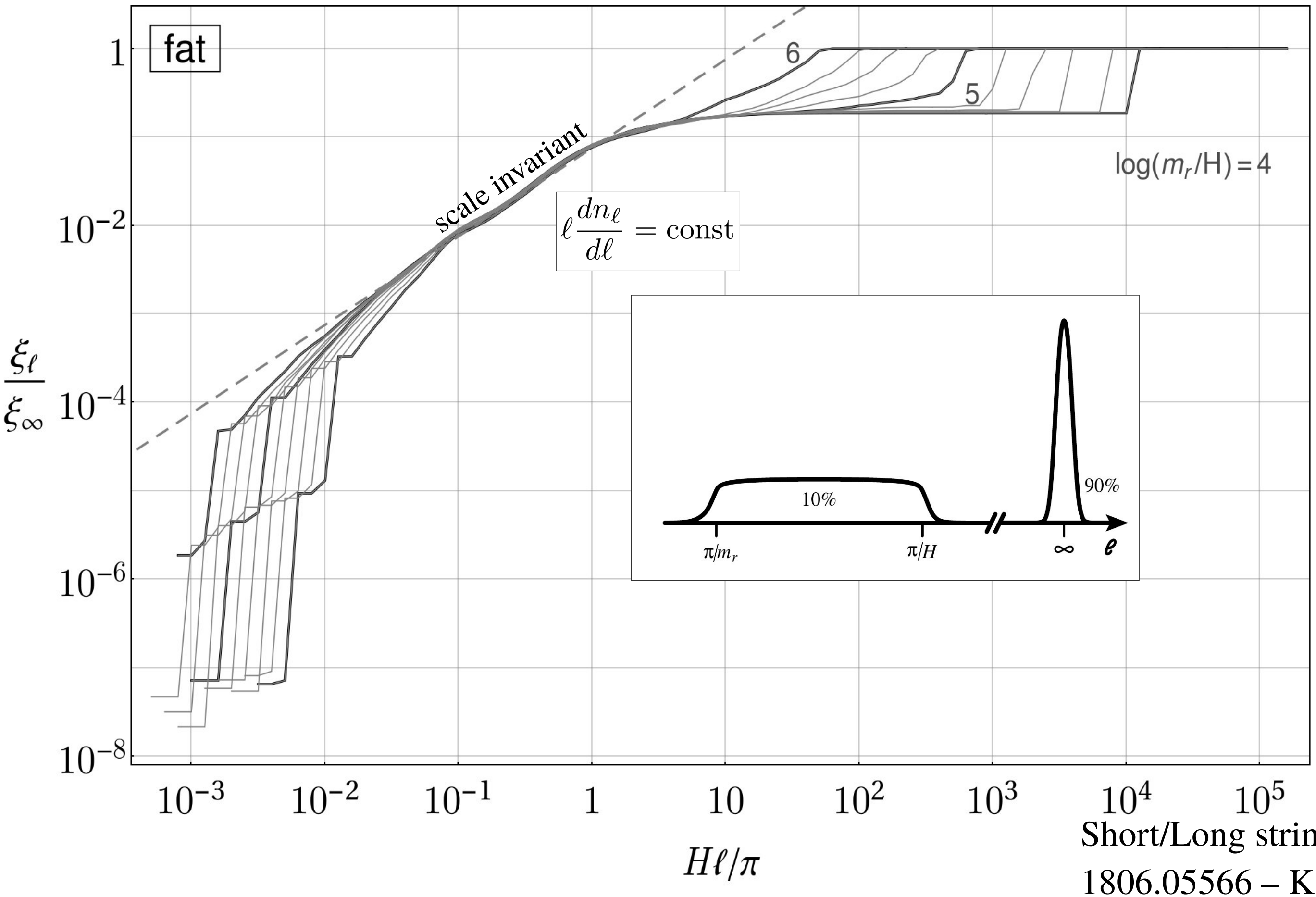
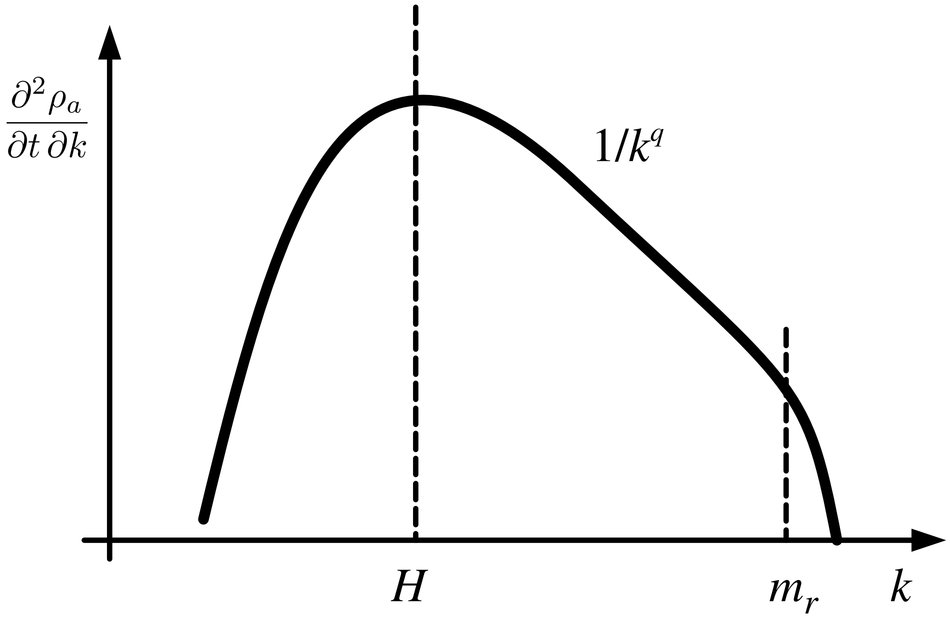


Short/Long string ratio also observed by:

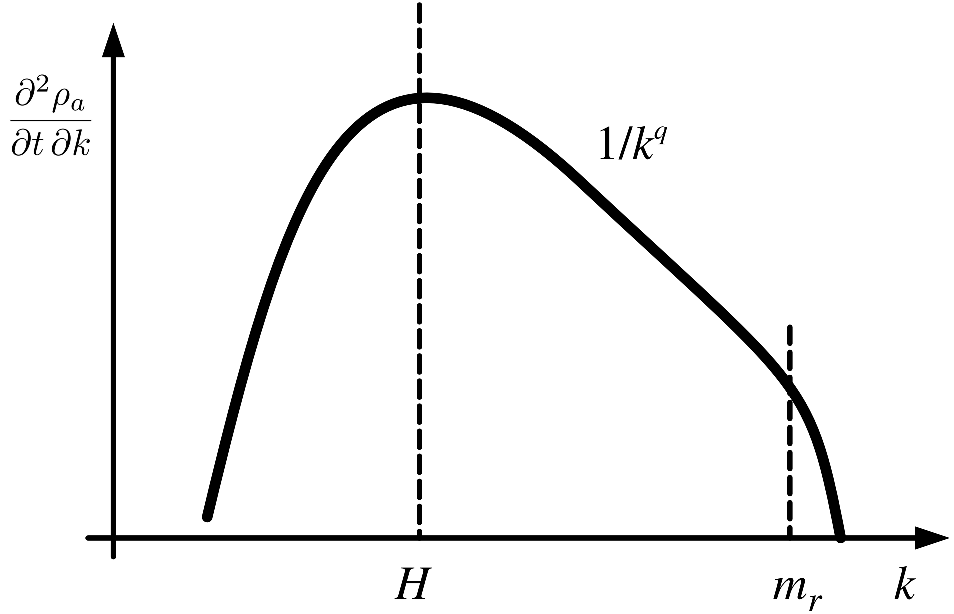
1806.05566 – Kawasaki et al.

1906.00967 – Buschmann, Foster, Safdi

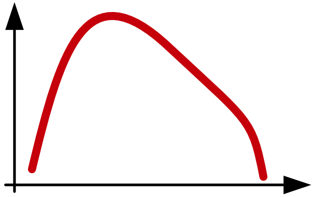
Axion Spectra VS Axion Number

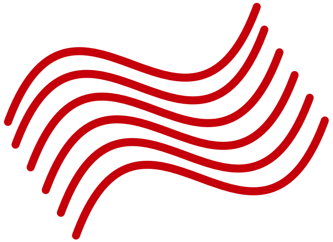


Axion Spectra VS Axion Number



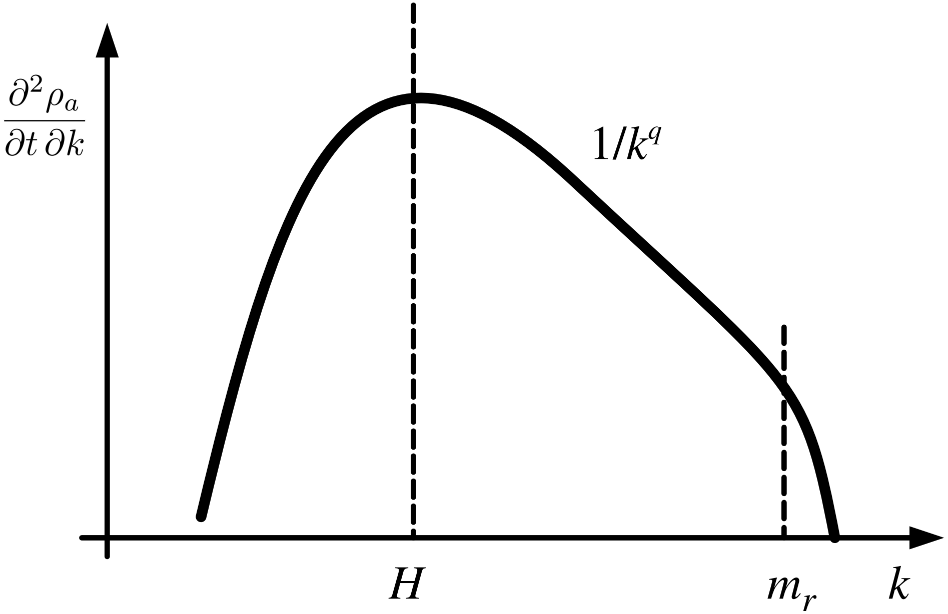
$q > 1$

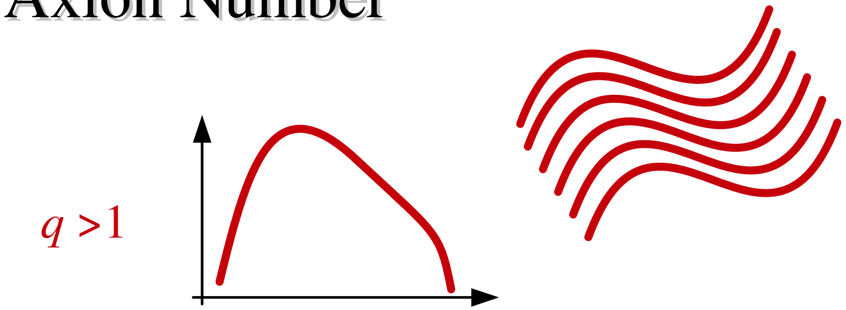




Davies, Shellard, ...

Axion Spectra VS Axion Number





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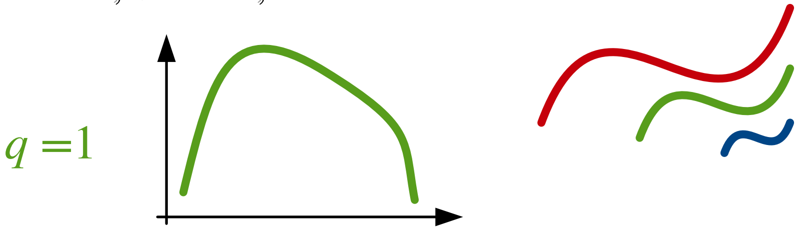

Sikivie, ...

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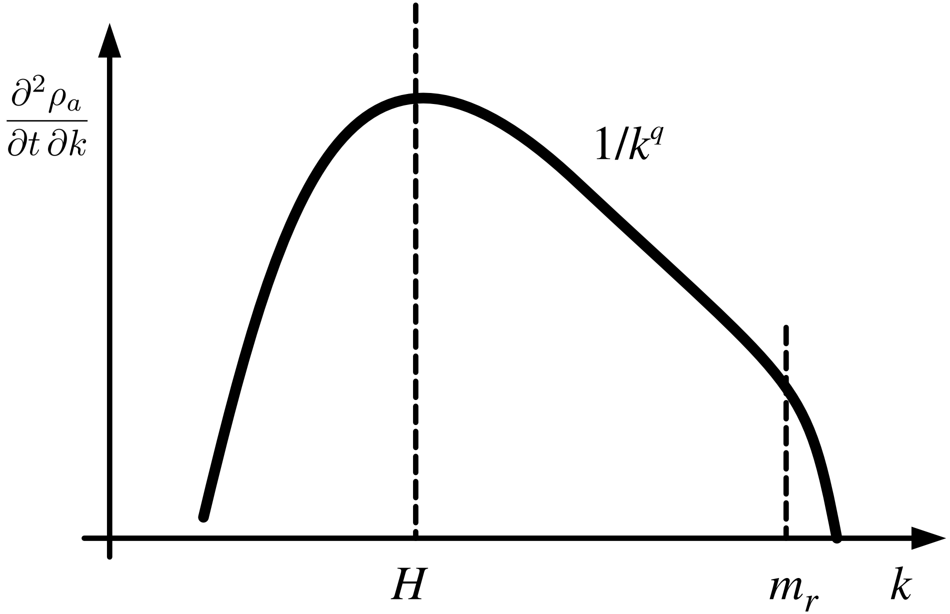

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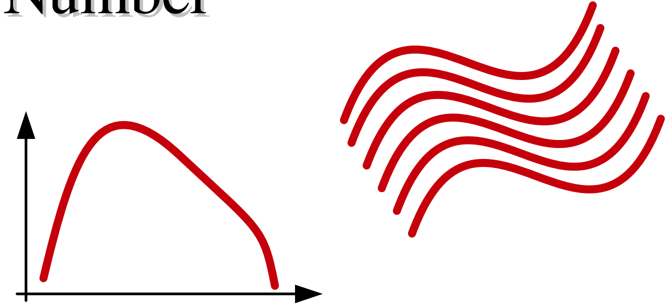
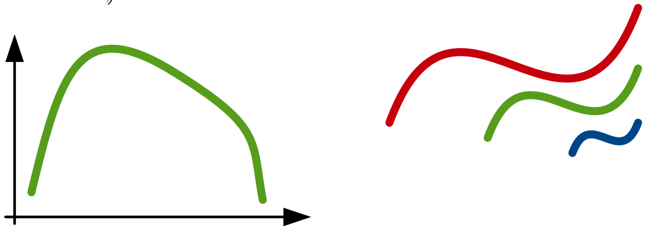


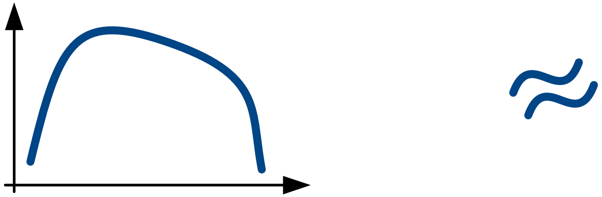
Davies, Shellard, ...

$q = 1$

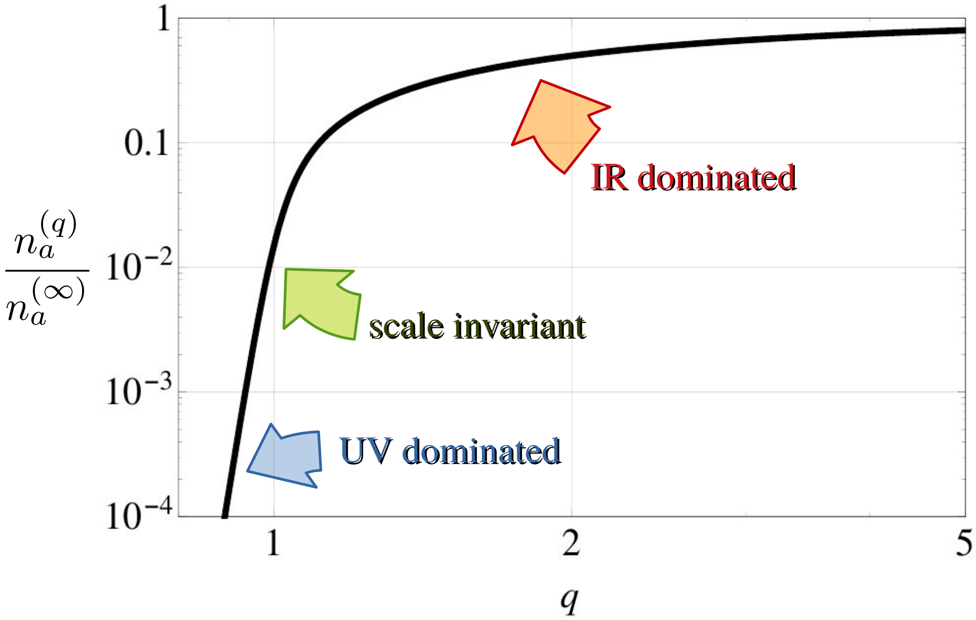


Sikivie, ...

$q < 1$

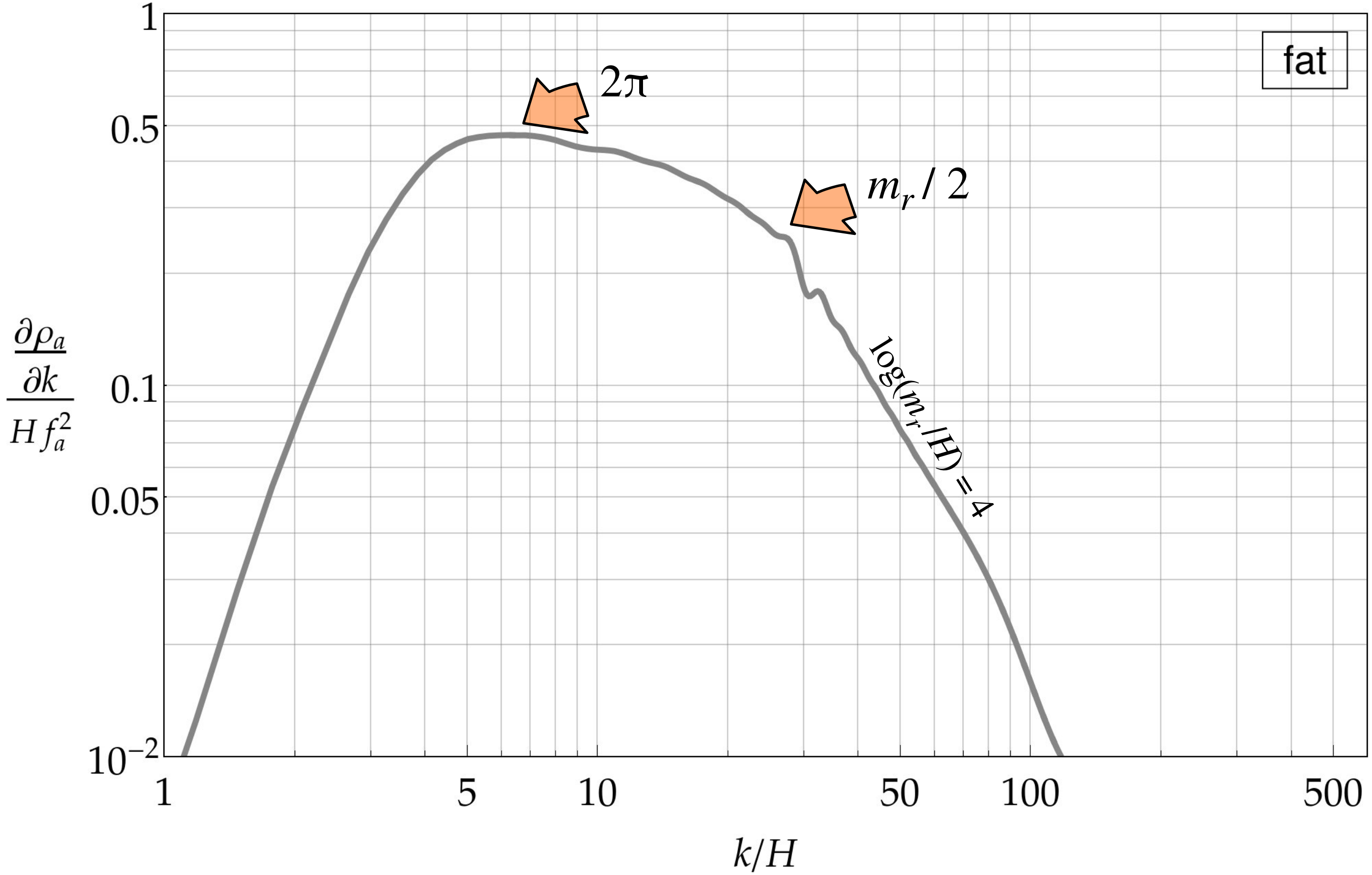


Axion Spectra VS Axion Number



Axion Spectrum

1806.04677 – Gorghetto, Hardy, GV



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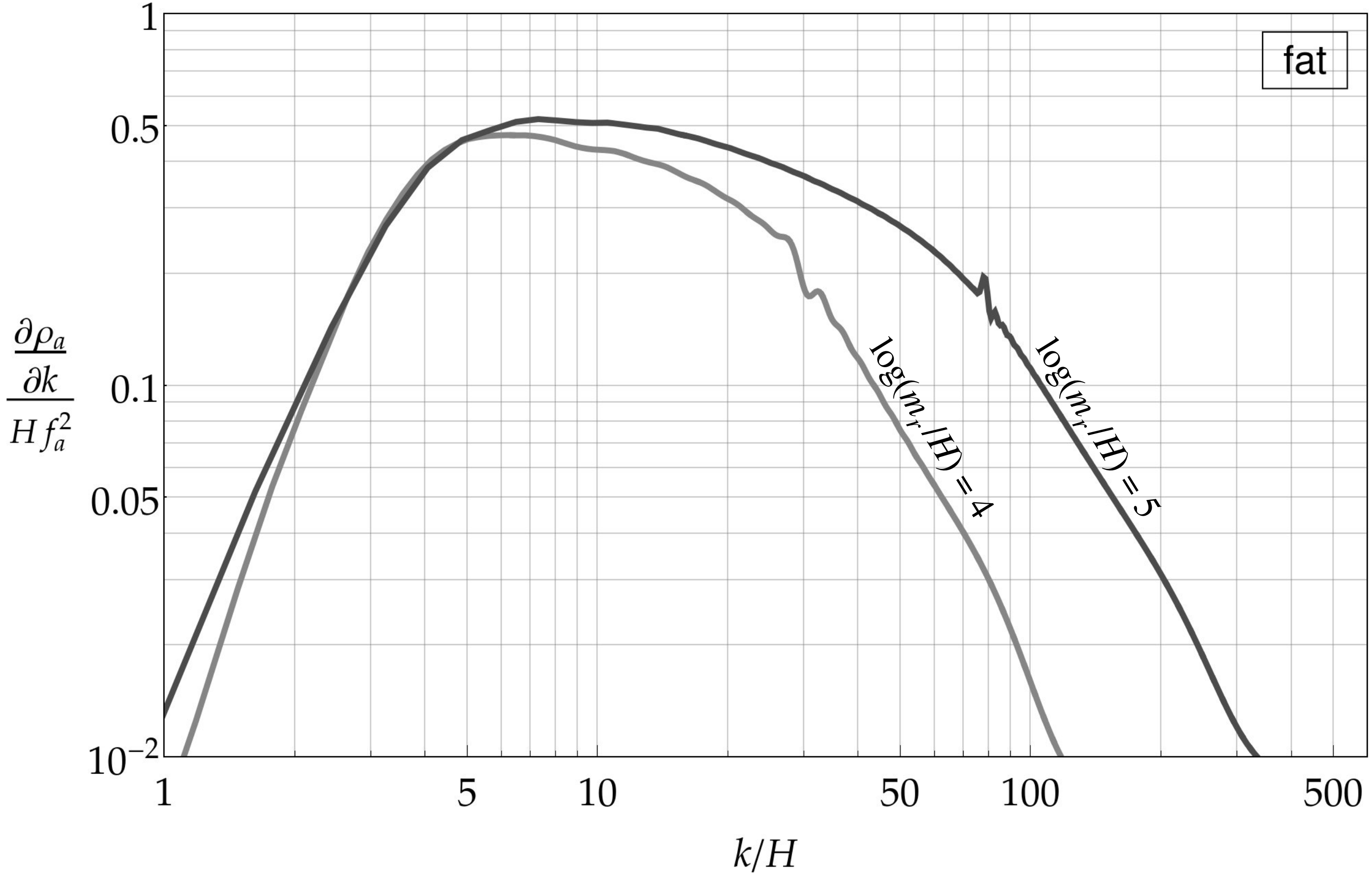

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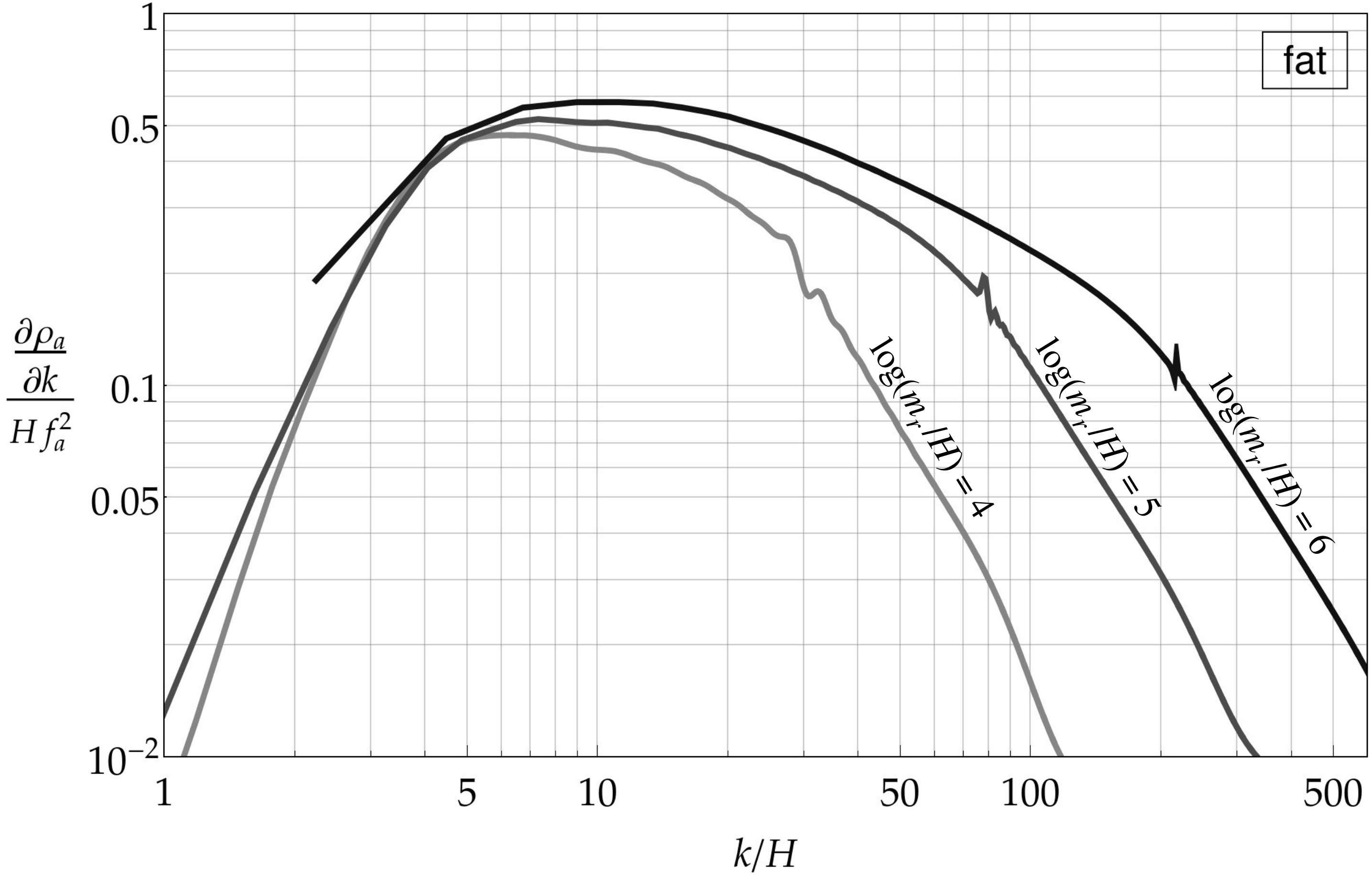

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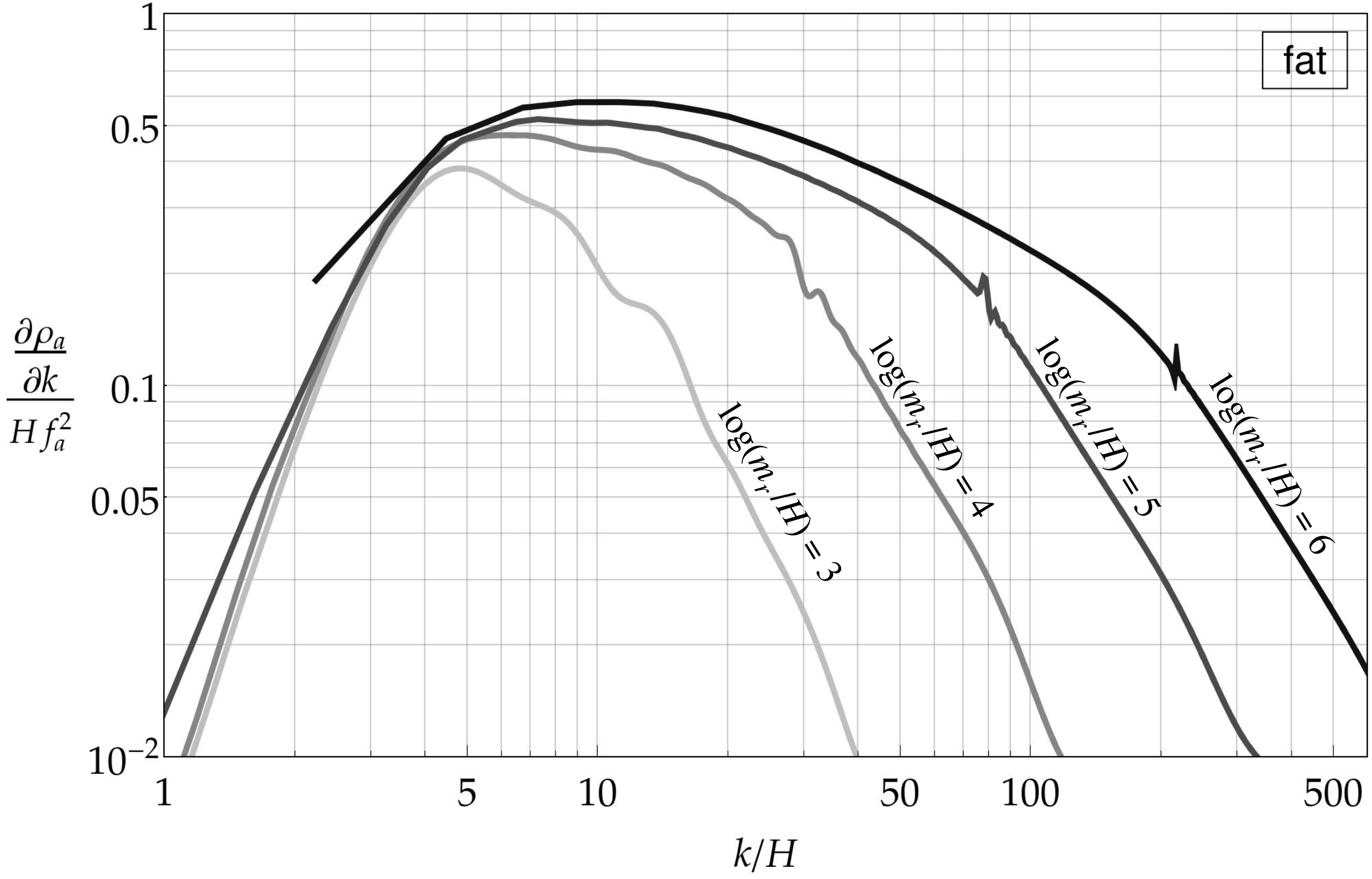
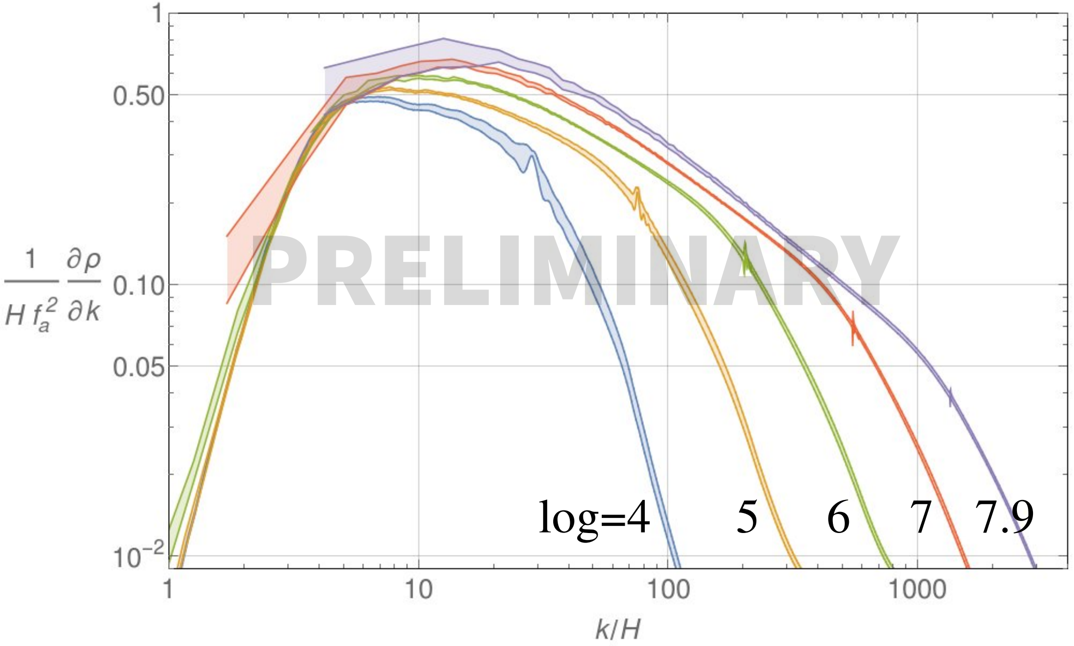
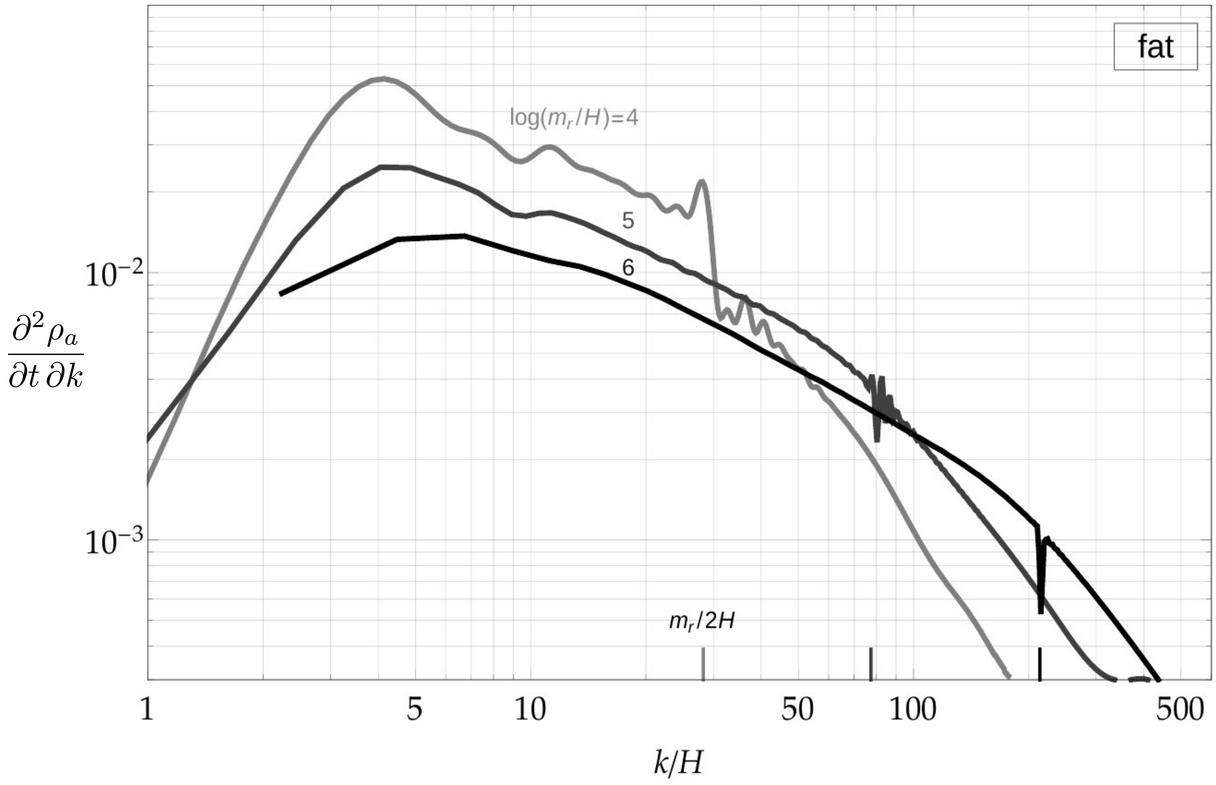


Axion Spectrum @ 4k



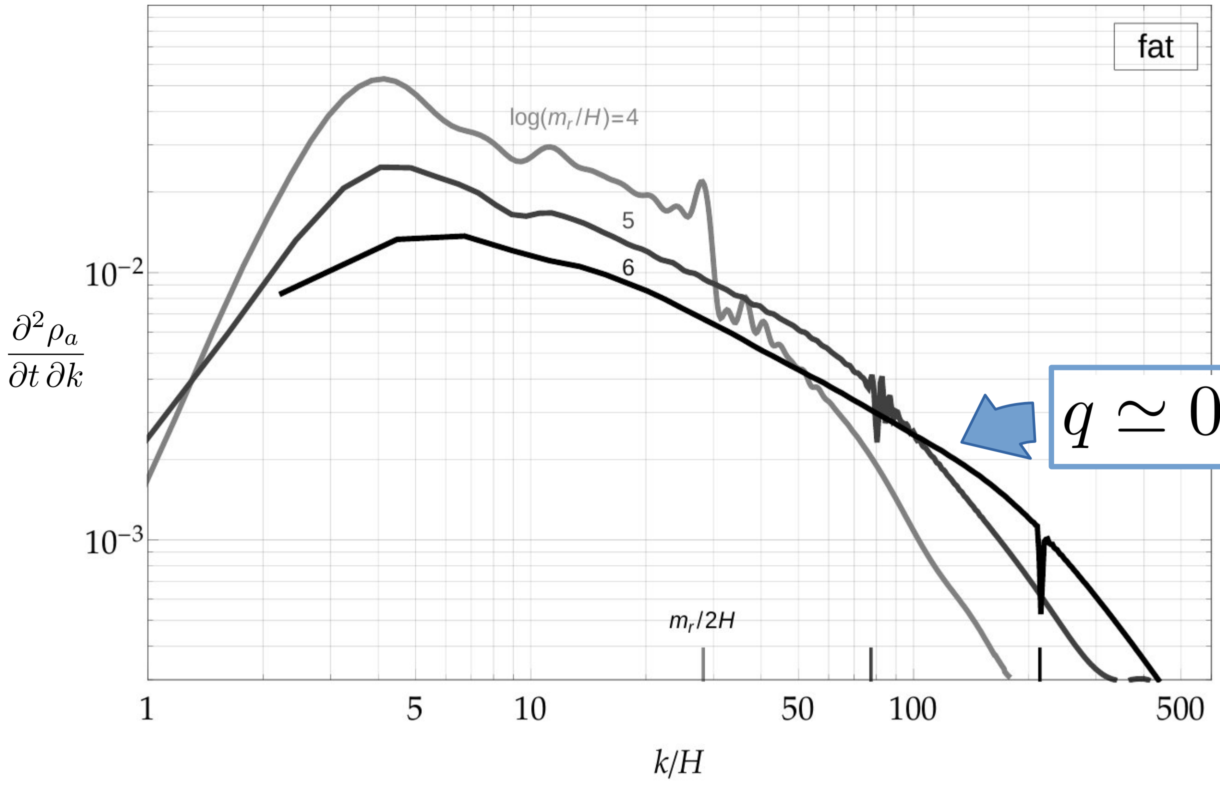
Gorghetto, Hardy, GV

Axion Instantaneous Spectrum



1806.04677 – Gorghetto, Hardy, GV

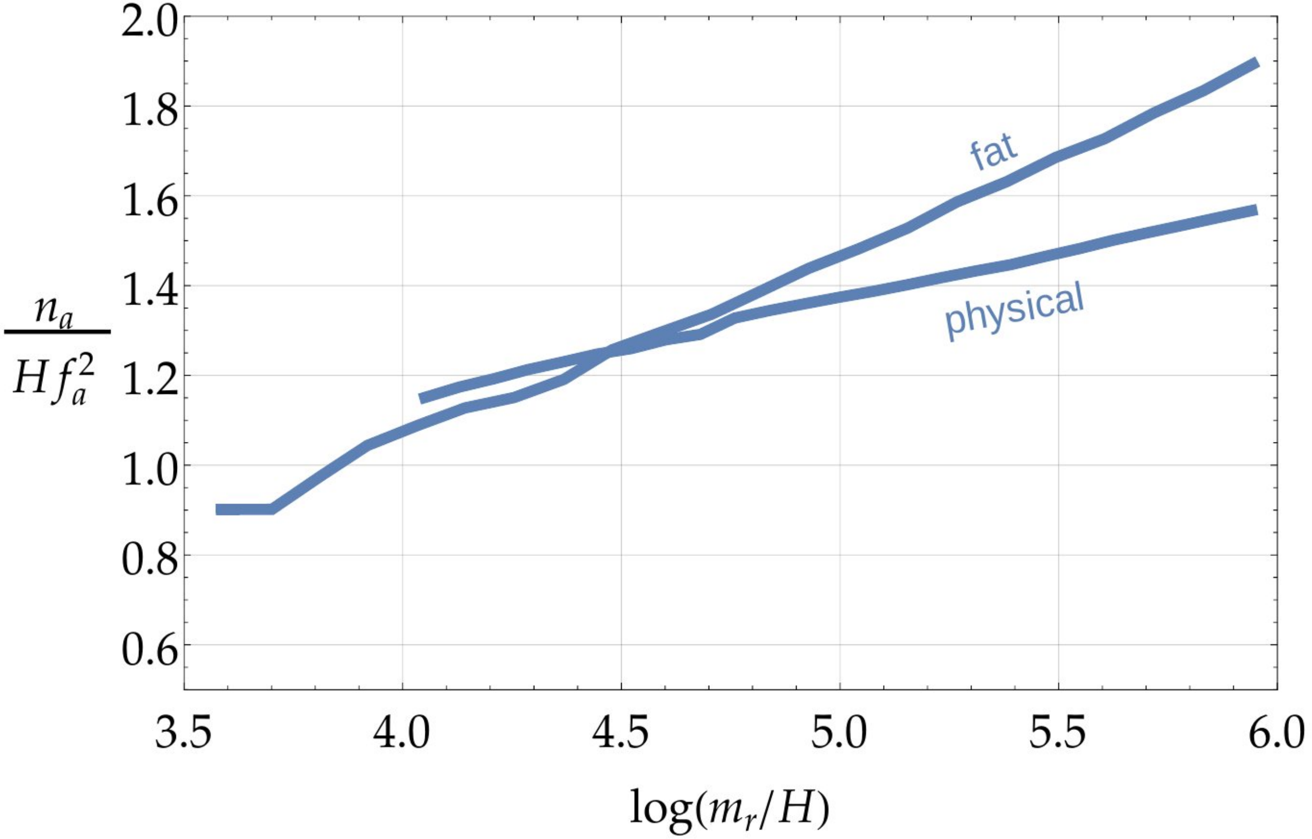
Axion Instantaneous Spectrum



$$q \simeq 0.75(5)$$

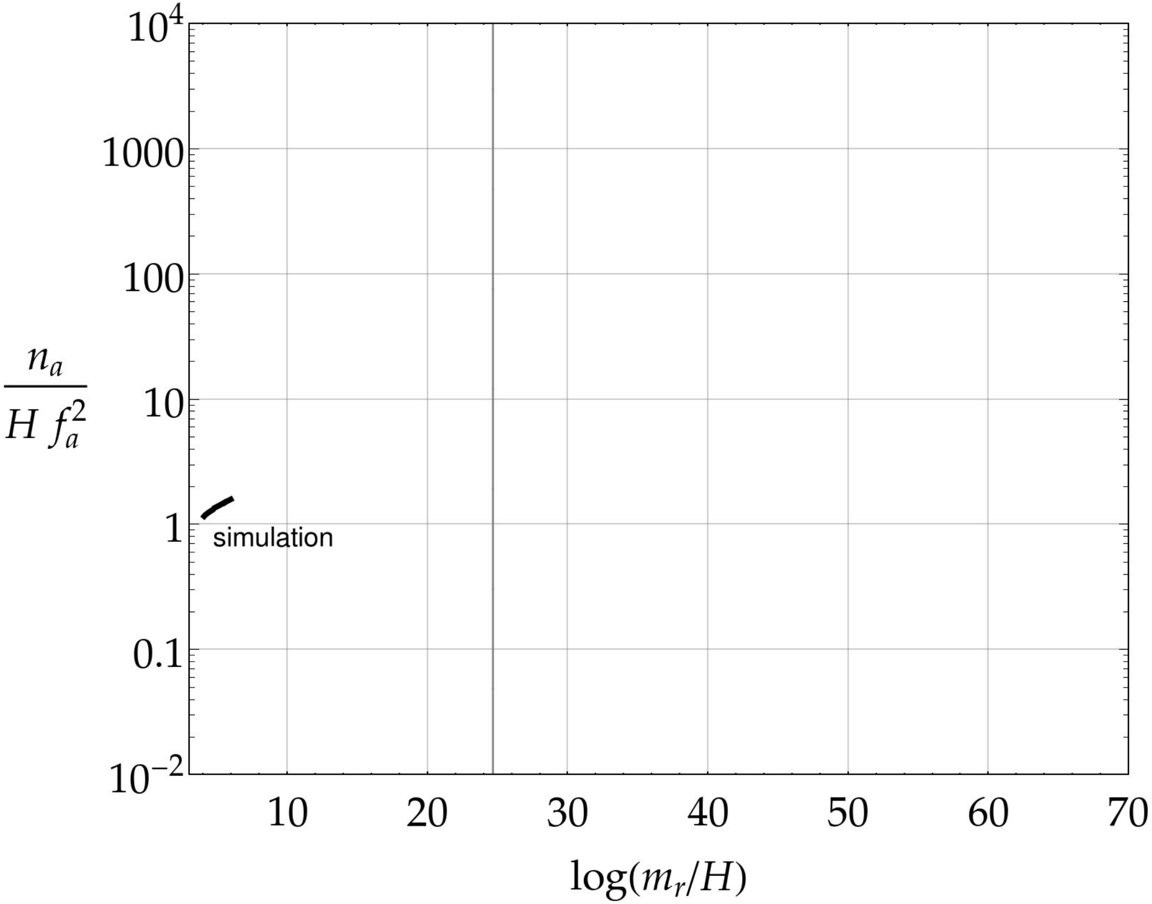
1806.04677 – Gorghetto, Hardy, GV

Axion Number Density



1806.04677 – Gorghetto, Hardy, GV

Axion Number Density → Extrapolation



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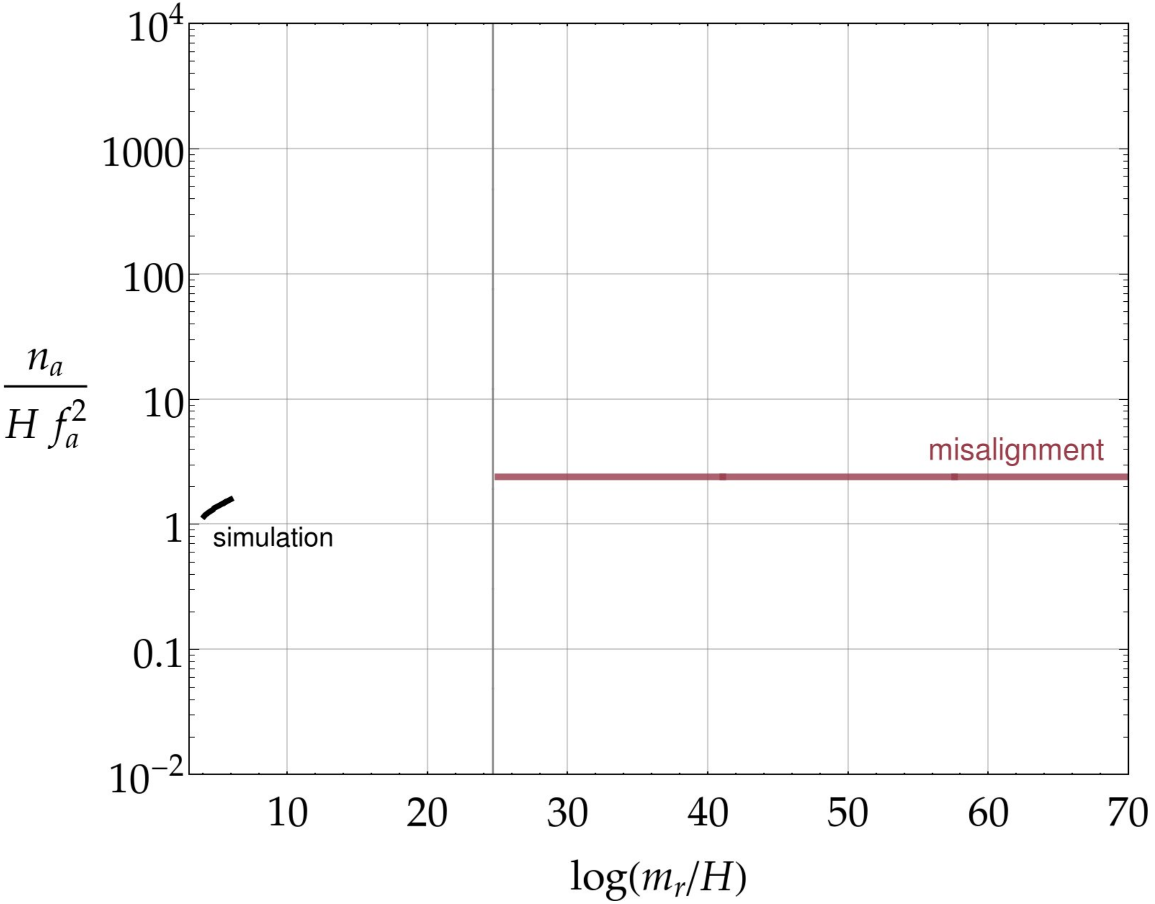

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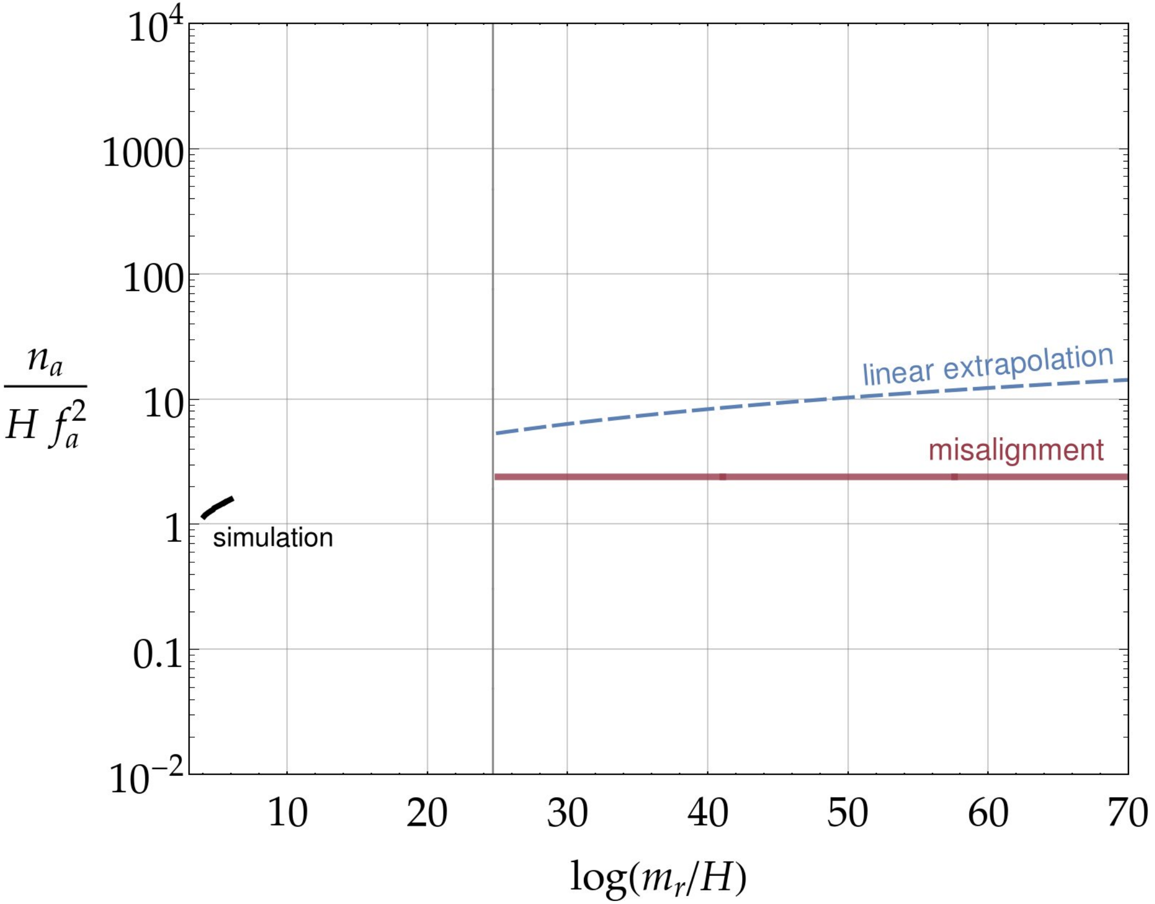

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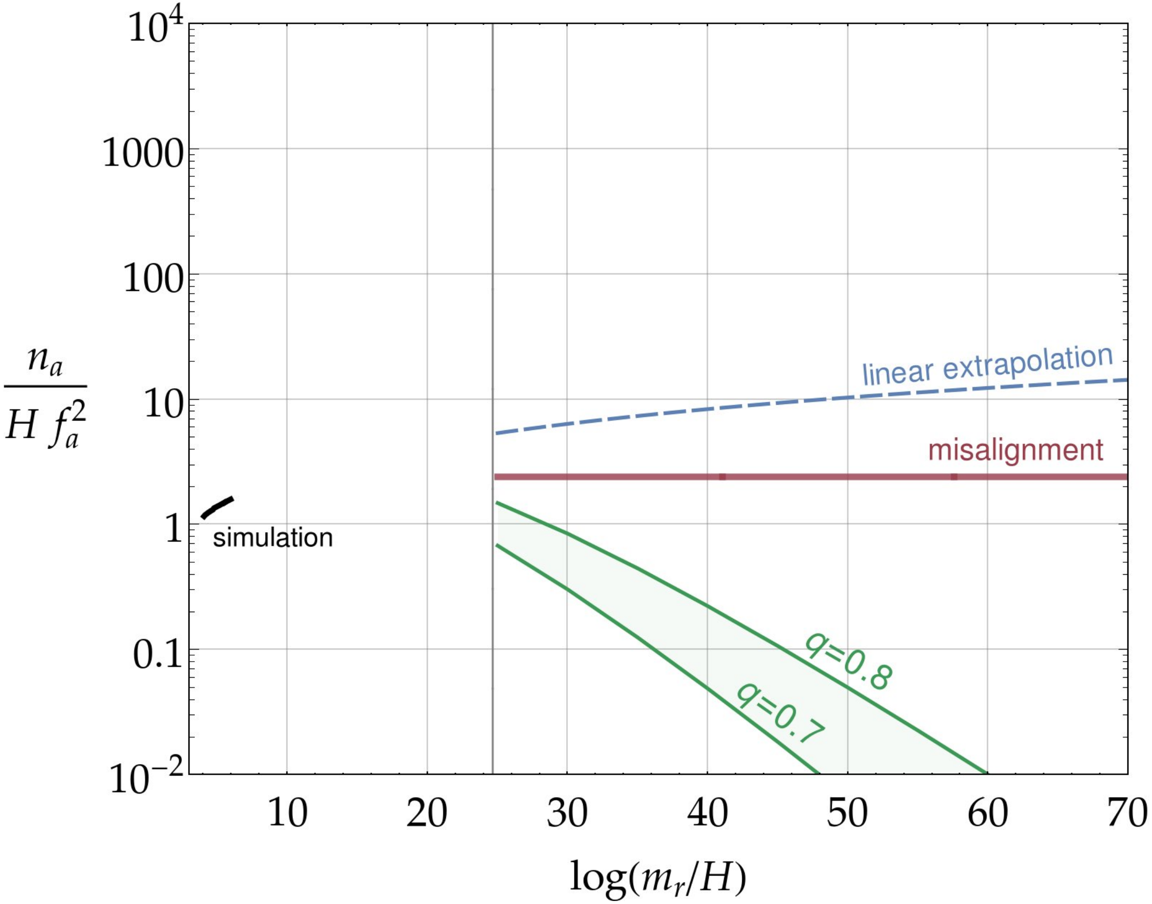

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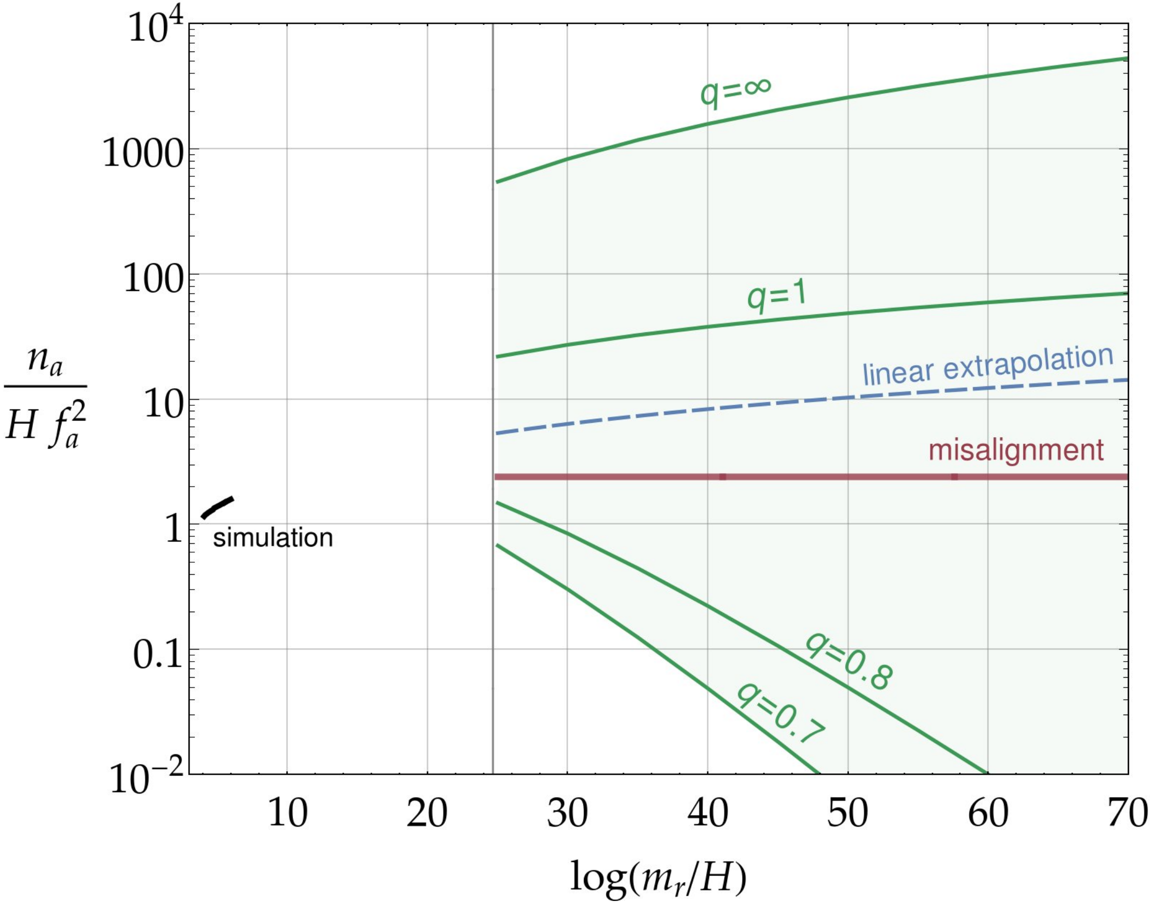

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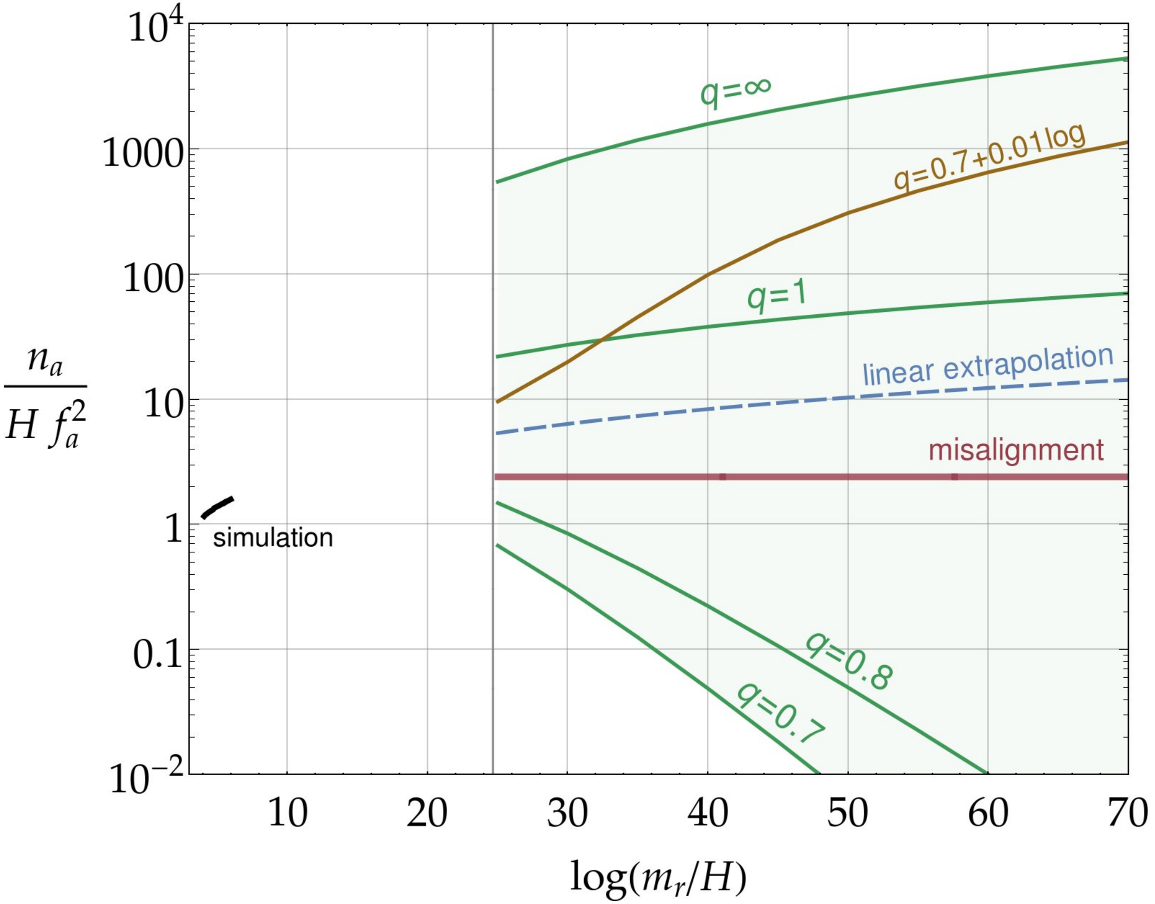

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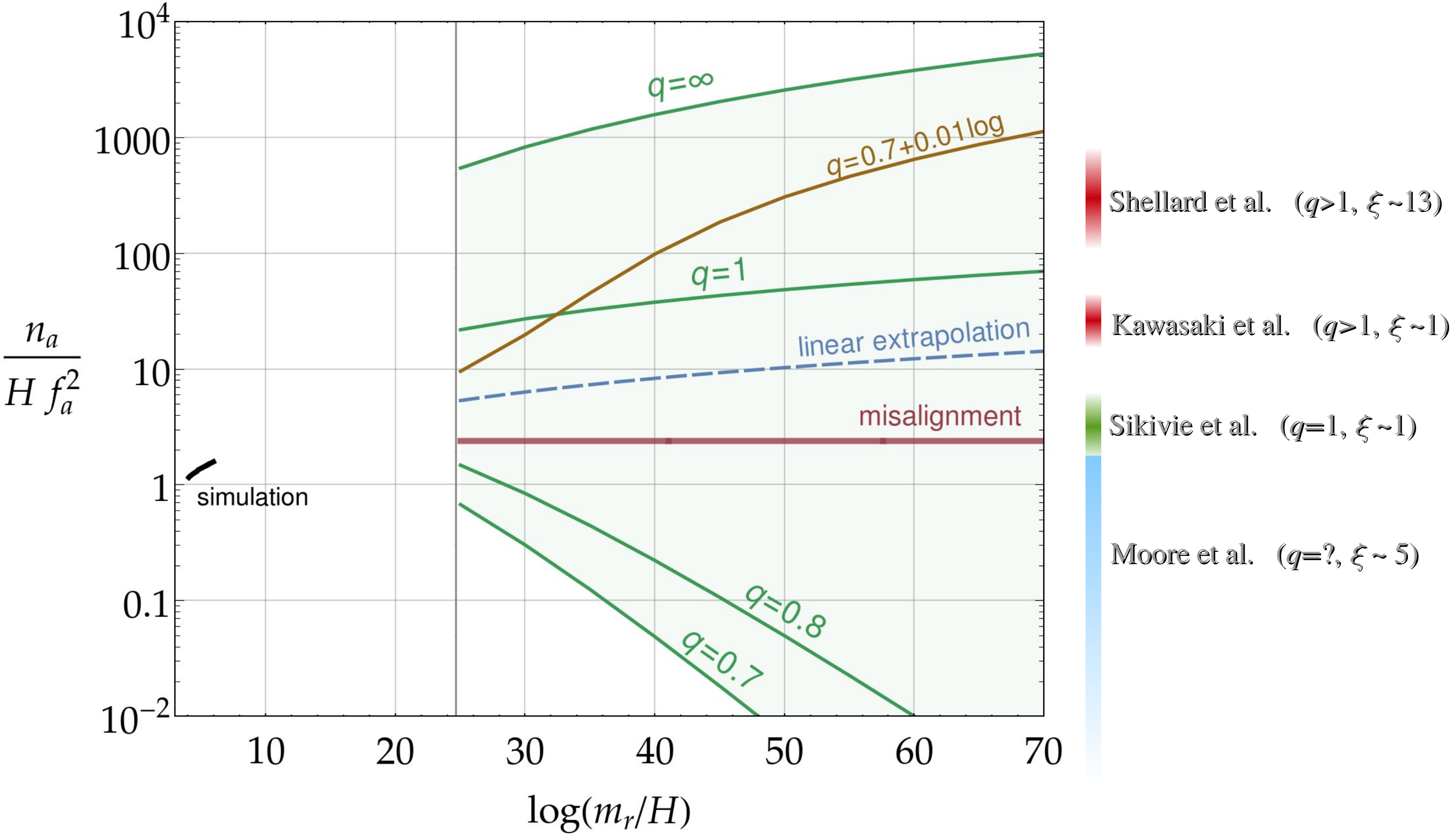
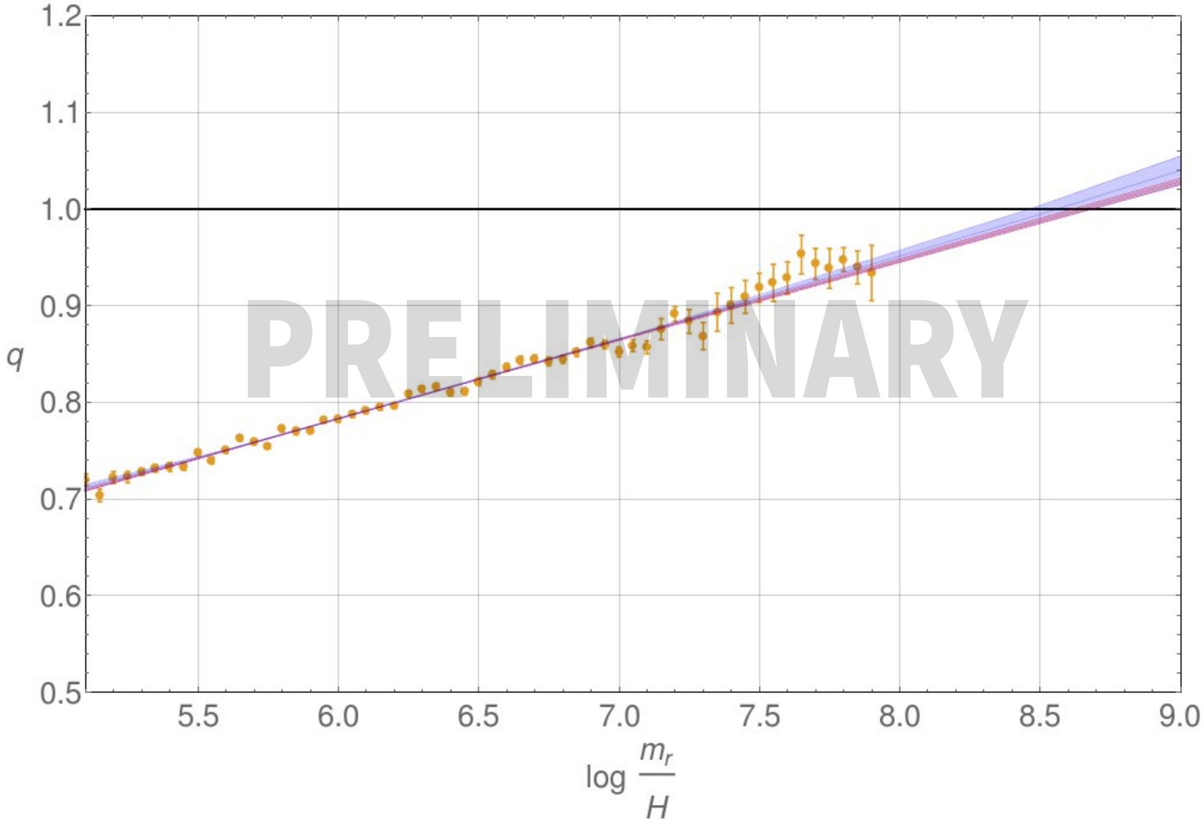


Running of the spectral index (in 4k)

Gorghetto, Hardy, GV



Fat Case: UV → IR dominated spectrum

Physical Case: similar?

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→ Large boost of axions from strings

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What happens when axions get a mass?

Probably some suppression of the boost because of non-linearities

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Physical Case: similar?

→ Large boost of axions from strings

What happens when axions get a mass?

Probably some suppression of the boost because of non-linearities

Very plausible that in this scenario:

$$\Omega_a^{(strings)} \gg \Omega_a^{(mis)} \qquad \Rightarrow \qquad f_a \ll 10^{11}\ \mathrm{GeV}$$

What happens next? Hard to tell reliably...

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Simulations with DW + Strings
very far from physical parameters [O(10^{30}) away]

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Dynamics of strings at log~few **very** different from that at log~70

May be same for DW

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Simulations with DW + Strings
very far from physical parameters [O(10^{30}) away]

Dynamics of strings at log~few **very** different from that at log~70

May be same for DW

Very rich pheno afterwards (oscillons, miniclusters, bose stars, etc…),
but very hard to estimate reliably

Conclusions:

Still a lot to be understood...

→

Far away from a reliable computation of the axion abundance

(feasible?) near-term goal:

lower bound from strings production

(after educated guesses of extrapolated scaling properties)